Terry’s Triangle Solution – Graydon Strachan

Determine all isosceles triangles with side lengths *a*, *b*, *c* such that

$$a = y - x$$

$$b = x + z$$

$$c = y - z$$

$$x + y + z < 10$$

$$x, y, z \subset \mathbb{Z}^{+}$$

Side lengths *a*, *b* and *c* must all be less than 8 and greater than 0. For any given side length *x*, where $x \subset Z^{+}$, $x \geq 1$ because $x \neq 0$. Assuming side lengths *a*, *b* = 1, then $c_{max} = 7$, as 1 + 1 + 7 = 9 < 10.

Integers *x*, *y* and *z* can be given similarly, with values less than 9. Side length *b* > *a*, *c*, as *b* is given by *x* + *z* as opposed to conjugate form *x* – *z*, similar to *a* = *y* – *x* or *c* = *y* - *z*. Assuming edge case *x* = 7, *z* = 1, b = 8.

The values of x, y and z can be computed such that *x*, *y* and *z* satisfy their given bounds. Given *a*, *b* < *c*, *a* + *b* > *c* in any proper triangle. Not all values a, b, c that satisfy these bounds are viable (fig. 1.1 – 1.3).

a + *b* > *c*

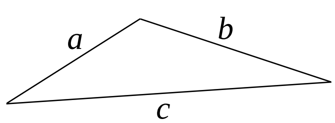

figure 1.1

a + *b* > *c*

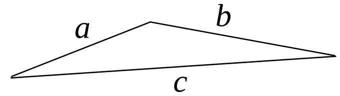

figure 1.2

a + *b* < *c*

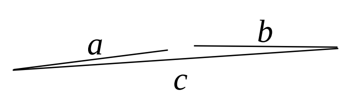

figure 1.3

A pruned exhaustive search approach can be taken when solving for viable *a*, *b*, *c* values. A general structure of this is given. The algorithm described below is of $O(n^3)$ time complexity.

```
triangles = 0
remain = 9 (Remain is the total sum x + y + z can equal)

for i in remain (Iterates for every possible value 0 <= i < 9)
   y = i + 1
   remain – i
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```
for j in remain (Iterates for the remaining values where x + y + z < 9)
    x = j + 1
    remain – j

    for k in remain (Iterates for the remaining values where x + y + z < 9)
        z = k + 1

        a = y – x (Calculate a, b, c values)
        b = x + z
        c = y – z

        if (a > 0) and (c > 0) (Checks if the side lengths are greater than 0)
            sides = [a, b, c]
            sort(sides)


            if sides1 + sides2 > sides3 (Checks if a triangle exists)
                    if sides1 = sides2 or sides2 = sides3 (Checks if the triangle is isosceles)
                        triangles + 1 (Adds to the number of possible triangles)
```

There are 12 possible triangles that satisfy the given specifications

The viable triangles are the T, where every column indicates a sequence of possible a, b, c values.

$$\boldsymbol{T} = \begin{pmatrix} \mathbf{2} & \mathbf{3} & \mathbf{3} & \mathbf{2} & \mathbf{4} & \mathbf{4} & \mathbf{4} & \mathbf{3} & \mathbf{3} & \mathbf{2} & \mathbf{5} & \mathbf{6} \\ \mathbf{2} & \mathbf{2} & \mathbf{3} & \mathbf{3} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{3} & \mathbf{4} & \mathbf{4} & \mathbf{2} & \mathbf{2} \\ \mathbf{2} & \mathbf{3} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{3} & \mathbf{2} & \mathbf{4} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} \end{pmatrix}$$

The corresponding x, y, z values are given by V, where every column indicates a possible sequence of x, y, z values.

$$\boldsymbol{V} = \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{2} & \mathbf{2} & \mathbf{3} & \mathbf{1} & \mathbf{1} \\ \mathbf{3} & \mathbf{4} & \mathbf{4} & \mathbf{4} & \mathbf{5} & \mathbf{5} & \mathbf{5} & \mathbf{5} & \mathbf{5} & \mathbf{5} & \mathbf{6} & \mathbf{7} \\ \mathbf{1} & \mathbf{1} & \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{1} & \mathbf{2} & \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$