HW1*

Sciences Po - International Economics (Spring 2025)

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1 Krugman (1980) and the gravity Equation

Redding and Venables (2004) is one of the early papers showing how to obtain a structural gravity equation from the CES-monopolistic competition setup.

A. Introducing vertical differentiation: the specific quality shifter

Vertical differentiation is defined by two characteristics:

1. The quality of the produced good varies along a strictly ranked hierarchy, where all consumers agree on the ranking.
2. Preferences, or willingness to pay, for the quality of the produced good varies among consumers.

In section A., we introduce this characteristic to the theoretical framework of Redding and Venables (2004). Unless otherwise stated, all assumptions from said paper are assumed throughout.

We define q_i the specific utility quality shifter. This vector captures the differentiation of goods by their respective qualities per origin country i. Introducing the quality shifter into the utility functions yields:

$$U_j = \left[\sum_i^R \int_{n_i} q_i(z)^{\frac{\sigma-1}{\sigma}} x_{ij}(z)^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}z \right]^{\frac{\sigma}{\sigma-1}} = \left[\sum_i^R n_i (q_i x_{ij})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \tag{1}$$

The additional innate assumption being made here is that goods of the same quality produced in each country i are demanded by country j in the same quantity. This allows us to move out of the integral into a summation.

Alternatively, one could introduce the quality shifter as q_i^{θ} rather than q_i, where $\theta \in [0, 1]$ captures the extent to which importers value quality as a whole. That said, we do not believe this adds

*This document presents the answers to homework 1 of 2 in Professor Thierry Mayer's M2 PhD Track class on International Economics. The corresponding GitHub repository with the relevant code for section 2 can be found here. All errors are our own.

extra value to the analysis. By construction, θ would be the same for all importers. This parameter would be of interest if we were trying to simulate the model and see how our variables of interest change with different θ's. This is not our case. In what follows, one may interpret q_i as q_i^θ such that $\theta = 1$, where importers give full weight to the quality of goods.

Next, we derive the bilateral trade equation, equation (8) in Redding and Venables (2004), starting with the Lagrangian of the demand side planner.

$$L = \left[\sum_i^R n_i \left(q_i x_{ij}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} + \lambda\left(E_j - \sum_i n_i p_{ij} x_{ij}\right)$$

E_j represents Country j's total expenditure on manufactures. Taking the FOC with respect to x_{ij} gives:

$$\implies U_j^{1/\sigma} q_i^{\frac{\sigma-1}{\sigma}} x_{ij}^{-\frac{1}{\sigma}} = \lambda p_{ij}$$

$$\implies x_{ij} = \lambda^{-\sigma} p_{ij}^{-\sigma} q_i^{\sigma-1} U_j$$

Replacing in the budget constraint, we get:

$$E_j = \sum_i n_i p_{ij}(\lambda^{-\sigma} p_{ij}^{-\sigma} q_i^{\sigma-1} U_j)$$

$$\lambda^{-\sigma} = \frac{E_j}{U_j \sum_i n_i p_{ij}^{1-\sigma} q_i^{\sigma-1}}$$

Replacing back into the formula for x_{ij}:

$$x_{ij} = \frac{E_j}{\sum_i n_i p_{ij}^{1-\sigma} q_i^{\sigma-1}} \cdot p_{ij}^{-\sigma} q_i^{\sigma-1}$$

Let us define the price index $G_j = \left[\sum_i^R n_i \left(\frac{p_{ij}}{q_i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$. We can now rewrite as:

$$x_{ij} = E_j G_j^{\sigma-1} p_{ij}^{-\sigma} q_i^{\sigma-1} \tag{2}$$

This corresponds to equation (3) in Redding and Venables (2004) with a specific utility shifter q_i. It characterizes demand, defining the volume of sales per firm i to each location j, where $E_j G_j^{\sigma-1}$ is country j's market capacity. An interesting thing to note is that the price index now accounts for quality preferences: a higher quality is subject to a higher price!

