CS1555 Recitation 11 Solution

Objective: to practice normalization, finding canonical forms, checking for lossless decompositions, and decomposing relations into BCNF.

Part 1: For each of the following relations R and sets of functional dependencies F, do the following:
1) Find the canonical cover (minimal cover) of F.
2) Using the canonical cover, find the keys of the R.

1. Consider the following set of functional dependencies F on a relation R (A, B, C, D, E):

$A \rightarrow BC$
$A \rightarrow D$
$B \rightarrow C$
$C \rightarrow D$
$DE \rightarrow C$
$BC \rightarrow D$

<u>Finding the canonical form:</u>

- Transform all FDs to canonical form (i.e., one attributes on the right):
  $A \rightarrow B$
  $A \rightarrow C$
  $A \rightarrow D$
  $B \rightarrow C$
  $C \rightarrow D$
  $DE \rightarrow C$
  $BC \rightarrow D$
- Drop extraneous attributes:
  B in $BC \rightarrow D$ is extraneous, since we already have $C \rightarrow D$. The set of FDs becomes:
  $A \rightarrow B$
  $A \rightarrow C$
  $A \rightarrow D$
  $B \rightarrow C$
  $C \rightarrow D$
  $DE \rightarrow C$
- Drop redundant FDs:
  $A \rightarrow B$ and $B \rightarrow C$ implies $A \rightarrow C$, so we drop $A \rightarrow C$.

$A \rightarrow B$, $B \rightarrow C$ and $C \rightarrow D$ implies $A \rightarrow D$, so we drop $A \rightarrow D$.

The set of FDs becomes:

$A \rightarrow B$
$B \rightarrow C$
$C \rightarrow D$
$DE \rightarrow C$

which is the canonical cover of F.

Finding the keys of R:

Observations:

- A and E do not appear in the right hand side of any FDs, so they have to appear in all keys of R.
- AE^+ : $AE \rightarrow AEB$ (since $A \rightarrow B$) $\rightarrow AEBC$ (since $B \rightarrow C$) $\rightarrow AEBCD$ (since $C \rightarrow D$). So AE is a key of R.
  In this case, we do not need to consider any other combination, because any other combination containing AE (e.g., AEB) is a super key and not minimal.

2. Consider the following set of functional dependencies F on relation R (A, B, C, D, E, H):

$A \rightarrow C$
$AC \rightarrow D$
$E \rightarrow AD$
$E \rightarrow H$
$A \rightarrow CD$
$E \rightarrow AH$

<u>Finding the canonical form:</u>

- Transform all FDs to canonical form (i.e., one attribute on the right):
  - $A \rightarrow C$
  - $AC \rightarrow D$
  - $E \rightarrow AD$ becomes $E \rightarrow A$ and $E \rightarrow D$
  - $E \rightarrow H$
  - $A \rightarrow CD$ becomes $A \rightarrow C$ and $A \rightarrow D$
  - $E \rightarrow AH$ becomes $E \rightarrow A$ and $E \rightarrow H$
- Remove redundant dependencies:
  - $A \rightarrow C$
  - $AC \rightarrow D$
  - $E \rightarrow A$
  - $E \rightarrow D$
  - $E \rightarrow H$
  - $A \rightarrow D$
- Drop extraneous attributes:
  $AC \rightarrow D$ can be removed because we have $A \rightarrow D$ so C is redundant:
  - $A \rightarrow C$
  - $E \rightarrow A$
  - $E \rightarrow D$
  - $E \rightarrow H$
  - $A \rightarrow D$
- Drop redundant FDs: Try removing some dependencies in F and still have a set of dependencies equivalent to F.
  $E \rightarrow D$ can be deduced from $E \rightarrow A$ and $A \rightarrow D$ so we can remove $E \rightarrow D$.
  The set of FDs becomes:
  - $A \rightarrow C$
  - $E \rightarrow A$
  - $E \rightarrow H$
  - $A \rightarrow D$

  which is the canonical cover of F.

Finding the keys of R:

Observations:

- B and E do not appear in the right hand side of any FDs, so they have to appear in all keys of R.
- BE+: BE → AEB (because E → A) → AEBC (because A → C) → AEBCD (because A → D) → AEBCDH (because E → H). So BE is a key of R.
  In this case, we do not need to consider any other combination, because any other combination containing BE (e.g., AEB) is a super key and not minimal.

Part 2:

1. Consider the following set of functional dependencies F on relation R (A, B, C, D, E, H):
   A → C
   AC → D
   E → AD
   E → H
   A → CD
   E → AH

The key for R is *EB* and the following set of functional dependencies constitutes the canonical cover:

A → C, E → A, E → H, A → D

1) Using Synthesis Method, construct a set of 3NF relations.

