STAT 1331 - Assignment 1

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Problem 1:

Consider an investor who bought an asset at time t for \$100. The investor hold the asset for two periods. The price of the asset was \$90 and \$105 at time t + 1 and t + 2, respectively.

Given:

$$P_t = 100$$
$$k = 2$$
$$P_{t+1} = 90$$
$$P_{t+2} = 105$$

1a) Compute the multiperiod simple return.

Solution:

$$\begin{aligned}
1 + R_t(2) &= \prod_{j=0}^{2-1}(1 + R_{t-j}) = \prod_{j=0}^{1}(1 + R_{t-j}) \\
&= (1 + R_{t+2})(1 + R_{t+1}) \\
&= (1 + \frac{P_{t+2} - P_{t+1}}{P_{t+1}})(1 + \frac{P_{t+1} - P_t}{P_t}) \\
&= (1 + \frac{105 - 90}{90})(1 + \frac{90 - 100}{100}) \\
&= (\frac{7}{6} + \frac{9}{10}) = \frac{31}{15} \approx 2.07
\end{aligned}$$

Thus the multiperiod simple return is approximately 2.07%.

1b) Compute the one period simple return at (t + 1) and (t + 2)

Solution:

The one-period simple return at time (t + 1) is:

$$\begin{aligned}
R_{t+1} &= (1 + \frac{P_{t+1} - P_t}{P_t}) \\
&= (1 + \frac{90 - 100}{100})
\end{aligned}$$

$$= \frac{9}{10} = 0.90$$

The one-period simple return at time (t + 2) is:

$$R_{t+2} = (1 + \frac{P_{t+2} - P_{t+1}}{P_{t+1}})$$

$$= (1 + \frac{105 - 90}{90})$$

$$= \frac{7}{6} \approx 1.67$$

Thus the one-period simple return at time (t + 1) is approximately 0.90%, and at time (t + 2) is approximately 1.67%.

1c) What is the relationship between the multiperiod simple return and the one period simple returns?

Solution:

The multiperiod simple return is the product of k one-period simple returns. The one-period simple return is a partial produce of the multiperiod simple return.

1d) Compute the multiperiod continuously compounded return.

Solution:

$$r_t(k) = \ln(1 + R_t(k)) = \sum_{i=0}^{k-1} r_{t-i}$$
$$r_t(2) = \ln(1 + R_t(2))$$
$$= \ln((1 + R_{t+2})(1 + R_{t+1}))$$
$$= \ln(1 + R_{t+2}) + \ln(1 + R_{t+1})$$
$$= \ln(1 + \frac{P_{t+2} - P_{t+1}}{P_{t+1}}) + \ln(1 + \frac{P_{t+1} - P_t}{P_t})$$
$$= \ln(\frac{7}{6}) + \ln(\frac{9}{10}) \approx 0.049$$

Thus, the multiperiod continously compounded return is approximately 0.049%

1e) Compute the one period continuously compounded return at time (t + 1) and (t + 2).

Solution:

The one-period continuously compounded return at time (t + 1) is:

$$r_{t+1} = \ln(1 + R_{t+1})$$

$$= \ln(1 + \frac{P_{t+1} - P_t}{P_t})$$

$$= \ln(\frac{9}{10}) \approx -0.105$$

The one-period continuously compounded return at time (t + 2) is:

$$r_{t+2} = \ln(1 + R_{t+2})$$

$$= \ln(1 + \frac{P_{t+2} - P_{t+1}}{P_{t+1}})$$

$$= \ln(\frac{7}{6}) \approx 0.154$$

Thus the one-period simple return at time (t + 1) is approximately -0.105%, and at time (t + 2) is approximately 0.154%.

1f) What is the relationship between the multiperiod continuouslycompounded return and the one period continuously compounded returns?

Solution:

The multiperiod continuously componded return is the sum of k one-period continuously compounded returns. The one-period continuously componded return is a partial sum of the multiperiod continuously compounded return.