Next, we can define the bilateral trade equation by setting an iceberg trading cost $p_{ij} = p_i T_{ij}$ and multiplying the entire equation by $p_i n_i$.

$$x_{ij} = E_j G_j^{\sigma-1} p_{ij}^{-\sigma} qi^{\sigma-1} \iff p_i n_i x_{ij} = p_i n_i E_j G_j^{\sigma-1} (p_i T_{ij})^{-\sigma} qi^{\sigma-1}$$

$$\iff p_i n_i x_{ij} = (p_i T_{ij})^{1-\sigma} n_i E_j G_j^{\sigma-1} q_i^{\sigma-1}$$

$$\iff \underbrace{p_i n_i x_{ij}}_{X_{ij}} = \left(\frac{T_{ij}}{q_i}\right)^{1-\sigma} p_i^{1-\sigma} n_i E_j G_j^{\sigma-1} \tag{3}$$

This is the bilateral trade equation with a specific quality shifter. It characterizes exports from country i to country j, denoted X_{ij}, and is structurally similar to the bilateral trade equation in Redding and Venables (2004). The right-hand side decomposes into several components: $E_j G_j^{\sigma-1}$, capturing country j's market capacity; $n_i p_i^{1-\sigma}$, reflecting country i's supply capacity; $T_{ij}^{1-\sigma}$, representing bilateral trade costs; and most importantly, $q_i^{\sigma-1}$, the quality shifter.

The quality shifter $q_i^{\sigma-1}$ plays a central role: it enhances the attractiveness of exports from country i by scaling up their perceived value. Higher q_i leads to disproportionately larger exports, and this effect is amplified by the elasticity parameter σ. Remember, $\sigma > 1$ is the price elasticity of demand. A higher σ implies that consumers are more sensitive to differences in price-adjusted quality. In particular, the exponent $\sigma - 1$ governs the strength of this response: when σ is large, small differences in quality q_i translate into large differences in trade flows. Thus, the interaction between quality and elasticity determines how much quality differences drive trade patterns across countries.

Alternatively, had one introduced the quality shifter as q_i^θ the bilateral trade equation would slightly change to:

$$X_{ij} = \left(\frac{T_{ij}}{q_i^\theta}\right)^{1-\sigma} p_i^{1-\sigma} n_i E_j G_j^{\sigma-1}$$

where θ is a parameter allowing us to experiment with different preferences over quality. In such a case, seeing as θ holds no subscript, the preferences over quality are the same for all importers.

B. Introducing vertical differentiation: bilateral preferences

In this section, we introduce bilateral preferences over quality such that quality is assessed differently in each country. In order to make the quality preferences bilateral, we add the j subscript to q_i. This implies that each importer j has their own quality assessment of each exporter i's products.

$$X_{ij} = \left(\frac{T_{ij}}{q_{ij}}\right)^{1-\sigma} \underbrace{p_i^{1-\sigma} n_i}_{s_i} \underbrace{E_j G_j^{\sigma-1}}_{m_j} \tag{4}$$

Just as in Redding and Venables (2004), $p_i^{1-\sigma} n_i$ is country i's supply capacity s_i, and $E_j G_j^{\sigma-1}$ is country j's market capacity m_j.

It is not possible to separately identify bilateral preferences from bilateral trade costs using bilateral friction variables such as, for example, distance or common language. The regression equation is as follows:

$$\ln X_{ij} = \beta\left[\ln(T_{ij}) - \ln(q_{ij})\right] + \underbrace{\ln(SC_i)}_{\nu_i} + \underbrace{\ln(MC_j)}_{\delta_j} + \varepsilon_{ij} \tag{5}$$

With the θ for importer quality preferences, this becomes:

$$\ln X_{ij} = \beta\left[\ln(T_{ij}) - \theta\ln(q_{ij})\right] + \ln(SC_i) + \ln(MC_j) + \varepsilon_{ij} \tag{6}$$

In these regressions, ν_i and δ_j are exporter and importer fixed effects respectively. The β coefficient only identifies the net effect of trade costs and bilateral preferences. Furthermore, intuitively speaking, the two may be correlated. For example, one may prefer buying products from a country similar to theirs, which is also more likely to be closer.

C. Does the gravity equation hold when introducing additive trade costs?

The previously modeled iceberg trade costs are multiplicative. Introducing additive trade costs gives $p_{ij} = p_i + T_{ij}$. Using this definition of p_{ij} when deriving the bilateral trade equation yields:

$$X_{ij} = (p_i + T_{ij}) n_i E_j G_j^{\sigma-1} (p_i + T_{ij})^{-\sigma} q_{ij}^{\sigma-1)}$$

$$\implies X_{ij} = (p_i + T_{ij})^{1-\sigma} n_i E_j G_j^{\sigma-1} q_{ij}^{(\sigma-1)}$$

$$\implies X_{ij} = \left(\frac{p_i + T_{ij}}{q_{ij}}\right)^{1-\sigma} n_i E_j G_j^{\sigma-1} \tag{7}$$

