## **Update 3 Question 1**

# **Loading Data**

```
In [ ]: import pandas as pd
        import numpy as np
        # file_path = "/Users/goyolozano/Desktop/Mini 4/Value/Update 2/final_clean_data.parquet" #change path
        # file_path = "final_clean_data.parquet"
        # Attempt to load the Parquet file
        # final_clean_data = None
        # try:
              final_clean_data = pd.read_parquet(file_path)
        #
              print(f"Data loaded successfully from: {file_path}")
              print(f"Data shape: {final_clean_data.shape}")
              print("\nFirst 5 rows of the loaded data:")
             print(final_clean_data.head())
              print("\nBasic data info:")
             final_clean_data.info()
        # except FileNotFoundError:
             print(f"ERROR: File not found at {file_path}.")
              print("Please update the 'file_path' variable to the correct location of your 'final_clean_data.parquet' file
        # except Exception as e:
            print(f"ERROR Loading data: {e}")
In []: final_clean_data = pd.read_parquet('/Users/goyolozano/Desktop/Mini 4/Value/Update 3/Deliverables/final_clean_data.pd
```

# **Defining Target Variables**

```
In [ ]: print("Defining target variables...")
           # Define default_label based on loan_status
           default_statuses = ['Charged Off', 'Default'] # Check if these match your data exactly
           if 'loan_status' in final_clean_data.columns:
                final_clean_data['default_label'] = final_clean_data['loan_status'].apply(
                      lambda x: 1 if x in default_statuses else 0
                print("Target variable 'default_label' created.")
                print(final_clean_data['default_label'].value_counts(normalize=True))
           else:
                print("ERROR: 'loan_status' column not found. Cannot create 'default_label'.")
                # Handle error appropriately (e.g., raise ValueError or exit)
           return_target_col = 'ret_INTb'
           if return_target_col not in final_clean_data.columns:
                  print(f"ERROR: Return target column '{return_target_col}' not found in the DataFrame.")
                 print("Available columns:", final_clean_data.columns.tolist())
print("Please update the 'return_target_col' variable.")
                  # Handle error appropriately
           else:
                print(f"Identified return target variable: '{return_target_col}'")
                print(f"Basic stats for '{return_target_col}':")
                print(final_clean_data[return_target_col].describe())
           print("\nIdentifying feature columns...")
           # --- Identify Feature Columns ---
           numerical_features_potential = [
                 'loan_amnt', 'funded_amnt', 'installment', 'annual_inc', 'dti', 'delinq_2yrs',
'open_acc', 'pub_rec', 'revol_bal', 'revol_util', 'total_acc', 'collections_12_mths_ex_med',
'acc_now_delinq', 'tot_coll_amt', 'tot_cur_bal', 'total_rev_hi_lim', 'bc_util',
                 'chargeoff_within_12_mths', 'mort_acc', 'num_accts_ever_120_pd', 'num_actv_bc_tl',
                'num_actv_rev_tl', 'num_bc_sats', 'num_bc_tl', 'num_il_tl', 'num_op_rev_tl', 'num_ev_actcs', 'num_bc_sats', 'num_bc_tl', 'num_il_tl', 'num_op_rev_tl', 'num_rev_acts', 'num_rev_tl_bal_gt_0', 'num_sats', 'num_tl_30dpd', 'num_tl_90g_dpd_24m', 'num_tl_op_past_12m', 'pct_tl_nvr_dlq', 'pub_rec_bankruptcies', 'tax_liens', 'total_bal_ex_mort', 'total_bc_limit', 'total_il_high_credit_limit',
                 'loan_length', 'term_num' # Check if these engineered features were used
```

```
categorical_features_potential = [
      'grade', 'sub_grade', 'emp_length', 'home_ownership', 'verification_status',
      'purpose', 'addr_state'
 # Define features to exclude (IDs, target variables, other calculated returns, etc.)
 features_to_exclude = [
     'id', 'member_id', # Identifiers
     'loan_status', # Source of default_label
'default_label', # Target variable 1
     return_target_col, # Target variable 2
     'ret_PESS', 'ret_OPT', 'ret_INTa', 'ret_INTc', # Other return calculations if present
     # Add any other columns dropped during Update 1 or 2 feature selection/cleaning
 # Filter potential features: keep only those present in the DataFrame and not excluded
 numerical_features = sorted([
     f for f in numerical_features_potential
     if f in final_clean_data.columns and f not in features_to_exclude
 1)
 categorical_features = sorted([
     f for f in categorical_features_potential
     if f in final_clean_data.columns and f not in features_to_exclude
 all_features = numerical_features + categorical_features
 # Final check for missing features from the lists (should be empty if lists are accurate)
 missing_features = [f for f in all_features if f not in final_clean_data.columns]
 if missing_features:
     print(f"\nERROR: The following identified features are NOT in the loaded DataFrame: {missing_features}")
     print("This likely means the feature lists above need adjustment.")
     # Handle error
     print(f"\nIdentified {len(numerical features)} numerical features:")
     # print(numerical_features) # Uncomment to see the full list
     print(f"Identified {len(categorical_features)} categorical features:")
     # print(categorical_features) # Uncomment to see the full list
     print(f"Total features identified: {len(all_features)}")
Defining target variables...
Target variable 'default_label' created.
default_label
     0.814544
     0.185456
Name: proportion, dtype: float64
Identified return target variable: 'ret_INTb'
Basic stats for 'ret_INTb':
         648349.000000
count
mean
              0.044809
std
              0.059373
min
             -0.196276
25%
              0.043313
50%
              0.056383
75%
              0.072959
              0.223935
max
Name: ret_INTb, dtype: float64
Identifying feature columns...
Identified 18 numerical features:
Identified 5 categorical features:
Total features identified: 23
```

# Train Test Split and Class Imbalance

```
In []: from sklearn.model_selection import train_test_split
import pandas as pd

# --- Train/Test Split ---
print("Performing train/test split...")

# Check if necessary variables/columns exist before splitting
if 'final_clean_data' in locals() and \
    'all_features' in locals() and \
    'default_label' in final_clean_data.columns and \
    return_target_col in final_clean_data.columns:
```

```
# Select the features (X) and target variables (y)
            X = final_clean_data[all_features]
            y_clf = final_clean_data['default_label'] # Classification target (for stratification)
            y_reg = final_clean_data[return_target_col] # Regression target (return)
            # Perform the split
            # test_size=0.2 and random_state=42 match Update 2 notebook
            # stratify=y_clf ensures the proportion of defaults (class imbalance)
            # is approximately the same in both train and test sets.
            # This is crucial for representative testing and was done in Update 2.
                X\_train, \ X\_test, \ y\_clf\_train, \ y\_clf\_test, \ y\_reg\_train, \ y\_reg\_test = train\_test\_split(
                    X, y_clf, y_reg,
                    test size=0.2.
                                           # 20% of data for testing
                    random_state=42,
                                          # Ensures reproducibility
                    stratify=y_clf
                                           # Preserves class proportions (handles imbalance at split)
                print("Train/Test split completed successfully.")
                print(f"X_train shape: {X_train.shape}, X_test shape: {X_test.shape}")
                print(f"y_clf_train distribution:\n{y_clf_train.value_counts(normalize=True)}")
                print(f"y_clf_test distribution:\n{y_clf_test.value_counts(normalize=True)}")
                print(f"y_reg_train shape: {y_reg_train.shape}, y_reg_test shape: {y_reg_test.shape}")
            except Exception as e:
                print(f"ERROR during train/test split: {e}")
                print("Please check if X, y_clf, and y_reg are defined correctly.")
                # Handle error appropriately
        else:
            print("ERROR: Prerequisite data or variables not found.")
            print("Please ensure 'final_clean_data', 'all_features', 'default_label', and 'return_target_col' are defined."
       Performing train/test split...
       Train/Test split completed successfully.
       X_train shape: (518679, 23), X_test shape: (129670, 23)
       y_clf_train distribution:
       default_label
            0.814544
           0.185456
       Name: proportion, dtype: float64
       y_clf_test distribution:
       default label
       0
            0.814545
           0.185455
       Name: proportion, dtype: float64
       y_reg_train shape: (518679,), y_reg_test shape: (129670,)
In [ ]: from sklearn.preprocessing import StandardScaler, OneHotEncoder
        from sklearn.compose import ColumnTransformer
        from sklearn.pipeline import Pipeline
        from sklearn.impute import SimpleImputer
        import pickle
        print("Defining preprocessing pipeline...")
        # Define transformers (ensure these match your choices from Update 2)
        numeric_transformer = Pipeline(steps=[
            ('imputer', SimpleImputer(strategy='median')),
('scaler', StandardScaler())
        1)
        categorical_transformer = Pipeline(steps=[
             ('imputer', SimpleImputer(strategy='most_frequent')),
             ('onehot', OneHotEncoder(handle_unknown='ignore', sparse_output=False)) # sparse_output=False often easier with
        1)
        # Create the preprocessor ColumnTransformer
        preprocessor = ColumnTransformer(
            transformers=[
                 ('num', numeric_transformer, numerical_features),
                 ('cat', categorical_transformer, categorical_features)
            remainder='passthrough' # Keep other columns if any, or use 'drop'
        # Fit the preprocessor on the TRAINING DATA ONLY
        print("Fitting preprocessor on X_train...")
        try:
        preprocessor.fit(X_train)
```

```
print("Preprocessor fitted successfully.")
            # Transform both training and testing data
            print("Transforming X_train and X_test...")
            X_train_processed = preprocessor.transform(X_train)
            X test processed = preprocessor.transform(X test)
            print("Data transformation complete.")
            print(f"X_train_processed shape: {X_train_processed.shape}")
            print(f"X_test_processed shape: {X_test_processed.shape}")
        except Exception as e:
            print(f"ERROR during preprocessing fitting or transformation: {e}")
            # Set processed variables to None or handle error
            X train processed = None
            X_test_processed = None
        # --- Next Steps ---
        # Now that we have X_train_processed and y_clf_train (or y_reg_train),
        # we can apply SMOTE to X_train_processed and the relevant y_train set.
       Defining preprocessing pipeline...
       Fitting preprocessor on X_train...
       Preprocessor fitted successfully.
       Transforming X_train and X_test...
       Data transformation complete.
       X_train_processed shape: (518679, 58)
       X_test_processed shape: (129670, 58)
In [ ]: from imblearn.over_sampling import SMOTE
        import numpy as np
        print("\nApplying SMOTE to the processed training data...")
        # Check if preprocessing was successful
        if X_train_processed is not None:
            # Initialize SMOTE
            # random_state for reproducibility
            # k_neighbors is an important parameter, default is 5
            smote = SMOTE(random_state=42, k_neighbors=5)
            print(f"Original training class distribution:\n{y_clf_train.value_counts()}")
                # Apply SMOTE - Pass processed training features and original classification target
                X_train_smote, y_train_smote = smote.fit_resample(X_train_processed, y_clf_train)
                print("SMOTE applied successfully.")
                print(f"Shape of training features after SMOTE: {X_train_smote.shape}")
                print(f"Distribution of training target after SMOTE:\n{np.bincount(y_train_smote)}") # Use np.bincount for
            except Exception as e:
                print(f"ERROR applying SMOTE: {e}")
                print("Please check the input data (X_train_processed, y_clf_train).")
                # Set smote variables to indicate failure
                X_train_smote = None
                y_train_smote = None
        else:
            print("Skipping SMOTE because preprocessing failed.")
            X_train_smote = None
            y_train_smote = None
       Applying SMOTE to the processed training data...
       Original training class distribution:
       default_label
            422487
       1
            96192
       Name: count, dtype: int64
       SMOTE applied successfully.
       Shape of training features after SMOTE: (844974, 58)
       Distribution of training target after SMOTE:
       [422487 422487]
In []: import pandas as pd
        import numpy as np
        from sklearn.linear_model import Lasso
        from sklearn.metrics import mean_squared_error, r2_score
        import pickle # Or joblib
        print("Extracting correct target variable 'ret_INTb' based on train/test indices...")
        target_return_column_actual = 'ret_INTb'
```

```
if 'final_clean_data' in locals() and \
   target_return_column_actual in final_clean_data.columns and \
   'X_train' in locals() and \
   'X_test' in locals():
    try:
        # Use the index from X_train/X_test to get the corresponding 'ret_INTb' values
        y_reg_train_actual = final_clean_data.loc[X_train.index, target_return_column_actual]
        y_reg_test_actual = final_clean_data.loc[X_test.index, target_return_column_actual]
        print(f"Successfully extracted '{target_return_column_actual}' for train and test sets.")
        print(f"y_reg_train_actual shape: {y_reg_train_actual.shape}")
        print(f"y_reg_test_actual shape: {y_reg_test_actual.shape}")
    except Exception as e:
        print(f"ERROR extracting target variable '{target_return_column_actual}': {e}")
        y_reg_train_actual = None
        y_reg_test_actual = None
else:
    print(f"ERROR: Prerequisite data ('final_clean_data', 'X_train', 'X_test', or column '{target_return_column_act
    y_reg_train_actual = None
    y_reg_test_actual = None
# --- 2. Define/Load the Lasso Return Prediction Model ---
print("\nDefining return prediction model (Lasso)...")
return_predictor = Lasso(alpha=0.001, random_state=42) # Placeholder alpha
# --- 3. Train the Lasso Model ---
print(f"Training {type(return_predictor).__name__} model on X_train_processed...")
# Check if processed data and actual target variable exist
if 'X_train_processed' in locals() and X_train_processed is not None and \
   y_reg_train_actual is not None:
    try:
        # Train the predictor on the processed training data and CORRECT regression target
        return_predictor.fit(X_train_processed, y_reg_train_actual)
        print("Return prediction model (Lasso) trained successfully.")
    except Exception as e:
        print(f"ERROR training return prediction model (Lasso): {e}")
        return_predictor = None # Indicate failure
    print("ERROR: X_train_processed or y_reg_train_actual not available. Cannot train model.")
    return_predictor = None
# --- 4. Predict Returns ---
y_pred_reg_train = None
y_pred_reg_test = None
if return_predictor is not None:
    print("\nGenerating return predictions for train and test sets using Lasso...")
        # Predict on the processed training set
        y_pred_reg_train = return_predictor.predict(X_train_processed)
        # Predict on the processed test set
        y_pred_reg_test = return_predictor.predict(X_test_processed)
        print("Return predictions generated.")
        # --- Optional: Evaluate the Lasso Regression Model ---
        if y_reg_test_actual is not None:
            print("\nEvaluating Lasso return prediction model performance (on test set):")
            rmse_test = np.sqrt(mean_squared_error(y_reg_test_actual, y_pred_reg_test))
            r2_test = r2_score(y_reg_test_actual, y_pred_reg_test)
            print(f"Test Set RMSE: {rmse_test:.4f}")
            print(f"Test Set R-squared: {r2_test:.4f}")
            # Quick check of prediction distribution vs actual
            print("\nPredicted Return (Test Set) Stats:")
            print(pd.Series(y_pred_reg_test).describe())
            print(f"\nActual '{target_return_column_actual}' (Test Set) Stats:")
            print(y_reg_test_actual.describe())
            print("\nSkipping evaluation because actual test target (y_reg_test_actual) is missing.")
    except Exception as e:
        print(f"ERROR predicting returns with Lasso: {e}")
```

```
print("Skipping return prediction because Lasso model training failed.")
Extracting correct target variable 'ret INTb' based on train/test indices...
Successfully extracted 'ret_INTb' for train and test sets.
y_reg_train_actual shape: (518679,)
y_reg_test_actual shape: (129670,)
Defining return prediction model (Lasso)...
Training Lasso model on X_train_processed...
Return prediction model (Lasso) trained successfully.
Generating return predictions for train and test sets using Lasso...
Return predictions generated.
Evaluating Lasso return prediction model performance (on test set):
Test Set RMSE: 0.0494
Test Set R-squared: 0.3070
Predicted Return (Test Set) Stats:
count
         129670.000000
              0.044938
mean
              0.031777
std
min
             -0.068069
25%
              0.019125
50%
              0.045872
75%
              0.066683
              0.142818
max
dtype: float64
Actual 'ret_INTb' (Test Set) Stats:
        129670.000000
mean
              0.044912
std
              0.059361
min
             -0.195200
              0.043409
25%
50%
              0.056474
75%
              0.072960
              0.209550
max
Name: ret_INTb, dtype: float64
```

## **Clustering Analysis**

```
In [ ]: import pandas as pd
        import numpy as np
        from sklearn.decomposition import PCA
        import pickle # Or joblib
        print("Setting up PCA features for clustering by fitting a new PCA model...")
        # --- 1. Define PCA Transformation ---
        n_pca_components = 18 # Based on Update 1 Slides (explaining 90% variance) - VERIFY THIS!
        print(f"Defining new PCA(n_components={n_pca_components}).")
        # Define the PCA object - it will be fitted below
        pca_model = PCA(n_components=n_pca_components, random_state=42)
        # --- 2. Extract Scaled Numerical Features from Training Set ---
        X_train_numerical_scaled = None
        print("\nExtracting and scaling numerical features from X_train...")
        # Ensure prerequisites are available
        if 'preprocessor' in locals() and hasattr(preprocessor, 'named_transformers_') and \
    'X_train' in locals() and 'numerical_features' in locals():
                 # Access the fitted numeric transformer pipeline from the preprocessor
                if 'num' in preprocessor.named_transformers_:
                      numeric_transformer = preprocessor.named_transformers_['num']
                      # Apply the numeric transformer (impute + scale) to the numerical features of X_train
                      X_train_numerical_scaled = numeric_transformer.transform(X_train[numerical_features])
                      print(f"Scaled numerical training data extracted. Shape: {X_train_numerical_scaled.shape}")
                      print("ERROR: Transformer named 'num' not found in the preprocessor.")
                      print(f"Available transformers: {list(preprocessor.named_transformers_.keys())}")
                      X_train_numerical_scaled = None # Ensure it's None if transformer missing
            except Exception as e:
                print(f"ERROR applying numeric transformer: {e}")
                X_train_numerical_scaled = None # Ensure it's None on error
```

```
print("ERROR: Prerequisite object ('preprocessor', 'X_train', or 'numerical_features') not found or preprocesso
 # --- 3. Fit PCA and Transform Training Data ---
 X_train_pca = None
 # Check if pca_model was defined and scaled data exists
 if pca_model is not None and X_train_numerical_scaled is not None:
    print("\nProcessing PCA...")
     try:
         # Fit the new PCA model
         print(f"Fitting PCA(n_components={n_pca_components})...")
         pca_model.fit(X_train_numerical_scaled)
         print("PCA model fitted successfully.")
         # Optional: Save the newly fitted PCA model
         # with open('pca_model_fitted.pkl', 'wb') as f:
              pickle.dump(pca_model, f)
         # print("Fitted PCA model saved (optional).")
         # Transform the data using the fitted model
         print("Transforming scaled numerical training data using fitted PCA model...")
         X_train_pca = pca_model.transform(X_train_numerical_scaled)
         print("PCA transformation complete.")
         print(f"Resulting X_train_pca shape: {X_train_pca.shape}")
         # Print explained variance
         print(f"Total explained variance by {pca_model.n_components_} components: {pca_model.explained_variance_rat
    except Exception as e:
         # Catch any error during fitting or transforming
         print(f"ERROR during PCA processing: {e}")
         X_train_pca = None # Ensure X_train_pca is None if error occurred
    print("\nSkipping PCA processing due to missing PCA model definition or scaled data.")
Setting up PCA features for clustering by fitting a new PCA model...
Defining new PCA(n_components=18).
Extracting and scaling numerical features from X_train...
Scaled numerical training data extracted. Shape: (518679, 18)
Processing PCA...
Fitting PCA(n_components=18)...
PCA model fitted successfully.
Transforming scaled numerical training data using fitted PCA model...
PCA transformation complete.
Resulting X_train_pca shape: (518679, 18)
Total explained variance by 18 components: 1.0000
```

# **Optimal Number of Clusters**

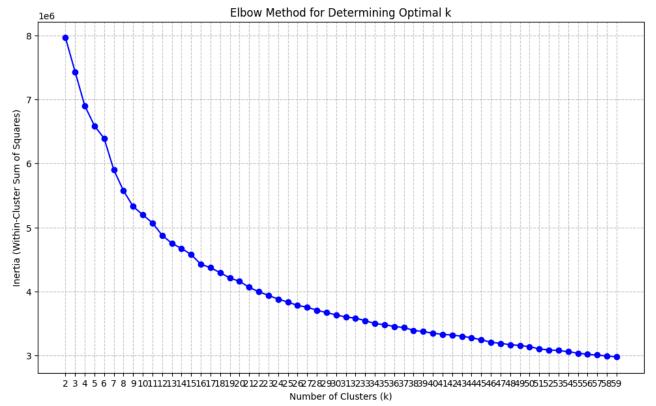
```
In [ ]: import numpy as np
        import matplotlib.pyplot as plt
        from sklearn.cluster import KMeans
        import time
        print("Determining optimal k for K-Means using the Elbow Method...")
        # Check if X_train_pca exists and is not None
        if 'X_train_pca' in locals() and X_train_pca is not None:
            # --- Elbow Method ---
            inertia_values = []
            # Define the range of k values to test
            # Consider adjusting the upper limit based on expectations or computational time
            k_{range} = range(2, 60)
            print(f"Testing k values from {k_range.start} to {k_range.stop - 1}...")
            start_time_elbow = time.time()
            for k in k_range:
                print(f" Fitting KMeans for k={k}...")
                # Initialize KMeans
                kmeans = KMeans(n_clusters=k,
                               n_init='auto',
```

```
random_state=42,
                       max_iter=300) # Default max_iter is usually sufficient
       # Fit KMeans on the PCA-transformed training data
       kmeans.fit(X_train_pca)
       # Store the inertia (Within-Cluster Sum of Squares)
       inertia_values.append(kmeans.inertia_)
       print(f"
                   Inertia for k={k}: {kmeans.inertia_:.2f}")
   end_time_elbow = time.time()
   print(f"\nElbow method calculations finished in {end_time_elbow - start_time_elbow:.2f} seconds.")
   # Plot the Elbow curve
   plt.figure(figsize=(12, 7))
   plt.plot(k_range, inertia_values, marker='o', linestyle='-', color='b')
   plt.xlabel('Number of Clusters (k)')
   plt.ylabel('Inertia (Within-Cluster Sum of Squares)')
   plt.title('Elbow Method for Determining Optimal k')
   plt.xticks(k_range)
   plt.grid(True, linestyle='--', alpha=0.7)
   plt.show()
   print("\nACTION REQUIRED: Examine the plot above.")
   print("Look for the 'elbow' point where the line bends and the rate of decrease")
   print("in inertia significantly slows down. This point suggests a suitable value for k.")
   print("Choose the value of 'k' based on this plot.")
else:
   print("ERROR: X_train_pca not available. Cannot determine optimal k.")
```

Determining optimal k for K-Means using the Elbow Method... Testing k values from 2 to 59... Fitting KMeans for k=2... Inertia for k=2: 7966613.52 Fitting KMeans for k=3... Inertia for k=3: 7432021.96 Fitting KMeans for k=4...Inertia for k=4: 6902111.59 Fitting KMeans for k=5... Inertia for k=5: 6588470.33 Fitting KMeans for k=6... Inertia for k=6: 6392364.45 Fitting KMeans for k=7... Inertia for k=7: 5900973.97 Fitting KMeans for k=8... Inertia for k=8: 5574542.40 Fitting KMeans for k=9... Inertia for k=9: 5327783.39 Fitting KMeans for k=10... Inertia for k=10: 5197121.74 Fitting KMeans for k=11... Inertia for k=11: 5071314.96 Fitting KMeans for k=12... Inertia for k=12: 4877745.40 Fitting KMeans for k=13... Inertia for k=13: 4752044.58 Fitting KMeans for k=14... Inertia for k=14: 4671411.64 Fitting KMeans for k=15... Inertia for k=15: 4576624.74 Fitting KMeans for k=16... Inertia for k=16: 4424733.12 Fitting KMeans for k=17... Inertia for k=17: 4371802.92 Fitting KMeans for k=18... Inertia for k=18: 4291823.14 Fitting KMeans for k=19... Inertia for k=19: 4210932.47 Fitting KMeans for k=20...Inertia for k=20: 4157083.13 Fitting KMeans for k=21... Inertia for k=21: 4064103.14 Fitting KMeans for k=22... Inertia for k=22: 3994398.42 Fitting KMeans for k=23... Inertia for k=23: 3937466.69 Fitting KMeans for k=24... Inertia for k=24: 3879594.04 Fitting KMeans for k=25... Inertia for k=25: 3833827.37 Fitting KMeans for k=26... Inertia for k=26: 3781201.45 Fitting KMeans for k=27... Inertia for k=27: 3752950.61 Fitting KMeans for k=28... Inertia for k=28: 3706327.56 Fitting KMeans for k=29... Inertia for k=29: 3668136.91 Fitting KMeans for k=30.. Inertia for k=30: 3633789.59 Fitting KMeans for k=31... Inertia for k=31: 3598335.98 Fitting KMeans for k=32... Inertia for k=32: 3583056.03 Fitting KMeans for k=33... Inertia for k=33: 3542749.78 Fitting KMeans for k=34... Inertia for k=34: 3497577.71 Fitting KMeans for k=35... Inertia for k=35: 3479802.87 Fitting KMeans for k=36... Inertia for k=36: 3451889.25 Fitting KMeans for k=37... Inertia for k=37: 3435718.65 Fitting KMeans for k=38... Inertia for k=38: 3391827.56 Fitting KMeans for k=39... Inertia for k=39: 3371652.85 Fitting KMeans for k=40... Inertia for k=40: 3348002.53

Fitting KMeans for k=41... Inertia for k=41: 3328347.75 Fitting KMeans for k=42... Inertia for k=42: 3317667.73 Fitting KMeans for k=43...Inertia for k=43: 3300128.02 Fitting KMeans for k=44... Inertia for k=44: 3276381.42 Fitting KMeans for k=45... Inertia for k=45: 3244114.15 Fitting KMeans for k=46... Inertia for k=46: 3206083.41 Fitting KMeans for k=47... Inertia for k=47: 3188234.78 Fitting KMeans for k=48... Inertia for k=48: 3167380.03 Fitting KMeans for k=49... Inertia for k=49: 3154948.57 Fitting KMeans for k=50... Inertia for k=50: 3135873.29 Fitting KMeans for k=51... Inertia for k=51: 3101942.24 Fitting KMeans for k=52... Inertia for k=52: 3083984.28 Fitting KMeans for k=53... Inertia for k=53: 3076999.49 Fitting KMeans for k=54... Inertia for k=54: 3056979.73 Fitting KMeans for k=55... Inertia for k=55: 3035469.65 Fitting KMeans for k=56...Inertia for k=56: 3016498.17 Fitting KMeans for k=57... Inertia for k=57: 3005174.02 Fitting KMeans for k=58... Inertia for k=58: 2989787.57 Fitting KMeans for k=59... Inertia for k=59: 2977291.65

Elbow method calculations finished in 75.37 seconds.



ACTION REQUIRED: Examine the plot above. Look for the 'elbow' point where the line bends and the rate of decrease in inertia significantly slows down. This point suggests a suitable value for k. Choose the value of 'k' based on this plot.

