CS1555 Recitation 11 - Solution

Objective: to practice normalization, canonical forms and finding keys, decomposing relations into BCNF.

Part 1: For each of the following relations R and sets of functional dependencies F, do the following:

1) Find the canonical cover (minimal cover) of F.
2) Using the canonical cover, find the keys of the R.

1. Consider the following set of functional dependencies F on a relation R (A, B, C, D, E):

$A \rightarrow BC$
$A \rightarrow D$
$B \rightarrow C$
$C \rightarrow D$
$DE \rightarrow C$
$BC \rightarrow D$

Finding the canonical form:

- Transform all FDs to canonical form (i.e., one attributes on the right):

  $A \rightarrow B$
  $A \rightarrow C$
  $A \rightarrow D$
  $B \rightarrow C$
  $C \rightarrow D$
  $DE \rightarrow C$
  $BC \rightarrow D$

- Drop extraneous attributes:
  B in $BC \rightarrow D$ is extraneous, since we already have $C \rightarrow D$. The set of FDs becomes:

  $A \rightarrow B$
  $A \rightarrow C$
  $A \rightarrow D$
  $B \rightarrow C$
  $C \rightarrow D$
  $DE \rightarrow C$

- Drop redundant FDs:
  $A \rightarrow B$ and $B \rightarrow C$ implies $A \rightarrow C$, so we drop $A \rightarrow C$.
  $A \rightarrow B$, $B \rightarrow C$ and $C \rightarrow D$ implies $A \rightarrow D$, so we drop $A \rightarrow D$.
  The set of FDs becomes:

$A \rightarrow B$
$B \rightarrow C$
$C \rightarrow D$
$DE \rightarrow C$

which is the canonical cover of F.

Finding the keys of R:

Observations:

- A and E do not appear in the right hand side of any FDs, so they have to appear in all keys of R.
- AE+ : $AE \rightarrow AEB$ (since $A \rightarrow B$) $\rightarrow AEBC$ (since $B \rightarrow C$) $\rightarrow AEBCD$ (since $C \rightarrow D$). So AE is a key of R.
  In this case, we do not need to consider any other combination, because any other combination containing AE (e.g., AEB) is a super key and not minimal.

2. Consider the following set of functional dependencies F on relation R (A, B, C, D, E, H):

$A \rightarrow C$
$AC \rightarrow D$
$E \rightarrow AD$
$E \rightarrow H$
$A \rightarrow CD$
$E \rightarrow AH$

Finding the canonical form:

- Transform all FDs to canonical form (i.e., one attribute on the right):
  $A \rightarrow C$
  $AC \rightarrow D$
  $E \rightarrow AD$ becomes $E \rightarrow A$ and $E \rightarrow D$
  $E \rightarrow H$
  $A \rightarrow CD$ becomes $A \rightarrow C$ and $A \rightarrow D$
  $E \rightarrow AH$ becomes $E \rightarrow A$ and $E \rightarrow H$

- Remove redundant dependencies:

  $A \rightarrow C$
  $AC \rightarrow D$
  $E \rightarrow A$
  $E \rightarrow D$
  $E \rightarrow H$
  $A \rightarrow D$

- Drop extraneous attributes:
  $AC \rightarrow D$ can be removed because we have $A \rightarrow D$ so C is redundant:

  $A \rightarrow C$
  $E \rightarrow A$
  $E \rightarrow D$
  $E \rightarrow H$
  $A \rightarrow D$

- Drop redundant FDs: Try removing some dependencies in F and still have a set of dependencies equivalent to F.
  $E \rightarrow D$ can be deduced from $E \rightarrow A$ and $A \rightarrow D$ so we can remove $E \rightarrow D$.
  The set of FDs becomes:

  $A \rightarrow C$
  $E \rightarrow A$
  $E \rightarrow H$
  $A \rightarrow D$

  which is the canonical cover of F.

<u>Finding the keys of R:</u>

Observations:

- B and E do not appear in the right hand side of any FDs, so they have to appear in all keys of R.
- BE+: BE $\rightarrow$ AEB (because $E \rightarrow A$) $\rightarrow$ AEBC (because $A \rightarrow C$) $\rightarrow$ AEBCD (because $A \rightarrow D$) $\rightarrow$ AEBCDH (because $E \rightarrow H$). So BE is a key of R.
  In this case, we do not need to consider any other combination, because any other combination containing BE (e.g., AEB) is a super key and not minimal.