Problem 2: Solve exercises 1,2,3 and 4 of Chapter 1 from Tsay(2013). The required dataset is available on Canvas.

Exercise I:

1a) Compute the sample mean, standard deviation, skewness, excess kurtosis, minimum, and maximum of each simple return series.

Solution:

```
library(fBasics)
```

```
## Warning: package 'fBasics' was built under R version 3.6.3
```

```
## Loading required package: timeDate
```

```
## Loading required package: timeSeries
```

```
## Warning: package 'timeSeries' was built under R version 3.6.3
```

```
rm(list = ls()) # clear the data environment
simple_return_series <- read.table("C:/Users/gordo/Desktop/Fall 2020 Classes/STAT 1331 - Financial Econ
#display 5 number summary
summary(simple_return_series[, c(seq(2,5,1))]) #get five number summary for all return series, but disr
```

```
##       axp                    vw                    ew
##  Min.   :-0.175949   Min.   :-0.0897620   Min.   :-0.0782400
##  1st Qu.:-0.009671   1st Qu.:-0.0054735   1st Qu.:-0.0046300
##  Median : 0.000000   Median : 0.0008480   Median : 0.0014290
##  Mean   : 0.000534   Mean   : 0.0002241   Mean   : 0.0006258
##  3rd Qu.: 0.010540   3rd Qu.: 0.0062125   3rd Qu.: 0.0064020
##  Max.   : 0.206485   Max.   : 0.1148890   Max.   : 0.1074220
##        sp
##  Min.   :-9.035e-02
##  1st Qu.:-5.798e-03
##  Median : 7.000e-04
##  Mean   : 9.423e-05
##  3rd Qu.: 6.117e-03
##  Max.   : 1.158e-01
```

```r
#display skewness for all return series, disregarding the date column
skewness(simple_return_series[, c(seq(2,5,1))])
```

```
##          axp           vw           ew           sp
##  0.459773364 -0.098318371 -0.247410170  0.008151943
```

```r
#display kurtosis for all return series, disregarding the date column
kurtosis(simple_return_series[, c(seq(2,5,1))])
```

```
##      axp       vw       ew       sp
## 9.592053 7.982134 8.108428 8.532667
```

```r
# simple_return_series
```

1b) Transform the simple returns to log returns. Compute the sample mean, standard deviation, skewness, excess kurtosis, minimum, and maximum of each log return series.

Solution:

```r
log_returns_1 <- log(simple_return_series[, c(seq(2,5,1))]+1)
summary(log_returns_1)
```

```
##       axp                    vw                    ew
##  Min.   :-0.193523   Min.   :-0.0940492   Min.   :-0.0814704
##  1st Qu.:-0.009719   1st Qu.:-0.0054885   1st Qu.:-0.0046407
##  Median : 0.000000   Median : 0.0008476   Median : 0.0014280
##  Mean   : 0.000188   Mean   : 0.0001308   Mean   : 0.0005526
##  3rd Qu.: 0.010484   3rd Qu.: 0.0061933   3rd Qu.: 0.0063816
##  Max.   : 0.187711   Max.   : 0.1087549   Max.   : 0.1020348
##        sp
##  Min.   :-9.470e-02
##  1st Qu.:-5.815e-03
##  Median : 6.998e-04
##  Mean   :-7.500e-07
##  3rd Qu.: 6.098e-03
##  Max.   : 1.096e-01
```

```r
#display skewness for all return series, disregarding the date column
skewness(log_returns_1)
```

```
##         axp          vw          ew          sp
##  0.02099179 -0.30035228 -0.42731505 -0.20635690
```

```r
#display kurtosis for all return series, disregarding the date column
kurtosis(log_returns_1)
```

```
##      axp       vw       ew       sp
## 9.020499 7.880082 8.017712 8.322826
```

1c) Test the null hypothesis that the mean of the log returns of AXP stock is zero. Use 5% significance level to draw your conclusion.