## Fitting KMeans Model

```
In [ ]: import numpy as np
        import pandas as pd
        from sklearn.cluster import KMeans
        import pickle # Or joblib
        # Assumes X_train_pca exists from the previous steps
        # The number of clusters chosen by the user
        chosen_k = 50
        print(f"Proceeding with k = {chosen_k} clusters.")
        # Check if X_train_pca exists
        if 'X_train_pca' in locals() and X_train_pca is not None:
            print(f"\n--- Step 14: Fitting final KMeans model with {chosen_k} clusters ---")
            # Initialize the final KMeans model
            kmeans_final = KMeans(n_clusters=chosen_k,
                                  n_init='auto', # Or explicitly n_init=10
                                  random_state=42,
                                  max_iter=300)
                # Fit the model on the PCA-transformed training data
                kmeans_final.fit(X_train_pca)
                print("Final KMeans model fitted successfully.")
                # Optional: Save the fitted final KMeans model
                # with open(f'kmeans_final_k{chosen_k}.pkl', 'wb') as f:
                      pickle.dump(kmeans_final, f)
                # print(f"Fitted KMeans model (k={chosen_k}) saved (optional).")
                # --- Step 15: Assign Training Clusters -
                print(f"\n--- Step 15: Assigning training data points to clusters ---")
                # Predict cluster labels for the training data
                cluster_labels_train = kmeans_final.predict(X_train_pca)
                print("Cluster assignments generated for training data.")
                print(f"Shape of cluster_labels_train: {cluster_labels_train.shape}")
                # Display the distribution of training samples across clusters
                print("\nDistribution of training samples per cluster:")
                print(pd.Series(cluster_labels_train).value_counts().sort_index())
            except Exception as e:
                print(f"ERROR during final KMeans fitting or prediction: {e}")
                kmeans_final = None
                cluster_labels_train = None
            print("ERROR: X_train_pca not available. Cannot fit final KMeans model.")
            kmeans_final = None
            cluster_labels_train = None
```

```
Proceeding with k = 50 clusters.
 --- Step 14: Fitting final KMeans model with 50 clusters ---
Final KMeans model fitted successfully.
  -- Step 15: Assigning training data points to clusters ---
Cluster assignments generated for training data.
Shape of cluster_labels_train: (518679,)
Distribution of training samples per cluster:
      10057
      14660
1
2
      18615
      13529
3
4
      24861
5
        569
6
      15129
7
      19993
8
      17631
9
         13
10
       2840
      18378
11
      12213
12
13
        131
      10365
14
      14673
15
      12815
16
      10459
17
18
       6615
19
       2359
20
       7893
21
      11846
22
       6512
23
       7883
24
      15627
25
      13568
26
       4037
27
       6467
28
       2011
29
       1788
30
        322
31
      11882
      22878
32
33
      10617
34
      11877
35
      12681
      18959
36
37
      16421
38
         27
      19952
39
40
      19714
41
      8365
42
      15484
43
       9247
44
       9848
45
       5601
46
       2295
47
       5000
48
       5270
49
       8702
Name: count, dtype: int64
```

# Calculate risk score for each cluster

```
import pandas as pd
import numpy as np

print("Calculating risk score (std dev of predicted returns) for each cluster...")

cluster_risk_scores = None # Initialize

# Check if prerequisites are available
if 'cluster_labels_train' in locals() and cluster_labels_train is not None and \
    'y_pred_reg_train' in locals() and y_pred_reg_train is not None:

    try:
        # Create a temporary DataFrame to easily group labels and predictions
        # Assumes cluster_labels_train and y_pred_reg_train correspond to the same samples in order
```

```
train_cluster_data = pd.DataFrame({
        'cluster': cluster_labels_train,
        'predicted_return': y_pred_reg_train
    })
    # Group by cluster and calculate the standard deviation of predicted returns
    # ddof=1 for sample standard deviation (default)
    cluster_risk_scores = train_cluster_data.groupby('cluster')['predicted_return'].std(ddof=1)
    # Handle cases where a cluster might have only one member (std dev = NaN)
    # Replace NaN with 0, as a single point has no deviation
    cluster_risk_scores = cluster_risk_scores.fillna(0)
    print("\nRisk Score (Std Dev of Predicted Return) per Cluster:")
    print(cluster_risk_scores)
except Exception as e:
    print(f"ERROR calculating cluster risk scores: {e}")
    cluster_risk_scores = None
print("ERROR: Prerequisites ('cluster_labels_train', 'y_pred_reg_train') not available.")
```

Calculating risk score (std dev of predicted returns) for each cluster...

```
Risk Score (Std Dev of Predicted Return) per Cluster:
cluster
      0.024840
      0.021440
1
2
      0.013072
      0.018574
3
      0.023398
5
      0.031648
6
      0.020525
      0.024165
8
      0.023653
9
      0.029418
10
      0.032525
11
      0.024020
12
      0.025658
13
      0.033718
14
      0.039527
15
      0.018869
      0.018955
16
17
      0.023772
18
      0.029148
      0.029387
19
20
      0.030774
21
      0.023898
22
      0.027761
23
      0.026096
      0.020680
24
25
      0.024990
26
      0.029212
27
      0.030564
28
      0.032655
29
      0.031287
30
      0.029291
31
      0.025205
32
      0.010535
33
      0.025153
34
      0.024341
35
      0.019751
36
      0.023072
37
      0.020958
38
      0.035180
39
      0.022766
40
      0.025253
41
      0.027060
42
      0.023649
43
      0.028439
44
      0.025467
45
      0.028719
46
      0.031687
47
      0.033902
48
      0.026350
      0.036635
Name: predicted_return, dtype: float64
```

### **Test Features**

```
In [ ]: import numpy as np
        import pandas as pd
        print("Preparing test set features for clustering...")
        X_test_numerical_scaled = None
        X_test_pca = None
        # Check if prerequisites are available
        if 'preprocessor' in locals() and hasattr(preprocessor, 'named_transformers_') and \
           'pca_model' in locals() and pca_model is not None and \
           'X_test' in locals() and 'numerical_features' in locals():
            # --- 1. Apply Numerical Transformer --
            print("Applying fitted numeric transformer (scaling/imputing) to X_test...")
                # Access the fitted numeric transformer pipeline
                if 'num' in preprocessor.named_transformers_:
                     numeric_transformer = preprocessor.named_transformers_['num']
                     # Apply the FITTED transformer to the numerical features of X_test
                     X_test_numerical_scaled = numeric_transformer.transform(X_test[numerical_features])
                     print(f"Scaled numerical test data extracted. Shape: {X_test_numerical_scaled.shape}")
                     print("ERROR: Transformer named 'num' not found in the preprocessor.")
                     X_test_numerical_scaled = None # Ensure it's None if transformer missing
            except Exception as e:
                print(f"ERROR applying numeric transformer to test data: {e}")
                X_test_numerical_scaled = None # Ensure it's None on error
            # --- 2. Apply Fitted PCA Model --
            if X_test_numerical_scaled is not None:
                print("\nApplying fitted PCA model to scaled numerical test data...")
                    # Apply the FITTED PCA model (fitted on training data) to the scaled numerical test data
                    X_test_pca = pca_model.transform(X_test_numerical_scaled)
                    print("PCA transformation complete for test data.")
                    print(f"Resulting X_test_pca shape: {X_test_pca.shape}")
                    # Sanity check: Number of columns should match X_train_pca
                    if 'X_train_pca' in locals() and X_train_pca is not None:
                        if X_test_pca.shape[1] != X_train_pca.shape[1]:
                            print(f"WARNING: X_test_pca columns ({X_test_pca.shape[1]}) do not match X_train_pca columns ({
                            print(f"X_test_pca has {X_test_pca.shape[1]} components, matching X_train_pca.")
                except Exception as e:
                    print(f"ERROR applying PCA transformation to test data: {e}")
                    X_test_pca = None
            el se:
                print("\nSkipping PCA transformation for test data because scaled numerical test data is missing.")
            print("ERROR: Prerequisite object ('preprocessor', 'pca_model', 'X_test', or 'numerical_features') not found.")
       Preparing test set features for clustering...
       Applying fitted numeric transformer (scaling/imputing) to X_test...
       Scaled numerical test data extracted. Shape: (129670, 18)
       Applying fitted PCA model to scaled numerical test data...
       PCA transformation complete for test data.
       Resulting X_test_pca shape: (129670, 18)
       X_test_pca has 18 components, matching X_train_pca.
```

# Assign Test Data Points to Clusters

```
In []: import numpy as np
import pandas as pd

print("Assigning test data points to clusters using the fitted KMeans model...")
```

```
cluster_labels_test = None # Initialize
 # Check if prerequisites are available
 if 'kmeans_final' in locals() and kmeans_final is not None and \
    'X_test_pca' in locals() and X_test_pca is not None:
         # Predict the cluster for each point in the test set PCA data
         cluster_labels_test = kmeans_final.predict(X_test_pca)
         print("Cluster assignments generated for test data.")
         print(f"Shape of cluster_labels_test: {cluster_labels_test.shape}")
         # Display the distribution of test samples across clusters
         print("\nDistribution of test samples per cluster:")
         print(pd.Series(cluster_labels_test).value_counts().sort_index())
     except Exception as e:
         print(f"ERROR predicting clusters for test data: {e}")
         cluster_labels_test = None
     print("ERROR: Prerequisites ('kmeans_final', 'X_test_pca') not available.")
Assigning test data points to clusters using the fitted KMeans model...
Cluster assignments generated for test data.
Shape of cluster_labels_test: (129670,)
Distribution of test samples per cluster:
0
      2475
1
      3695
      4696
2
3
      3390
      6201
4
5
      151
6
      3793
7
      4961
8
      4370
10
      720
11
      4676
12
      3118
13
        35
14
      2718
      3605
15
16
      3361
17
      2577
      1693
18
19
      619
      1962
20
21
      2944
22
      1590
      1902
23
24
      3907
25
      3365
26
      990
27
      1603
28
      536
29
       453
30
       70
      2878
31
32
      5732
33
      2605
34
      3007
35
      3202
36
      4633
37
      4092
38
        3
      4956
39
40
      4990
      2042
41
      3783
42
43
      2354
44
      2452
45
      1369
46
      571
47
      1240
48
      1308
49
      2275
Name: count, dtype: int64
```

## Assigning cluster risk scores to test data points

```
In [ ]: import pandas as pd
        import numpy as np
        print("Assigning cluster risk scores to test data points...")
        risk_scores_test = None # Initialize
        # Check if prerequisites are available
        if 'cluster_labels_test' in locals() and cluster_labels_test is not None and \
   'cluster_risk_scores' in locals() and cluster_risk_scores is not None:
                # Create a pandas Series from the test cluster labels
                # Using the index from X_test ensures alignment if needed later
                cluster_labels_test_series = pd.Series(cluster_labels_test, index=X_test.index, name='cluster')
                # Map the cluster labels to the pre-calculated risk scores
                # The .map() function looks up each value in cluster_labels_test_series
                 # using the index of cluster_risk_scores
                risk_scores_test = cluster_labels_test_series.map(cluster_risk_scores)
                 # Check if any values failed to map (shouldn't happen if all clusters 0-8 have risk scores)
                if risk_scores_test.isnull().any():
                     print("WARNING: Some test samples could not be mapped to a risk score!")
                     print(f"Number of NaNs: {risk_scores_test.isnull().sum()}")
                     # Optional: Fill NaNs with a default value, e.g., average risk, or investigate
                     # risk_scores_test = risk_scores_test.fillna(cluster_risk_scores.mean())
                print("Risk scores assigned to test data successfully.")
                print("\nFirst 10 assigned risk scores for the test set:")
                print(risk_scores_test.head(10))
                print("\nSummary statistics for assigned risk scores:")
                print(risk_scores_test.describe())
            except Exception as e:
                print(f"ERROR assigning risk scores to test data: {e}")
                 risk_scores_test = None
            print("ERROR: Prerequisites ('cluster_labels_test', 'cluster_risk_scores', 'X_test') not available.")
       Assigning cluster risk scores to test data points...
       Risk scores assigned to test data successfully.
       First 10 assigned risk scores for the test set:
       179685
                0.023649
       373414
                 0.020525
       348264
                 0.023398
       370100
                 0.036635
       31026
                 0.024020
       80359
                 0.024165
       152532
                0.023653
       179943
                 0.024020
       270206
                 0.024165
       579935
                 0.024990
       Name: cluster, dtype: float64
       Summary statistics for assigned risk scores:
                129670.000000
       count
       mean
                     0.023597
                      0.005417
       std
                     0.010535
       min
       25%
                      0.020958
       50%
                      0.023772
       75%
                     0.025253
                      0.039527
       Name: cluster, dtype: float64
```

### **Final Results Test**

```
In []: import pandas as pd
print("Combining predicted returns, risk scores, and cluster assignments for the test set...")
```

```
final_results_test = None # Initialize
# Check if prerequisites are available
if 'X_test' in locals() and X_test is not None and \
   'y_pred_reg_test' in locals() and y_pred_reg_test is not None and \
   'risk_scores_test' in locals() and risk_scores_test is not None and \
   'cluster_labels_test' in locals() and cluster_labels_test is not None:
        # Ensure all components have the same length as the X_test index
        if not (len(y_pred_reg_test) == len(X_test.index) and \
                len(risk_scores_test) == len(X_test.index) and \
                len(cluster_labels_test) == len(X_test.index)):
            raise ValueError("Mismatch in lengths of components for the final DataFrame.")
        # Create the final DataFrame including the cluster assignment
        final_results_test = pd.DataFrame({
            'predicted_return': y_pred_reg_test,
                                                      # From Step 11
            'risk_score': risk_scores_test, # From Step 19
'cluster': cluster_labels_test # From Step 18
                                                      # From Step 19
        }, index=X_test.index) # Align using the index from X_test
        # Convert 'cluster' column to categorical dtype
        if 'cluster' in final_results_test.columns:
            final_results_test['cluster'] = final_results_test['cluster'].astype('category')
            print("Converted 'cluster' column to categorical data type.")
        # Optional: Rename the index if it has a specific meaning like 'loan_id'
        # final_results_test.index.name = 'loan_id'
        print("\nFinal results DataFrame created successfully for the test set.")
        print("\nFirst 10 rows of the final results:")
        print(final_results_test.head(10))
        # --- MODIFIED: Use describe() default to only show numerical stats ---
        print("\nSummary statistics for numerical columns in final results:")
        print(final_results_test.describe()) # Removed include='all'
        # --- End of Modification
        print("\nData types of final results columns:")
        print(final_results_test.dtypes)
    except Exception as e:
        print(f"ERROR creating final results DataFrame: {e}")
        final_results_test = None
    print("ERROR: Prerequisites ('X_test', 'y_pred_reg_test', 'risk_scores_test', 'cluster_labels_test') not availa
```

```
Combining predicted returns, risk scores, and cluster assignments for the test set... Converted 'cluster' column to categorical data type.
```

Final results DataFrame created successfully for the test set.

```
First 10 rows of the final results:
       predicted_return risk_score cluster
179685
               0.068349
                           0.023649
               -0.000414
373414
                           0.020525
                                          6
348264
               0.064083
                           0.023398
                                          4
370100
               0.035245
                           0.036635
               0.038243
                           0.024020
31026
                                          11
80359
               0.066956
                           0.024165
               0.013884
                           0.023653
                                          8
152532
179943
               0.011841
                           0.024020
                                          11
270206
               0.019652
                           0.024165
                                          7
579935
               0.051069
                                          25
                           0.024990
```

Summary statistics for numerical columns in final results:

	predicted_return	risk_score
count	129670.000000	129670.000000
mean	0.044938	0.023597
std	0.031777	0.005417
min	-0.068069	0.010535
25%	0.019125	0.020958
50%	0.045872	0.023772
75%	0.066683	0.025253
max	0.142818	0.039527

Data types of final results columns:

dtype: object

## **Explanation of Update 3 Question 1 Work**

#### Goal

The primary objective of Question 1 was to assign two key metrics to each loan in the test set:

- 1. A **predicted return**, based on the best predictive model identified in Update 2.
- 2. A risk score, derived from a cluster analysis performed on the training set, as suggested by the case study reference.

### Methodology & Steps

### Phase 1: Data Preparation & Pre-computation

- 1. Loaded Data: Started by loading the final\_clean\_data.parquet dataset generated from previous cleaning steps.
- 2. Defined Target Variables:
  - default\_label: Created a binary (0/1) indicator for loan default based on the loan\_status column. This was primarily used for stratification during the data split.
  - ret\_INTb : Identified and confirmed ret\_INTb as the specific target variable representing the calculated intermediate return that we needed to predict for the regression task.
- 3. **Identified Features:** Defined lists of numerical\_features and categorical\_features based on the columns available in the dataset (excluding IDs, the target variables, and other calculated return columns). (Note: It's assumed these lists were verified against the final feature set used in Update 2).
- 4. **Performed Train/Test Split:** The data was split into training (80%) and testing (20%) sets ( X\_train , X\_test , y\_clf\_train , y\_clf\_test ). Crucially, this split was stratified based on the default\_label to maintain similar default proportions in both sets, and a random\_state was used for reproducibility.
- 5. **Extracted Correct Return Target:** Using the indices generated by the train/test split, extracted the corresponding ret\_INTb values from the original dataset to create the accurate y\_reg\_train\_actual and y\_reg\_test\_actual Series for the regression task.
- 6. Defined & Fitted Preprocessor: A ColumnTransformer (preprocessor) was defined to handle preprocessing consistently.
  - Numerical Features: Imputed missing values using the median and scaled using StandardScaler.
  - $\bullet \quad \text{Categorical Features: Imputed missing values using the most frequent value and encoded using} \quad \textbf{0neHotEncoder} \; .$
  - This preprocessor was fitted only on X\_train to prevent data leakage.

- 7. **Applied Preprocessor:** The fitted preprocessor was used to transform both the training and test sets, resulting in X\_train\_processed and X\_test\_processed.
- 8. **(Context Only) Applied SMOTE:** For potential use in classification tasks (though not directly required for the Q1 return/risk calculation), SMOTE was applied to the X\_train\_processed and y\_clf\_train data to create a balanced training set (X\_train\_smote, y\_train\_smote).

#### Phase 2: Return Prediction

- 9. **Selected Return Model:** Based on your input, we identified Lasso regression as the "best model from Week 4" for the task of predicting the ret\_INTb return. (*Note: The specific alpha parameter for Lasso should ideally match the one optimized in Update 2*).
- 10. **Trained Return Model:** The Lasso model was trained using the preprocessed training data (X\_train\_processed) and the actual corresponding return values (y\_reg\_train\_actual).
- 11. Predicted Returns: The trained Lasso model was used to generate return predictions for:
  - The training set: y\_pred\_reg\_train (predicting on X\_train\_processed ).
  - The test set: y\_pred\_reg\_test (predicting on X\_test\_processed ).

#### Phase 3: Clustering for Risk Assessment (Performed on Training Set)

- 12. **Prepared Clustering Features (PCA):** To cluster loans based on their characteristics (as suggested by Q1), we reduced the dimensionality of the numerical features using PCA:
  - The fitted numeric\_transformer (imputer + scaler) from the preprocessor was applied to X\_train[numerical\_features] .
  - A new PCA model was defined (with n\_components=18, based on Update 1 analysis suggesting this captures ~90% variance) and fitted on this scaled numerical training data.
  - The fitted pca\_model was used to transform the scaled numerical training data into X\_train\_pca.
- 13. **Determined Optimal Number of Clusters (k):** The Elbow method was used on X\_train\_pca to find the optimal number of clusters. Based on your decision after reviewing the plot, k=50 was chosen. Professor suggested this
- 14. Fitted Final K-Means Model: A KMeans model (kmeans\_final) was initialized with n\_clusters=50 and fitted on the X\_train\_pca data.
- 15. **Assigned Training Clusters:** The fitted kmeans\_final model was used to predict the cluster assignment (0-8) for each sample in the training set ( X train pca ), resulting in cluster labels train .

#### Phase 4: Risk Calculation & Assignment

- 16. Calculated Risk per Cluster: This is the core of the risk definition in Q1.
  - We combined the <code>cluster\_labels\_train</code> and the predicted returns for the training set (<code>y\_pred\_reg\_train</code>).
  - We grouped this data by cluster label (0-8).
  - For each cluster, we calculated the **standard deviation** of the <code>y\_pred\_reg\_train</code> values belonging to that cluster.
  - These standard deviation values were stored as the risk score for each cluster in cluster\_risk\_scores . (NaNs for singlemember clusters were filled with 0).
- 17. **Prepared Test Features for Clustering:** To assign test loans to these clusters, we applied the *same* transformations learned from the training data:
  - The **fitted** numeric\_transformer was applied to X\_test[numerical\_features] .
  - The **fitted** pca\_model was applied to the scaled numerical test data, resulting in X\_test\_pca.
- 18. **Assigned Test Clusters:** The **fitted** kmeans\_final model (trained on X\_train\_pca) was used to predict the closest cluster (0-8) for each sample in X\_test\_pca, resulting in cluster\_labels\_test.
- 19. **Assigned Risk to Test Loans:** We mapped the cluster\_labels\_test to the cluster\_risk\_scores . Each test loan was assigned the risk score corresponding to the cluster it fell into. This resulted in the risk\_scores\_test Series.

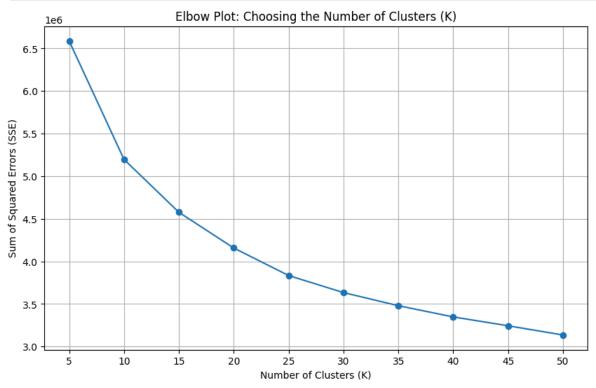
#### Phase 5: Final Output

- 20. Combined Results for Test Set: A final DataFrame (final\_results\_test) was created for the test set. It includes:
  - The loan identifier (index from X\_test ).
  - The **predicted return** ( y\_pred\_reg\_test from the Lasso model).
  - The assigned risk score ( risk\_scores\_test derived from the clustering).

#### Outcome

This final\_results\_test DataFrame contains the required outputs for Question 1, providing both a predicted return and a cluster-based risk score for each loan in the held-out test set. This information can now potentially be used in Question 2 for developing or evaluating investment strategies.

```
In [ ]: from sklearn.cluster import KMeans
        import matplotlib.pyplot as plt
        # Scale raw numeric features (from original X_train before OneHot)
        from sklearn.preprocessing import StandardScaler
        numeric_features = numerical_features # already defined
        scaler = StandardScaler()
        X_numeric = scaler.fit_transform(X_train[numeric_features]) # only numeric features for clustering
        # Try K values from 5 to 50
        sse = []
        K_{range} = range(5, 51, 5)
        for k in K_range:
            kmeans = KMeans(n_clusters=k, random_state=42)
            kmeans.fit(X_numeric)
            sse.append(kmeans.inertia_)
        # Plot the Elbow Curve
        plt.figure(figsize=(10, 6))
        plt.plot(K_range, sse, marker='o')
        plt.title("Elbow Plot: Choosing the Number of Clusters (K)")
        plt.xlabel("Number of Clusters (K)")
        plt.ylabel("Sum of Squared Errors (SSE)")
        plt.grid(True)
        plt.xticks(K_range)
        plt.show()
```



```
In []: # Choose K = 20 based on elbow plot
K_final = 20
kmeans_final = KMeans(n_clusters=K_final, random_state=42)
X_train['cluster'] = kmeans_final.fit_predict(X_numeric)
# Compute standard deviation of predicted returns in each cluster (risk score)
cluster_risk = y_reg_train.groupby(X_train['cluster']).std().rename('cluster_risk')

# Scale test set numeric features
X_test_numeric = scaler.transform(X_test[numeric_features])
X_test['cluster'] = kmeans_final.predict(X_test_numeric)

# Join cluster_risk onto test set
X_test = X_test.join(cluster_risk, on='cluster')
X_test['risk'] = X_test['cluster_risk']

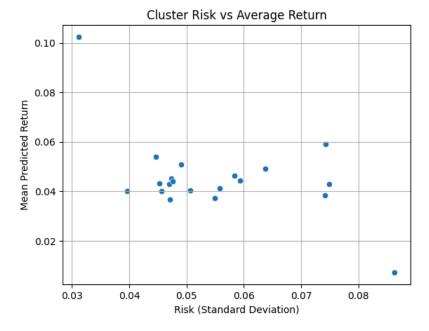
print("Sample of assigned clusters and risk scores:")
X_test[['cluster', 'risk']].head(10)
```

Sample of assigned clusters and risk scores:

t[]:		cluster	risk
	179685	13	0.046952
	373414	10	0.074189
	348264	3	0.055767
	370100	1	0.074259
	31026	13	0.046952
	80359	3	0.055767
	152532	13	0.046952
	179943	5	0.039567
	270206	14	0.045590
	579935	13	0.046952

We evaluated different numbers of clusters (K = 5 to 50) and used the elbow method to identify K = 20 as a strong balance point. This value gave us meaningful similarity groups without overfitting. We then calculated the standard deviation of predicted returns within each cluster to estimate risk, and assigned these risk scores to test loans based on cluster membership.

```
In [ ]: import seaborn as sns # Import the seaborn library and alias it as 'sns'
        import pandas as pd
        import numpy as np
        from sklearn.linear_model import Lasso # Import Lasso
        from sklearn.metrics import mean_squared_error, r2_score
        import pickle # Or joblib
        cluster_means = y_reg_train.groupby(X_train['cluster']).mean()
        cluster_std = y_reg_train.groupby(X_train['cluster']).std()
        cluster_summary = pd.DataFrame({
             'mean_return': cluster_means,
             'risk_std': cluster_std
        })
        sns.scatterplot(x='risk_std', y='mean_return', data=cluster_summary) # Now 'sns' is recognized
        plt.title("Cluster Risk vs Average Return")
        plt.xlabel("Risk (Standard Deviation)")
        plt.ylabel("Mean Predicted Return")
        plt.grid(True)
        plt.show()
```



Observations: Most clusters are in the low-to-mid risk range (std dev between  $\sim$ 0.04 and  $\sim$ 0.07)

Higher return clusters don't always have higher risk

There's one standout cluster: low risk (~0.03) and high return (~0.10) - very interesting!

A few clusters have high risk but average or low return (bottom-right corner) → those are not good investments

We used clustering to group loans with similar characteristics, then calculated each group's average return and risk. Surprisingly, we found that higher risk doesn't always mean higher return in this dataset. One cluster in particular shows a rare combination of low risk and high return, which is very promising. Our optimization strategy in the next step can prioritize loans from this cluster.

### Optimizing the Portfolio

```
In [ ]: !pip install gurobipy
        import gurobipy as gp
        from gurobipy import GRB
       Requirement already satisfied: gurobipy in /Users/goyolozano/Desktop/Mini 4/Value/Update 2/bvenv/lib/python3.10/site
       -packages (12.0.1)
In [ ]: params = {
         'WLSACCESSID": 'f5cc9cd8-d763-4a52-90d7-39c0994137e2',
        "WLSSECRET": 'aab9b930-f1f2-4d73-bdd6-e5c8244ee44c',
        "LICENSEID": 2608044,
        env = gp.Env(params=params)
       Set parameter WLSAccessID
       Set parameter WLSSecret
       Set parameter WLSSecret
       Set parameter LicenseID to value 2608044
       Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
In [ ]: loan_amounts = final_clean_data['loan_amnt']
        final_results_test['loan_amnt'] = final_results_test.index.map(loan_amounts)
        final_results_test.head()
                 predicted_return risk_score cluster loan_amnt
         179685
                        0.068349
                                  0.023649
                                                      21000.0
         373414
                        -0.000414
                                   0.020525
                                                      23275.0
         348264
                        0.064083
                                  0.023398
                                                 4
                                                       6000.0
         370100
                        0.035245
                                  0.036635
                                                49
                                                      30000.0
          31026
                        0.038243
                                  0.024020
                                                11
                                                      15950.0
In [ ]: m = final_results_test['predicted_return']
        r = final_results_test['risk_score']
        c = final_results_test['cluster']
        a = final_results_test['loan_amnt']
```

### Maximize Total Revenue

```
In []: indices = final_results_test.index

modelA = gp.Model(env=env)

selection = modelA.addVars(indices, vtype=GRB.BINARY, name='selection')
    invest = modelA.addVars(indices, lb=0, name="inv_amnt")

modelA.setObjective(sum(selection[i] * r[i] * invest[i] for i in indices), GRB.MAXIMIZE)

modelA.addConstr(sum(selection[i] for i in indices) == 100, name="num_loans")
    modelA.addConstrs((invest[i] <= a.loc[i] for i in indices), name="inv_max")
    modelA.addConstrs((invest[i] >= 25 * selection[i] for i in indices), name="inv_min")
    modelA.addConstrs((invest[i] <= selection[i] * a.loc[i] for i in indices), name="inv_select")</pre>
```

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In [ ]: modelA.optimize()

```
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
       CPU model: Apple M4
       Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
       Academic license 2608044 - for non-commercial use only - registered to tc @andrew.cmu.edu
       Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
       Model fingerprint: 0xb48f18ee
       Model has 129670 quadratic objective terms
       Variable types: 129670 continuous, 129670 integer (129670 binary)
       Coefficient statistics:
       CPU model: Apple M4
       Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
       Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
       Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
       Model fingerprint: 0xb48f18ee
       Model has 129670 quadratic objective terms
       Variable types: 129670 continuous, 129670 integer (129670 binary)
       Coefficient statistics:
        Matrix range
                         [1e+00, 4e+04]
         Objective range [0e+00, 0e+00]
         QObjective range [2e-02, 8e-02]
                         [1e+00, 1e+00]
         Bounds range
         RHS range
                          [1e+02, 4e+04]
       Presolve removed 129670 rows and 243 columns (presolve time = 5s)...
       Presolve removed 130156 rows and 243 columns
       Presolve time: 5.55s
       Presolved: 388282 rows, 388524 columns, 1035659 nonzeros
       Variable types: 0 continuous, 388524 integer (129670 binary)
       Found heuristic solution: objective 37591.209388
       Found heuristic solution: objective 128223.90764
       Explored 1 nodes (0 simplex iterations) in 6.53 seconds (4.59 work units)
       Thread count was 10 (of 10 available processors)
       Solution count 2: 128224 37591.2
       Optimal solution found (tolerance 1.00e-04)
       Best objective 1.282239076351e+05, best bound 1.282239076351e+05, gap 0.0000%
In []: print("Objective = ", modelA.ObjVal)
        modelA.printAttr('X')
```

Objective = 128223.90763506704

Variable	Х
Variable	 X
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selection[637991]	1
selection[349878]	1
selection[555527]	1
selection[414087]	1
selection[538202]	1
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selection[597441]	1
selection[345726]	1
selection[222766]	1
selection[438168]	1
selection[26765]	1
selection[515146]	1
selection[444353]	1
selection[633077]	1
selection[377409]	1
selection[416561]	1
selection[502553]	1
selection[49140]	1
selection[77654]	1
selection[257003]	1
selection[398658]	1
selection[178700]	1
selection[356174]	1
selection[432414]	1
selection[398942]	1
selection[625073]	1
selection[56450]	1
selection[328194]	1
selection[147234]	1
selection[271880]	1
selection[199622] selection[172389]	1
selection[461868]	1
selection[246640]	1
selection[287283]	1
selection[609075]	1
selection[412760]	1
selection[550697]	1
selection[398246]	1
selection[73616]	1
selection[244016]	1
selection[26305]	1
selection[611269]	1
selection[527168]	1
selection[420139]	1
selection[33432]	1
selection[589064]	1
selection[434653]	1
selection[558702]	1
selection[305739]	1
selection[348879]	1
selection[228325]	1
selection[428694]	1
selection[240187]	1
selection[63927] selection[414875]	1
selection[16214] selection[292990]	1
selection[228132]	1
selection[522598]	1
selection[160563]	1
selection[444770]	1
selection[330591]	1
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selection[633276]	1
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selection[573967]	1
selection[281921]	1
selection[552532]	1
selection[316475]	1
selection[392652]	1
selection[80659]	1
selection[80458]	1
selection[303503]	1
selection[356952]	1
selection[263908]	1
selection[380343]	1
selection[515685]	1
selection[2363]	1
selection[263118]	1
selection[367492]	1
selection[444849]	1
selection[7746]	1
selection[410072]	1
selection[13906]	1
selection[90941]	1
selection[316753]	1
coloc+ion[262107]	1
selection[363197]	
selection[59101]	1
selection[520483]	1
selection[326324]	1
inv_amnt[37188]	35000
inv_amnt[637991]	35000
inv_amnt[349878]	35000
inv_amnt[555527]	35000
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inv_amnt[414087]	35000
inv_amnt[538202]	35000
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inv_amnt[76704]	35000
inv_amnt[296843]	
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inv_amnt[624803]	35000
inv_amnt[102941]	35000
inv_amnt[530881]	35000
inv_amnt[308027]	35000
inv_amnt[398553]	35000
inv_amnt[334767]	35000
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inv_amnt[279824]	35000
inv_amnt[597441]	35000
inv_amnt[345726]	35000
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inv_amnt[438168]	35000
inv_amnt[26765]	35000
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inv_amnt[377409]	35000
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inv_amnt[398942]	35000
111V_a11111 [390942]	
inv_amnt[625073]	35000
inv_amnt[56450]	35000
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inv_amnt[328194]	35000
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inv_amnt[461868]	35000
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inv_amnt[550697] inv_amnt[398246]	35000 35000 35000
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inv_amnt[550697] inv_amnt[398246] inv_amnt[73616] inv_amnt[244016] inv_amnt[26305]	35000 35000 35000 35000 35000 35000
inv_amnt[550697] inv_amnt[398246] inv_amnt[73616] inv_amnt[244016] inv_amnt[26305]	35000 35000 35000 35000 35000
inv_amnt[550697] inv_amnt[398246] inv_amnt[73616] inv_amnt[244016] inv_amnt[26305] inv_amnt[611269]	35000 35000 35000 35000 35000 35000
inv_amnt[550697] inv_amnt[398246] inv_amnt[73616] inv_amnt[244016] inv_amnt[26305]	35000 35000 35000 35000 35000 35000