Part 2:

1. Consider the following set of functional dependencies F on relation R (A, B, C, D, E, H):
   $A \rightarrow C$
   $AC \rightarrow D$
   $E \rightarrow AD$
   $E \rightarrow H$
   $A \rightarrow CD$
   $E \rightarrow AH$

The key for R is *EB* and the following set of functional dependencies constitutes the canonical cover:

$A \rightarrow C$, $E \rightarrow A$, $E \rightarrow H$, $A \rightarrow D$

a) Using Synthesis Method, construct a set of 3NF relations.

We are starting from a canonical cover. → We can skip steps 1, 2 and 3.

Step 4: Group FDs with same determinant:

$A \rightarrow CD$

$E \rightarrow AH$

Step 5: Construct a relation for each group:

R1(<u>A</u>,C,D)

R2(<u>E</u>,A,H)

Step 6: If none of the relations contain the key for the original relation, add a relation with the key.

R3(<u>E,B</u>)

R1, R2 and R3 are in 3NF and in BCNF.

b) Using Universal Method, decompose R into a set of BCNF relations.

Using A $\rightarrow$ C to decompose R, we get:
    R1(A,B,D,E,H)   in 1NF
    R2(A,C)   is already in BCNF form
Using A $\rightarrow$ D to decompose R1, we get:
    R11(A,B,E,H)   in 1NF
    R12(A,D)   is already in BCNF form
Using E $\rightarrow$ A to decompose R11, we get:
    R111(B,E,H)   in 1NF
    R112(E,A)   is already in BCNF form
Using E $\rightarrow$ H to decompose R111, we get:
    R1111(B,E)   in BCNF form
    R1112(E,H)   is already in BCNF form
Group the relations with the same key:
    R1(A,C,D)
    R2(E,A,H)
    R3(E,B)
R1, R2 and R3 are in BCNF form.

2. Consider the following set of functional dependencies F on relation R (A, B, C, D, E):
    A $\rightarrow$ BC
    A $\rightarrow$ D
    B $\rightarrow$ C
    C $\rightarrow$ D
    DE $\rightarrow$ C
    BC $\rightarrow$ D

The key for R is *AE* and the following set of functional dependencies constitutes the canonical cover:

A $\rightarrow$ B, B $\rightarrow$ C, C $\rightarrow$ D, DE $\rightarrow$ C

a) Using Synthesis Method, construct a set of 3NF relations.

We are starting from a canonical cover. $\rightarrow$ We can skip steps 1, 2 and 3.
We can skip Step 4: no FDs with same determinant

Step 5: Construct a relation for each group:

R1($\underline{A}$,B)
R2($\underline{B}$,C)
R3($\underline{C}$,D)
R4($\underline{D, E}$,C)

Step 6: If none of the relations contain the key for the original relation, add a relation with the key.

R5($\underline{A,E}$)

R1, R2, R3, R4, and R5 are in 3NF and in BCNF.

b) Using Universal Method, decompose R into a set of BCNF relations.

Using B → C to decompose R, we get:

R1($\underline{A}$,B,C,D,$\underline{E}$) in 1NF
R2($\underline{B}$,C) is already in BCNF form

Using C → D to decompose R1, we get:

R11($\underline{A}$,B,C,D,$\underline{E}$) in 1NF
R12($\underline{C}$,D) is already in BCNF form

Using A → B to decompose R11, we get:

R111($\underline{A}$,C,D,$\underline{E}$) in 1NF
R112($\underline{A}$,B) is already in BCNF form

Using DE → C to decompose R111, we get:

R1111($\underline{A,E}$) in BCNF form
R1112($\underline{D,E}$,C) is already in BCNF form

Group the relations with the same key:

R1($\underline{B}$,C)
R2($\underline{C}$,D)
R3($\underline{A}$,B)
R4($\underline{D,E}$,C)
R5($\underline{A,E}$)

R1, R2, R3, R4, and R5 are in 3NF and in BCNF.