

What Exponent – Graydon Strachan

Determine the value of x , a two-digit number such that the expression $10^x - x$ when evaluated gives a sum whose digits can be added to yield 300.

Determining the sum of all digits in a decimal number can be done using the modulo operator '%' and the division operator '/' successively. Because we assume the given number x is a signed integer, division will always truncate any floating points. For example, $11/10$ will yield 1, assuming 11 and 10 are integers. This algorithm completes in linear time and has an $O(n)$ time complexity, where n is the number of digits contained in any number x . A simple implementation is shown below.

```
x = 245
sum = 0
while x / 10 ≥ 1
    sum = sum + x % 10
    x = x / 10
```

This problem can be efficiently solved through the use of a binary search, a method of determining a desired output by strategically eliminating half of all values in each search iteration. This algorithm has $O(\log n)$, time complexity. An algorithm implementing both of these sub algorithms completes in logarithmic time. We can assume that x can be given by $\{x \mid 0 \leq x \leq 99\}$, as x has two digits. A completed algorithm is shown below.

```
x = 50
found = false;

while found = false
    y = 10x - x

    sum = 0;
    while y / 10 >= 1
        sum = sum + x % 10
        y = y / 10

    if sum = 300
        found = true
    else if sum > 300
        x = x + x / 2
    else
        x = x - x / 2
```

The output of this algorithm yields 34.