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inv_amnt[420139]
                                35000
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       inv_amnt[33432]
       inv_amnt[589064]
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       inv_amnt[434653]
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       inv_amnt[558702]
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       inv amnt[305739]
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       inv_amnt[348879]
       inv_amnt[228325]
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       inv_amnt[428694]
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       inv_amnt[240187]
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       inv_amnt[63927]
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       inv_amnt[414875]
       inv_amnt[16214]
                               35000
       inv amnt[292990]
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       inv_amnt[228132]
       inv_amnt[522598]
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       inv_amnt[444770]
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       inv amnt[330591]
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       inv_amnt[281921]
       inv_amnt[552532]
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       inv_amnt[303503]
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       inv_amnt[356952]
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       inv_amnt[515685]
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       inv_amnt[263118]
                                35000
       inv_amnt[367492]
       inv_amnt[444849]
                                35000
       inv_amnt[7746]
                              35000
       inv_amnt[410072]
                                35000
                               35000
       inv_amnt[13906]
                               35000
       inv_amnt[90941]
       inv_amnt[316753]
                                35000
       inv_amnt[363197]
                                35000
       inv_amnt[59101]
                               35000
       inv_amnt[520483]
                                35000
       inv_amnt[326324]
                                35000
In [ ]: # sensitivity
        constraint_name = "num_loans" # Replace with the name of the constraint
        \verb|original_rhs| = \verb|modelA.getConstrByName(constraint_name). RHS| \textit{# Get the original RHS value}
        rhs_changes = np.linspace(-10, 10, 50) # Define a range of RHS changes
        objective_values = []
        for change in rhs_changes:
            # Update the RHS value
            modelA.getConstrByName(constraint_name).RHS = original_rhs + change
            # Re-optimize the model
            modelA.optimize()
            # Store the objective function value
            if modelA.status == GRB.OPTIMAL:
                objective_values.append(modelA.ObjVal)
                objective_values.append(np.nan) # Store NaN if not optimal
        # Restore the original RHS value
        modelA.getConstrByName(constraint_name).RHS = original_rhs
        # Analyze the results (e.g., plot objective values vs. RHS changes)
        import matplotlib.pyplot as plt
        plt.plot(rhs_changes, objective_values, marker='o')
        plt.xlabel("Change in RHS")
        plt.ylabel("Objective Function Value")
        plt.title(f"Sensitivity Analysis for Constraint: {constraint_name}")
        plt.grid(True)
        plt.show()
```

```
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc @andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x210382e6
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
  RHS range
                   [9e+01, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num loans by 10.000000000
Presolve removed 129670 rows and 243 columns (presolve time = 5s)...
Presolve removed 130156 rows and 243 columns
Presolve time: 5.57s
Presolved: 388282 rows, 388524 columns, 1035659 nonzeros
Variable types: 0 continuous, 388524 integer (129670 binary)
Found heuristic solution: objective 32643.825309
Found heuristic solution: objective 115401.51687
Explored 1 nodes (0 simplex iterations) in 6.54 seconds (4.57 work units)
Thread count was 10 (of 10 available processors)
Solution count 2: 115402 32643.8
Optimal solution found (tolerance 1.00e-04)
Best objective 1.154015168716e+05, best bound 1.154015168716e+05, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x0d3b2238
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [9e+01, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 0.408163265
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x51cc0721
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
```

```
RHS range
                   [9e+01, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 0.816326531
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xec56d885
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
  RHS range
                   [9e+01, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 1.224489796
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.12 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x5dc46b29
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [9e+01, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 1.632653061
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
```

```
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x9d533263
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [9e+01, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 2.040816327
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xdf00ee51
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [9e+01, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 2.448979592
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.11 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xc32c4006
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
                   [9e+01, 4e+04]
  RHS range
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 2.857142857
Presolve removed 129670 rows and 0 columns
Presolve time: 0.03s
Explored 0 nodes (0 simplex iterations) in 0.11 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
```

```
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x9811d841
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [9e+01, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 3.265306122
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc__@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x6ddc4df2
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [9e+01, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 3.673469388
Presolve removed 129670 rows and 0 columns
Presolve time: 0.03s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xad013a06
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
```

```
RHS range
                   [9e+01, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 4.081632653
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x25471db1
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
  RHS range
                   [9e+01, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 4.489795918
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.11 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xc1b66eaa
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [9e+01, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 4.897959184
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
```

```
Academic license 2608044 - for non-commercial use only - registered to tc__@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xe9428ea4
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 5.306122449
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xe80646b3
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 5.714285714
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.14 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x465635b6
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
                   [1e+02, 4e+04]
  RHS range
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 6.122448980
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.12 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
```

```
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x188aa0ea
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
Bounds range [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 6.530612245
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.11 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x42aed811
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 6.938775510
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.13 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x3a757f0b
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
```

```
RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 7.346938776
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.11 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x24c58cda
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 7.755102041
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xf9629c14
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 8.163265306
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.12 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
```

```
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xec87fb61
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 8.571428571
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.11 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x018eb414
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 8.979591837
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x7f9f979f
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
                   [1e+02, 4e+04]
  RHS range
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 9.387755102
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.14 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
```

```
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x11a558f6
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
Bounds range [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 9.795918367
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.12 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc__@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x9ecb6782
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 10.204081633
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x76d05978
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
```

```
RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 10.612244898
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.14 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x898a12d0
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 11.020408163
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.14 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x272c9775
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 11.428571429
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.13 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
```

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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xd9e1b43b
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 11.836734694
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x6800947b
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 12.244897959
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x753df75f
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
                   [1e+02, 4e+04]
  RHS range
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 12.653061224
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.12 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
```

```
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x9c5a17b2
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
Bounds range [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 13.061224490
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc__@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x079fa93b
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 13.469387755
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x773a4f37
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
```

```
RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 13.877551020
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.16 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x2bf7e537
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 14.285714286
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.12 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xd927c4e3
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 14.693877551
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
```

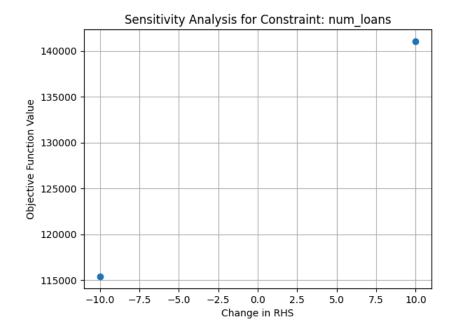
```
Academic license 2608044 - for non-commercial use only - registered to tc__@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x9d5e4293
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 15.102040816
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.12 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x62c8799a
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 15.510204082
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.11 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xec9e5c98
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
                   [1e+02, 4e+04]
  RHS range
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 15.918367347
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.14 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
```

```
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x97e022ea
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
Bounds range [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 16.326530612
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.14 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc__@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x71f5aa9d
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 16.734693878
Presolve removed 129670 rows and 0 columns
Presolve time: 0.03s
Explored 0 nodes (0 simplex iterations) in 0.12 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xe461c188
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
```

```
RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 17.142857143
Presolve removed 129670 rows and 0 columns
Presolve time: 0.03s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xdd8083a1
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 17.551020408
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.14 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xd55be20f
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 17.959183673
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.11 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
```

```
Academic license 2608044 - for non-commercial use only - registered to tc__@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x79cba5b1
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 18.367346939
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.10 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xaadc9a7c
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 18.775510204
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.14 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xe51cf483
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
                   [1e+02, 4e+04]
  RHS range
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 19.183673469
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.12 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
```

```
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xd5889ec9
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
Bounds range [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 19.591836735
Presolve removed 129670 rows and 0 columns
Presolve time: 0.04s
Explored 0 nodes (0 simplex iterations) in 0.12 seconds (0.30 work units)
Thread count was 1 (of 10 available processors)
Solution count 0
Model is infeasible or unbounded
Best objective -, best bound -, gap -
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc__@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x80dc4ed0
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [1e+02, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint num_loans by 20.0000000000
Presolve removed 129670 rows and 243 columns (presolve time = 6s)...
Presolve removed 130156 rows and 243 columns
Presolve time: 5.73s
Presolved: 388282 rows, 388524 columns, 1035659 nonzeros
Variable types: 0 continuous, 388524 integer (129670 binary)
Found heuristic solution: objective 41509.689168
Found heuristic solution: objective 141046.29840
Explored 1 nodes (0 simplex iterations) in 6.71 seconds (4.60 work units)
Thread count was 10 (of 10 available processors)
Solution count 2: 141046 41509.7
Optimal solution found (tolerance 1.00e-04)
Best objective 1.410462983986e+05, best bound 1.410462983986e+05, gap 0.0000%
```



## Maximize Total Revenue w/ Budget

```
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc @andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xa2a18287
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
 Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                  [1e+00, 1e+00]
  Bounds range
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xa2a18287
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
                   [1e+03, 4e+04]
  RHS range
Found heuristic solution: objective -0.0000000
Presolve removed 129670 rows and 0 columns
Presolve time: 0.48s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
 AA' NZ
          : 4.463e+03
 Factor NZ : 9.362e+03 (roughly 5 MB of memory)
 Factor Ops: 2.815e+04 (less than 1 second per iteration)
 Threads
           : 1
                  Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                                     Time
                                                 Dual
  0 -1.08641952e+06 3.50196118e+09 2.27e+01 6.98e+02
                                                         2.45e+05
                                                                      2s
  1 -3.48107110e+05 4.71232876e+08 1.54e+01 1.44e-12
                                                         2.72e+04
                                                                       2s
  2 -1.44246578e+05 7.19267153e+07 6.99e+00 2.73e-12 3.80e+03
                                                                       25
   3 -3.65364331e+04 1.60783674e+07 1.98e+00 6.37e-12 8.76e+02
Barrier performed 3 iterations in 1.79 seconds (3.42 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Use crossover to convert LP symmetric solution to basic solution...
Root relaxation: objective 3.952677e+02, 7173 iterations, 0.25 seconds (0.36 work units)
                                        Objective Bounds
    Nodes
                 Current Node
                                                                    Work
 Expl Unexpl |
              Obj Depth IntInf | Incumbent
                                                 BestBd
                                                         Gap | It/Node Time
                                   -0.00000 395.26769
          0
             395.26769
     0
                           0
                               1
                                                                        15
Н
     0
          0
                                 379.4569832
                                             395.26769
                                                        4.17%
                                                                        1s
Н
     0
          0
                                395.2676908 395.26769 0.00%
                                                                       1s
     0
             395.26769
                                1 395.26769 395.26769 0.00%
Explored 1 nodes (7173 simplex iterations) in 1.95 seconds (3.55 work units)
Thread count was 10 (of 10 available processors)
Solution count 3: 395.268 379.457 -0
```

Optimal solution found (tolerance 1.00e-04) Best objective 3.952676907954e+02, best bound 3.952676907954e+02, gap 0.0000% In [ ]: print("Objective = ", modelB.ObjVal) modelB.printAttr('X') Objective = 395.26769079540406 Variable Variable Χ selection[563331] 10000 inv\_amnt[563331] In [ ]: # sensitivity constraint\_name = "budget" # Replace with the name of the constraint original\_rhs = modelB.getConstrByName(constraint\_name).RHS # Get the original RHS value rhs\_changes = np.linspace(-10000, 10000, 50) # Define a range of RHS changes objective\_values = [] for change in rhs\_changes: # Update the RHS value modelB.getConstrByName(constraint\_name).RHS = original\_rhs + change # Re-optimize the model modelB.optimize() # Store the objective function value if modelB.status == GRB.OPTIMAL: objective\_values.append(modelB.ObjVal) else: objective\_values.append(np.nan) # Store NaN if not optimal # Restore the original RHS value modelB.getConstrByName(constraint\_name).RHS = original\_rhs # Analyze the results (e.g., plot objective values vs. RHS changes) import matplotlib.pyplot as plt plt.plot(rhs\_changes, objective\_values, marker='o') plt.xlabel("Change in RHS") plt.ylabel("Objective Function Value") plt.title(f"Sensitivity Analysis for Constraint: {constraint\_name}") plt.grid(True) plt.show()

```
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc @andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x363a916d
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
  RHS range
                   [1e+03, 4e+04]
MIP start from previous solve did not produce a new incumbent solution
MIP start from previous solve violates constraint budget by 10000.000000000
Found heuristic solution: objective -0.0000000
Presolve removed 389011 rows and 259340 columns
Presolve time: 0.04s
Presolve: All rows and columns removed
Explored 0 nodes (0 simplex iterations) in 0.11 seconds (0.23 work units)
Thread count was 1 (of 10 available processors)
Solution count 1: -0
No other solutions better than -0
Optimal solution found (tolerance 1.00e-04)
Best objective -0.00000000000000e+00, best bound -0.00000000000e+00, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xa2b06a76
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [4e+02, 4e+04]
Loaded MIP start from previous solve with objective −0
Presolve removed 129670 rows and 0 columns
Presolve time: 0.50s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Use crossover to convert LP symmetric solution to basic solution...
Root relaxation: objective 1.613338e+01, 5492 iterations, 0.36 seconds (0.50 work units)
                  Current Node
                                        Objective Bounds
                                                                    Work
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                 BestBd Gap | It/Node Time
                                  16.1333751 16.13338 0.00%
     0
Explored 1 nodes (5492 simplex iterations) in 2.04 seconds (3.67 work units)
Thread count was 10 (of 10 available processors)
Solution count 2: 16.1334 -0
Optimal solution found (tolerance 1.00e-04)
Best objective 1.613337513451e+01, best bound 1.613337513451e+01, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
```

```
Model fingerprint: 0x91d89e69
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [8e+02, 4e+04]
Loaded MIP start from previous solve with objective 16.1334
Presolve removed 129670 rows and 0 columns
Presolve time: 0.50s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 32.2667503
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
AA' NZ
          : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops : 5.290e+05 (less than 1 second per iteration)
 Threads
          : 8
                 Objective
                                          Residual
                                       Primal Dual
Iter
          Primal
                          Dual
                                                          Compl
                                                                    Time
  0 -2.75875793e+05 5.93892226e+06 3.54e+00 2.29e-01 2.03e+01
                                                                      25
   1 -1.33140943e+04 1.44430581e+06 1.34e+00 3.33e-16 2.59e+00
   2 -7.63139359e+03 9.12622963e+03 4.85e-01 5.00e-16 2.00e-01
                                                                      2s
      2.82736351e+01 4.76592340e+03 2.55e-03 2.22e-16 8.24e-03
                                                                      2s
      2.43925005e+01 7.51045795e+01 0.00e+00 4.44e-16 7.82e-05
      2.61482616e+01 3.44807494e+01 0.00e+00 4.44e-16 1.29e-05
   5
                                                                      25
      3.17533919e+01 3.29030157e+01 0.00e+00 2.22e-16
                                                         1.77e-06
                                                                      2s
      3.22586324e+01 3.22678599e+01 1.29e-07 2.22e-16 1.42e-08
Barrier performed 7 iterations in 2.23 seconds (3.89 work units)
Objective cutoff exceeded
Concurrent spin time: 0.01s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.63 seconds (0.73 work units)
                 Current Node
                                       Objective Bounds
   Nodes
                                                                   Work
 Expl Unexpl |
               Obj Depth IntInf | Incumbent
                                              BestBd
                                                         Gap | It/Node Time
     0
                                   32.26675 32.26786 0.00%
Explored 1 nodes (0 simplex iterations) in 2.26 seconds (3.91 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 32.2668
Optimal solution found (tolerance 1.00e-04)
Best objective 3.226675026901e+01, best bound 3.226785988788e+01, gap 0.0034%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x1d04ffdf
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [1e+03, 4e+04]
```

Loaded MIP start from previous solve with objective 32.2668

```
Presolve removed 129670 rows and 0 columns
Presolve time: 0.49s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 48.4001254
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.02s
Barrier statistics:
 AA' NZ : 1.297e+05
Factor NZ : 2.598e+05 (roughly 160 MB of memory)
AA' NZ
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
                 Objective
                                           Residual
Iter
           Primal
                          Dual
                                       Primal Dual
                                                           Compl
                                                                      Time
  0 -8.50931630e+05 1.00636811e+07 1.15e+01 2.06e-01 3.20e+01 1 -5.33876184e+04 2.12814798e+06 2.52e+00 4.44e-16 3.79e+00
                                                                        2s
                                                                        2s
  2 -2.00328263e+04 1.85920499e+04 7.54e-01 4.44e-16 1.88e-01
      2.30454268e+01 8.04207369e+03 4.65e-03 4.44e-16 1.32e-02
                                                                        25
      4.09420611e+01 9.06628959e+01 0.00e+00 2.22e-16 7.67e-05
                                                                        2s
      4.35864412e+01 4.94183897e+01 0.00e+00 4.44e-16 9.00e-06
                                                                        2s
      4.83989549e+01 4.84095507e+01 2.05e-08 2.22e-16 1.63e-08
                                                                        2s
      4.84001254e+01 4.84001254e+01 0.00e+00 3.33e-16 1.72e-14
                                                                        25
Barrier performed 7 iterations in 2.04 seconds (3.91 work units)
Objective cutoff exceeded
Concurrent spin time: 0.00s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.60 seconds (0.74 work units)
                  Current Node
                                        Objective Bounds
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                 BestBd Gap | It/Node Time
                                    48.40013 48.40013 0.00%
Explored 1 nodes (0 simplex iterations) in 2.06 seconds (3.92 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 48.4001
Optimal solution found (tolerance 1.00e-04)
Best objective 4.840012540352e+01, best bound 4.840012541303e+01, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xb3db403d
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                [1e+00, 1e+00]
                   [1e+03, 4e+04]
  RHS range
Loaded MIP start from previous solve with objective 48.4001
Presolve removed 129670 rows and 0 columns
Presolve time: 0.51s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 64.5335005
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
```

5s

```
Ordering time: 0.02s
Barrier statistics:
        : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
           . 8
                 Objective
                                          Residual
          Primal
                         Dual
                                      Primal
Iter
                                               Dual
                                                         Compl
                                                                   Time
   0 -5.17731670e+05 1.19463116e+07 1.37e+01 2.29e-01 3.94e+01
   1 -2.01253734e+04 2.74888133e+06 2.69e+00 4.44e-16 4.83e+00
                                                                     25
   2 -9.78866343e+03 1.47398590e+04 7.59e-01 4.44e-16 1.84e-01
                                                                     2s
  3 8.87801984e+01 6.26625789e+03 4.72e-03 3.33e-16 1.05e-02
                                                                     2s
      5.43125832e+01 1.12435670e+02 0.00e+00 3.33e-16 8.96e-05
                                                                     25
      5.86178041e+01 6.53404201e+01 0.00e+00 2.22e-16 1.04e-05
                                                                     2s
     6.45330973e+01 6.45387079e+01 0.00e+00 2.22e-16 8.65e-09
                                                                     25
Barrier performed 6 iterations in 2.19 seconds (3.85 work units)
Objective cutoff exceeded
Concurrent spin time: 0.01s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.58 seconds (0.69 work units)
                                       Objective Bounds
                 Current Node
                                                                  Work
 Expl Unexpl | Obj Depth IntInf | Incumbent BestBd Gap | It/Node Time
                                   64.53350 64.53871 0.01%
Explored 1 nodes (0 simplex iterations) in 2.22 seconds (3.87 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 64.5335
Optimal solution found (tolerance 1.00e-04)
Best objective 6.453350053803e+01, best bound 6.453870791321e+01, gap 0.0081%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xef82cd54
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                 [1e+00, 4e+04]
 Matrix range
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
                  [1e+00, 1e+00]
 Bounds range
 RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 64.5335
Presolve removed 129670 rows and 0 columns
Presolve time: 0.52s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 80.6668757
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
AA' NZ : 3.290e+02
 Factor NZ : 1.094e+03
 Factor Ops: 1.161e+04 (less than 1 second per iteration)
 Threads
          : 1
                 {\tt Objective}
                                          Residual
Iter
          Primal
                         Dual
                                       Primal Dual
                                                         Compl
                                                                   Time
      2.72212396e+03 4.10947062e+07 4.26e+01 1.21e+02 4.72e+04
                                                                     5s
   1 9.59185320e+03 7.77566078e+06 1.32e+01 3.29e-13 6.71e+03
```

```
2 1.32533455e+03 1.72831987e+06 4.13e+00 2.27e-13 1.30e+03
Barrier performed 2 iterations in 4.52 seconds (3.45 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
AA' NZ
           : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
           : 8
                                          Residual
                 Objective
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                          Compl
                                                                     Time
     -1.59278168e+04 1.39808391e+07 1.58e+01 2.54e-01 4.73e+01
                                                                       55
      1.72419056e+04 3.38299888e+06 2.78e+00 5.55e-16 5.88e+00
   1
                                                                       5s
      3.47300027e+02 1.85730493e+04
                                      7.41e-01 3.33e-16
                                                                       5s
                                                         1.90e-01
      1.60080871e+02 5.99355003e+03
                                      5.70e-03 4.44e-16 1.03e-02
                                                                       5s
      6.51262025e+01 1.40907078e+02 0.00e+00 6.66e-16 1.17e-04
                                                                       5s
      7.10956005e+01 8.16180271e+01
                                      0.00e+00 3.33e-16 1.62e-05
                                                                       5s
      8.06663908e+01 8.06736736e+01 0.00e+00 2.22e-16 1.12e-08
                                                                       5s
Barrier performed 6 iterations in 4.97 seconds (3.86 work units)
Objective cutoff exceeded
Concurrent spin time: 0.00s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.60 seconds (0.69 work units)
                                       Objective Bounds
    Nodes
                 Current Node
                                                                    Work
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd
                                                         Gap | It/Node Time
     0
                           0
                                   80.66688 80.67367 0.01%
                                                                       45
Explored 1 nodes (0 simplex iterations) in 5.00 seconds (3.87 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 80.6669
Optimal solution found (tolerance 1.00e-04)
Best objective 8.066687567253e+01, best bound 8.067367357457e+01, gap 0.0084%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xa479b2dd
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
                  [0e+00, 0e+00]
  Objective range
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
  RHS range
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 80.6669
Presolve removed 129670 rows and 0 columns
Presolve time: 0.52s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 96.8002508
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
```

```
Ordering time: 0.00s
Barrier statistics:
          : 4.550e+02
 AA' NZ
 Factor NZ : 1.346e+03 (roughly 1 MB of memory)
 Factor Ops : 1.212e+04 (less than 1 second per iteration)
          : 1
                 Objective
                                          Residual
                                       Primal
Iter
          Primal
                          Dual
                                                 Dual
                                                          Compl
                                                                    Time
   0 -2.35924751e+05 7.02700214e+07 2.35e+01 1.68e+02 5.29e+04
                                                                      25
   1 -2.70774959e+04 1.28157787e+07 9.35e+00 4.04e-13 7.22e+03
                                                                      2s
  2 -1.37035514e+04 1.95922047e+06 3.60e+00 2.27e-13 1.01e+03
                                                                      2s
Barrier performed 2 iterations in 1.70 seconds (3.45 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
           : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
           : 8
                 Objective
                                          Residual
Iter
          Primal
                                       Primal
                          Dual
                                                 Dual
                                                          Compl
                                                                    Time
   0 -1.61633090e+06 2.02384608e+07 1.06e+01 4.12e-01 6.29e+01
   1 -9.39454210e+04 4.11290692e+06 2.35e+00 8.88e-16 7.24e+00
                                                                      25
   2 -1.92517429e+04 1.38472972e+05
                                      3.66e-01 8.88e-16 3.29e-01
                                                                      2s
   3 -1.50352117e+03 2.68493256e+04 2.76e-02 1.33e-15 6.18e-02
     5.91926722e+01 2.70993954e+02 0.00e+00 8.88e-16 3.27e-04
                                                                      25
      6.10398870e+01 1.02482881e+02
                                      0.00e+00 6.66e-16 6.39e-05
                                                                      2s
   5
      9.63112003e+01 1.64654613e+02 0.00e+00 7.77e-16 1.05e-04
                                                                      2s
      9.65872773e+01 9.68007702e+01 0.00e+00 4.44e-16 3.29e-07
Barrier performed 7 iterations in 2.18 seconds (3.91 work units)
Objective cutoff exceeded
Concurrent spin time: 0.03s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.63 seconds (0.74 work units)
                 Current Node
                                       Objective Bounds
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd Gap | It/Node Time
                                   96.80025 96.80077 0.00%
Explored 1 nodes (0 simplex iterations) in 2.21 seconds (3.92 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 96.8003
Optimal solution found (tolerance 1.00e-04)
Best objective 9.680025080704e+01, best bound 9.680077018315e+01, gap 0.0005%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x1f79c9b0
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
```

```
Bounds range
                  [1e+00, 1e+00]
 RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 96.8003
Presolve removed 129670 rows and 0 columns
Presolve time: 0.46s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 112.9336259
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
AA' NZ
          : 6.130e+02
Factor NZ : 1.662e+03 (roughly 1 MB of memory)
 Factor Ops: 1.275e+04 (less than 1 second per iteration)
Threads
         : 1
                 Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                          Compl
                                                                    Time
  0 -1.40969689e+05
                      9.27842714e+07 2.15e+01 1.76e+02 5.17e+04
                                                                      2s
  1 -1.03256493e+04 1.71199316e+07 8.55e+00 4.80e-13 7.07e+03
                                                                      25
  2 -9.03137183e+03 3.09768706e+06 3.56e+00 3.41e-13 1.18e+03
                                                                      25
Barrier performed 2 iterations in 1.57 seconds (3.42 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.02s
Barrier statistics:
AA' NZ
          : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
Threads
          : 8
                 Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                                    Time
                                                          Compl
  0 -1.35029547e+06 2.20540664e+07 1.17e+01 4.34e-01 7.01e+01
                                                                      2s
  1 -6.57247214e+04 4.72349293e+06 2.44e+00 8.88e-16 8.26e+00
                                                                      2s
  2 -1.48911419e+04 1.64870338e+05 3.77e-01 8.88e-16 3.74e-01
                                                                      25
  3 -1.21721022e+03 2.75131999e+04 3.27e-02 1.11e-15 6.85e-02
                                                                      25
      6.96576599e+01 3.33134998e+02
                                      0.00e+00 8.88e-16 4.06e-04
                                                                      2s
      7.13629563e+01 1.18275182e+02 0.00e+00 8.88e-16 7.24e-05
                                                                      2s
      1.12203256e+02 2.15955554e+02 0.00e+00 8.88e-16 1.60e-04
                                                                      2s
      1.12559143e+02 1.12934534e+02 0.00e+00 6.66e-16 5.79e-07
                                                                      25
Barrier performed 7 iterations in 2.05 seconds (3.88 work units)
Objective cutoff exceeded
Concurrent spin time: 0.01s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.63 seconds (0.74 work units)
                 Current Node
                                       Objective Bounds
                                                                   Work
Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd Gap | It/Node Time
    0
                                  112.93363 112.93453 0.00%
Explored 1 nodes (0 simplex iterations) in 2.08 seconds (3.89 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 112.934
Optimal solution found (tolerance 1.00e-04)
Best objective 1.129336259415e+02, best bound 1.129345342002e+02, gap 0.0008%
```

```
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc @andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x80e07a26
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range
                  [1e+00, 4e+04]
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
                  [1e+00, 1e+00]
 Bounds range
 RHS range
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 112.934
Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 129.0670011
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
AA' NZ : 8.010e+02
 Factor NZ : 2.038e+03 (roughly 1 MB of memory)
 Factor Ops: 1.350e+04 (less than 1 second per iteration)
 Threads
         : 1
                                          Residual
                 Objective
Iter
          Primal
                          Dual
                                       Primal
                                                Dual
                                                          Compl
                                                                    Time
  0 -2.00050564e+05 1.46563096e+08 2.48e+01 2.24e+02 6.51e+04
                                                                      25
  1 -3.12242550e+04 2.63755973e+07 1.13e+01 4.49e-13 8.73e+03
                                                                      25
  2 -1.50446343e+04 5.20083235e+06 3.81e+00 3.41e-13 1.52e+03
                                                                      2s
  3 -2.57715685e+03 8.84746675e+05 8.95e-01 9.09e-13 2.65e+02
                                                                      2s
Barrier performed 3 iterations in 1.74 seconds (3.41 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
AA' NZ : 1.297e+05
Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
         : 8
                 Objective
                                          Residual
                                      Primal Dual
          Primal
Iter
                         Dual
                                                          Compl
                                                                    Time
   0 -1.00192401e+06 2.39458383e+07 1.27e+01 4.58e-01 7.75e+01
  1 -3.49243844e+04 5.34161242e+06 2.51e+00 1.33e-15 9.30e+00
                                                                      25
   2 -1.02116120e+04 1.84629596e+05 3.86e-01 6.66e-16 4.10e-01
                                                                      2s
   3 -5.45211832e+02 1.93356738e+04 3.08e-02 1.11e-15 5.20e-02
                                                                      2s
     8.23885764e+01 3.50731694e+02 0.00e+00 1.11e-15 4.14e-04
                                                                      2s
      8.47972856e+01 1.32929465e+02 0.00e+00 6.66e-16 7.42e-05
                                                                      2s
      1.28950316e+02 1.50087612e+02 0.00e+00 6.66e-16 3.26e-05
                                                                      2s
      1.29053501e+02 1.29067200e+02 7.27e-07 4.44e-16 2.11e-08
Barrier performed 7 iterations in 2.22 seconds (3.87 work units)
Objective cutoff exceeded
Concurrent spin time: 0.01s
Solved with barrier
```

```
Root relaxation: interrupted, 0 iterations, 0.65 seconds (0.74 work units)
    Nodes
                 Current Node
                                        Objective Bounds
                                                                    Work
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd
                                                         Gap | It/Node Time
                                  129.06700 129.06720 0.00%
     0
Explored 1 nodes (0 simplex iterations) in 2.26 seconds (3.88 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 129.067
Optimal solution found (tolerance 1.00e-04)
Best objective 1.290670010761e+02, best bound 1.290671998320e+02, gap 0.0002%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x9d9d69a3
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
  Bounds range
  RHS range
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 129.067
Presolve removed 129670 rows and 0 columns
Presolve time: 0.48s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 145.2003762
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
 AA' NZ
           : 9.840e+02
 Factor NZ : 2.404e+03 (roughly 1 MB of memory)
 Factor Ops: 1.423e+04 (less than 1 second per iteration)
 Threads
           : 1
                                          Residual
                 Objective
                          Dual
                                       Primal
          Primal
                                                 Dual
Tter
                                                           Compl
                                                                     Time
   0 -1.09066016e+05 1.86824891e+08 2.51e+01 2.34e+02 6.70e+04
                                                                       2s
  1 -1.20256922e+04 3.31361611e+07 1.07e+01 4.32e-13 8.91e+03
                                                                       25
   2 -8.60878387e+03 7.04030940e+06 3.83e+00 3.41e-13 1.67e+03
                                                                       2s
   3 -1.17435968e+03 1.29095923e+06 9.89e-01 2.05e-12 3.18e+02
                                                                       2s
Barrier performed 3 iterations in 1.79 seconds (3.41 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.02s
Barrier statistics:
           : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
           : 8
                  Objective
                                           Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                           Compl
                                                                     Time
```

```
0 -5.75492778e+05 2.59007872e+07 1.38e+01 4.83e-01 8.51e+01
   1 -2.31448823e+03 5.96675280e+06 2.57e+00 9.99e-16 1.03e+01
                                                                       25
      -5.28276027e+03 2.04301441e+05 3.93e-01 8.88e-16 4.45e-01
                                                                       25
      3.10617513e+01 3.64203582e+04 2.72e-02 1.33e-15 7.59e-02
                                                                       25
      9.54319270e+01 4.12205431e+02 0.00e+00 1.11e-15 4.89e-04
                                                                       2s
      9.87456386e+01 1.51692902e+02 0.00e+00 8.88e-16 8.17e-05 1.44879554e+02 1.98557730e+02 0.00e+00 6.66e-16 8.28e-05
                                                                       2s
   6
                                                                       2s
      1.45101164e+02 1.45200719e+02 0.00e+00 8.88e-16 1.54e-07
Barrier performed 7 iterations in 2.29 seconds (3.87 work units)
Objective cutoff exceeded
Concurrent spin time: 0.02s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.66 seconds (0.74 work units)
   Nodes
                  Current Node
                                        Objective Bounds
                                                                    Work
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                 BestBd Gap | It/Node Time
                                   145.20038 145.20072 0.00%
Explored 1 nodes (0 simplex iterations) in 2.32 seconds (3.88 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 145.2
Optimal solution found (tolerance 1.00e-04)
Best objective 1.452003762106e+02, best bound 1.452007194923e+02, gap 0.0002%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x4e5a63ad
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
                   [1e+03, 4e+04]
  RHS range
Loaded MIP start from previous solve with objective 145.2
Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 161.3337513
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
          : 1.167e+03
 AA' NZ
 Factor NZ : 2.770e+03 (roughly 1 MB of memory)
 Factor Ops : 1.496e+04 (less than 1 second per iteration)
           : 1
                  Objective
                                           Residual
Iter
           Primal
                           Dual
                                        Primal
                                                  Dual
                                                           Compl
                                                                     Time
  0 -1.61908678e+04 2.25870473e+08 2.53e+01 2.45e+02 6.80e+04
                                                                       25
   1 8.09143339e+03 3.98537256e+07
                                       1.01e+01 4.79e-13 9.03e+03
                                                                       2s
  2 -4.37742998e+02 8.88647905e+06 3.75e+00 3.41e-13 1.79e+03
                                                                       2s
   3 8.81669998e+02 1.87936661e+06 1.03e+00 1.48e-12 3.85e+02
Barrier performed 3 iterations in 1.60 seconds (3.41 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
```

```
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
 AA' NZ : 1.297e+05
Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
           : 8
                                            Residual
                  Objective
           Primal
                                        Primal
Iter
                           Dual
                                                 Dual
                                                            Compl
                                                                      Time
   0 -7.43924743e+04 2.79052678e+07 1.48e+01 5.08e-01 9.29e+01
                                                                        25
   1 3.15892077e+04 6.59724869e+06 2.61e+00 1.33e-15 1.14e+01
                                                                        25
   2 -1.82251349e+02 2.24348945e+05 3.98e-01 8.88e-16 4.80e-01
                                                                        2s
      6.14836180e+02 6.59437939e+04
                                       3.52e-02 1.78e-15 1.29e-01
                                                                        25
      1.02083286e+02 5.61626102e+02 0.00e+00 1.33e-15 7.09e-04
                                                                        25
      1.04334212e+02 1.73827336e+02 0.00e+00 1.11e-15 1.07e-04
                                                                        2s
      1.58971649e+02 3.63280953e+02 0.00e+00 8.88e-16 3.15e-04 1.59869052e+02 1.61337181e+02 0.00e+00 8.88e-16 2.26e-06
   6
                                                                        2s
                                                                        25
Barrier performed 7 iterations in 2.10 seconds (3.87 work units)
Objective cutoff exceeded
Concurrent spin time: 0.02s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.66 seconds (0.74 work units)
                  Current Node
                                         Objective Bounds
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                  BestBd Gap | It/Node Time
                                   161.33375 161.33718 0.00%
     0
Explored 1 nodes (0 simplex iterations) in 2.12 seconds (3.88 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 161.334
Optimal solution found (tolerance 1.00e-04)
Best objective 1.613337513451e+02, best bound 1.613371814047e+02, gap 0.0021%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x3f35396f
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 161.334
Presolve removed 129670 rows and 0 columns
Presolve time: 0.48s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 177.4671265
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
AA' NZ
           : 1.361e+03
 Factor NZ : 3.158e+03 (roughly 2 MB of memory)
 Factor Ops: 1.574e+04 (less than 1 second per iteration)
```