Applying the same steps as to derive the regression equation (5), we find:

$$X_{ij} = \beta\left[\ln(p_i + T_{ij}) - \ln(q_{ij})\right] + \ln(n_i) - \underbrace{\ln(MC_j)}_{\delta_j} + \varepsilon_{ij} \tag{8}$$

In this new model, we lose the importer fixed effects, ν_i, and therefore cannot control for “supply capacity”. With the exporter fixed effects, our gravity equation does not hold. Our β can still not identify the effects separately. A key takeaway is that multiplicative trade costs are an essential requirement when looking to derive and estimate a structural gravity model.

D. Does the gravity equation hold when firms are oligopolists?

Under monopolistic competition, firms are price-takers. We explore two kinds of oligopoly market structures: Bertrand and Cournot competitions, and ultimately come to the same conclusion for both: the gravity equation becomes biased due to endogeneity issues.

Bertrand Competition

In Bertrand competition, firms choose quantities based on prevailing prices. For simplicity, we express $x_{ij}(p)$ as the demand function, depending on both own price p_{ij} and others p_{-ij}. Our new profit function for representative country i:

$$\pi_i = \sum_j^R \frac{p_{ij}x_{ij}(p)}{T_{ij}} - G_i^\alpha w_i^\beta v_i^\gamma c_i[F + \sum_j^R x_{ij}(p)].$$

Taking the FOC with respect to p_{ij} gives:

$$\left(\frac{p_{ij}}{T_{ij}} - G_i^\alpha w_i^\beta v_i^\gamma c_i\right)\frac{\partial x_{ij}(p)}{\partial p_{ij}} + \frac{x_{ij}(p)}{T_{ij}} = 0$$

Let $\varepsilon_{ij} = \frac{\partial x_{ij}(p)}{dp_{ij}} \times \frac{p_{ij}}{x_{ij}(p)}$ be the price elasticity of demand. Replacing, we get:

$$p_{ij} = \frac{\varepsilon_{ij}}{1+\varepsilon_{ij}} T_{ij} G_i^\alpha w_i^\beta v_i^\gamma c_i$$

Earlier, $\frac{\sigma}{\sigma-1}$ was constant. Now, price depends on quantities produced, as well as how all prices evolve along with those quantities. We could make go further and calculate market shares based on these elasticities, but that is unnecessary to make our point - the gravity equation now suffers from endogeneity.

Cournot competition

In Cournot competition, firms choose prices based on quantities being produced. We can express $p_{ij}(x)$ as the pricing function, depending on both own quantity produced x_{ij} and others x_{-ij}. Our new profit function for representative country i:

$$\pi_i = \sum_j^R \frac{p_{ij}(x)\cdot x_{ij}}{T_{ij}} - G_i^\alpha w_i^\beta v_i^\gamma c_i[F + \sum_j^R x_{ij}].$$

Taking FOC with respect to x_{ij}, we ultimately get:

$$p_{ij}(x) + x_{ij}\frac{\partial p_{ij}(x)}{\partial x_{ij}} = T_{ij} G_i^\alpha w_i^\beta v_i^\gamma c_i$$

The markup is going to depend on both own quantity produced, and how others' quantities evolve alongside each other.

E. Krugman (1980) is a homogeneous firms/representative consumer model. Keeping with CES demand and monopolistic competition, what is the main additional condition needed or gravity to hold if firms' productivity is heterogeneous? What is a model of heterogeneous consumers that can be used while keeping a gravity prediction? In both cases, be precise about needed functional forms.

Keeping with CES demand and monopolistic competition, but adding heterogeneity in firm productivities needs an assumption on the distribution of said productivities. The most widely used in the literature is the Pareto distribution. Following Melitz (2003) and Chaney (2008), this is due to its analytical tractability and the observed distribution of firm size in the United States (which closely follows a Pareto distribution. The specific functional form needed is:

$$P(\tilde{\varphi}_h < \varphi) = G_h(\varphi) = 1 - \varphi^{-\gamma_h}$$

Where $G_h(\varphi)$ is the CDF, and φ is unit labor productivity.

As for heterogeneous consumers, we could use the adaptation of Head and Mayer (2011) we derived above. Bilateral quality preferences imply heterogeneity in demand per importer. To complement this, we may also need to make an assumption of homotheticity of preferences (how preferences change as a consumer grows richer or poorer).