We are starting from a canonical cover. → We can skip steps 1, 2 and 3.

Step 4: Group FDs with same determinant:

$A \rightarrow CD$
$E \rightarrow AH$

Step 5: Construct a relation for each group:

R1($\underline{A}$,C,D)
R2($\underline{E}$,A,H)

Step 6: If none of the relations contain the key for the original relation, add a relation with the key.

R3($\underline{E,B}$)

R1, R2 and R3 are in 3NF and in BCNF.

2) Using Universal Method, decompose R into a set of BCNF relations.

Using A → C to decompose R, we get:

R1(A,$\underline{B}$,D,$\underline{E}$,H) in 1NF

R2($\underline{A}$,C) is already in BCNF form

Using A → D to decompose R1, we get:

R11(A,$\underline{B}$,$\underline{E}$,H) in 1NF

R12($\underline{A}$,D) is already in BCNF form

Using E → A to decompose R11, we get:

R111($\underline{B,E}$,H) in 1NF

R112($\underline{E}$,A) is already in BCNF form

Using E → H to decompose R111, we get:

R1111($\underline{B,E}$) in BCNF form

R1112($\underline{E}$,H) is already in BCNF form

Group the relations with the same key:

R1($\underline{A}$,C,D)

R2($\underline{E}$,A,H)

R3($\underline{E,B}$)

R1, R2 and R3 are in BCNF form.

2. Consider the following set of functional dependencies F on relation R (A, B, C, D, E):

   $A \rightarrow BC$
   $A \rightarrow D$
   $B \rightarrow C$
   $C \rightarrow D$
   $DE \rightarrow C$
   $BC \rightarrow D$

The key for R is *AE* and the following set of functional dependencies constitutes the canonical cover:

$A \rightarrow B$, $B \rightarrow C$, $C \rightarrow D$, $DE \rightarrow C$

a) Using Synthesis Method, construct a set of 3NF relations.

We are starting from a canonical cover. → We can skip steps 1, 2 and 3.

We can skip Step 4: no FDs with same determinant

Step 5: Construct a relation for each group:

R1(<u>A</u>,B)
R2(<u>B</u>,C)
R3(<u>C</u>,D)
R4(<u>D, E</u>,C)

Step 6: If none of the relations contain the key for the original relation, add a relation with the key.

R5(<u>A,E</u>)

R1, R2, R3, R4, and R5 are in 3NF and in BCNF.

b) Using Universal Method, decompose R into a set of BCNF relations.

Using B → C to decompose R, we get:

R1($\underline{A}$,B,C,D,$\underline{E}$) in 1NF
R2($\underline{B}$,C) is already in BCNF form

Using C → D to decompose R1, we get:

R11($\underline{A}$,B,C,D,$\underline{E}$) in 1NF
R12($\underline{C}$,D) is already in BCNF form

Using A → B to decompose R11, we get:

R111($\underline{A}$,C,D,$\underline{E}$) in 1NF
R112($\underline{A}$,B) is already in BCNF form

Using DE → C to decompose R111, we get:

R1111($\underline{A,E}$) in BCNF form
R1112($\underline{D,E}$,C) is already in BCNF form

Group the relations with the same key:

R1($\underline{B}$,C)
R2($\underline{C}$,D)
R3($\underline{A}$,B)
R4($\underline{D,E}$,C)
R5($\underline{A,E}$)

R1, R2, R3, R4, and R5 are in 3NF and in BCNF.

Part 3: Assume that R is decomposed into:

R1 (A, B), F1 = {A → B}
R2 (B, C), F2 = {B → C}
R3 (C, D, E), F3 = {C → D, DE → C)
Is this decomposition a lossless-join decomposition? Use the table method.

Checking for lossless-join:

Initially the Table looks like this:

	A	B	C	D	E
R1(A,B)	K	K	U	U	U
R2(B,C)	U	K	K	U	U
R3(C,D,E)	U	U	K	K	K

Using B → C: we can replace U13 by K

	A	B	C	D	E
R1(A,B)	K	K	**K**	U	U
R2(B,C)	U	K	K	U	U
R3(C,D,E)	U	U	K	K	K

Using C → D: we can replace U14 and U24 by K

	A	B	C	D	E
R1(A,B)	K	K	**K**	**K**	U
R2(B,C)	U	K	K	**K**	U
R3(C,D,E)	U	U	K	K	K

We cannot proceed and there is no row of all known values → the decomposition is lossy.