Threads : 1

```
Objective
                                          Residual
          Primal
                                       Primal
Tter
                                                                    Time
                          Dual
                                                 Dual
                                                          Compl
     -5.33942436e+05 3.97538866e+08 1.68e+01 3.30e+02 9.46e+04
                                                                      25
   a
   1 -8.45938201e+04
                      6.60695407e+07
                                      8.57e+00 7.07e-13 1.19e+04
                                                                      2s
   2 -3.78827004e+04 8.17083808e+06 3.49e+00 6.82e-13 1.35e+03
                                                                      2s
   3 -1.02123925e+04 1.66491624e+06 9.75e-01 2.05e-12 2.85e+02
Barrier performed 3 iterations in 1.69 seconds (3.41 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.04s
Barrier statistics:
          : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
           : 8
                 Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                                    Time
                                                          Compl
     -3.22306906e+06 3.77532237e+07 2.26e+01 4.02e-01 1.15e+02
   0
                                                                      25
                      7.35121269e+06
     -1.88761808e+05
                                      4.29e+00 1.22e-15
                                                         1.29e+01
                                                                      25
   2 -1.92883989e+04 2.17895633e+05 3.54e-01 8.88e-16 4.29e-01
                                                                      25
      8.58567977e+02 1.31151501e+05 8.77e-02 1.33e-15 2.50e-01
      9.41787522e+01 5.98800624e+02
                                      3.46e-04 8.88e-16 9.83e-04
                                                                      2s
                                      0.00e+00 1.33e-15 1.20e-04
      1.15253610e+02 1.93024017e+02
                                                                      2s
      1.76568587e+02 2.04701245e+02 0.00e+00 6.66e-16 4.34e-05
                                                                      2s
      1.77378904e+02 1.77469033e+02 0.00e+00 8.88e-16 1.39e-07
                                                                      25
Barrier performed 7 iterations in 2.17 seconds (3.87 work units)
Objective cutoff exceeded
Concurrent spin time: 0.03s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.64 seconds (0.74 work units)
    Nodes
                 Current Node
                                       Objective Bounds
                                                                   Work
 Expl Unexpl |
               Obj Depth IntInf | Incumbent
                                                BestBd
                                                         Gap | It/Node Time
                                  177.46713 177.46903 0.00%
Explored 1 nodes (0 simplex iterations) in 2.19 seconds (3.88 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 177.467
Optimal solution found (tolerance 1.00e-04)
Best objective 1.774671264796e+02, best bound 1.774690334452e+02, gap 0.0011%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xc338bad5
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
 Matrix range
 Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
 Bounds range
                  [1e+00, 1e+00]
                   [1e+03, 4e+04]
 RHS range
Loaded MIP start from previous solve with objective 177.467
Presolve removed 129670 rows and 0 columns
```

file:///Users/goyolozano/Desktop/Mini 4/Value/Update 3/Deliverables/Update\_3\_Optimization.html

Presolve time: 0.47s Presolved: 389011 rows, 389010 columns, 1037360 nonzeros Variable types: 259340 continuous, 129670 integer (129670 binary) Found heuristic solution: objective 193.6005016 Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier Showing barrier log only... Root barrier log... Ordering time: 0.00s Barrier statistics: AA' NZ : 1.577e+03 Factor NZ : 3.590e+03 (roughly 2 MB of memory) Factor Ops: 1.660e+04 (less than 1 second per iteration) Threads : 1 Objective Residual Iter Primal Dual Primal Dual Compl Time 0 -4.99564946e+05 4.75201306e+08 1.71e+01 3.37e+02 9.72e+04 25 1 -8.03005275e+04 7.84924976e+07 8.84e+00 7.76e-13 1.21e+04 2s 2 -4.04365652e+04 1.03700215e+07 3.75e+00 9.09e-13 1.49e+03 3 -1.11614622e+04 2.19992184e+06 1.10e+00 2.05e-12 3.34e+02 2s 25 Barrier performed 3 iterations in 1.61 seconds (3.41 work units) Barrier solve interrupted - model solved by another algorithm Concurrent spin time: 0.01s Solved with primal simplex Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier Showing barrier log only... Root barrier log... Ordering time: 0.03s Barrier statistics: AA' NZ : 1.297e+05 Factor NZ : 2.598e+05 (roughly 160 MB of memory) Factor Ops: 5.290e+05 (less than 1 second per iteration) Threads : 8 Objective Residual Iter Primal Dual Primal Dual Compl 0 -3.04883856e+06 3.94174396e+07 2.48e+01 4.12e-01 1.22e+02 25 1 -1.67223663e+05 7.93220665e+06 4.41e+00 9.99e-16 1.39e+01 2 -1.70266568e+04 2.29824550e+05 3.47e-01 8.88e-16 4.46e-01 25 3 -6.88792559e+01 8.97478280e+04 8.44e-02 1.33e-15 1.84e-01 2s 8.76925284e+01 6.14197381e+02 7.78e-04 1.11e-15 1.27e-03 5 1.25439209e+02 2.10002639e+02 0.00e+00 1.11e-15 1.30e-04 2s 1.70212234e+02 1.95016261e+02 0.00e+00 8.88e-16 3.83e-05 2s 1.93593938e+02 1.93677000e+02 0.00e+00 4.44e-16 1.28e-07 25 1.93601017e+02 1.93600502e+02 2.04e-04 4.44e-16 1.79e-13 25 Barrier performed 8 iterations in 2.15 seconds (3.92 work units) Objective cutoff exceeded Concurrent spin time: 0.06s Solved with barrier Root relaxation: interrupted, 0 iterations, 0.71 seconds (0.79 work units) Current Node Objective Bounds Work Obj Depth IntInf | Incumbent Expl Unexpl | BestBd Gap | It/Node Time 0 193.60050 193.60050 0.00% Explored 1 nodes (0 simplex iterations) in 2.18 seconds (3.93 work units) Thread count was 10 (of 10 available processors) Solution count 1: 193.601 Optimal solution found (tolerance 1.00e-04) Best objective 1.936005016141e+02, best bound 1.936005016953e+02, gap 0.0000% Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81) CPU model: Apple M4 Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

```
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x93fc7b68
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 193.601
Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 209.7338767
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
           : 1.814e+03
 Factor NZ : 4.064e+03 (roughly 2 MB of memory)
 Factor Ops: 1.755e+04 (less than 1 second per iteration)
 Threads
                 Objective
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                          Compl
                                                                    Time
   0 -4.49306835e+05 5.68862567e+08 1.80e+01 3.46e+02 1.01e+05
                                                                      2s
  1 -7.77326052e+04 9.30331354e+07 9.42e+00 7.08e-13 1.26e+04
                                                                      2s
   2 -3.85540054e+04 1.45138719e+07 4.00e+00 1.14e-12 1.81e+03
                                                                      25
   3 -1.26842789e+04 3.89684970e+06 1.39e+00 1.36e-12 5.10e+02
                                                                      2s
Barrier performed 3 iterations in 2.38 seconds (3.41 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
           : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops : 5.290e+05 (less than 1 second per iteration)
 Threads
                 Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal Dual
                                                          Compl
                                                                    Time
   0 -2.84221669e+06 4.10915456e+07 2.70e+01 4.23e-01 1.29e+02
                                                                      35
   1 -1.43948096e+05 8.51272373e+06 4.51e+00 9.99e-16 1.49e+01
   2 -1.51135126e+04 2.42386898e+05 3.48e-01 6.66e-16 4.67e-01
                                                                      35
      6.33304811e+02 1.60250205e+05 1.01e-01 1.55e-15 3.04e-01
                                                                      3s
      3.91605781e+01 7.62587135e+02 1.92e-03 1.55e-15 2.36e-03
                                                                      3s
      1.28757212e+02 3.27381236e+02 0.00e+00 1.11e-15 3.06e-04
   5
                                                                      3s
      1.40310311e+02 2.11494359e+02 0.00e+00 6.66e-16 1.10e-04
                                                                      3s
      2.08085878e+02 2.18571662e+02 0.00e+00 8.88e-16 1.62e-05
                                                                      3s
      2.09715776e+02 2.09733995e+02 0.00e+00 8.88e-16 2.81e-08
                                                                      35
Barrier performed 8 iterations in 2.91 seconds (3.92 work units)
Objective cutoff exceeded
Concurrent spin time: 0.07s
Solved with barrier
```

Root relaxation: interrupted, 0 iterations, 0.70 seconds (0.79 work units)

```
Current Node
                                       Objective Bounds
                                                                   Work
   Nodes
                                                         Gap | It/Node Time
 Expl Unexpl |
               Obj Depth IntInf | Incumbent
                                                BestBd
                                  209.73388 209.73399 0.00%
     0
Explored 1 nodes (0 simplex iterations) in 2.94 seconds (3.93 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 209.734
Optimal solution found (tolerance 1.00e-04)
Best objective 2.097338767486e+02, best bound 2.097339948998e+02, gap 0.0001%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x8d0b0124
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
 Matrix range
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
 Bounds range
                  [1e+00, 1e+00]
                   [1e+03, 4e+04]
 RHS range
Loaded MIP start from previous solve with objective 209.734
Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 225.8672519
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
 AA' NZ
          : 2.034e+03
 Factor NZ : 4.504e+03 (roughly 2 MB of memory)
 Factor Ops : 1.843e+04 (less than 1 second per iteration)
 Threads
          : 1
                                          Residual
                 Objective
Iter
          Primal
                          Dual
                                       Primal
                                                Dual
                                                                    Time
                                                          Compl
   0 -3.92428814e+05 6.68413266e+08 1.89e+01 3.55e+02 1.06e+05
                                                                      2s
  1 -6.71350567e+04 1.07949728e+08 9.64e+00 7.32e-13 1.30e+04
                                                                      25
  2 -3.64006104e+04 1.89666074e+07 4.29e+00 6.82e-13 2.12e+03
                                                                      25
   3 -1.23156861e+04 5.31137092e+06 1.55e+00 2.27e-12 6.04e+02
                                                                      25
Barrier performed 3 iterations in 1.61 seconds (3.40 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
 AA' NZ
          : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
          : 8
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                          Compl
                                                                    Time
   0 -2.60097711e+06 4.27907365e+07 2.92e+01 4.34e-01 1.36e+02
                                                                      2s
   1 -1.19536343e+05 9.09485680e+06 4.60e+00 1.33e-15 1.59e+01
                                                                      2s
   2 -1.32266564e+04 2.54719145e+05 3.48e-01 6.66e-16 4.87e-01
                                                                      2s
```

```
5.61893277e+02 1.56317485e+05 1.02e-01 1.78e-15 2.93e-01
      3.86105677e+01 9.51799262e+02 2.53e-03 1.11e-15 2.94e-03
                                                                      25
      1.39179264e+02 3.63882843e+02 0.00e+00 8.88e-16 3.47e-04
                                                                      25
      1.50863203e+02 2.27558531e+02 0.00e+00 8.88e-16 1.18e-04
                                                                      25
      2.23934121e+02 2.35125096e+02 1.18e-06 8.88e-16 1.73e-05
                                                                      2s
      2.25847506e+02 2.25867386e+02 0.00e+00 8.88e-16 3.06e-08
                                                                      25
Barrier performed 8 iterations in 2.14 seconds (3.91 work units)
Objective cutoff exceeded
Concurrent spin time: 0.05s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.69 seconds (0.79 work units)
                                       Objective Bounds
                 Current Node
                                                                   Work
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd
                                                         Gap | It/Node Time
                                  225.86725 225.86739 0.00%
Explored 1 nodes (0 simplex iterations) in 2.16 seconds (3.92 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 225,867
Optimal solution found (tolerance 1.00e-04)
Best objective 2.258672518831e+02, best bound 2.258673859925e+02, gap 0.0001%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x63495750
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
 Matrix range
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
 Bounds range
                  [1e+00, 1e+00]
 RHS range
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 225.867
Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 242.0006270
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
           : 2.277e+03
 Factor NZ : 4.990e+03 (roughly 3 MB of memory)
 Factor Ops: 1.940e+04 (less than 1 second per iteration)
 Threads
           : 1
                 Objective
                                          Residual
          Primal
                                       Primal
                                                          Compl
Iter
                          Dual
                                                 Dual
                                                                    Time
   0 -3.71193664e+05 7.84477233e+08 1.98e+01 3.66e+02 1.11e+05
                                                                      2s
  1 -7.16031361e+04 1.12422525e+08 1.05e+01 7.31e-13 1.23e+04
                                                                      2s
   2 -3.53168799e+04 2.18858909e+07 4.41e+00 6.82e-13 2.20e+03
                                                                      25
   3 -1.01994691e+04 4.90619416e+06 1.47e+00 3.41e-12 5.32e+02
                                                                      2s
Barrier performed 3 iterations in 2.42 seconds (3.40 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
```

```
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
AA' NZ : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads : 8
                                         Residual
                Objective
Iter
          Primal
                         Dual
                                      Primal Dual
                                                         Compl
                                                                   Time
  0 -2.32779350e+06 4.45084988e+07 3.13e+01 4.46e-01 1.43e+02
                                                                     3s
   1 -9.43006395e+04 9.67821298e+06 4.67e+00 1.11e-15 1.68e+01
                                                                     3s
   2 -1.13509055e+04 2.66984728e+05 3.49e-01 8.88e-16 5.08e-01
                                                                     3s
  3 4.33342190e+02 1.52261505e+05 1.03e-01 1.33e-15 2.84e-01
                                                                     3s
   4 5.04616623e+01 1.66251655e+03 3.57e-03 1.33e-15 4.61e-03
                                                                     3s
      1.55656620e+02 4.18549831e+02 0.00e+00 9.99e-16 4.05e-04
                                                                     3s
      1.71069793e+02 2.43342701e+02 0.00e+00 8.88e-16 1.11e-04
                                                                     35
      2.41233093e+02 2.45147182e+02 2.08e-06 8.88e-16 6.04e-06
                                                                     3s
   8 2.41999725e+02 2.42000635e+02 5.35e-06 1.33e-15 1.42e-09
                                                                     3s
Barrier performed 8 iterations in 2.94 seconds (3.91 work units)
Objective cutoff exceeded
Concurrent spin time: 0.04s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.68 seconds (0.79 work units)
                                      Objective Bounds
                Current Node
                                                                  Work
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                               BestBd Gap | It/Node Time
                                  242.00063 242.00063 0.00%
Explored 1 nodes (0 simplex iterations) in 2.97 seconds (3.92 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 242.001
Optimal solution found (tolerance 1.00e-04)
Best objective 2.420006270176e+02, best bound 2.420006345521e+02, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x3d9b3540
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range [1e+00, 4e+04]
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
 Bounds range
                  [1e+00, 1e+00]
 RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 242.001
Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 258.1340022
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
 AA' NZ : 2.515e+03
 Factor NZ : 5.466e+03 (roughly 3 MB of memory)
 Factor Ops: 2.036e+04 (less than 1 second per iteration)
 Threads
          : 1
```

```
Objective
                                          Residual
                                                          Compl
Iter
           Primal
                          Dual
                                       Primal
                                                 Dual
                                                                    Time
     -3.09058849e+05 9.05271888e+08 2.07e+01 3.75e+02 1.15e+05
                                                                      25
   0
   1 -6.01918693e+04 1.30030168e+08 1.09e+01 8.30e-13 1.29e+04
                                                                      2s
   2 -3.10640095e+04 2.61570447e+07 4.67e+00 1.14e-12 2.40e+03
                                                                      2s
   3 -8.46890893e+03 5.65528404e+06 1.61e+00 3.01e-12 5.63e+02
                                                                      25
Barrier performed 3 iterations in 1.74 seconds (3.40 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
AA' NZ
           : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
            : 8
                 Objective
                                          Residual
          Primal
                                       Primal
Iter
                          Dual
                                                 Dual
                                                          Compl
                                                                    Time
   0 -2.02865743e+06 4.62270654e+07 3.33e+01 4.58e-01 1.50e+02
                                                                      2s
   1 -6.87289846e+04 1.02609035e+07 4.74e+00 1.33e-15 1.78e+01
                                                                      25
     -9.50349038e+03 2.79306001e+05
                                      3.50e-01 8.88e-16 5.28e-01
                                                                      2s
      3.22607995e+02 1.60728790e+05 1.04e-01 1.33e-15 3.06e-01
                                                                      25
      7.77109605e+01 1.21746581e+03 3.12e-03 1.11e-15 3.96e-03
   5
      1.61729199e+02 4.59059443e+02
                                      0.00e+00 1.33e-15 4.59e-04
                                                                      2s
   6
      1.76005937e+02 2.60111899e+02
                                      0.00e+00 8.88e-16 1.30e-04
                                                                      2s
      2.55536282e+02 2.69649414e+02 0.00e+00 8.88e-16 2.18e-05
                                                                      25
   8
      2.58106257e+02 2.58134184e+02 5.54e-07 8.88e-16 4.31e-08
                                                                      25
Barrier performed 8 iterations in 2.27 seconds (3.91 work units)
Objective cutoff exceeded
Concurrent spin time: 0.06s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.70 seconds (0.79 work units)
    Nodes
                  Current Node
                                       Objective Bounds
                                                                   Work
               Obj Depth IntInf | Incumbent
                                                BestBd
                                                         Gap | It/Node Time
 Expl Unexpl |
                                  258.13400 258.13418 0.00%
Explored 1 nodes (0 simplex iterations) in 2.29 seconds (3.92 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 258.134
Optimal solution found (tolerance 1.00e-04)
Best objective 2.581340021521e+02, best bound 2.581341840095e+02, gap 0.0001%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xa79e3809
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 258.134
Presolve removed 129670 rows and 0 columns
```

Presolve time: 0.49s Presolved: 389011 rows, 389010 columns, 1037360 nonzeros Variable types: 259340 continuous, 129670 integer (129670 binary) Found heuristic solution: objective 274.2673773 Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier Showing barrier log only... Root barrier log... Ordering time: 0.00s Barrier statistics: AA' NZ : 2.726e+03 Factor NZ : 5.888e+03 (roughly 3 MB of memory) Factor Ops : 2.120e+04 (less than 1 second per iteration) Threads Residual Objective Iter Primal Dual Primal Dual Compl Time 0 -2.39025998e+05 1.00479565e+09 2.10e+01 3.83e+02 1.18e+05 25 1 -4.60729124e+04 1.63379819e+08 1.08e+01 9.09e-13 1.48e+04 2s 2 -2.57028851e+04 3.15070748e+07 4.92e+00 1.59e-12 2.67e+03 2s 3 -7.49331928e+03 8.16439340e+06 1.91e+00 2.16e-12 7.31e+02 25 Barrier performed 3 iterations in 1.75 seconds (3.40 work units) Barrier solve interrupted - model solved by another algorithm Concurrent spin time: 0.00s Solved with primal simplex Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier Showing barrier log only... Root barrier log... Ordering time: 0.03s Barrier statistics: AA' NZ : 1.297e+05 Factor NZ : 2.598e+05 (roughly 160 MB of memory) Factor Ops: 5.290e+05 (less than 1 second per iteration) Threads : 8 Objective Residual Primal Iter Primal Dual Dual Compl 0 -1.69922238e+06 4.79641613e+07 3.54e+01 4.70e-01 1.57e+02 25 1 -4.25634198e+04 1.08449370e+07 4.79e+00 1.33e-15 1.88e+01 2 -7.65439458e+03 2.93068976e+05 3.52e-01 8.88e-16 5.51e-01 25 3.01069909e+02 1.57440450e+05 1.04e-01 1.55e-15 3.02e-01 2s 6.00960837e+01 1.76943852e+03 5.11e-03 1.33e-15 6.37e-03 1.69711433e+02 5.65630317e+02 0.00e+00 1.33e-15 6.11e-04 2s 1.79434694e+02 2.76005599e+02 0.00e+00 8.88e-16 1.49e-04 2s 2.69825849e+02 2.89182537e+02 2.07e-07 8.88e-16 2.99e-05 25 2.74205351e+02 2.74267866e+02 0.00e+00 6.66e-16 9.64e-08 25 Barrier performed 8 iterations in 2.25 seconds (3.91 work units) Objective cutoff exceeded Concurrent spin time: 0.04s Solved with barrier Root relaxation: interrupted, 0 iterations, 0.70 seconds (0.80 work units) Current Node Objective Bounds Obj Depth IntInf | Incumbent Expl Unexpl | BestBd Gap | It/Node Time 0 274.26738 274.26787 0.00% Explored 1 nodes (0 simplex iterations) in 2.29 seconds (3.92 work units) Thread count was 10 (of 10 available processors) Solution count 1: 274.267 Optimal solution found (tolerance 1.00e-04) Best objective 2.742673772866e+02, best bound 2.742678662803e+02, gap 0.0002% Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81) CPU model: Apple M4

Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

```
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x58ec2645
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 274.267
Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 290.4007524
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
           : 2.956e+03
 Factor NZ : 6.348e+03 (roughly 4 MB of memory)
 Factor Ops: 2.212e+04 (less than 1 second per iteration)
 Threads
                 Objective
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                          Compl
                                                                    Time
   0 -2.81317145e+05 1.29902373e+09 2.47e+01 4.90e+02 1.44e+05
                                                                      2s
  1 -1.19529296e+05 2.03949945e+08 1.44e+01 8.71e-13 1.83e+04
                                                                      2s
   2 -3.53800472e+04 4.05050130e+07 5.40e+00 4.55e-13 3.22e+03
                                                                      25
   3 -8.60834361e+03 1.13260657e+07 2.43e+00 6.14e-12 9.09e+02
                                                                      2s
Barrier performed 3 iterations in 1.69 seconds (3.40 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
           : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
                 Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal Dual
                                                          Compl
                                                                    Time
   0 -1.34499780e+06 4.97029120e+07 3.74e+01 4.83e-01 1.64e+02
                                                                      25
   1 -1.62065212e+04 1.14283908e+07 4.84e+00 1.55e-15 1.98e+01
   2 -5.81784808e+03 3.06942130e+05 3.53e-01 8.88e-16 5.74e-01
                                                                      25
      6.69300037e+02 1.51396601e+05 1.01e-01 1.55e-15 2.94e-01
                                                                      2s
      1.56797855e+02 1.03222330e+03 1.77e-03 1.33e-15 2.54e-03
      1.84019449e+02 3.79182166e+02 0.00e+00 8.88e-16 3.01e-04
   5
                                                                      25
      2.11792273e+02 2.92813940e+02 0.00e+00 6.66e-16 1.25e-04
                                                                      2s
   6
      2.89887781e+02 2.93950704e+02 1.27e-05 6.66e-16 6.27e-06
                                                                      25
      2.90400092e+02 2.90400761e+02 5.91e-05 8.88e-16 1.26e-09
                                                                      25
Barrier performed 8 iterations in 2.22 seconds (3.91 work units)
Objective cutoff exceeded
Concurrent spin time: 0.03s
Solved with barrier
```

Root relaxation: interrupted, 0 iterations, 0.69 seconds (0.80 work units)

```
Current Node
                                       Objective Bounds
                                                                   Work
   Nodes
                                                         Gap | It/Node Time
 Expl Unexpl |
               Obj Depth IntInf | Incumbent
                                                BestBd
                                  290.40075 290.40076 0.00%
     0
Explored 1 nodes (0 simplex iterations) in 2.24 seconds (3.92 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 290,401
Optimal solution found (tolerance 1.00e-04)
Best objective 2.904007524211e+02, best bound 2.904007607728e+02, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x3a10fecf
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
 Matrix range
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
 Bounds range
                  [1e+00, 1e+00]
 RHS range
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 290.401
Presolve removed 129670 rows and 0 columns
Presolve time: 0.49s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 306.5341276
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
 AA' NZ
          : 3.186e+03
 Factor NZ : 6.808e+03 (roughly 4 MB of memory)
 Factor Ops : 2.304e+04 (less than 1 second per iteration)
 Threads
          : 1
                                          Residual
                 Objective
Iter
          Primal
                          Dual
                                       Primal
                                                Dual
                                                                    Time
                                                          Compl
                     1.45267447e+09 2.55e+01 5.02e+02 1.49e+05
   0 -2.07522851e+05
                                                                      2s
  1 -1.01674100e+05 2.26913357e+08 1.44e+01 9.45e-13 1.90e+04
                                                                      25
  2 -2.58502950e+04 4.59373281e+07 6.29e+00 9.09e-13 3.40e+03
                                                                      25
  3 -5.23183725e+03 1.20400528e+07 2.50e+00 3.75e-12 9.14e+02
                                                                      25
Barrier performed 3 iterations in 1.82 seconds (3.39 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
 AA' NZ
          : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
          : 8
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                          Compl
                                                                    Time
   0 -9.65218777e+05 5.14469525e+07 3.94e+01 4.95e-01 1.71e+02
                                                                      2s
     1.03723344e+04 1.20114568e+07 4.88e+00 1.33e-15
                                                         2.07e+01
                                                                      2s
   2 -3.97332933e+03 3.20827053e+05 3.55e-01 7.77e-16 5.98e-01
                                                                      2s
```

```
7.78171757e+02 1.43862304e+05 9.65e-02 1.78e-15 2.81e-01
      1.76096033e+02 1.47649932e+03 2.43e-03 1.33e-15 3.67e-03
                                                                      25
      1.95351368e+02 4.48489280e+02 0.00e+00 8.88e-16 3.90e-04
                                                                      25
      2.15500304e+02 3.08755200e+02 0.00e+00 6.66e-16 1.44e-04
   6
                                                                      25
      3.05151079e+02 3.14229867e+02 0.00e+00 8.88e-16 1.40e-05
                                                                      2s
      3.06526211e+02 3.06534180e+02 0.00e+00 8.88e-16 1.22e-08
                                                                      2s
Barrier performed 8 iterations in 2.36 seconds (3.90 work units)
Objective cutoff exceeded
Concurrent spin time: 0.05s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.71 seconds (0.80 work units)
                 Current Node
                                       Objective Bounds
                                                                   Work
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd
                                                         Gap | It/Node Time
                                  306.53413 306.53418 0.00%
Explored 1 nodes (0 simplex iterations) in 2.38 seconds (3.92 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 306.534
Optimal solution found (tolerance 1.00e-04)
Best objective 3.065341275556e+02, best bound 3.065341796374e+02, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x3862ed3c
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
 Matrix range
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
 Bounds range
                  [1e+00, 1e+00]
 RHS range
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 306.534
Presolve removed 129670 rows and 0 columns
Presolve time: 0.48s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 322.6675027
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
           : 3.426e+03
 Factor NZ : 7.288e+03 (roughly 4 MB of memory)
 Factor Ops: 2.400e+04 (less than 1 second per iteration)
 Threads
           : 1
                 Objective
                                          Residual
          Primal
                                       Primal
                                                          Compl
Iter
                          Dual
                                                 Dual
                                                                    Time
   0 -1.32616709e+05 1.62408261e+09 2.65e+01 5.17e+02 1.55e+05
                                                                      2s
  1 -6.87426005e+04 2.92915062e+08 1.55e+01 1.16e-12 2.30e+04
                                                                      2s
   2 -1.64180745e+04 5.73813536e+07 5.90e+00 1.14e-12 3.99e+03
                                                                      25
   3 -2.66891130e+03 1.74154905e+07 2.83e+00 1.05e-11 1.24e+03
                                                                      2s
Barrier performed 3 iterations in 2.00 seconds (3.39 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
```

```
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
AA' NZ : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops : 5.290e+05 (less than 1 second per iteration)
 Threads : 8
                                           Residual
                 Objective
                          Dual
Iter
          Primal
                                       Primal Dual
                                                            Compl
                                                                      Time
  0 -5.64128324e+05 5.31842690e+07 4.13e+01 5.07e-01 1.78e+02
                                                                       2s
   1 3.68997579e+04 1.25926575e+07 4.91e+00 1.33e-15 2.17e+01
                                                                        25
  2 -2.12882108e+03 3.34578689e+05 3.58e-01 6.66e-16 6.21e-01 3 6.20969173e+02 7.53453320e+04 9.34e-02 1.78e-15 1.82e-01
                                                                        25
                                                                        25
   4 1.87952192e+02 1.02441243e+03 8.72e-04 1.33e-15 1.95e-03
                                                                        2s
      1.98132764e+02 3.32398932e+02 0.00e+00 1.33e-15 2.07e-04 3.02975486e+02 5.71844291e+02 0.00e+00 8.88e-16 4.15e-04
                                                                        25
                                                                        25
  7 3.12976196e+02 3.22786495e+02 0.00e+00 8.88e-16 1.51e-05
                                                                        2s
  8 3.22665984e+02 3.22668572e+02 1.38e-05 8.88e-16 4.05e-09
                                                                        3s
Barrier performed 8 iterations in 2.51 seconds (3.90 work units)
Objective cutoff exceeded
Concurrent spin time: 0.04s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.72 seconds (0.80 work units)
                                        Objective Bounds
                 Current Node
                                                                     Work
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                 BestBd Gap | It/Node Time
                                   322.66750 322.66857 0.00%
Explored 1 nodes (0 simplex iterations) in 2.55 seconds (3.91 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 322.668
Optimal solution found (tolerance 1.00e-04)
Best objective 3.226675026901e+02, best bound 3.226685720191e+02, gap 0.0003%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x7c1ec210
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 322.668
Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 338.8008778
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
 AA' NZ : 3.648e+03
 Factor NZ : 7.732e+03 (roughly 4 MB of memory)
 Factor Ops: 2.489e+04 (less than 1 second per iteration)
 Threads
```

```
Objective
                                          Residual
                                                          Compl
Tter
           Primal
                          Dual
                                       Primal
                                                 Dual
                                                                    Time
    -1.26417929e+06 2.72398366e+09 2.17e+01 6.71e+02 2.33e+05
                                                                      25
   0
   1 -3.64020594e+05 3.61596132e+08 1.41e+01 1.66e-12 2.53e+04
                                                                      2s
   2 -1.33869637e+05 4.79068163e+07
                                      5.74e+00 1.82e-12 3.01e+03
                                                                      2s
   3 -3.89278167e+04 1.15149416e+07 1.80e+00 3.64e-12 7.52e+02
                                                                      25
Barrier performed 3 iterations in 1.72 seconds (3.39 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.04s
Barrier statistics:
AA' NZ
           : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
            : 8
                 Objective
                                          Residual
          Primal
                                       Primal
Iter
                          Dual
                                                 Dual
                                                          Compl
                                                                    Time
   0 -5.64597229e+06 7.55305602e+07 2.16e+01 7.79e-01 2.26e+02
                                                                      2s
   1 -4.36888716e+05 1.41708046e+07 4.95e+00 2.00e-15 2.54e+01
                                                                      25
     -2.66832095e+04 3.80547414e+05
                                      2.62e-01 1.11e-15 7.12e-01
                                                                      2s
   3 -4.84020982e+03 2.46103842e+04 5.28e-02 2.22e-15 1.15e-01
                                                                      25
      2.08516971e+02 1.48686324e+03 3.25e-05 2.22e-15 1.97e-03
   5
      2.13596624e+02 4.01558422e+02
                                      0.00e+00 2.66e-15 2.90e-04
                                                                      2s
      3.34414754e+02 4.63402567e+02 6.12e-06 8.88e-16 1.99e-04
   6
                                                                      2s
      3.37655804e+02 3.38814459e+02 3.20e-06 8.88e-16 1.79e-06
                                                                      25
Barrier performed 7 iterations in 2.25 seconds (3.85 work units)
Objective cutoff exceeded
Concurrent spin time: 0.00s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.71 seconds (0.75 work units)
    Nodes
                 Current Node
                                       Objective Bounds
                                                                   Work
 Expl Unexpl |
               Obj Depth IntInf | Incumbent
                                                BestBd
                                                         Gap | It/Node Time
     0
                          0
                                  338.80088 338.81446 0.00%
                                                                       2s
Explored 1 nodes (0 simplex iterations) in 2.28 seconds (3.86 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 338.801
Optimal solution found (tolerance 1.00e-04)
Best objective 3.388008778246e+02, best bound 3.388144593004e+02, gap 0.0040%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xc7661096
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 338.801
Presolve removed 129670 rows and 0 columns
Presolve time: 0.48s
```