2 Empirical exercise : market potential and development

In this section, we look to replicate regressions and graphs from Redding and Venables (2004) for a more recent set of years. For this, we use an updated version of the data from Head and Mayer (2011) with international trade only, such that Foreign Market Potential (FMP henceforth), may be calculated.

A. Calculate the FMP for each country in 2016 and regress on GDP per capita as a log-log regression.

For calculating FMP, we follow the methodology presented in Head and Mayer (2011). The log-linearized gravity equation is written as:

$$\ln X_{ij} = \ln A_i + \ln \phi_{ij} + \ln\left(\frac{X_j}{\Phi_j}\right)$$

With X_{ij} representing bilateral trade flows, A_i being exporter i's capabilities, and the ratio of X_j and Φ_j reflecting market potential in country j. The ϕ_{ij} are estimated by specifying a vector of observed trade costs (distance, contiguity, common currency and language in our case), and the $\ln\left(\frac{X_j}{\Phi_j}\right)$ are estimated as fixed effects for each of the importing countries, denoted FE_j. Given these estimates, we can calculate FMP as:

$$\text{FMP}_i \equiv \sum_{h \neq i} \widehat{\phi}_{ih} \times \exp\left(\widehat{\text{FE}}_j\right)$$

The first-stage regression equation is as follows:

$$\ln(\text{Flow}_{ij}) = \beta \ln(\text{Distance}_{ij}) + \gamma Z_{ij} + \gamma_i + \delta_j + \epsilon_{ij}$$

However, following Silva and Tenreyro (2006), an OLS estimation is likely to be inaccurate given the large number of zero values as well as heteroskedasticity of trade data. We add a Poisson specification in addition to see how the estimation differs once these issues are accounted for. Next, we estimate $\widehat{\phi}_{ih}$ like Redding and Venables (2004):

$$\widehat{\phi}_{ih} \equiv \sum_{j \neq i} \text{Distance}_{ij}^{\beta} * Z^{\gamma}$$

Table 1 shows the results of the first-stage gravity regression. The OLS and Poisson specifications yield significantly different results. The variation in the OLS estimate from Redding and Venables (2004) is caused by the addition of the dummies listed in the table, having chosen a more recent time and also having almost 3 times the observations.

Table 1: Gravity Model Estimation Results

	OLS	**Poisson**
log(Distance)	−1.689***	−0.749***
	(0.055)	(0.048)
Contiguity	0.841***	0.419***
	(0.155)	(0.069)
Common language	0.922***	0.053
	(0.073)	(0.086)
Common currency	0.300	0.011
	(0.199)	(0.126)
FTA Agreement	0.443***	0.409***
	(0.075)	(0.060)
Num. Observations	28 563	31 150
R^2	0.747	0.944
Adjusted R^2	0.743	0.944
FE: Exporter (iso_o)	X	X
FE: Importer (iso_d)	X	X

Notes: Standard errors in parentheses. *** p< 0.01. Standard errors clustered at exporter level.

Table 2 presents the results of regressing GDP per capita on FMP. The regression equation is:

$$\ln(\text{GDPpc}_i) = \beta_0 + \beta_1 \ln(\widehat{\text{FMP}_i}) + \delta_j + \epsilon_i$$

Under OLS, A 1% increase in FMP is associated with approximately a 0.732% increase in GDP per capita. This decreases to 0.725% when including country fixed effects (for importers). However, a 1% increase in FMP is associated with a 0.241% increase in GDP per capita in the Poisson specification. This conservative estimate comes from the handling of zero values and greater weight to smaller trade flows compared to OLS.

Table 2: Second-Stage Regression: GDP per Capita and Fundamental Market Potential (2016)

	OLS	**OLS**	**Poisson**	**Poisson**
ln FMP	0.732***	0.725***	0.241***	0.239***
	(0.071)	(0.071)	(0.027)	(0.027)
Num. Observations	28 230	28 230	28 230	28 230
R^2	0.334	0.339	0.017	0.017
Adjusted R^2	0.334	0.335	0.017	0.014
FE: Importer (iso_d)		X		X

Notes: Standard errors in parentheses. *** p< 0.01. Standard errors are clustered at the exporter level (iso_o).