Presolved: 389011 rows, 389010 columns, 1037360 nonzeros Variable types: 259340 continuous, 129670 integer (129670 binary) Found heuristic solution: objective 354.9342530 Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier Showing barrier log only... Root barrier log... Ordering time: 0.00s Barrier statistics: : 3.871e+03 AA' N7 Factor NZ : 8.178e+03 (roughly 5 MB of memory) Factor Ops: 2.578e+04 (less than 1 second per iteration) Threads : 1 Objective Residual Iter Primal Dual Primal Dual Compl Time 0 -1.20892955e+06 2.92014313e+09 2.18e+01 6.78e+02 2.36e+05 2s 1 -3.50125422e+05 3.88863354e+08 1.41e+01 1.69e-12 2.56e+04 25 2 -1.36282037e+05 5.33750824e+07 6.03e+00 9.09e-13 3.18e+03 2s 3 -3.79601462e+04 1.26131375e+07 1.83e+00 7.73e-12 7.81e+02 25 Barrier performed 3 iterations in 1.88 seconds (3.39 work units) Barrier solve interrupted - model solved by another algorithm Concurrent spin time: 0.00s Solved with primal simplex Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier Showing barrier log only... Root barrier log... Ordering time: 0.03s Barrier statistics: : 1.297e+05 AA' NZ Factor NZ : 2.598e+05 (roughly 160 MB of memory) Factor Ops: 5.290e+05 (less than 1 second per iteration) Threads : 8 Residual Objective Primal Dual Tter Primal Dual Compl Time 0 -5.53146221e+06 7.69249664e+07 2.27e+01 7.89e-01 2.32e+02 25 1 -4.19748826e+05 1.47295183e+07 5.03e+00 2.44e-15 2.64e+01 25 2 -2.59473734e+04 3.95121424e+05 2.67e-01 8.88e-16 7.37e-01 3 -7.38328947e+03 2.03557538e+04 7.90e-02 2.89e-15 1.51e-01 2s 2.13150879e+02 1.85117945e+03 0.00e+00 2.00e-15 2.53e-03 2s 2.18038587e+02 4.88807402e+02 0.00e+00 1.78e-15 4.18e-04 6 2.75322448e+02 4.26693051e+02 0.00e+00 8.88e-16 2.33e-04 2s 3.40638216e+02 3.56664694e+02 0.00e+00 1.78e-15 2.47e-05 2s 8 3.54933571e+02 3.54937690e+02 7.91e-06 8.88e-16 6.41e-09 Barrier performed 8 iterations in 2.40 seconds (3.90 work units) Objective cutoff exceeded Concurrent spin time: 0.03s Solved with barrier Root relaxation: interrupted, 0 iterations, 0.73 seconds (0.80 work units) Nodes Current Node Objective Bounds Work Expl Unexpl | Obj Depth IntInf | Incumbent BestBd Gap | It/Node Time 354.93425 354.93769 0.00% Explored 1 nodes (0 simplex iterations) in 2.42 seconds (3.91 work units) Thread count was 10 (of 10 available processors) Solution count 1: 354.934 Optimal solution found (tolerance 1.00e-04) Best objective 3.549342529591e+02, best bound 3.549376896766e+02, gap 0.0010% Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81) CPU model: Apple M4 Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

```
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x9ce30c52
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 354.934
Presolve removed 129670 rows and 0 columns
Presolve time: 0.52s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 371.0676281
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
 AA' NZ
           : 4.110e+03
 Factor NZ : 8.656e+03 (roughly 5 MB of memory)
 Factor Ops: 2.674e+04 (less than 1 second per iteration)
 Threads
          : 1
                 Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
   0
    -1.16001676e+06 3.14599801e+09 2.22e+01 6.86e+02 2.39e+05
                                                                      2s
  1 -3.55411887e+05 4.21697241e+08 1.47e+01 1.62e-12
                                                         2.63e+04
                                                                      2s
   2 -1.39762513e+05 6.07398118e+07 6.42e+00 9.09e-13 3.44e+03
                                                                      2s
   3 -4.34005685e+04 1.55801955e+07 2.15e+00 5.46e-12 9.18e+02
                                                                      25
Barrier performed 3 iterations in 1.76 seconds (3.38 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
 AA' NZ : 1.297e+05
Factor NZ : 2.598e+05 (roughly 160 MB of memory)
AA' N7
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
          : 8
                 Objective
                                          Residual
                                       Primal
Iter
          Primal
                          Dual
                                               Dual
                                                          Compl
                                                                    Time
    -5.40456805e+06 7.83201779e+07 2.37e+01 7.99e-01 2.38e+02
                                                                      2s
  1 -4.01543922e+05 1.52858684e+07 5.10e+00 2.00e-15 2.73e+01
                                                                      2s
   2 -2.51896878e+04 4.09455636e+05 2.72e-01 1.11e-15 7.63e-01
   3 -8.42986461e+03 2.33329858e+04 9.38e-02 2.22e-15 1.78e-01
                                                                      25
      2.20838545e+02
                      2.43611239e+03 0.00e+00 2.55e-15
                                                         3.42e-03
                                                                      2s
                                      0.00e+00 1.78e-15 5.38e-04
      2.25890334e+02 5.74428563e+02
                                                                      25
      3.02230185e+02 4.62867108e+02
                                      0.00e+00 1.78e-15 2.48e-04
   6
                                                                      25
      3.57028315e+02 3.72763125e+02
                                      5.18e-06 1.78e-15
                                                         2.43e-05
                                                                      2s
      3.71066218e+02 3.71075856e+02 7.65e-06 8.88e-16 1.49e-08
                                                                      25
Barrier performed 8 iterations in 2.30 seconds (3.90 work units)
Objective cutoff exceeded
Concurrent spin time: 0.05s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.71 seconds (0.80 work units)
   Nodes
                 Current Node
                                       Objective Bounds
                                                                   Work
```

```
Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd Gap | It/Node Time
                                  371.06763 371.07586 0.00%
Explored 1 nodes (0 simplex iterations) in 2.32 seconds (3.91 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 371.068
Optimal solution found (tolerance 1.00e-04)
Best objective 3.710676280936e+02, best bound 3.710758563495e+02, gap 0.0022%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x7d05c2e4
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range
                  [1e+00, 4e+04]
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
 Bounds range
                  [1e+00, 1e+00]
 RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 371.068
Presolve removed 129670 rows and 0 columns
Presolve time: 0.50s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 387.2010032
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
 AA' NZ
          : 4.341e+03
 Factor NZ : 9.118e+03 (roughly 5 MB of memory)
 Factor Ops: 2.766e+04 (less than 1 second per iteration)
 Threads
           : 1
                 Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                                    Time
                                                          Compl
  0 -1.11240745e+06 3.38338985e+09 2.26e+01 6.94e+02 2.43e+05
                                                                      2s
  1 -3.53522493e+05 4.54855214e+08 1.52e+01 1.49e-12
                                                         2.70e+04
                                                                      25
  2 -1.43024223e+05 6.83120784e+07 6.82e+00 1.59e-12 3.70e+03
                                                                      25
   3 -3.71944332e+04 1.54552234e+07 1.96e+00 8.64e-12 8.64e+02
                                                                      25
Barrier performed 3 iterations in 1.90 seconds (3.39 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
          : 1.297e+05
 AA' NZ
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
          : 8
                 Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                                    Time
                                                          Compl
  0 -5.26398195e+06 7.97246104e+07 2.48e+01 8.09e-01
                                                         2.44e+02
                                                                      2s
  1 -3.82368919e+05 1.58413627e+07
                                      5.17e+00 2.00e-15
                                                         2.83e+01
                                                                      2s
   2 -2.43901514e+04 4.23657894e+05 2.77e-01 8.88e-16 7.88e-01
                                                                      2s
   3 -8.54489242e+03 2.63042052e+04 1.01e-01 2.22e-15 1.94e-01
                                                                      2s
```

```
2.31031110e+02 3.75610487e+03 6.59e-05 3.55e-15 5.44e-03
      2.36670988e+02 7.54337786e+02 1.96e-05 1.78e-15 7.98e-04
                                                                      25
      3.53169462e+02 7.01312356e+02 1.28e-05 1.33e-15 5.37e-04
                                                                      25
      3.70739947e+02 3.89559064e+02 2.67e-06 1.33e-15 2.90e-05
                                                                      25
      3.87198583e+02 3.87205962e+02 0.00e+00 8.88e-16 1.14e-08
                                                                      25
Barrier performed 8 iterations in 2.43 seconds (3.90 work units)
Objective cutoff exceeded
Concurrent spin time: 0.05s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.77 seconds (0.80 work units)
                 Current Node
                                       Objective Bounds
                                                                   Work
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd
                                                         Gap | It/Node Time
                                  387.20100 387.20596 0.00%
Explored 1 nodes (0 simplex iterations) in 2.47 seconds (3.91 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 387.201
Optimal solution found (tolerance 1.00e-04)
Best objective 3.872010032282e+02, best bound 3.872059620269e+02, gap 0.0013%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x0e06c278
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range
                  [1e+00, 4e+04]
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
 Bounds range
                  [1e+00, 1e+00]
 RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 387.201
Presolve removed 129670 rows and 0 columns
Presolve time: 0.48s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 403.3343784
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
          : 4.648e+03
 Factor NZ : 9.732e+03 (roughly 6 MB of memory)
 Factor Ops: 2.889e+04 (less than 1 second per iteration)
Threads
                 Objective
                                          Residual
                                       Primal Dual
Iter
          Primal
                          Dual
                                                          Compl
  0 -1.08484415e+06 3.40235147e+09 2.35e+01 6.48e+02 2.29e+05
                                                                      25
   1 -3.99947354e+05 4.68099274e+08 1.84e+01 1.36e-12
                                                                      2s
   2 -1.46486679e+05 7.36637088e+07 7.23e+00 1.36e-12 3.76e+03
                                                                      25
   3 -3.74394147e+04 1.62435050e+07 2.06e+00 9.09e-12 8.58e+02
Barrier performed 3 iterations in 1.93 seconds (3.37 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
```

```
Root barrier log...
Ordering time: 0.02s
Barrier statistics:
 AA' NZ
         : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
          : 8
                 Objective
                                         Residual
                       Dual
Tter
          Primal
                                      Primal Dual
                                                         Compl
                                                                   Time
   0 -5.12186995e+06 8.10929637e+07 2.58e+01 8.20e-01 2.50e+02
                                                                     2s
  1 -3.63694228e+05 1.63908665e+07 5.23e+00 2.44e-15 2.92e+01
                                                                     2s
   2 -2.36724610e+04 4.37665724e+05 2.83e-01 1.78e-15 8.14e-01
                                                                     25
   3 -8.38206424e+03 2.66383963e+04 1.04e-01 3.55e-15 2.17e-01
                                                                     25
   4 2.37118869e+02 4.40458864e+03 0.00e+00 2.89e-15 6.43e-03
                                                                     25
      2.42440210e+02 8.14379321e+02 2.40e-05 2.22e-15 8.82e-04
                                                                     2s
   6
      3.46350871e+02 6.69241324e+02 4.23e-06 1.78e-15 4.98e-04
                                                                     25
      3.77266328e+02 4.07212319e+02 5.39e-06 8.88e-16 4.62e-05
                                                                     25
   8 4.03327820e+02 4.03378665e+02 0.00e+00 8.88e-16 7.84e-08
                                                                     2s
     4.03334333e+02 4.03334423e+02 0.00e+00 8.88e-16 7.84e-11
                                                                     25
Barrier performed 9 iterations in 2.48 seconds (3.95 work units)
Objective cutoff exceeded
Concurrent spin time: 0.10s (can be avoided by choosing Method=3)
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.80 seconds (0.87 work units)
                                      Objective Bounds
                Current Node
                                                                  Work
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                               BestBd Gap | It/Node Time
                                  403.33438 403.33442 0.00%
Explored 1 nodes (0 simplex iterations) in 2.54 seconds (3.96 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 403.334
Optimal solution found (tolerance 1.00e-04)
Best objective 4.033343783627e+02, best bound 4.033344226499e+02, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x76022859
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range [1e+00, 4e+04]
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
 Bounds range
                  [1e+00, 1e+00]
 RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 403.334
Presolve removed 129670 rows and 0 columns
Presolve time: 0.48s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 419.4677535
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
 AA' NZ
         : 4.981e+03
 Factor NZ : 1.040e+04 (roughly 6 MB of memory)
 Factor Ops: 3.022e+04 (less than 1 second per iteration)
 Threads
```

```
Objective
                                          Residual
                                                          Compl
Iter
           Primal
                          Dual
                                       Primal
                                                 Dual
                                                                    Time
   0 -1.04212997e+06 3.60614361e+09 2.39e+01 6.38e+02 2.27e+05
                                                                      25
   1 -3.88213120e+05 5.76458459e+08 1.90e+01 1.77e-12 3.03e+04
                                                                      2s
   2 -1.56014473e+05 9.03858788e+07 7.98e+00 3.64e-12 4.34e+03
                                                                      2s
   3 -4.01009317e+04 1.97627032e+07 2.29e+00 9.55e-12 9.80e+02
                                                                      25
Barrier performed 3 iterations in 1.75 seconds (3.37 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
AA' NZ
           : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
           : 8
                 Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                          Compl
                                                                    Time
    -4.97936048e+06 8.24258833e+07 2.68e+01 8.31e-01 2.56e+02
                                                                      2s
   1 -3.45604039e+05 1.69343707e+07 5.29e+00 2.22e-15 3.02e+01
                                                                      25
     -2.29798862e+04 4.51901480e+05
                                      2.89e-01 1.78e-15 8.40e-01
                                                                      2s
   3 -8.36144598e+03 9.26835058e+03 1.09e-01 3.11e-15 2.05e-01
                                                                      25
      2.48918056e+02 2.44409849e+03 0.00e+00 2.66e-15 3.39e-03
   5
      2.54347618e+02
                      5.53350062e+02 0.00e+00 1.78e-15 4.61e-04
                                                                      2s
   6
      4.10392383e+02 6.99479720e+02 9.49e-06 1.78e-15 4.46e-04
                                                                      2s
      4.16168741e+02 4.19726290e+02 8.00e-06 1.78e-15 5.49e-06
                                                                      25
   8
      4.19467627e+02 4.19467903e+02 9.25e-06 8.88e-16 4.98e-10
                                                                      25
Barrier performed 8 iterations in 2.28 seconds (3.88 work units)
Objective cutoff exceeded
Concurrent spin time: 0.05s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.72 seconds (0.80 work units)
    Nodes
                 Current Node
                                       Objective Bounds
                                                                   Work
              Obj Depth IntInf | Incumbent
                                                BestBd
                                                         Gap | It/Node Time
 Expl Unexpl |
                                  419.46775 419.46790 0.00%
Explored 1 nodes (0 simplex iterations) in 2.31 seconds (3.89 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 419.468
Optimal solution found (tolerance 1.00e-04)
Best objective 4.194677534972e+02, best bound 4.194679029120e+02, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x6320aa5c
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
 Matrix range
 Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
 Bounds range
                  [1e+00, 1e+00]
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 419.468
```

Presolve removed 129670 rows and 0 columns

Presolve time: 0.50s Presolved: 389011 rows, 389010 columns, 1037360 nonzeros Variable types: 259340 continuous, 129670 integer (129670 binary) Found heuristic solution: objective 435.6011286 Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier Showing barrier log only... Root barrier log... Ordering time: 0.01s Barrier statistics: AA' NZ : 5.298e+03 Factor NZ : 1.103e+04 (roughly 6 MB of memory) Factor Ops: 3.149e+04 (less than 1 second per iteration) Threads Residual Objective Iter Primal Dual Primal Dual Compl Time 0 -1.00250431e+06 3.78607426e+09 2.44e+01 6.26e+02 2.24e+05 25 1 -3.81804884e+05 6.10281096e+08 1.99e+01 1.36e-12 3.02e+04 2s 2 -1.58332526e+05 1.09141696e+08 8.42e+00 2.73e-12 4.94e+03 2s 3 -5.18968063e+04 2.90552098e+07 3.02e+00 7.05e-12 1.36e+03 25 Barrier performed 3 iterations in 1.86 seconds (3.37 work units) Barrier solve interrupted - model solved by another algorithm Concurrent spin time: 0.01s Solved with primal simplex Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier Showing barrier log only... Root barrier log... Ordering time: 0.04s Barrier statistics: AA' NZ : 1.297e+05 Factor NZ : 2.598e+05 (roughly 160 MB of memory) Factor Ops: 5.290e+05 (less than 1 second per iteration) Threads : 8 Objective Residual Primal Iter Primal Dual Dual Compl 0 -4.82667930e+06 8.37633053e+07 2.78e+01 8.41e-01 2.62e+02 25 1 -3.27181731e+05 1.74765510e+07 5.35e+00 2.44e-15 3.11e+01 2 -2.22345542e+04 4.66196821e+05 2.95e-01 1.33e-15 8.66e-01 25 3 -8.78091759e+03 3.10155733e+04 1.23e-01 2.89e-15 2.16e-01 2s 2.63917824e+02 7.20884170e+03 0.00e+00 1.78e-15 1.07e-02 2.72496240e+02 1.39298238e+03 0.00e+00 1.78e-15 1.73e-03 2s 3.92269675e+02 5.79477678e+02 0.00e+00 8.88e-16 2.89e-04 2s 4.30019251e+02 4.39445514e+02 2.80e-07 1.33e-15 1.45e-05 25 4.35600931e+02 4.35601809e+02 0.00e+00 8.88e-16 1.35e-09 25 Barrier performed 8 iterations in 2.45 seconds (3.88 work units) Objective cutoff exceeded Concurrent spin time: 0.06s Solved with barrier Root relaxation: interrupted, 0 iterations, 0.80 seconds (0.81 work units) Current Node Objective Bounds Obj Depth IntInf | Incumbent Expl Unexpl | BestBd Gap | It/Node Time 0 435.60113 435.60181 0.00% Explored 1 nodes (0 simplex iterations) in 2.48 seconds (3.89 work units) Thread count was 10 (of 10 available processors) Solution count 1: 435.601 Optimal solution found (tolerance 1.00e-04) Best objective 4.356011286317e+02, best bound 4.356018090715e+02, gap 0.0002% Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81) CPU model: Apple M4 Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

```
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xfe6023d7
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 435.601
Presolve removed 129670 rows and 0 columns
Presolve time: 0.49s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 451.7345038
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
           : 5.639e+03
 Factor NZ : 1.171e+04 (roughly 7 MB of memory)
 Factor Ops: 3.285e+04 (less than 1 second per iteration)
 Threads
                 Objective
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                          Compl
                                                                    Time
   0 -9.55737290e+05 3.96044352e+09 2.46e+01 6.15e+02 2.20e+05
                                                                      2s
  1 -3.79490136e+05 6.45790540e+08 2.02e+01 1.36e-12 3.02e+04
                                                                      2s
   2 -1.59537504e+05 1.17865002e+08 8.85e+00 2.27e-12 5.05e+03
                                                                      25
   3 -4.49282095e+04 2.81953215e+07 2.89e+00 1.05e-11 1.24e+03
                                                                      2s
Barrier performed 3 iterations in 1.85 seconds (3.36 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
           : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
                 Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                          Compl
                                                                    Time
   0 -4.66699145e+06 8.50945822e+07 2.88e+01 8.53e-01 2.68e+02
                                                                      2s
   1 -3.08702661e+05 1.80163388e+07 5.40e+00 2.66e-15 3.20e+01
   2 -2.14830322e+04 4.80442987e+05 3.01e-01 1.78e-15 8.92e-01
                                                                      25
   3 -8.51179199e+03 3.13781955e+04 1.25e-01 2.66e-15 2.18e-01
                                                                      2s
      2.70814204e+02 7.83971107e+03 0.00e+00 1.89e-15 1.17e-02
                                                                      25
      2.79011844e+02 1.53548336e+03 0.00e+00 1.78e-15 1.94e-03
   5
                                                                      25
      3.91645272e+02 6.19905113e+02 0.00e+00 8.88e-16 3.52e-04
                                                                      2s
      4.44357686e+02 4.58715301e+02 1.07e-06 1.78e-15 2.21e-05
                                                                      25
      4.51732588e+02 4.51746028e+02 0.00e+00 1.78e-15 2.07e-08
                                                                      25
Barrier performed 8 iterations in 2.38 seconds (3.88 work units)
Objective cutoff exceeded
Concurrent spin time: 0.05s
Solved with barrier
```

Root relaxation: interrupted, 0 iterations, 0.75 seconds (0.81 work units)

```
Current Node
                                        Objective Bounds
                                                                     Work
   Nodes
 Expl Unexpl |
               Obj Depth IntInf | Incumbent
                                                 BestBd
                                                          Gap | It/Node Time
                                   451.73450 451.74603 0.00%
     0
Explored 1 nodes (0 simplex iterations) in 2.42 seconds (3.89 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 451.735
Optimal solution found (tolerance 1.00e-04)
Best objective 4.517345037662e+02, best bound 4.517460275800e+02, gap 0.0026%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xff1887d9
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 451.735
Presolve removed 129670 rows and 0 columns
Presolve time: 0.49s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 467.8678789
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
 AA' NZ
          : 5.923e+03
 Factor NZ : 1.228e+04 (roughly 7 MB of memory)
 Factor Ops: 3.399e+04 (less than 1 second per iteration)
 Threads
          : 1
                                           Residual
                  Objective
Iter
           Primal
                           Dual
                                        Primal
                                                 Dual
                                                                      Time
                                                            Compl
                      4.16585968e+09 2.47e+01 6.13e+02 2.20e+05
   0 -9.05227555e+05
                                                                        2s
  1 -3.70086794e+05 6.82815206e+08 2.04e+01 1.36e-12 3.05e+04
                                                                        25
  2 -1.57414490e+05 1.36296069e+08 9.40e+00 1.36e-12 5.54e+03 

3 -5.71843312e+04 3.84504664e+07 3.84e+00 1.34e-11 1.62e+03
                                                                        25
                                                                        25
Barrier performed 3 iterations in 1.72 seconds (3.37 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
 AA' NZ
          : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
          : 8
                                           Residual
Iter
           Primal
                           Dual
                                        Primal
                                                  Dual
                                                            Compl
                                                                      Time
   0 -4.49781167e+06 8.64298113e+07 2.97e+01 8.64e-01
                                                          2.74e+02
                                                                        2s
   1 -2.89944051e+05 1.85549728e+07
                                       5.45e+00 2.22e-15
                                                           3.30e+01
                                                                        2s
   2 -2.07460673e+04 4.95514380e+05 3.08e-01 1.33e-15 9.20e-01
                                                                        2s
```

```
3 -7.87859420e+03 1.86519642e+04 1.23e-01 2.66e-15 1.96e-01
   4 2.81648146e+02 5.49095083e+03 0.00e+00 2.66e-15 8.03e-03
                                                                      25
      2.88675052e+02 1.13254473e+03 1.20e-05 1.78e-15 1.30e-03
                                                                      25
      4.11663788e+02 1.00515556e+03 0.00e+00 1.33e-15 9.15e-04
                                                                      25
      4.40859783e+02 4.88428776e+02 4.13e-06 1.33e-15 7.34e-05
                                                                      2s
   8
      4.67660283e+02 4.68399949e+02 0.00e+00 1.78e-15 1.14e-06
                                                                      2s
      4.67867682e+02 4.67868411e+02 2.05e-06 1.78e-15 1.14e-09
                                                                      25
Barrier performed 9 iterations in 2.29 seconds (3.95 work units)
Objective cutoff exceeded
Concurrent spin time: 0.13s (can be avoided by choosing Method=3)
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.81 seconds (0.87 work units)
                 Current Node
                                       Objective Bounds
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd
                                                         Gap | It/Node Time
     0
                                  467.86788 467.86841 0.00%
Explored 1 nodes (0 simplex iterations) in 2.36 seconds (3.96 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 467.868
Optimal solution found (tolerance 1.00e-04)
Best objective 4.678678789007e+02, best bound 4.678684110002e+02, gap 0.0001%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
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Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x3f517c73
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range
                  [1e+00, 4e+04]
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
 Bounds range
                   [1e+00, 1e+00]
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 467.868
Presolve removed 129670 rows and 0 columns
Presolve time: 0.49s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 474.3212290
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
           : 6.264e+03
 Factor NZ : 1.296e+04 (roughly 8 MB of memory)
 Factor Ops: 3.535e+04 (less than 1 second per iteration)
 Threads
           : 1
                                          Residual
                 Objective
Iter
                          Dual
                                       Primal
                                                 Dual
                                                                    Time
          Primal
                                                          Compl
   0 -1.00280948e+06 4.32534624e+09 2.53e+01 5.44e+02 2.16e+05
                                                                      25
  1 -4.53473711e+05 7.30250193e+08 2.29e+01 1.18e-12 3.18e+04
                                                                      25
   2 -1.88768250e+05 1.86268402e+08
                                      9.69e+00 1.82e-12
                                                         7.17e+03
                                                                      2s
  3 -6.84894879e+04 5.73184337e+07 4.07e+00 2.00e-11 2.19e+03
                                                                      2s
Barrier performed 3 iterations in 1.75 seconds (3.35 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
```

Use crossover to convert LP symmetric solution to basic solution...

Root relaxation: objective 4.840013e+02, 8084 iterations, 0.24 seconds (0.37 work units)

```
Current Node
                                      Objective Bounds
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                               BestBd Gap | It/Node Time
             484.00125
                               1 474.32123 484.00125 2.04%
                                                                      1s
    0
          0
                                483.2147520 484.00125 0.16%
Н
                                                                     1s
Н
    0
          0
                                484.0012540 484.00125 0.00%
                                                                     2s
                               1 484.00125 484.00125 0.00%
             484,00125
                                                                     2s
```

Explored 1 nodes (8084 simplex iterations) in 2.07 seconds (4.01 work units) Thread count was 10 (of 10 available processors)

Solution count 3: 484.001 483.215 474.321

Optimal solution found (tolerance 1.00e-04)
Best objective 4.840012540352e+02, best bound 4.840012540352e+02, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)

CPU model: Apple M4

Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Academic license 2608044 - for non-commercial use only - registered to tc\_\_\_@andrew.cmu.edu Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros Model fingerprint: 0xb00a4892 Model has 129670 quadratic objective terms Variable types: 129670 continuous, 129670 integer (129670 binary) Coefficient statistics:

Matrix range [1e+00, 4e+04]
Objective range [0e+00, 0e+00]
QObjective range [2e-02, 8e-02]
Bounds range [1e+00, 1e+00]
RHS range [1e+03, 4e+04]

Loaded MIP start from previous solve with objective 484.001

Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...

Root barrier log...

Ordering time: 0.01s

Barrier statistics: AA' NZ : 6.594e+03

Factor NZ : 1.362e+04 (roughly 8 MB of memory)

Factor Ops: 3.667e+04 (less than 1 second per iteration)

Threads : 1

Objective Residual Primal Dual Primal Iter Dual Compl Time 0 -9.62386701e+05 4.58452936e+09 2.56e+01 5.52e+02 2.18e+05 2s 1 -4.21880332e+05 7.69784500e+08 2.24e+01 1.37e-12 3.18e+04 25 2 -1.82095711e+05 1.37386889e+08 1.02e+01 2.96e-12 5.19e+03 2s 3 -4.28047644e+04 2.71816414e+07 2.89e+00 1.71e-11 1.05e+03 25

Barrier performed 3 iterations in 1.86 seconds (3.46 work units) Barrier solve interrupted – model solved by another algorithm

Concurrent spin time: 0.01s

Solved with primal simplex

Use crossover to convert LP symmetric solution to basic solution...

Root relaxation: objective 5.001346e+02, 8353 iterations, 0.28 seconds (0.37 work units)

	Nodes		Cu	rrent	Node		(	Objecti	ve Bou	nds	- 1	Work	<
E	Expl Unex	φl	Obj	Depth	Intl	[nf	Incun	nbent	Bestl	3d G	ap	It/Node	Time
	0	0	500.13	463	0	1	484.00	0125 5	00.134	3.	33%	_	1s
Н	0	0				5	00.0136	5289 5	00.134	63 0.	02%	-	1s
Н	0	0				5	00.1346	5292 5	00.134	63 0.	00%	-	2s
	0	0	500.13	463	0	1	500.13	3463 5	00.134	63 0.	00%	_	2s

```
Explored 1 nodes (8353 simplex iterations) in 2.12 seconds (4.13 work units)
Thread count was 10 (of 10 available processors)
Solution count 3: 500.135 500.014 484.001
Optimal solution found (tolerance 1.00e-04)
Best objective 5.001346291697e+02, best bound 5.001346291697e+02, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x428d2a33
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
 Matrix range
 Objective range [0e+00, 0e+00]
 OObjective range [2e-02, 8e-02]
 Bounds range
                  [1e+00, 1e+00]
 RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 500.135
Presolve removed 129670 rows and 0 columns
Presolve time: 0.49s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
AA' N7
         : 6.887e+03
 Factor NZ : 1.421e+04 (roughly 8 MB of memory)
 Factor Ops: 3.784e+04 (less than 1 second per iteration)
 Threads
         : 1
                 Objective
                                          Residual
          Primal
                          Dual
                                       Primal Dual
Iter
                                                                    Time
                                                          Compl
   0 -9.24010266e+05 4.79124798e+09 2.60e+01 5.61e+02 2.18e+05
                                                                      2s
   1 -4.18168439e+05 8.16397812e+08 2.29e+01 1.19e-12 3.25e+04
                                                                      2s
   2 -1.80076255e+05 1.48624509e+08 1.10e+01 3.41e-12 5.41e+03
                                                                      2s
   3 -4.06066785e+04 2.86214811e+07 2.95e+00 9.55e-12 1.07e+03
Barrier performed 3 iterations in 1.90 seconds (3.46 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
Use crossover to convert LP symmetric solution to basic solution...
Root relaxation: objective 5.162680e+02, 8572 iterations, 0.27 seconds (0.37 work units)
                 Current Node
                                       Objective Bounds
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                              BestBd Gap | It/Node Time
                               1 500.13463 516.26800 3.23%
             516,26800
                                                                       1s
                                515.8243365 516.26800 0.09%
Н
     0
          0
                                                                       25
Н
     0
          0
                                516.2680043 516.26800 0.00%
                                                                       2s
           0 516.26800
                               1 516.26800 516.26800 0.00%
Explored 1 nodes (8572 simplex iterations) in 2.22 seconds (4.13 work units)
Thread count was 10 (of 10 available processors)
Solution count 3: 516.268 515.824 500.135
Optimal solution found (tolerance 1.00e-04)
Best objective 5.162680043042e+02, best bound 5.162680043042e+02, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
```

Update\_3\_Optimization Thread count: 10 physical cores, 10 logical processors, using up to 10 threads Academic license 2608044 - for non-commercial use only - registered to tc\_\_\_@andrew.cmu.edu Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros Model fingerprint: 0x8118110f Model has 129670 quadratic objective terms Variable types: 129670 continuous, 129670 integer (129670 binary) Coefficient statistics: Matrix range [1e+00, 4e+04] Objective range [0e+00, 0e+00] QObjective range [2e-02, 8e-02] [1e+00, 1e+00] Bounds range [1e+03, 4e+04] RHS range Loaded MIP start from previous solve with objective 516.268 Presolve removed 129670 rows and 0 columns Presolve time: 0.48s Presolved: 389011 rows, 389010 columns, 1037360 nonzeros Variable types: 259340 continuous, 129670 integer (129670 binary) Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier Showing barrier log only... Root barrier log... Ordering time: 0.00s Barrier statistics: : 7.166e+03 Factor NZ : 1.477e+04 (roughly 9 MB of memory) Factor Ops: 3.896e+04 (less than 1 second per iteration) Threads Objective Iter Primal Dual Primal Dual Compl Time 0 -8.78934330e+05 5.00386555e+09 2.62e+01 5.68e+02 2.19e+05 2s 1 -4.11009593e+05 8.55749766e+08 2.32e+01 1.36e-12 3.30e+04 2s 2 -1.79398672e+05 2.29192364e+08 1.03e+01 1.82e-12 7.82e+03 25 3 -6.26221627e+04 5.96670715e+07 4.49e+00 1.11e-11 2.11e+03 2s Barrier performed 3 iterations in 2.45 seconds (3.46 work units) Barrier solve interrupted - model solved by another algorithm Concurrent spin time: 0.01s Solved with primal simplex Use crossover to convert LP symmetric solution to basic solution... Root relaxation: objective 5.324014e+02, 8811 iterations, 0.24 seconds (0.38 work units) Nodes Current Node Objective Bounds Expl Unexpl | Obj Depth IntInf | Incumbent BestBd Gap | It/Node Time 0 532.40138 0 1 516.26800 532.40138 3.13% 2s 529.6587057 532.40138 0.52% Н 0 0 25 531.2996274 532.40138 0.21% Н 0 0 2s 532.0044247 Н 0 0 532.40138 0.07% 2s 1 532.00442 532.40138 0.07% 0 0 532.40138 0 3s 532.4013794 532.40138 0.00% Explored 1 nodes (76719 simplex iterations) in 4.51 seconds (7.30 work units) Thread count was 10 (of 10 available processors) Solution count 6: 532.401 532.004 531.413 ... 516.268 Optimal solution found (tolerance 1.00e-04) Best objective 5.324013794387e+02, best bound 5.324013794387e+02, gap 0.0000% Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81) CPU model: Apple M4 Thread count: 10 physical cores, 10 logical processors, using up to 10 threads Academic license 2608044 - for non-commercial use only - registered to tc\_\_\_@andrew.cmu.edu

 $file: ///Users/goyolozano/Desktop/Mini~4/Value/Update~3/Deliverables/Update\_3\_Optimization.html$ 

[1e+00, 4e+04]

Model has 129670 quadratic objective terms

Model fingerprint: 0x227e4b88

Coefficient statistics: Matrix range [1e+

Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros

Variable types: 129670 continuous, 129670 integer (129670 binary)

```
Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
                   [1e+03, 4e+04]
 RHS range
Loaded MIP start from previous solve with objective 532.401
Presolve removed 129670 rows and 0 columns
Presolve time: 0.49s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 548.5347546
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
AA' NZ : 7.457e+03
Factor NZ : 1.535e+04 (roughly 10 MB of memory)
Factor Ops: 4.012e+04 (less than 1 second per iteration)
                  Objective
                                           Residual
Iter
           Primal
                         Dual
                                       Primal Dual
                                                            Compl
                                                                      Time
  0 -8.37533713e+05 5.28975344e+09 2.65e+01 5.75e+02 2.22e+05
                                                                        25
  1 -4.05283276e+05 1.04806099e+09 2.36e+01 1.32e-12 3.87e+04 2 -1.79082460e+05 2.64304985e+08 1.07e+01 2.27e-12 8.71e+03
                                                                        2s
                                                                        2s
  3 -6.39650923e+04 7.87234821e+07 4.88e+00 1.46e-11 2.59e+03
Barrier performed 3 iterations in 2.27 seconds (3.35 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.02s
Barrier statistics:
AA' NZ : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
                 Objective
                                           Residual
                                       Primal Dual
Iter
           Primal
                         Dual
                                                           Compl
  0 -3.63412446e+06 9.28000117e+07 3.44e+01 9.22e-01 3.04e+02 1 -2.02047431e+05 2.11919425e+07 5.66e+00 2.66e-15 3.75e+01
                                                                        25
                                                                        3s
  2 -1.68998821e+04 5.80352754e+05 3.46e-01 1.78e-15 1.08e+00
                                                                        3s
  3 -6.49331860e+03 1.02685668e+04 1.35e-01 2.66e-15 1.92e-01
                                                                        3s
      3.35112160e+02 3.77271623e+03 0.00e+00 2.66e-15 5.30e-03
                                                                        3s
      3.43881724e+02 1.03658427e+03 0.00e+00 2.66e-15 1.07e-03
                                                                        3s
      4.70297417e+02 5.82003702e+02 0.00e+00 1.78e-15 1.72e-04
                                                                        3s
      5.48186689e+02 5.50095525e+02 1.71e-05 1.78e-15 2.94e-06
                                                                        3s
  8 5.48534569e+02 5.48534783e+02 0.00e+00 1.78e-15 7.59e-11
                                                                        35
Barrier performed 8 iterations in 2.78 seconds (3.86 work units)
Objective cutoff exceeded
Concurrent spin time: 0.08s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.74 seconds (0.81 work units)
                  Current Node
                                        Objective Bounds
Expl Unexpl | Obj Depth IntInf | Incumbent BestBd Gap | It/Node Time
                                   548.53475 548.53478 0.00%
Explored 1 nodes (0 simplex iterations) in 2.84 seconds (3.88 work units)
Thread count was 10 (of 10 available processors)
```

```
Update_3_Optimization
Solution count 1: 548.535
Optimal solution found (tolerance 1.00e-04)
Best objective 5.485347545732e+02, best bound 5.485347832161e+02, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x1b9942c2
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                   [1e+00, 1e+00]
[1e+03, 4e+04]
  Bounds range
  RHS range
Loaded MIP start from previous solve with objective 548.535
Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 564.6681297
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
           : 7.732e+03
 Factor NZ : 1.590e+04 (roughly 10 MB of memory)
 Factor Ops : 4.122e+04 (less than 1 second per iteration)
                  Objective
                                           Residual
                                       Primal
Iter
           Primal
                          Dual
                                                 Dual
                                                                     Time
                                                           Compl
   0 -8.61046768e+05 5.71585298e+09 2.68e+01 5.82e+02 2.32e+05
                                                                       25
   1 -4.01308444e+05 1.14184784e+09 2.34e+01 1.21e-12
                                                          4.06e+04
                                                                       25
   2 -1.82318628e+05 2.30412377e+08 1.26e+01 1.82e-12 7.50e+03
                                                                       25
   3 -5.57598664e+04 5.81055173e+07 4.36e+00 7.05e-12 1.91e+03
                                                                       2s
Barrier performed 3 iterations in 2.03 seconds (3.35 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
          : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops : 5.290e+05 (less than 1 second per iteration)
           : 8
                  Objective
                                           Residual
Iter
           Primal
                          Dual
                                       Primal
                                                 Dual
                                                           Compl
                                                                     Time
   0 -3.44765318e+06 9.40537062e+07 3.52e+01 9.34e-01 3.10e+02
                                                                       25
   1 -1.84687831e+05 2.17127080e+07 5.70e+00 2.66e-15 3.84e+01
                                                                       2s
  2 -1.62742055e+04 5.98538661e+05 3.56e-01 1.78e-15 1.11e+00
                                                                       2s
                                                                       2s
```

3 -6.11438156e+03 1.13624856e+04 1.37e-01 2.66e-15 1.93e-01 3.45293349e+02 4.12172886e+03 2.45e-05 2.66e-15 5.82e-03 2s 3.54387585e+02 1.12143128e+03 0.00e+00 2.22e-15 1.18e-03 25 4.81545730e+02 6.10432213e+02 2.52e-06 1.78e-15 1.99e-04 2s 5.64203874e+02 5.66605480e+02 5.39e-05 1.78e-15 3.70e-06 3s 5.64669258e+02 5.64668139e+02 2.23e-04 1.78e-15 1.79e-11 3s

Barrier performed 8 iterations in 2.55 seconds (3.86 work units)

```
Objective cutoff exceeded
Concurrent spin time: 0.09s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.87 seconds (0.82 work units)
    Nodes
                 Current Node
                                       Objective Bounds
                                                                   Work
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd Gap | It/Node Time
     0
                                  564.66813 564.66814 0.00%
                          0
Explored 1 nodes (0 simplex iterations) in 2.62 seconds (3.87 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 564,668
Optimal solution found (tolerance 1.00e-04)
Best objective 5.646681297077e+02, best bound 5.646681391612e+02, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xf1260c6d
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range
                  [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                  [1e+00, 1e+00]
  RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 564.668
Presolve removed 129670 rows and 0 columns
Presolve time: 0.48s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 580.8015048
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
AA' NZ : 8.014e+03
 Factor NZ : 1.646e+04 (roughly 10 MB of memory)
 Factor Ops: 4.235e+04 (less than 1 second per iteration)
 Threads
          : 1
                 Objective
                                          Residual
          Primal
                                       Primal Dual
Iter
                          Dual
                                                          Compl
                                                                    Time
  0 -8.21806561e+05 5.94047162e+09 2.73e+01 5.92e+02 2.32e+05
                                                                      2s
  1 -3.92604066e+05 1.18947224e+09 2.37e+01 1.36e-12 4.11e+04
                                                                      2s
  2 -1.75961089e+05 2.42450104e+08 1.33e+01 2.73e-12 7.65e+03
                                                                      25
  3 -5.21991588e+04 6.05286530e+07 4.44e+00 1.52e-11 1.92e+03
Barrier performed 3 iterations in 1.81 seconds (3.35 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.02s
Barrier statistics:
 AA' NZ : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
```

```
Factor Ops : 5.290e+05 (less than 1 second per iteration)
Threads : 8
```

	0bje	ctive	Residual		
Iter	Primal	Dual	Primal Dual	Compl	Time
0	-3.25840088e+06	9.52968208e+07	3.61e+01 9.45e-01	3.15e+02	2s
1	-1.67522601e+05	2.22308999e+07	5.73e+00 2.44e-15	3.93e+01	2s
2	-1.54166182e+04	6.17089682e+05	3.60e-01 1.78e-15	1.14e+00	2s
3	-5.85025406e+03	2.02169022e+04	1.40e-01 3.55e-15	2.32e-01	2s
4	3.48089974e+02	6.78262044e+03	0.00e+00 2.66e-15	9.92e-03	2s
5	3.56790537e+02	1.42996244e+03	0.00e+00 2.66e-15	1.66e-03	2s
6	4.54624276e+02	6.75263297e+02	0.00e+00 1.78e-15	3.40e-04	2s
7	5.62229729e+02	5.91224101e+02	4.15e-05 1.78e-15	4.47e-05	2s
8	5.80689486e+02	5.80852680e+02	0.00e+00 1.78e-15	2.51e-07	2s

Barrier performed 8 iterations in 2.35 seconds (3.86 work units) Objective cutoff exceeded

Concurrent spin time: 0.09s

Solved with barrier

Root relaxation: interrupted, 0 iterations, 0.81 seconds (0.82 work units)

```
Nodes | Current Node | Objective Bounds | Work Expl Unexpl | Obj Depth IntInf | Incumbent BestBd Gap | It/Node Time

0 0 - 0 580.80150 580.85268 0.01% - 2s
```

Explored 1 nodes (0 simplex iterations) in 2.44 seconds (3.87 work units) Thread count was 10 (of 10 available processors)

Solution count 1: 580.802

Optimal solution found (tolerance 1.00e-04)
Best objective 5.808015048422e+02, best bound 5.808526802259e+02, gap 0.0088%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)

CPU model: Apple M4

Thread count: 10 physical cores, 10 logical processors, using up to 10 threads

Academic license 2608044 - for non-commercial use only - registered to tc\_\_\_@andrew.cmu.edu Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros

Model fingerprint: 0xb1af6486

Model has 129670 quadratic objective terms

Variable types: 129670 continuous, 129670 integer (129670 binary)

Coefficient statistics:

Matrix range [1e+00, 4e+04]
Objective range [0e+00, 0e+00]
Objective range [2e-02, 8e-02]
Bounds range [1e+00, 1e+00]
RHS range [1e+03, 4e+04]

Loaded MIP start from previous solve with objective 580.802

Presolve removed 129670 rows and 0 columns

Presolve time: 0.53s

Presolved: 389011 rows, 389010 columns, 1037360 nonzeros

Variable types: 259340 continuous, 129670 integer (129670 binary)

Found heuristic solution: objective 596.9348800

Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier Showing barrier log only...

Root barrier log...

Ordering time: 0.00s

Barrier statistics:

AA' NZ : 8.326e+03

Factor NZ : 1.709e+04 (roughly 10 MB of memory)

Factor Ops : 4.360e+04 (less than 1 second per iteration)

Threads : 1

	0bje	ctive	Residual		
Iter	Primal	Dual	Primal Dual	Compl	Time
0	-7.94635916e+05	6.25593790e+09	2.85e+01 6.10e+02	2.37e+05	2s
1	-4.29215999e+05	1.32220863e+09	2.65e+01 1.82e-12	4.55e+04	2s
2	-1.62651630e+05	2.57334488e+08	1.18e+01 1.82e-12	7.76e+03	2s
3	-4.56624358e+04	6.36861066e+07	4.13e+00 2.21e-11	1.96e+03	2s
4	-2.68698291e+03	4.88865980e+06	2.91e-01 3.00e-11	1.49e+02	2s

```
Barrier performed 4 iterations in 1.73 seconds (3.33 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.00s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
           : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
                                          Residual
                 Objective
Iter
          Primal
                          Dual
                                       Primal
                                                Dual
                                                          Compl
                                                                    Time
   0 -3.07053765e+06 9.65188187e+07 3.70e+01 9.57e-01 3.21e+02
                                                                      2s
   1 -1.50864250e+05 2.27454602e+07 5.76e+00 2.66e-15 4.02e+01
                                                                      25
   2 -1.42950188e+04 6.35363509e+05 3.61e-01 1.78e-15 1.17e+00
                                                                      25
   3 -5.44151500e+03 2.01217666e+04 1.41e-01 4.00e-15 2.35e-01
                                                                      2s
   4 3.58054530e+02 6.94468725e+03 0.00e+00 3.11e-15 1.02e-02
                                                                      25
      3.66965988e+02 1.44883004e+03 0.00e+00 2.22e-15 1.67e-03
                                                                      2s
                      7.12564266e+02
                                      1.30e-05 1.78e-15 3.72e-04
      4.71421225e+02
                                                                      25
      5.84354447e+02 5.98416955e+02 0.00e+00 1.78e-15 2.17e-05
                                                                      2s
     5.96932352e+02 5.96944656e+02 8.40e-06 1.78e-15 1.90e-08
                                                                      25
Barrier performed 8 iterations in 2.26 seconds (3.85 work units)
Objective cutoff exceeded
Concurrent spin time: 0.08s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.77 seconds (0.82 work units)
                 Current Node
                                       Objective Bounds
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                              BestBd Gap | It/Node Time
                                  596.93488 596.94466 0.00%
Explored 1 nodes (0 simplex iterations) in 2.33 seconds (3.86 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 596.935
Optimal solution found (tolerance 1.00e-04)
Best objective 5.969348799767e+02, best bound 5.969446562954e+02, gap 0.0016%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xc1a4a240
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range
                  [1e+00, 4e+04]
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
                  [1e+00, 1e+00]
 Bounds range
                  [1e+03, 4e+04]
 RHS range
Loaded MIP start from previous solve with objective 596.935
Presolve removed 129670 rows and 0 columns
Presolve time: 0.46s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 613.0682551
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
```

```
Root barrier log...
Ordering time: 0.01s
Barrier statistics:
 AA' NZ
           : 8.577e+03
 Factor NZ : 1.759e+04 (roughly 10 MB of memory)
 Factor Ops: 4.460e+04 (less than 1 second per iteration)
 Threads
          : 1
                 Objective
                                          Residual
          Primal
                                       Primal
Tter
                          Dual
                                               Dual
                                                                    Time
                                                          Compl
   0 -7.54639628e+05
                     6.52358278e+09 2.87e+01 6.17e+02 2.40e+05
                                                                      2s
  1 -4.13733922e+05 1.38458246e+09 2.65e+01 2.27e-12 4.64e+04
                                                                      2s
   2 -1.70064906e+05 3.34662318e+08 1.14e+01 1.36e-12 9.82e+03
                                                                      25
   3 -5.99616523e+04 1.04136444e+08 5.06e+00 1.36e-11 3.01e+03
                                                                      2s
Barrier performed 3 iterations in 1.68 seconds (3.33 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
          : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
          : 8
                 Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                Dual
                                                          Compl
                                                                    Time
   0 -2.89777302e+06 9.76838906e+07 3.78e+01 9.69e-01 3.27e+02
                                                                      2s
   1 -1.35749101e+05 2.32525777e+07 5.79e+00 2.66e-15 4.11e+01
                                                                      25
   2 -1.32789722e+04 6.53623010e+05 3.61e-01 1.78e-15 1.20e+00
                                                                      25
   3 -5.06904482e+03 2.00474346e+04 1.42e-01 3.55e-15 2.38e-01
                                                                      2s
      3.68083383e+02 7.10564570e+03 4.32e-05 3.11e-15 1.04e-02
                                                                      2s
      3.77209812e+02 1.46822173e+03 0.00e+00 1.78e-15 1.68e-03
                                                                      2s
                      7.50733193e+02
      4.88081072e+02
                                      0.00e+00 1.78e-15
                                                         4.05e-04
                                                                      25
      5.95281930e+02 6.18771196e+02 6.55e-05 1.78e-15 3.62e-05
                                                                      25
      6.13052653e+02 6.13120599e+02 2.11e-06 1.78e-15 1.05e-07
                                                                      2s
Barrier performed 8 iterations in 2.23 seconds (3.84 work units)
Objective cutoff exceeded
Concurrent spin time: 0.09s
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.78 seconds (0.82 work units)
                 Current Node
                                       Objective Bounds
 Expl Unexpl |
               Obj Depth IntInf | Incumbent
                                                         Gap | It/Node Time
                                                BestBd
                                  613.06826 613.12060 0.01%
Explored 1 nodes (0 simplex iterations) in 2.28 seconds (3.85 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 613.068
Optimal solution found (tolerance 1.00e-04)
Best objective 6.130682551112e+02, best bound 6.131205989062e+02, gap 0.0085%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xf4a68db2
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
```

```
[1e+00, 4e+04]
 Matrix range
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
                  [1e+00, 1e+00]
 Bounds range
 RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 613.068
Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 629.2016302
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
AA' NZ
           : 8.813e+03
Factor NZ : 1.806e+04 (roughly 10 MB of memory)
Factor Ops: 4.555e+04 (less than 1 second per iteration)
Threads
           : 1
                 Objective
                                          Residual
                                       Primal
Iter
          Primal
                          Dual
                                                Dual
                                                          Compl
                                                                    Time
                      6.76494600e+09 2.89e+01 6.23e+02
     -7.12942788e+05
                                                         2.42e+05
                                                                      2s
  1 -3.95607813e+05 1.44200459e+09 2.63e+01 1.59e-12 4.72e+04
                                                                      2s
  2 -1.63531502e+05 3.50843186e+08 1.16e+01 3.64e-12 1.01e+04
                                                                      2s
  3 -5.28453674e+04 1.06667870e+08 4.97e+00 1.61e-11
                                                        3.02e+03
                                                                      2s
  4 -4.03133356e+03 8.65724604e+06 4.71e-01 3.64e-11 2.51e+02
                                                                      25
Barrier performed 4 iterations in 1.82 seconds (3.33 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.04s
Barrier statistics:
AA' NZ
           : 1.297e+05
Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
           : 8
                 Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                                    Time
                                                          Compl
  0 -2.72076868e+06 9.88448750e+07 3.87e+01 9.82e-01 3.33e+02
                                                                      25
  1 -1.20645239e+05 2.37572321e+07 5.82e+00 2.66e-15 4.20e+01
                                                                      2s
  2 -1.22683767e+04 6.71936644e+05
                                      3.62e-01 1.78e-15
                                                        1.23e+00
                                                                      2s
  3 -4.69665747e+03 2.00412766e+04 1.42e-01 3.11e-15 2.41e-01
                                                                      25
     3.78088289e+02 7.27758982e+03 0.00e+00 3.11e-15 1.06e-02
                                                                      2s
      3.87383991e+02
                      1.49011609e+03
                                      0.00e+00 2.22e-15 1.70e-03
                                                                      2s
      5.03496854e+02 7.84104178e+02 0.00e+00 1.78e-15 4.33e-04
                                                                      25
      6.07960758e+02 6.36955560e+02 1.74e-05 1.78e-15 4.47e-05
  8
      6.29179196e+02 6.29284077e+02 0.00e+00 1.78e-15 1.62e-07
                                                                      25
      6.29201399e+02 6.29201713e+02 0.00e+00 1.78e-15 1.62e-10
                                                                      2s
Barrier performed 9 iterations in 2.39 seconds (3.91 work units)
Objective cutoff exceeded
Concurrent spin time: 0.16s (can be avoided by choosing Method=3)
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.85 seconds (0.89 work units)
                 Current Node
                                       Objective Bounds
Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd Gap | It/Node Time
                                  629.20163 629.20171 0.00%
                                                                       2s
```

```
Explored 1 nodes (0 simplex iterations) in 2.49 seconds (3.92 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 629.202
Optimal solution found (tolerance 1.00e-04)
Best objective 6.292016302457e+02, best bound 6.292017126953e+02, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xfa5bd3e3
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
  RHS range
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 629.202
Presolve removed 129670 rows and 0 columns
Presolve time: 0.49s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 645.3350054
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
AA' NZ : 9.046e+03
 Factor NZ : 1.853e+04 (roughly 10 MB of memory)
 Factor Ops: 4.648e+04 (less than 1 second per iteration)
 Threads
          : 1
                  Objective
                                            Residual
           Primal
                           Dual
                                        Primal Dual
Iter
                                                                       Time
                                                             Compl
   0 -6.81460642e+05 7.05646366e+09 2.97e+01 6.32e+02 2.47e+05
                                                                         2s
  1 -4.00471651e+05 1.54104814e+09 2.76e+01 2.27e-12 5.00e+04 2 -1.32492348e+05 3.07429857e+08 1.32e+01 2.50e-12 8.52e+03
                                                                         2s
                                                                         2s
   3 -3.48341338e+04 7.45780928e+07 4.24e+00 1.64e-11 2.12e+03
   4 -1.84687902e+03 6.04486433e+06 2.71e-01 1.27e-11 1.66e+02
                                                                         2s
Barrier performed 4 iterations in 1.78 seconds (3.32 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
 AA' NZ : 1.297e+05
Factor NZ : 2.598e+05 (roughly 160 MB of memory)
AA' NZ
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
            : 8
                                            Residual
                  Objective
Iter
           Primal
                           Dual
                                        Primal
                                                 Dual
                                                             Compl
                                                                       Time
  0 -2.54992298e+06 9.99743706e+07 3.95e+01 9.94e-01 3.38e+02
1 -1.06328490e+05 2.42570774e+07 5.84e+00 2.66e-15 4.29e+01
                                                                         2s
                                                                         25
   2 -1.13152567e+04 6.90212967e+05 3.62e-01 2.66e-15 1.26e+00
   3 -4.34262161e+03 2.00269538e+04 1.43e-01 4.44e-15 2.44e-01
                                                                         2s
   4 3.87950141e+02 7.44273219e+03 0.00e+00 3.55e-15 1.09e-02
                                                                         2s
      3.97364561e+02 1.51206190e+03 0.00e+00 2.66e-15 1.72e-03
                                                                         2s
      5.17426744e+02 8.32491635e+02 0.00e+00 1.78e-15 4.86e-04
                                                                         2s
```

Update\_3\_Optimization

```
7 6.21756307e+02 6.56573194e+02 0.00e+00 1.78e-15 5.37e-05
  8 6.45299495e+02 6.45477015e+02 4.24e-06 1.78e-15 2.74e-07
                                                                      25
      6.45334662e+02 6.45335147e+02 0.00e+00 1.78e-15 2.74e-10
                                                                      25
Barrier performed 9 iterations in 2.34 seconds (3.90 work units)
Objective cutoff exceeded
Concurrent spin time: 0.14s (can be avoided by choosing Method=3)
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.83 seconds (0.89 work units)
                 Current Node
                                       Objective Bounds
                                                                  Work
 Expl Unexpl | Obj Depth IntInf | Incumbent BestBd Gap | It/Node Time
                                  645.33501 645.33515 0.00%
Explored 1 nodes (0 simplex iterations) in 2.43 seconds (3.91 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 645.335
Optimal solution found (tolerance 1.00e-04)
Best objective 6.453350053803e+02, best bound 6.453351473981e+02, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x204e5dde
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range [1e+00, 4e+04]
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
 Bounds range [1e+00, 1e+00]
 RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 645.335
Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 661.4683805
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
 AA' NZ : 9.254e+03
 Factor NZ : 1.894e+04 (roughly 11 MB of memory)
 Factor Ops: 4.731e+04 (less than 1 second per iteration)
 Threads
          : 1
                 Objective
                                         Residual
Tter
          Primal
                         Dual
                                      Primal Dual
                                                         Compl
                                                                   Time
   0 -1.46384476e+06 6.67758740e+09 2.71e+01 4.44e+02 2.23e+05
                                                                     4s
   1 -7.44449176e+05 1.51842931e+09 2.77e+01 1.36e-12 4.41e+04
  2 -3.70438295e+05 2.74082276e+08 1.53e+01 1.82e-12 7.42e+03
                                                                      45
   3 -1.16088468e+05 7.20645790e+07 5.19e+00 7.73e-12 1.87e+03
                                                                      4s
   4 -1.50373811e+04 5.14035787e+06 7.41e-01 1.32e-11 1.64e+02
                                                                      4s
Barrier performed 4 iterations in 3.62 seconds (3.32 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
```

```
Ordering time: 0.02s
Barrier statistics:
          : 1.297e+05
 AA' NZ
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops : 5.290e+05 (less than 1 second per iteration)
                 Objective
                                          Residual
          Primal
                                       Primal
Iter
                          Dual
                                                 Dual
                                                          Compl
   0 -7.36029231e+06 1.21650671e+08 4.57e+01 1.00e+00 3.93e+02
                                                                      45
   1 -4.18174198e+05 2.64699990e+07 6.09e+00 2.66e-15 4.66e+01
                                                                      4s
   2 -3.10366553e+04 7.75005859e+05 3.89e-01 1.78e-15 1.41e+00
                                                                      4s
   3 -1.10480556e+04 5.39070025e+04 1.41e-01 3.55e-15 2.85e-01
                                                                      45
      3.53709184e+02 1.19988900e+04 4.63e-04 4.00e-15 1.88e-02
                                                                      4s
      4.05228614e+02 1.53393010e+03 0.00e+00 2.66e-15 1.74e-03
                                                                      45
      4.49434246e+02 1.64528964e+03 0.00e+00 1.78e-15 1.84e-03
                                                                      45
      4.91608948e+02 7.51137436e+02 0.00e+00 1.78e-15 4.00e-04
                                                                      4s
      6.59266498e+02 6.82294201e+02 3.34e-06 1.78e-15 3.55e-05
   8
                                                                      45
      6.61466182e+02 6.61489209e+02 4.18e-07 1.78e-15 3.55e-08
                                                                      4s
Barrier performed 9 iterations in 4.21 seconds (3.90 work units)
Objective cutoff exceeded
Concurrent spin time: 0.14s (can be avoided by choosing Method=3)
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.85 seconds (0.89 work units)
   Nodes
                 Current Node
                                       Objective Bounds
 Expl Unexpl | Obj Depth IntInf | Incumbent BestBd
                                                         Gap | It/Node Time
                                  661.46838 661.48921 0.00%
Explored 1 nodes (0 simplex iterations) in 4.29 seconds (3.91 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 661.468
Optimal solution found (tolerance 1.00e-04)
Best objective 6.614683805148e+02, best bound 6.614892091048e+02, gap 0.0031%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x1520ddc2
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range
                   [1e+00, 4e+04]
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
 Bounds range
                   [1e+00, 1e+00]
                  [1e+03, 4e+04]
 RHS range
Loaded MIP start from previous solve with objective 661.468
Presolve removed 129670 rows and 0 columns
Presolve time: 0.48s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 677.6017556
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
          : 9.493e+03
 Factor NZ: 1.942e+04 (roughly 12 MB of memory)
 Factor Ops: 4.827e+04 (less than 1 second per iteration)
 Threads
          : 1
```