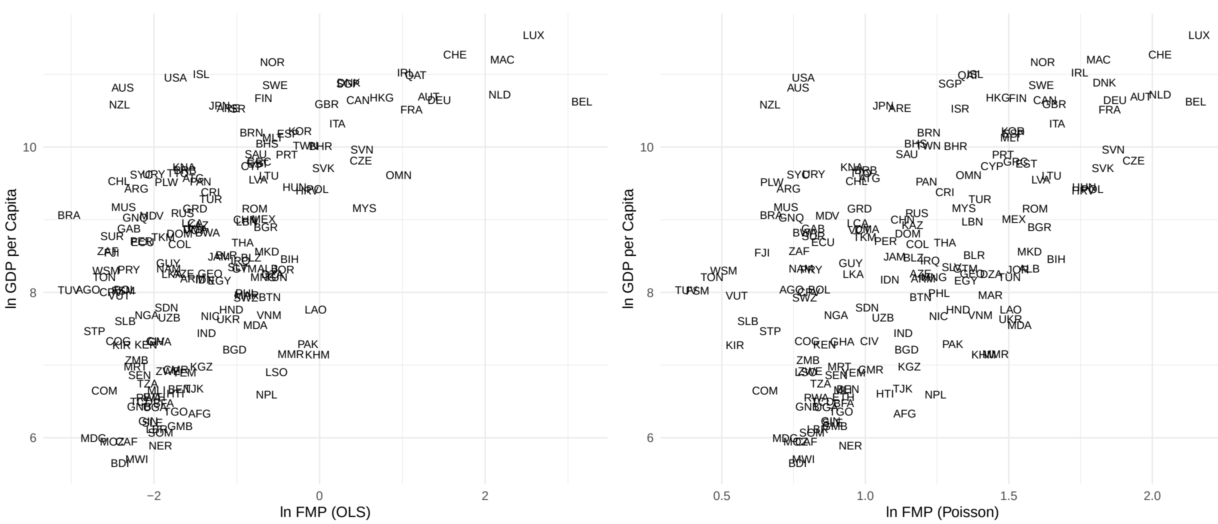

Figure 1: FMP and GDP per Capita

Figure 1 plots FMP with GDP per capita. The distribution changes across estimation strategies. The panel looks similar to the original graphs produced by Redding and Venables (2004). It is worth noting that there are negative values for the log of FMP - suggesting some very small values. This is a discrepancy from the original.

B. Replicate the exercise for all years since 2004. Which fixed-effects can be introduced in the second step? does it change results?

Now, we move to a panel dataset for the years 2004-2016. We can now introduce time fixed effects into the regression.

Table 3: Gravity Model Estimation Results

	OLS	**Poisson**
log(Distance)	-1.670^{***}	-0.765^{***}
	(0.050)	(0.051)
Contiguity	0.880^{***}	0.435^{***}
	(0.142)	(0.068)
Common Language	0.930^{***}	0.070
	(0.066)	(0.083)
Common Currency	0.313	-0.033
	(0.202)	(0.113)
FTAs	0.470^{***}	0.459^{***}
	(0.064)	(0.051)
Num. Observations	361 496	393 234
R^2	0.728	0.924
Adjusted R^2	0.728	0.924
FE: Exporter (iso_o)	X	X
FE: Importer (iso_d)	X	X

Notes: Standard errors in parentheses. *** p< 0.01. Fixed effects included for exporters (iso_o) and importers (iso_d).

The first stage results remain very similar to before. The new second stage regression equation is (with time fixed effects):

$$\ln(\text{GDPpc}_i) = \beta_0 + \beta_1 \ln(\widehat{\text{FMP}_i}) + \delta_j + \theta_t + \epsilon_i$$

Table 4: Second-Stage Regression Results: GDP per Capita on Market Potential (2004-2016)

	OLS	**OLS**	**Poisson**	**Poisson**
ln FMP	0.768***	0.760***	0.247***	0.244***
	(0.072)	(0.072)	(0.026)	(0.026)
Num. Observations	374 273	374 273	374 273	374 273
R^2	0.328	0.333	0.020	0.020
Adjusted R^2	0.328	0.333	0.020	0.020
FE: Year	X	X	X	X
FE: Importer (iso_d)		X		X

Notes: Standard errors in parentheses. *** $p< 0.01$. All models include year fixed effects. Standard errors clustered at the exporter level (iso_o).

Apart from a slight increase across the columns, adding year fixed effects does not change much from the initial estimation. This suggests that this relationship is robust across time. The bottom line is that countries with higher foreign market potential tend to have higher GDP per capita.

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