Residual

Objective

```
Tter
          Primal
                          Dual
                                       Primal
                                                Dual
                                                          Compl
                                                                    Time
   0 -1.45578940e+06 6.94321674e+09 2.75e+01 4.52e+02
                                                         2.26e+05
                                                                      25
  1 -7.38698950e+05 1.63034727e+09 2.61e+01 1.16e-12 4.66e+04
                                                                      25
   2 -3.73916246e+05 3.04531484e+08 1.67e+01 1.82e-12 8.16e+03
                                                                      2s
   3 -1.15786425e+05 9.17612509e+07
                                      5.34e+00 8.19e-12
                                                         2.27e+03
                                                                      2s
   4 -1.78523298e+04 7.47517410e+06 8.96e-01 9.55e-12 2.27e+02
                                                                      25
Barrier performed 4 iterations in 1.71 seconds (3.32 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.05s
Barrier statistics:
AA' NZ
           : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
           : 8
                 Objective
                                          Residual
          Primal
Iter
                          Dual
                                       Primal
                                                 Dual
                                                          Compl
                                                                    Time
   0 -7.34983411e+06 1.22142241e+08 4.66e+01 1.00e+00 3.97e+02
                                                                      2s
   1 -4.13083001e+05 2.68368478e+07 6.13e+00 2.66e-15 4.72e+01
                                                                      2s
     -3.07937999e+04 7.90784844e+05
                                      3.93e-01 1.78e-15 1.43e+00
                                                                      2s
   3 -1.09421500e+04 5.46295464e+04 1.42e-01 2.66e-15 2.86e-01
                                                                      25
      3.58226364e+02 1.24202380e+04 5.36e-04 2.66e-15 1.95e-02
   5
      4.15460476e+02 1.47347866e+03 0.00e+00 3.55e-15 1.63e-03
                                                                      2s
      5.88281285e+02 1.56392589e+03 0.00e+00 1.78e-15 1.50e-03
   6
                                                                      2s
      6.26111070e+02 7.04726830e+02 0.00e+00 1.78e-15 1.21e-04
                                                                      25
      6.77324663e+02 6.77930105e+02 0.00e+00 1.78e-15 9.34e-07
   8
                                                                      25
      6.77601482e+02 6.77602084e+02 7.43e-07 1.78e-15 9.34e-10
                                                                      2s
Barrier performed 9 iterations in 2.29 seconds (3.90 work units)
Objective cutoff exceeded
Concurrent spin time: 0.12s (can be avoided by choosing Method=3)
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.83 seconds (0.89 work units)
                 Current Node
                                       Objective Bounds
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd
                                                        Gap | It/Node Time
                                  677.60176 677.60208 0.00%
Explored 1 nodes (0 simplex iterations) in 2.36 seconds (3.91 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 677.602
Optimal solution found (tolerance 1.00e-04)
Best objective 6.776017556493e+02, best bound 6.776020840021e+02, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x8638e25a
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
 RHS range
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 677.602
```

```
Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 693.7351308
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
AA' NZ
         : 9.711e+03
 Factor NZ : 1.986e+04 (roughly 12 MB of memory)
 Factor Ops: 4.914e+04 (less than 1 second per iteration)
 Threads
          : 1
                  Objective
                                            Residual
           Primal
                                        Primal Dual
Tter
                           Dual
                                                            Compl
                                                                       Time
   0 -1.43627435e+06 7.15678775e+09 2.77e+01 4.58e+02 2.28e+05
                                                                         2s
  1 -7.41661063e+05 1.70253562e+09 2.66e+01 1.13e-12 4.77e+04 2 -3.76112011e+05 3.19281171e+08 1.70e+01 1.14e-12 8.40e+03
                                                                         2s
                                                                         2s
   3 -1.14909784e+05 9.58424271e+07 5.39e+00 9.55e-12 2.32e+03
                                                                         25
   4 -1.73142923e+04 7.69199741e+06 8.83e-01 8.19e-12 2.28e+02
Barrier performed 4 iterations in 1.67 seconds (3.32 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.02s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
 AA' NZ : 1.297e+05
Factor NZ : 2.598e+05 (roughly 160 MB of memory)
AA' N7
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
           : 8
                  Objective
                                            Residual
Iter
           Primal
                           Dual
                                        Primal Dual
                                                            Compl
                                                                       Time
   0 -7.33762138e+06 1.22622648e+08 4.74e+01 1.00e+00 4.01e+02
1 -4.08009811e+05 2.71977715e+07 6.16e+00 2.66e-15 4.78e+01
                                                                         2s
                                                                         2s
   2 -3.05553678e+04 8.06448531e+05 3.97e-01 1.78e-15 1.46e+00
   3 -1.08282026e+04 5.52314832e+04 1.43e-01 3.55e-15 2.86e-01
                                                                         2s
   4 3.65291205e+02 1.27695022e+04 5.83e-04 2.66e-15 2.01e-02
                                                                         2s
      4.25755258e+02 1.45507597e+03 0.00e+00 3.11e-15 1.59e-03
                                                                         25
      5.99636515e+02 1.52104283e+03 0.00e+00 2.22e-15 1.42e-03
   6
                                                                         25
       6.41350552e+02 7.20564681e+02
                                        2.32e-06 1.78e-15 1.22e-04
                                                                         2s
      6.93610572e+02 6.94172387e+02 0.00e+00 1.78e-15 8.67e-07
                                                                         2s
      6.93735018e+02 6.93735568e+02 2.32e-06 1.78e-15 8.67e-10
                                                                         25
Barrier performed 9 iterations in 2.26 seconds (3.90 work units)
Objective cutoff exceeded
Concurrent spin time: 0.12s (can be avoided by choosing Method=3)
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.83 seconds (0.89 work units)
                  Current Node
                                         Objective Bounds
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                  BestBd
                                                           Gap | It/Node Time
                                    693.73513 693.73557 0.00%
Explored 1 nodes (0 simplex iterations) in 2.32 seconds (3.91 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 693.735
Optimal solution found (tolerance 1.00e-04)
Best objective 6.937351307838e+02, best bound 6.937355680423e+02, gap 0.0001%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
```

```
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x27c40039
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                  [1e+00, 4e+04]
 Matrix range
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
 Bounds range
                  [1e+00, 1e+00]
 RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 693.735
Presolve removed 129670 rows and 0 columns
Presolve time: 0.50s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 709.8685059
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
          : 9.914e+03
 Factor NZ : 2.026e+04 (roughly 12 MB of memory)
 Factor Ops: 4.995e+04 (less than 1 second per iteration)
 Threads
          : 1
                 Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                          Compl
                                                                    Time
   0 -2.19038279e+06 1.44674964e+10 2.83e+01 1.07e+03 4.53e+05
                                                                      2s
   1 -1.14896780e+06 3.32839113e+09 3.29e+01 2.37e-12 9.20e+04
                                                                      25
   2 -5.41167824e+05 5.77157054e+08 1.64e+01 5.46e-12 1.49e+04
                                                                      25
   3 -1.64951045e+05 1.63604968e+08 5.29e+00 1.64e-11 3.96e+03
                                                                      2s
   4 -2.35444259e+04 1.60599107e+07 8.03e-01 2.68e-11 4.44e+02
                                                                      2s
Barrier performed 4 iterations in 1.74 seconds (3.32 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.04s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
AA' NZ
          : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
          : 8
                 Objective
                                          Residual
          Primal
                                       Primal Dual
Iter
                          Dual
                                                          Compl
                                                                    Time
   0 -7.32322145e+06 1.23094907e+08 4.83e+01 1.00e+00 4.04e+02
                                                                      25
   1 -4.02908887e+05 2.75531543e+07 6.20e+00 2.66e-15 4.85e+01
                                                                      25
   2 -3.03171838e+04 8.22003476e+05 4.00e-01 1.78e-15 1.48e+00
                                                                      2s
   3 -1.07126304e+04 5.58426534e+04 1.43e-01 2.66e-15 2.87e-01
                                                                      25
      3.72790970e+02 1.31538545e+04 6.30e-04 3.55e-15 2.08e-02
                                                                      2s
      4.36114719e+02 1.44320487e+03 0.00e+00 2.22e-15 1.55e-03
                                                                      2s
      6.11364659e+02 1.48761260e+03 0.00e+00 1.78e-15 1.35e-03
                                                                      2s
      6.56923240e+02 7.36416821e+02 0.00e+00 1.78e-15 1.23e-04
                                                                      2s
      7.09627150e+02 7.10395784e+02 0.00e+00 1.78e-15 1.19e-06
                                                                      2s
      7.09868287e+02 7.09869033e+02 4.50e-06 2.66e-15 1.19e-09
                                                                      25
Barrier performed 9 iterations in 2.31 seconds (3.90 work units)
```

Objective cutoff exceeded

Concurrent spin time: 0.13s (can be avoided by choosing Method=3)

Solved with barrier

```
Root relaxation: interrupted, 0 iterations, 0.86 seconds (0.89 work units)
                 Current Node
                                       Objective Bounds
               Obj Depth IntInf | Incumbent
 Expl Unexpl |
                                              BestBd
                                                         Gap | It/Node Time
     0
                                  709.86851 709.86903 0.00%
Explored 1 nodes (0 simplex iterations) in 2.40 seconds (3.91 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 709.869
Optimal solution found (tolerance 1.00e-04)
Best objective 7.098685059183e+02, best bound 7.098690332052e+02, gap 0.0001%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xa6cd2a0d
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
                   [1e+00, 4e+04]
  Matrix range
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
  Bounds range
                   [1e+00, 1e+00]
                   [1e+03, 4e+04]
  RHS range
Loaded MIP start from previous solve with objective 709.869
Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 726.0018811
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.01s
Barrier statistics:
          : 1.015e+04
 Factor NZ : 2.073e+04 (roughly 12 MB of memory)
 Factor Ops: 5.089e+04 (less than 1 second per iteration)
 Threads
           : 1
                  Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                          Compl
                                                                    Time
   0 -2.18625635e+06 1.47815144e+10 2.88e+01 1.07e+03 4.53e+05
                                                                      25
   1 -1.23400834e+06
                      3.49397312e+09
                                      3.54e+01 2.68e-12
                                                         9.61e+04
                                                                       2s
   2 -5.44715203e+05 6.64141710e+08 1.64e+01 4.55e-12 1.66e+04
                                                                      25
   3 -1.78387970e+05 1.81549995e+08 5.77e+00 3.55e-11 4.48e+03
                                                                      2s
   4 -1.74256630e+04 1.41988184e+07 6.09e-01 2.82e-11 3.84e+02
Barrier performed 4 iterations in 1.69 seconds (3.31 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.03s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
 AA' NZ
          : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
          : 8
```

```
Objective
                                          Residual
                                                          Compl
Tter
           Primal
                          Dual
                                       Primal
                                                 Dual
                                                                    Time
   0 -7.31215917e+06 1.23532804e+08 4.91e+01 1.00e+00 4.08e+02
                                                                      25
   1 -3.98217214e+05 2.79006323e+07 6.24e+00 2.66e-15 4.91e+01
                                                                      2s
   2 -3.00991075e+04 8.37365811e+05 4.04e-01 1.78e-15 1.51e+00
                                                                      2s
   3 -1.06008288e+04 5.63879004e+04 1.44e-01 3.11e-15 2.87e-01
                                                                      2s
      3.80530728e+02 1.34786089e+04 6.74e-04 3.55e-15 2.13e-02
      4.46193215e+02 1.48148427e+03 0.00e+00 2.66e-15 1.60e-03
                                                                      25
      5.75651455e+02 1.03416150e+03
                                      0.00e+00 1.78e-15 7.07e-04
                                                                      25
      6.84664994e+02 7.50403912e+02 0.00e+00 1.78e-15 1.01e-04
                                                                      2s
      7.25902669e+02 7.26289586e+02 0.00e+00 1.78e-15 5.97e-07
   8
                                                                      25
      7.26001797e+02 7.26002169e+02 3.05e-06 1.78e-15 5.97e-10
                                                                      2s
Barrier performed 9 iterations in 2.26 seconds (3.89 work units)
Objective cutoff exceeded
Concurrent spin time: 0.14s (can be avoided by choosing Method=3)
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.85 seconds (0.89 work units)
                 Current Node
                                       Objective Bounds
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd Gap | It/Node Time
                                  726.00188 726.00217 0.00%
Explored 1 nodes (0 simplex iterations) in 2.34 seconds (3.90 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 726.002
Optimal solution found (tolerance 1.00e-04)
Best objective 7.260018810528e+02, best bound 7.260021687595e+02, gap 0.0000%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x24a8c4fd
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range
                  [1e+00, 4e+04]
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
                  [1e+00, 1e+00]
  Bounds range
 RHS range
                  [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 726.002
Presolve removed 129670 rows and 0 columns
Presolve time: 0.47s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 742.1352562
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.01s
Barrier statistics:
           : 1.038e+04
 Factor NZ : 2.120e+04 (roughly 13 MB of memory)
 Factor Ops: 5.183e+04 (less than 1 second per iteration)
 Threads
                 Objective
                                          Residual
Iter
           Primal
                          Dual
                                       Primal
                                                 Dual
                                                          Compl
                                                                    Time
   0 -2.16992179e+06 1.52224063e+10 2.90e+01 1.08e+03 4.56e+05
                                                                      3s
   1 -1.24470838e+06 3.61151372e+09 3.58e+01 3.64e-12 9.75e+04
   2 -5.38164832e+05 6.57369299e+08 1.64e+01 3.18e-12 1.61e+04
                                                                      3s
   3 -1.81059925e+05 1.78793880e+08
                                      5.89e+00 1.82e-11 4.27e+03
                                                                      3s
   4 -2.05436618e+04 1.27087569e+07 7.09e-01 3.46e-11 3.55e+02
                                                                      3s
```

```
Barrier performed 4 iterations in 3.07 seconds (3.31 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
 AA' NZ : 1.297e+05
Factor NZ : 2.598e+05 (roughly 160 MB of memory)
AA' NZ
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
                  Objective
                                           Residual
Iter
           Primal
                           Dual
                                       Primal Dual
                                                           Compl
                                                                      Time
  0 -7.29962518e+06 1.23961193e+08 4.99e+01 1.00e+00 4.11e+02
1 -3.93539518e+05 2.82426823e+07 6.27e+00 2.89e-15 4.97e+01
                                                                        3s
                                                                        3s
   2 -2.98816782e+04 8.52562991e+05 4.08e-01 1.78e-15 1.54e+00
   3 -9.89026349e+03 2.65189640e+04 1.35e-01 4.88e-15 2.47e-01
                                                                        3s
      4.31872434e+02
                       9.40034693e+03 1.98e-04 4.00e-15 1.41e-02
                                                                        4s
      4.50346433e+02 3.46036485e+03 4.23e-05 2.66e-15 4.66e-03
                                                                        45
      4.79270821e+02 1.90804558e+03 0.00e+00 1.78e-15 2.20e-03
                                                                        45
       6.41789315e+02 8.35820717e+02
                                       0.00e+00 1.78e-15 2.99e-04
                                                                        4s
      7.40947912e+02 7.48049686e+02 1.77e-05 1.78e-15 1.10e-05
                                                                        4s
      7.42133861e+02 7.42141207e+02 0.00e+00 1.78e-15 1.13e-08
                                                                        45
Barrier performed 9 iterations in 3.74 seconds (3.89 work units)
Objective cutoff exceeded
Concurrent spin time: 0.17s (can be avoided by choosing Method=3)
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.96 seconds (0.89 work units)
                  Current Node
                                        Objective Bounds
 Expl Unexpl | Obj Depth IntInf | Incumbent BestBd Gap | It/Node Time
                                   742.13526 742.14121 0.00%
Explored 1 nodes (0 simplex iterations) in 3.83 seconds (3.90 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 742.135
Optimal solution found (tolerance 1.00e-04)
Best objective 7.421352561873e+02, best bound 7.421412069624e+02, gap 0.0008%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x58bbd2ba
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
  Matrix range
                   [1e+00, 4e+04]
  Objective range [0e+00, 0e+00]
  QObjective range [2e-02, 8e-02]
                  [1e+00, 1e+00]
  Bounds range
                   [1e+03, 4e+04]
Loaded MIP start from previous solve with objective 742.135
Presolve removed 129670 rows and 0 columns
Presolve time: 0.49s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 758.2686313
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
```

Root barrier log...

```
Ordering time: 0.00s
Barrier statistics:
 AA' NZ
          : 1.059e+04
 Factor NZ : 2.162e+04 (roughly 13 MB of memory)
 Factor Ops: 5.267e+04 (less than 1 second per iteration)
 Threads
          : 1
                 Objective
                                          Residual
          Primal
                                       Primal Dual
Tter
                          Dual
                                                                   Time
                                                          Compl
   0 -2.15199011e+06 1.55709408e+10 2.92e+01 1.08e+03 4.57e+05
                                                                     2s
  1 -1.25791100e+06 3.71046529e+09 3.63e+01 2.12e-12 9.86e+04
                                                                     2s
   2 -5.43095258e+05 6.83176112e+08 1.66e+01 5.91e-12 1.65e+04
                                                                     25
   3 -1.81957176e+05 1.85490703e+08 5.97e+00 4.50e-11 4.36e+03
                                                                      2s
   4 -2.00414719e+04 1.33165730e+07 7.05e-01 4.73e-11 3.60e+02
                                                                     25
Barrier performed 4 iterations in 1.77 seconds (3.31 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.03s
Barrier statistics:
AA' NZ : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
          : 8
                                          Residual
                 Objective
Iter
          Primal
                          Dual
                                      Primal Dual
                                                          Compl
                                                                   Time
    -7.28514607e+06 1.24382925e+08 5.07e+01 1.00e+00 4.14e+02
                                                                     25
   1 -3.88841438e+05 2.85797866e+07 6.31e+00 2.66e-15 5.03e+01
                                                                     25
   2 -2.96616408e+04 8.67593379e+05 4.11e-01 2.66e-15 1.56e+00
                                                                     2s
  3 -9.82276890e+03 2.65322726e+04 1.36e-01 3.11e-15 2.47e-01
                                                                     2s
      4.42412420e+02 9.56521997e+03 1.99e-04 4.44e-15 1.44e-02
                                                                     2s
      4.60968707e+02
                                      4.21e-05 3.55e-15 4.73e-03
                      3.51615144e+03
                                                                     25
      4.91168842e+02 1.94304668e+03 0.00e+00 1.78e-15 2.24e-03
                                                                     25
      6.62323660e+02 8.79330252e+02 0.00e+00 1.78e-15 3.35e-04
                                                                     2s
      7.56298219e+02 7.67028998e+02 6.75e-05 1.78e-15 1.66e-05
                                                                     2s
      7.58266187e+02 7.58277481e+02 5.46e-05 1.78e-15 1.78e-08
                                                                     25
Barrier performed 9 iterations in 2.37 seconds (3.89 work units)
Objective cutoff exceeded
Concurrent spin time: 0.15s (can be avoided by choosing Method=3)
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.86 seconds (0.89 work units)
                 Current Node
                                       Objective Bounds
 Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd Gap | It/Node Time
     0
                                  758.26863 758.27748 0.00%
Explored 1 nodes (0 simplex iterations) in 2.45 seconds (3.90 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 758.269
Optimal solution found (tolerance 1.00e-04)
Best objective 7.582686313218e+02, best bound 7.582774811657e+02, gap 0.0012%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0xd82a2cc2
Model has 129670 quadratic objective terms
```

```
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range
                  [1e+00, 4e+04]
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
 Bounds range
                  [1e+00, 1e+00]
                  [1e+03, 4e+04]
 RHS range
Loaded MIP start from previous solve with objective 758.269
Presolve removed 129670 rows and 0 columns
Presolve time: 0.48s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 774.4020065
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.00s
Barrier statistics:
         : 1.082e+04
 Factor NZ : 2.207e+04 (roughly 13 MB of memory)
 Factor Ops : 5.357e+04 (less than 1 second per iteration)
 Threads
          : 1
                 Objective
                                         Residual
Iter
          Primal
                          Dual
                                      Primal
                                                Dual
                                                                   Time
                                                         Compl
   0 -2.13777369e+06 1.58876206e+10 2.95e+01 1.08e+03 4.57e+05
                                                                     25
   1 -1.27715129e+06 3.80132363e+09 3.70e+01 2.08e-12 9.95e+04
                                                                     25
  2 -5.62671220e+05 7.09741028e+08 1.76e+01 5.46e-12 1.70e+04
                                                                     25
   3 -1.71632892e+05 1.85589941e+08 5.85e+00 2.64e-11 4.23e+03
   4 -2.04607941e+04 1.30001526e+07 7.43e-01 4.46e-11 3.53e+02
                                                                     2s
Barrier performed 4 iterations in 1.71 seconds (3.30 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.05s
Barrier statistics:
 AA' NZ
         : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
          : 8
                 Objective
                                         Residual
                                      Primal Dual
          Primal
                         Dual
Iter
                                                         Compl
                                                                   Time
   0 -7.26982591e+06 1.24793335e+08 5.15e+01 1.00e+00 4.18e+02
                                                                     2s
   1 -3.84196627e+05 2.89115502e+07 6.34e+00 2.89e-15 5.08e+01
                                                                     25
   2 -2.94449532e+04 8.82461879e+05 4.15e-01 1.78e-15 1.59e+00
   3 -9.75356186e+03 2.65198811e+04 1.37e-01 3.55e-15 2.47e-01
                                                                     25
     4.53284154e+02 9.70681126e+03 1.95e-04 2.66e-15 1.46e-02
                                                                     25
      4.71452031e+02 3.65321032e+03 4.32e-05 2.66e-15 4.93e-03
   6
      5.01848583e+02 1.99739643e+03 0.00e+00 1.78e-15 2.31e-03
                                                                     25
      6.72241415e+02 8.84782244e+02 0.00e+00 1.78e-15 3.28e-04
                                                                     2s
      7.72936018e+02 7.81815019e+02 3.06e-05 1.78e-15 1.37e-05
                                                                     25
      7.74400639e+02 7.74409470e+02 8.26e-05 1.78e-15 1.43e-08
Barrier performed 9 iterations in 2.32 seconds (3.88 work units)
Objective cutoff exceeded
Concurrent spin time: 0.14s (can be avoided by choosing Method=3)
Solved with barrier
Root relaxation: interrupted, 0 iterations, 0.86 seconds (0.90 work units)
                Current Node
                                      Objective Bounds
 Expl Unexpl | Obj Depth IntInf | Incumbent BestBd Gap | It/Node Time
```

```
774.40201 774.40947 0.00%
Explored 1 nodes (0 simplex iterations) in 2.38 seconds (3.89 work units)
Thread count was 10 (of 10 available processors)
Solution count 1: 774.402
Optimal solution found (tolerance 1.00e-04)
Best objective 7.744020064563e+02, best bound 7.744094701015e+02, gap 0.0010%
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
Academic license 2608044 - for non-commercial use only - registered to tc___@andrew.cmu.edu
Optimize a model with 389011 rows, 259340 columns and 778020 nonzeros
Model fingerprint: 0x661c317e
Model has 129670 quadratic objective terms
Variable types: 129670 continuous, 129670 integer (129670 binary)
Coefficient statistics:
 Matrix range
                  [1e+00, 4e+04]
 Objective range [0e+00, 0e+00]
 QObjective range [2e-02, 8e-02]
 Bounds range
                  [1e+00, 1e+00]
                  [1e+03, 4e+04]
 RHS range
Loaded MIP start from previous solve with objective 774.402
Presolve removed 129670 rows and 0 columns
Presolve time: 0.46s
Presolved: 389011 rows, 389010 columns, 1037360 nonzeros
Variable types: 259340 continuous, 129670 integer (129670 binary)
Found heuristic solution: objective 790.5353816
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.01s
Barrier statistics:
 AA' NZ
           : 1.103e+04
 Factor NZ : 2.250e+04 (roughly 14 MB of memory)
 Factor Ops: 5.443e+04 (less than 1 second per iteration)
                 Objective
                                          Residual
Iter
          Primal
                          Dual
                                       Primal
                                                 Dual
                                                          Compl
                                                                    Time
   0 -2.12066757e+06 1.62053963e+10 2.97e+01 1.08e+03 4.57e+05
                                                                      2s
   1 -1.28415311e+06 3.89470517e+09 3.73e+01 2.24e-12 1.00e+05
   2 -5.65644183e+05 7.33355760e+08 1.79e+01 7.28e-12 1.73e+04
                                                                      2s
   3 -1.70320301e+05 1.92083393e+08 5.92e+00 2.91e-11 4.29e+03
                                                                      2s
   4 -2.04384086e+04 1.35862314e+07 7.54e-01 4.18e-11 3.60e+02
Barrier performed 4 iterations in 1.70 seconds (3.32 work units)
Barrier solve interrupted - model solved by another algorithm
Concurrent spin time: 0.01s
Solved with primal simplex
Deterministic concurrent LP optimizer: primal simplex, dual simplex, and barrier
Showing barrier log only...
Root barrier log...
Ordering time: 0.05s
Barrier statistics:
           : 1.297e+05
 Factor NZ : 2.598e+05 (roughly 160 MB of memory)
 Factor Ops: 5.290e+05 (less than 1 second per iteration)
 Threads
           : 8
                 Objective
                                          Residual
          Primal
                                       Primal
Iter
                                                                    Time
                          Dual
                                                 Dual
                                                          Compl
   0 -7.25276443e+06 1.25197354e+08 5.23e+01 1.00e+00 4.21e+02
                                                                      2s
   1 -3.79541114e+05 2.92386420e+07
                                      6.37e+00 2.66e-15
                                                         5.14e+01
                                                                      2s
   2 -2.92274019e+04 8.97192925e+05
                                      4.19e-01 1.78e-15
                                                         1.61e+00
                                                                      2s
   3 -9.68153257e+03 2.64889254e+04 1.38e-01 4.00e-15 2.47e-01
                                                                      2s
     4.64328929e+02 9.83430700e+03 1.88e-04 2.66e-15 1.47e-02
                                                                      2s
```

```
      5
      4.81776906e+02
      3.86135978e+03
      4.50e-05
      2.66e-15
      5.24e-03
      2s

      6
      5.11535010e+02
      2.06883080e+03
      0.00e+00
      1.78e-15
      2.40e-03
      2s

      7
      6.74419195e+02
      1.02860567e+03
      0.00e+00
      1.78e-15
      5.46e-04
      2s

      8
      7.87627109e+02
      8.04078022e+02
      0.00e+00
      1.78e-15
      2.54e-05
      2s

      9
      7.90532804e+02
      7.90548955e+02
      5.99e-05
      1.78e-15
      2.54e-08
      2s
```

Barrier performed 9 iterations in 2.28 seconds (3.90 work units) Objective cutoff exceeded

Concurrent spin time: 0.13s (can be avoided by choosing Method=3)

Solved with barrier

Root relaxation: interrupted, 0 iterations, 0.83 seconds (0.89 work units)

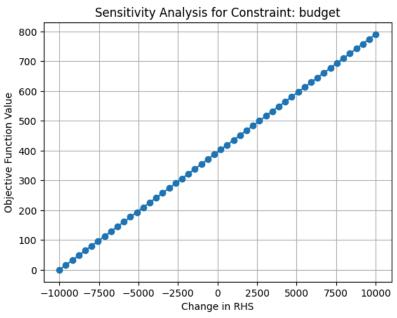
```
Nodes | Current Node | Objective Bounds | Work
Expl Unexpl | Obj Depth IntInf | Incumbent BestBd Gap | It/Node Time

0 0 - 0 790.53538 790.54896 0.00% - 2s
```

Explored 1 nodes (0 simplex iterations) in 2.36 seconds (3.91 work units) Thread count was 10 (of 10 available processors)

Solution count 1: 790.535

Optimal solution found (tolerance 1.00e-04) Best objective 7.905353815908e+02, best bound 7.905489553319e+02, gap 0.0017%



#### Minimize Risk

In [ ]: modelR.optimize()

```
In []: modelR = gp.Model(env=env)

selection = modelR.addVars(indices, vtype=GRB.BINARY, name='selection')
invest = modelR.addVars(indices, lb=0, name="inv_amnt")

modelR.setObjective(sum(selection[i] * r[i] * for i in indices), GRB.MINIMIZE)

# modelR.addConstr(sum(selection[i] * for i in indices) <= 100, name="num_loans_upper")
modelR.addConstr(sum(selection[i] * for i in indices) >= 90, name="num_loans_lower")
modelR.addConstrs((invest[i] <= a.loc[i] * for i in indices), name="inv_max")
modelR.addConstrs((invest[i] <= selection[i] * a.loc[i] * for i in indices), name="inv_selection")
modelR.addConstrs((invest[i] >= 25 * selection[i] * for i in indices), name="inv_min")
modelR.addConstr(sum(invest[i] * for i in indices) <= 10000, name="budget")</pre>
Out[]: <gurobi.Constr *Awaiting Model Update*>
```

```
Gurobi Optimizer version 12.0.1 build v12.0.1rc0 (mac64[arm] - Darwin 24.3.0 24D81)
       CPU model: Apple M4
       Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
       Academic license 2608044 - for non-commercial use only - registered to tc @andrew.cmu.edu
       Optimize a model with 389012 rows, 259340 columns and 907690 nonzeros
       Model fingerprint: 0x7b8cb362
       Variable types: 129670 continuous, 129670 integer (129670 binary)
       Coefficient statistics:
                         [1e+00, 4e+04]
         Matrix range
         Objective range [1e-02, 4e-02]
         Bounds range
                          [1e+00, 1e+00]
         RHS range
                          [9e+01, 4e+04]
       CPU model: Apple M4
       Thread count: 10 physical cores, 10 logical processors, using up to 10 threads
       Academic license 2608044 - for non-commercial use only - registered to tc @andrew.cmu.edu
       Optimize a model with 389012 rows, 259340 columns and 907690 nonzeros
       Model fingerprint: 0x7b8cb362
       Variable types: 129670 continuous, 129670 integer (129670 binary)
       Coefficient statistics:
                       [1e+00, 4e+04]
         Matrix range
         Objective range [1e-02, 4e-02]
         Bounds range
                          [1e+00, 1e+00]
        RHS range
                          [9e+01, 4e+04]
       Presolve removed 389010 rows and 129670 columns
       Presolve time: 0.17s
       Presolved: 2 rows, 129670 columns, 259340 nonzeros
       Variable types: 0 continuous, 129670 integer (129670 binary)
       Found heuristic solution: objective 0.9481942
       Performing another presolve...
       Presolve removed 1 rows and 129669 columns
       Presolve time: 0.06s
       Explored 1 nodes (0 simplex iterations) in 0.39 seconds (0.93 work units)
       Thread count was 10 (of 10 available processors)
       Solution count 2: 0.948194 0.948194
       Optimal solution found (tolerance 1.00e-04)
       Best objective 9.481942177986e-01, best bound 9.481942177986e-01, gap 0.0000%
In [ ]: print("Objective = ", modelR.ObjVal)
        modelR.printAttr('X')
```

Objective = 0.948194217798609

Objective - 0:940	194217790009
Variable	X
Variable	X
selection[500840]	1
selection[86479]	1
selection[543415]	1
selection[646559]	1
selection[387942] selection[420058]	1
selection[546455]	1
selection[342920]	1
selection[422041]	1
selection[219863] selection[469750]	1 1
selection[329646]	1
selection[365954]	1
selection[548753]	1
selection[616478] selection[91638]	1
selection[130970]	1
selection[87157]	1
selection[565845]	1 1
selection[58027] selection[229421]	1
selection[307232]	1
selection[49199]	1
selection[565548] selection[470745]	1 1
selection[23060]	1
selection[362488]	1
selection[580082]	1
selection[113086] selection[353454]	1
selection[156034]	1
selection[358563]	1
selection[616544]	1
selection[234025] selection[138977]	1
selection[170455]	1
selection[641327]	1
selection[311307] selection[183929]	1 1
selection[595619]	1
selection[419491]	1
selection[496741]	1
selection[242302] selection[309872]	1
selection[73297]	1
selection[368067]	1
selection[416410] selection[280215]	1
selection[243019]	1
selection[366160]	1
selection[586329]	1
selection[421245] selection[623905]	1
selection[75580]	1
selection[219720]	1
selection[382239] selection[417300]	1
selection[297285]	1
selection[343334]	1
selection[585203]	1
selection[299121] selection[643433]	1
selection[371937]	1
selection[53265]	1
selection[66882] selection[313780]	1
selection[513780]	1
selection[415084]	1
selection[36109]	1
selection[615536] selection[108692]	1
selection[634285]	1
selection[60127]	1
selection[321077] selection[362597]	1
20 (CC (TOH [302337]	1

selection[568943]	1
selection[198426]	1
selection[264597]	1
selection[506176]	1
selection[196801]	1
selection[325988]	1
selection[192194]	1
selection[145413]	1
selection[608393]	1
selection[333090]	1
selection[347718]	1
selection[449313]	1
selection[148063]	1
selection[110440]	1
selection[157954]	1
inv_amnt[500840]	25
inv_amnt[86479]	25
inv_amnt[543415]	25
inv_amnt[646559]	25
111V_dill11C[040339]	
inv_amnt[387942]	25
inv_amnt[420058]	25
inv_amnt[546455]	25
inv_amnt[342920]	25
1117 GHITT [342320]	
inv_amnt[422041]	25
inv_amnt[219863]	25
inv_amnt[469750]	25
inv_amnt[329646]	25
111V_alli11t [329040]	
inv_amnt[365954]	25
inv_amnt[548753]	25
inv_amnt[616478]	25
<del>_</del>	
inv_amnt[91638]	25
inv_amnt[130970]	25
inv_amnt[87157]	25
inv_amnt[565845]	25
inv_amnt[58027]	25
inv_amnt[229421]	25
inv_amnt[307232]	25
inv_amnt[49199]	25
<del>-</del>	
inv_amnt[565548]	25
inv_amnt[470745]	25
inv_amnt[23060]	25
inv_amnt[362488]	25
inv_amnt[580082]	25
inv_amnt[113086]	25
inv_amnt[353454]	25
inv_amnt[156034]	25
	25
inv_amnt[358563]	
inv_amnt[616544]	25
inv_amnt[234025]	25
inv_amnt[138977]	25
inv_amnt[170455]	25
inv_amnt[641327]	25
inv_amnt[311307]	25
inv_amnt[183929]	25
inv_amnt[595619]	25
inv_amnt[419491]	25
inv_amnt[496741]	25
inv_amnt[242302]	25
inv_amnt[309872]	25
	25
inv_amnt[73297]	
inv_amnt[368067]	25
inv_amnt[416410]	25
inv_amnt[280215]	25
inv_amnt[243019]	25
111V_d   11 [ [ 243019 ]	
inv_amnt[366160]	25
inv_amnt[586329]	25
inv_amnt[421245]	25
inv_amnt[623905]	
TIIV_dIIII1 [023903]	25
inv_amnt[75580]	25
inv_amnt[219720]	25
inv_amnt[382239]	25
inv_amnt[417300]	25
inv_amnt[297285]	25
inv_amnt[343334]	25
inv_amnt[585203]	25
inv_amnt[299121]	25
inv_amnt[643433]	25
inv_amn+[271027]	
inv_amnt[371937]	25
inv_amnt[53265]	25
inv_amnt[66882]	25

```
inv_amnt[313780]
                           25
                          25
inv_amnt[51499]
inv_amnt[415084]
                           25
inv_amnt[36109]
                          25
inv_amnt[615536]
                           25
inv amnt[108692]
                           25
inv_amnt[634285]
                           25
inv_amnt[60127]
                          25
inv_amnt[321077]
                           25
inv_amnt[362597]
                           25
inv_amnt[568943]
                           25
inv_amnt[198426]
                           25
inv_amnt[264597]
                           25
inv amnt[506176]
                           25
inv_amnt[196801]
                           25
inv_amnt[325988]
                           25
inv_amnt[192194]
                           25
inv_amnt[145413]
                           25
inv amnt[608393]
                           25
inv_amnt[333090]
                           25
inv_amnt[347718]
                           25
inv_amnt[449313]
                           25
inv_amnt[148063]
                           25
inv_amnt[110440]
                           25
inv_amnt[157954]
                           25
```

#### Returns

```
In [ ]: # maximum return
        import re # Import the regular expression module
        # Assuming your output is stored in a string called 'output_string'
        output_string = '''selection[226102]
        selection[222245]
        selection[208159]
                                     1 ....
        selection[176412]
        # Extract indices using regular expressions
        indices_string = re.findall(r'selection\[(\d+)\]', output_string)
        # Convert to list of integers
        selected_indices = [int(index) for index in indices_string]
        print(selected_indices)
        print(len(selected_indices))
       [226102, 222245, 208159, 176412]
In [ ]: final_clean_data.loc[selected_indices]['ret_INTb'].mean()
Out[]: np.float64(0.0039146638442535305)
In [ ]: # minimal risk
        import re # Import the regular expression module
        # Assuming your output is stored in a string called 'output_string'
        output_string = '''selection[500840]
                                                        1
        selection[86479]
                                    1
        selection[543415]
                                     1
        selection[646559]
                                     1
        selection[387942]
                                     1
        selection[420058]
                                     1
        selection[546455]
        selection[342920]
                                     1
        selection[422041]
                                     1
        selection[219863]
        selection[469750]
                                     1
        selection[329646]
                                     1
        selection[365954]
                                     1
        selection[548753]
                                     1
        selection[616478]
                                     1
        selection[91638]
                                    1
        selection[130970]
                                     1
        selection[87157]
                                    1
        selection[565845]
                                     1
        selection[58027]
                                    1
        selection[229421]
                                     1
        selection[307232]
                                     1
```

```
selection[49199]
selection[565548]
                             1
selection[470745]
                             1
selection[23060]
                            1
selection[362488]
                             1
selection[580082]
                             1
selection[113086]
                             1
selection[353454]
                            1
selection[156034]
                             1
selection[358563]
                             1
selection[616544]
                             1
selection[234025]
                             1
selection[138977]
                             1
selection[170455]
                            1
selection[641327]
                            1
selection[311307]
selection[183929]
                            1
selection[595619]
                            1
selection[419491]
                             1
selection[496741]
                             1
selection[242302]
                            1
selection[309872]
                            1
selection[73297]
                            1
selection[368067]
                            1
selection[416410]
                             1
selection[280215]
                             1
selection[243019]
                            1
selection[366160]
                            1
selection[586329]
                             1
selection[421245]
                            1
selection[623905]
                            1
selection[75580]
                            1
selection[219720]
                            1
selection[382239]
selection[417300]
                             1
selection[297285]
                             1
selection[343334]
selection[585203]
                             1
selection[299121]
selection[643433]
                            1
selection[371937]
                            1
selection[53265]
                            1
selection[66882]
                           1
selection[313780]
                            1
selection[51499]
selection[415084]
                            1
selection[36109]
                            1
selection[615536]
                             1
selection[108692]
                             1
selection[634285]
selection[60127]
                            1
selection[321077]
                             1
selection[362597]
                            1
selection[568943]
                             1
selection[198426]
                             1
selection[264597]
                            1
selection[506176]
                             1
selection[196801]
                             1
selection[325988]
                             1
selection[192194]
                             1
selection[145413]
                             1
selection[608393]
                            1
selection[333090]
selection[347718]
                             1
selection[449313]
                             1
selection[148063]
                             1
selection[110440]
                             1
                            1111
selection[157954]
# Extract indices using regular expressions
indices\_string = re.findall(r'selection\setminus[(\d+)\]', output\_string)
# Convert to list of integers
selected_indices = [int(index) for index in indices_string]
print(selected_indices)
print(len(selected_indices))
```

[500840, 86479, 543415, 646559, 387942, 420058, 546455, 342920, 422041, 219863, 469750, 329646, 365954, 548753, 6164 78, 91638, 130970, 87157, 565845, 58027, 229421, 307232, 49199, 565548, 470745, 23060, 362488, 580082, 113086, 35345 4, 156034, 358563, 616544, 234025, 138977, 170455, 641327, 311307, 183929, 595619, 419491, 496741, 242302, 309872, 7 3297, 368067, 416410, 280215, 243019, 366160, 586329, 421245, 623905, 75580, 219720, 382239, 417300, 297285, 343334, 585203, 299121, 643433, 371937, 53265, 66882, 313780, 51499, 415084, 36109, 615536, 108692, 634285, 60127, 321077, 3 62597, 568943, 198426, 264597, 506176, 196801, 325988, 192194, 145413, 608393, 333090, 347718, 449313, 148063, 11044 0, 157954]

```
In []: final_clean_data.loc[selected_indices]['ret_INTb'].mean()
```

Out[]: np.float64(0.057136079289583)

## Findings Update 3 Question

```
In []: import pandas as pd
import numpy as np

lc_data = pd.read_parquet("/Users/goyolozano/Desktop/Mini 4/Value/Update 3/Deliverables/final_clean_data.parquet")

display(lc_data.head(5))
print(lc_data.columns)
```

	loan_amnt	funded_amnt	term	int_rate	installment	grade	emp_length	home_ownership	annual_inc	verification_status	•••
0	15000.0	15000.0	60 months	12.39	336.64	С	10+ years	RENT	78000.0	Source Verified	
1	10400.0	10400.0	36 months	6.99	321.08	А	8 years	MORTGAGE	58000.0	Not Verified	
2	7650.0	7650.0	36 months	13.66	260.20	С	< 1 year	RENT	50000.0	Source Verified	
3	12800.0	12800.0	60 months	17.14	319.08	D	10+ years	MORTGAGE	125000.0	Verified	
4	21425.0	21425.0	60 months	15.59	516.36	D	6 years	RENT	63800.0	Source Verified	

5 rows × 34 columns

```
In []: # Cleaning Return Columns
    cols_to_exclude = ["ret_PESS", "ret_OPT", "ret_INTa", "ret_INTc"]
    lc_data = lc_data.drop(columns=cols_to_exclude)

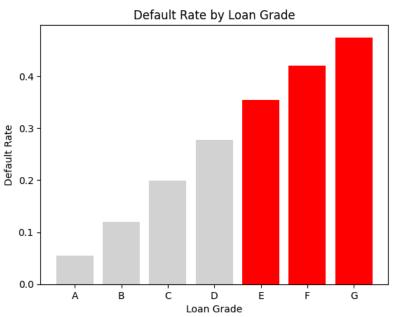
# Deafault Column
    default_statuses = ["Charged Off", "Default"]
    lc_data["default_label"] = lc_data["loan_status"].isin(default_statuses).astype(int)
    print(lc_data.columns)
```

# Deafult by Loan Grade

```
In []: import matplotlib.pyplot as plt

# 1. Compute default rate by grade
default_by_grade = (
    lc_data
    .groupby('grade')['default_label']
```

```
.reset_index(name='default_rate')
grades = default_by_grade['grade']
rates = default_by_grade['default_rate']
# 2. Define grade buckets
safe = {'A', 'B', 'C'} # ga
medium = {'D'} # orange
                            # green
# the rest (F, G) will be red
# 3. Map each grade to a color
colors = [
    'lightgrey' if g in safe
    else 'lightgrey' if g in medium
    else 'red'
    for g in grades
# 4. Plot
plt.figure()
plt.bar(grades, rates, color=colors)
plt.xlabel('Loan Grade')
plt.ylabel('Default Rate')
plt.title('Default Rate by Loan Grade')
plt.xticks(rotation=0)
plt.show()
```



# Returns by loan grade

```
In []: import pandas as pd
import matplotlib.pyplot as plt

# 1. Prepare grade list and corresponding return-series
grades = sorted(lc_data['grade'].unique())
ret_data = [lc_data.loc[lc_data['grade'] == g, 'ret_INTb'] for g in grades]

# 2. Define colors: lightgrey for A-D (first 4), red for F-G (last 2)
# (E will also get lightgrey since it's the 5th of 7)
colors = ['lightgrey']*4 + ['lightgrey']*(len(grades)-7) + ['red']*3

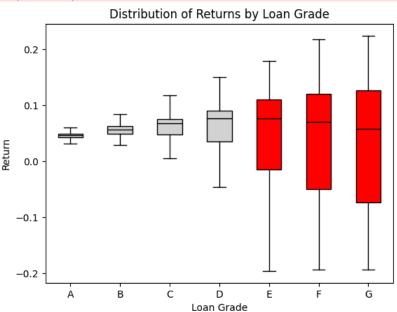
# 3. Draw boxplots without outliers, with patch_artist=True to fill boxes
fig, ax = plt.subplots()
bp = ax.boxplot(
    ret_data,
    labels=grades,
    showfliers=False,
    patch_artist=True
)
```

```
# 4. Apply our color scheme
for patch, color in zip(bp['boxes'], colors):
    patch.set_facecolor(color)
    patch.set_edgecolor('black')

# 5. Tweak medians to be visible
for median in bp['medians']:
    median.set_color('black')

ax.set_xlabel('Loan Grade')
ax.set_ylabel('Return')
ax.set_title('Distribution of Returns by Loan Grade')
plt.show()
```

/var/folders/kr/\_0jlkcjd3bz5k\_\_n\_t79wdnc0000gn/T/ipykernel\_68326/2225996352.py:14: MatplotlibDeprecationWarning: The
'labels' parameter of boxplot() has been renamed 'tick\_labels' since Matplotlib 3.9; support for the old name will b
e dropped in 3.11.
 bp = ax.boxplot(



### **Decision Tree**

In [ ]: lc\_data.dtypes

float64

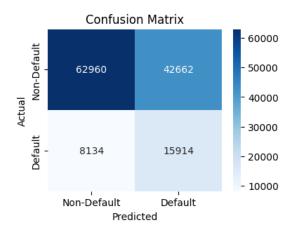
float64

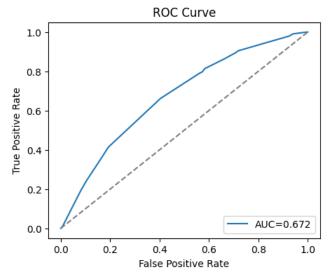
Out[]: loan\_amnt

funded\_amnt

```
term
                                   object
                                  float64
        int_rate
        installment
                                  float64
                                   object
        grade
        emp_length
                                   object
                                   object
        home_ownership
                                  float64
        annual_inc
        verification_status
                                   object
                                   object
        loan_status
        purpose
                                   object
        dti
                                  float64
        delinq_2yrs
                                  float64
        open_acc
                                  float64
        pub_rec
                                  float64
                                  float64
        revol_bal
        revol_util
                                  float64
                                  float64
        total_pymnt
        recoveries
                                  float64
        mths_since_last_delinq
                                  float64
                                  float64
        acc_now_delinq
        bc_util
                                  float64
        mort_acc
                                  float64
                                  float64
        num_tl_90g_dpd_24m
        pub_rec_bankruptcies
                                  float64
        total_bc_limit
                                  float64
        loan_length
                                  float64
        term_num
                                    int64
        ret_INTb
                                  float64
        default_label
                                    int64
        dtype: object
In [ ]: # ## Decision Tree with SMOTE (Speed-Optimized)
        import pandas as pd
        import matplotlib.pyplot as plt
        import seaborn as sns
        from sklearn.model_selection import train_test_split, GridSearchCV
        from imblearn.pipeline import Pipeline as ImbPipeline
        from imblearn.over_sampling import SMOTE
        from sklearn.impute import SimpleImputer
        from sklearn.preprocessing import StandardScaler, OneHotEncoder
        from sklearn.compose import ColumnTransformer
        from sklearn.tree import DecisionTreeClassifier
        from sklearn.metrics import (
            accuracy_score,
            classification_report,
            roc_auc_score,
            roc_curve,
            confusion_matrix
        # 1. Define target and features, drop leakage columns
        leakage_cols = ['loan_status','total_pymnt','recoveries','loan_length','ret_INTb']
        y = lc_data['default_label']
        X = lc_data.drop(columns=leakage_cols + ['default_label'])
        # 2. Train/test split
        X_train, X_test, y_train, y_test = train_test_split(
            X, y, test_size=0.2, stratify=y, random_state=42
        # 3. Identify feature types
        cat_cols = ['grade','term','emp_length','home_ownership','verification_status','purpose']
        num_cols = [c for c in X_train.columns if c not in cat_cols]
        # 4. Preprocessing pipelines
        num_transformer = ImbPipeline([
            ('imputer', SimpleImputer(strategy='mean')),
            ('scaler', StandardScaler())
        ])
        cat_transformer = ImbPipeline([
            ('imputer', SimpleImputer(strategy='most_frequent')),
            ('onehot', OneHotEncoder(handle_unknown='ignore', sparse_output=False))
        ])
        preprocessor = ColumnTransformer([
```

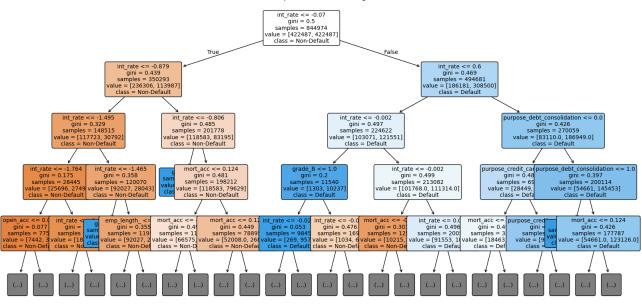
```
('num', num_transformer, num_cols),
     ('cat', cat_transformer, cat_cols)
 ], remainder='drop')
 # 5. Build Decision Tree pipeline with SMOTE
 pipeline_dt = ImbPipeline([
     ('prep', preprocessor),
('smote', SMOTE(random_state=42)),
     ('clf', DecisionTreeClassifier(class_weight='balanced', random_state=42))
 1)
 # 6. Speed-optimized hyperparameter grid
 param_grid_dt = {
     'clf__max_depth':
                              [3, 5, 10, 15],
     'clf_min_samples_leaf':[50, 100, 150]
 search_dt = GridSearchCV(
     pipeline dt,
     param_grid_dt,
     cv=5,
     scoring='roc_auc',
     n_{jobs=-1},
     verbose=1
 print("Tuning Decision Tree...")
 search_dt.fit(X_train, y_train)
 best_dt = search_dt.best_estimator_
 print("Best parameters:", search_dt.best_params_)
 print(f"Best CV ROC AUC: {search_dt.best_score_:.4f}")
 # 7. Evaluate on test set
 y_pred = best_dt.predict(X_test)
 y_proba = best_dt.predict_proba(X_test)[:, 1]
 print("\n=== Test Metrics ===")
 print("Accuracy :", accuracy_score(y_test, y_pred))
print("ROC AUC :", roc_auc_score(y_test, y_proba))
 print("Classification Report:\n", classification_report(y_test, y_pred, digits=4))
 # Confusion matrix
 cm = confusion_matrix(y_test, y_pred)
 plt.figure(figsize=(4,3))
 sns.heatmap(cm, annot=True, fmt='d', cmap='Blues',
             xticklabels=['Non-Default','Default'],
             yticklabels=['Non-Default','Default'])
 plt.title("Confusion Matrix")
 plt.xlabel("Predicted")
 plt.ylabel("Actual")
 plt.show()
 # ROC curve
 fpr, tpr, _ = roc_curve(y_test, y_proba)
 plt.figure(figsize=(5,4))
 plt.plot(fpr, tpr, label=f"AUC={roc_auc_score(y_test, y_proba):.3f}")
 plt.plot([0,1],[0,1],"--", color="gray")
 plt.title("ROC Curve")
 plt.xlabel("False Positive Rate")
 plt.ylabel("True Positive Rate")
 plt.legend(loc="lower right")
 plt.show()
Tuning Decision Tree...
Fitting 5 folds for each of 12 candidates, totalling 60 fits
Best parameters: {'clf__max_depth': 5, 'clf__min_samples_leaf': 100}
Best CV ROC AUC: 0.6724
=== Test Metrics ===
Accuracy : 0.6082671396622195
ROC AUC : 0.6718470137559045
Classification Report:
                             recall f1-score
               precision
                                                 support
                  0.8856
                            0.5961
                                                 105622
           0
                                      0.7126
           1
                 0.2717
                            0.6618
                                      0.3852
                                                  24048
                                      0.6083
                                                 129670
    accuracy
   macro avg
                 0.5786
                            0.6289
                                      0.5489
                                                 129670
weighted avg
                 0.7717
                            0.6083
                                      0.6518
                                                 129670
```





```
In [ ]: # ## Visualize Top 3 Levels of the Tuned Decision Tree (No Leakage)
         import matplotlib.pyplot as plt
         from sklearn.tree import plot_tree
         # 1. Extract the trained DecisionTreeClassifier and the ColumnTransformer
         clf = best_dt.named_steps['clf']
         preproc = best_dt.named_steps['prep']
         # 2. Re-declare the feature lists (must match what you used above)
cat_cols = ['grade','term','emp_length','home_ownership','verification_status','purpose']
         num_cols = [c for c in X_train.columns if c not in cat_cols]
         # 3. Build the full list of feature names after one-hot encoding
         ohe = preproc.named_transformers_['cat'].named_steps['onehot']
         cat_feature_names = ohe.get_feature_names_out(cat_cols)
         feature_names = list(num_cols) + list(cat_feature_names)
         # 4. Plot the tree (limit to top 3 levels for readability)
         plt.figure(figsize=(20, 10))
         plot_tree(
             clf,
             max_depth=4,
             feature_names=feature_names,
             class_names=['Non-Default', 'Default'],
             filled=True,
             rounded=True,
             fontsize=10
         plt.title("Decision Tree (Top 3 Levels) - No Leakage Features")
         plt.show()
```





```
In [ ]: # ## Return-Prediction Model (LassoCV)
         from sklearn.linear_model import LassoCV
         from sklearn.pipeline import Pipeline
        from sklearn.compose import ColumnTransformer
        from sklearn.impute import SimpleImputer
         from sklearn.preprocessing import StandardScaler, OneHotEncoder
         # 1. Use the exact same X_train/X_test you already defined for the tree
              so that the splits line up.
         X_ret_train, X_ret_test = X_train.copy(), X_test.copy()
        y_ret_train = lc_data.loc[X_ret_train.index, 'ret_INTb']
y_ret_test = lc_data.loc[X_ret_test .index, 'ret_INTb']
         # 2. Build preprocessing for returns (exclude default_label/leakage)
         ret_num_cols = num_cols
         ret_cat_cols = cat_cols
        num_pipe_ret = Pipeline([
             ('imputer', SimpleImputer(strategy='mean')),
             ('scaler', StandardScaler())
         ])
         cat_pipe_ret = Pipeline([
             ('imputer', SimpleImputer(strategy='most_frequent')),
             ('onehot', OneHotEncoder(handle_unknown='ignore', sparse_output=False))
         ])
        preproc_ret = ColumnTransformer([
             ('num', num_pipe_ret, ret_num_cols),
             ('cat', cat_pipe_ret, ret_cat_cols)
         ], remainder='drop')
        # 3. LassoCV pipeline
        pipeline_ret = Pipeline([
             ('prep',
                            preproc_ret),
             ('lasso',
                            LassoCV(cv=5, random_state=42, n_jobs=-1))
         ])
         # 4. Fit
         print("Training return model (LassoCV on ret_INTb)...")
        pipeline_ret.fit(X_ret_train, y_ret_train)
print("Done. R² on hold-out:", pipeline_ret.score(X_ret_test, y_ret_test))
       Training return model (LassoCV on ret_INTb)...
```

Done. R<sup>2</sup> on hold-out: 0.015032269859592828

```
In []: # ## Compare 6 Pick-100 Strategies on the Same Test Set

# %%
import numpy as np
import pandas as pd
```

```
N_LOANS_TO_PICK = 100
        idx = X_{test.index}
        # 1) Build DataFrames of tree-predicted default probs & actual returns
        proba_df = pd.DataFrame({'pr_default': y_proba}, index=idx)
        y_test_returns = lc_data.loc[idx, 'ret_INTb']
        # 2) Predict returns on X_test
        pred_ret = pipeline_ret.predict(X_test)
        pred_ret_series = pd.Series(pred_ret, index=idx)
        # Strategy 1: Random
        rng = np.random.RandomState(42)
        pick1 = rng.choice(idx, size=min(N_LOANS_TO_PICK, len(idx)), replace=False)
        print("Strat 1 Random avg ret:", y_test_returns.loc[pick1].mean())
        # Strategy 2: Lowest Pr(Default)
        pick2 = proba_df['pr_default'].sort_values().head(N_LOANS_TO_PICK).index
        print("Strat 2 Low PrD avg ret:", y_test_returns.loc[pick2].mean())
        # Strategy 3: Highest Predicted Return
        pick3 = pred_ret_series.sort_values(ascending=False).head(N_LOANS_TO_PICK).index
        print("Strat 3 High Ret avg ret:", y_test_returns.loc[pick3].mean())
        # Strategy 4: Highest Combined ER = (1-PrD)×PredRet
        er = (1 - proba_df['pr_default']) * pred_ret_series
        pick4 = er.sort values(ascending=False).head(N LOANS TO PICK).index
        print("Strat 4 Comb ER avg ret:", y_test_returns.loc[pick4].mean())
        # Strategy 5: "Edge" Grades C & D
        mask_cd = X_test['grade'].isin(['C','D'])
        pick5 = proba_df.loc[mask_cd, 'pr_default'].sort_values().head(N_LOANS_TO_PICK).index
print("Strat 5 Edge C/D avg ret:", y_test_returns.loc[pick5].mean())
        # Strategy 6: Tiered Grade (Risky→Safe)
        risky = proba_df.loc[X_test['grade'].isin(list('CDEFG')), 'pr_default'].sort_values()
        safe = proba_df.loc[X_test['grade'].isin(['A','B']), 'pr_default'].sort_values()
        pick6 = list(risky.head(N_LOANS_TO_PICK).index)
        if len(pick6) < N_LOANS_TO_PICK:</pre>
            pick6 += list(safe.head(N_LOANS_TO_PICK - len(pick6)).index)
        print("Strat 6 Tiered avg ret:", y_test_returns.loc[pick6].mean())
       Strat 1 Random
                         avg ret: 0.04066447432582983
       Strat 2 Low PrD avg ret: 0.038871904153984616
       Strat 3 High Ret avg ret: 0.05315078740074155
       Strat 4 Comb ER avg ret: 0.05415116499445733
       Strat 5 Edge C/D avg ret: 0.047417057838564015
       Strat 6 Tiered avg ret: 0.069166532151941
In [ ]: # ## How Avg Return Changes as You Pick More Loans
        # %%
        import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        # 1) Set up
        ks = list(range(20, 501, 20))
                                         # pick sizes from 20 to 1000
        idx = X_test.index
        rng = np.random.RandomState(42)
        # pre-existing structures:
        # proba_df['pr_default'], pred_ret_series, y_test_returns
        # 2) Containers for each strategy
        results = {
             'Random'
             'Low Pr Default'
                               : [],
             'High_Pred_Return' : [],
            'Combined_ER' : [],
            'Edge_C/D'
                               : [],
            'Tiered'
                                : []
        # 3) Loop over different portfolio sizes
        for k in ks:
            # --- Strat 1: Random --
            pick = rng.choice(idx, size=min(k, len(idx)), replace=False)
            results['Random'].append(y_test_returns.loc[pick].mean())
            # --- Strat 2: Lowest Pr(Default) ---
```

```
pick = proba_df['pr_default'].nsmallest(k).index
    results['Low_Pr_Default'].append(y_test_returns.loc[pick].mean())
    # --- Strat 3: Highest Predicted Return --
    pick = pred_ret_series.nlargest(k).index
    results['High_Pred_Return'].append(y_test_returns.loc[pick].mean())
    # --- Strat 4: Combined ER = (1-PrD) × PredRet ---
    er_scores = (1 - proba_df['pr_default']) * pred_ret_series
    pick = er_scores.nlargest(k).index
    results['Combined_ER'].append(y_test_returns.loc[pick].mean())
        - Strat 5: Edge C/D only
    mask_cd = X_test['grade'].isin(['C','D'])
    pick = proba_df.loc[mask_cd, 'pr_default'].nsmallest(k).index
    results['Edge_C/D'].append(y_test_returns.loc[pick].mean())
    # --- Strat 6: Tiered (Risky→Safe) --
    risky = proba_df.loc[X_test['grade'].isin(list('CDEFG')), 'pr_default'].nsmallest(k)
    pick = list(risky.index)
    if len(pick) < k:</pre>
       needed = k - len(pick)
        safe = proba_df.loc[X_test['grade'].isin(['A','B']), 'pr_default'].nsmallest(needed)
        pick += list(safe.index)
    results['Tiered'].append(y_test_returns.loc[pick].mean())
# 4) Build DataFrame
df_ret = pd.DataFrame(results, index=ks) * 100 # convert to percentage
# 5) Plot
plt.figure(figsize=(10,6))
for strat in df_ret.columns:
    plt.plot(df_ret.index, df_ret[strat], label=strat)
plt.xlabel("Number of Loans Selected")
plt.ylabel("Average Return (%)")
plt.title("How Avg. Return Varies with Portfolio Size")
plt.legend(loc="lower right")
plt.grid(False)
plt.show()
```