Solution:

```r
t.test(log_returns_1$axp, mean = 0, conf.level = 0.95)
```

```
##
##  One Sample t-test
##
## data:  log_returns_1$axp
## t = 0.35999, df = 2534, p-value = 0.7189
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -0.0008360686  0.0012120714
## sample estimates:
##    mean of x
## 0.0001880014
```

Conclusion:

Since $p > 0.05$, we fail to reject the null hypothesis. In other words, we do not have sufficient evidence to conclude that the mean of the log returns of AXP stock differs significantly from zero.

Exercise II:

2a) Compute the sample mean, standard deviation, skewness, excess kurtosis, minimum, and maximum of each simple return series.

Solution:

```r
simple_return_series <- read.table("C:/Users/gordo/Desktop/Fall 2020 Classes/STAT 1331 - Financial Econ
# simple_return_series$sp
# display 5 number summary
summary(simple_return_series[, c(seq(2,5,1))]) #get five number summary for all return series, but disr
```

```
##        ge                   vw                   ew                  sp
##  Min.   :-0.272877  Min.   :-0.225363  Min.   :-0.27225  Min.   :-0.239541
##  1st Qu.:-0.030648  1st Qu.:-0.016655  1st Qu.:-0.01879  1st Qu.:-0.018381
##  Median : 0.007117  Median : 0.013354  Median : 0.01497  Median : 0.009019
##  Mean   : 0.010519  Mean   : 0.009316  Mean   : 0.01218  Mean   : 0.006178
##  3rd Qu.: 0.048684  3rd Qu.: 0.038534  3rd Qu.: 0.04315  3rd Qu.: 0.035150
##  Max.   : 0.251236  Max.   : 0.165585  Max.   : 0.29926  Max.   : 0.163047
##                                                          NA's   :1
```

```r
#display skewness for all return series, disregarding the date column
skewness(simple_return_series[, c(seq(2,5,1))], na.rm = TRUE)
```

```
##         ge         vw         ew         sp
##  0.0516182 -0.6608207 -0.3069642 -0.5894543
```

```r
#display kurtosis for all return series, disregarding the date column
kurtosis(simple_return_series[, c(seq(2,5,1))], na.rm = TRUE)
```

```
##       ge       vw       ew       sp
## 1.239488 2.355320 3.138812 2.361272
```

2b) Transform the simple returns to log returns. Compute the sample mean, standard deviation, skewness, excess kurtosis, minimum, and maximum of each log return series.

Solution:

```r
log_returns_2 <- log(simple_return_series[, c(seq(2,5,1))]+1)
summary(log_returns_2)
```

```
##        ge                   vw                   ew                  sp
##  Min.   :-0.318660  Min.   :-0.255361  Min.   :-0.31779  Min.   :-0.273833
##  1st Qu.:-0.031127  1st Qu.:-0.016795  1st Qu.:-0.01897  1st Qu.:-0.018552
##  Median : 0.007092  Median : 0.013266  Median : 0.01486  Median : 0.008979
##  Mean   : 0.008318  Mean   : 0.008331  Mean   : 0.01061  Mean   : 0.005240
##  3rd Qu.: 0.047536  3rd Qu.: 0.037810  3rd Qu.: 0.04224  3rd Qu.: 0.034546
##  Max.   : 0.224132  Max.   : 0.153223  Max.   : 0.26179  Max.   : 0.151043
##                                                          NA's   :1
```

```r
#display skewness for all return series, disregarding the date column
skewness(log_returns_2)
```

```
##         ge         vw         ew         sp
## -0.2907818 -0.9430518 -0.7457118         NA
```

```r
#display kurtosis for all return series, disregarding the date column
kurtosis(log_returns_2)
```

```
##       ge       vw       ew       sp
## 1.778316 3.517912 4.169659       NA
```

2c) Test the null hypothesis that the mean of the log returns of AXP stock is zero. Use 5% significance level to draw your conclusion.

Solution:

```
t.test(log_returns_2$ge, mean = 0, conf.level = 0.95)
```

```
##
##  One Sample t-test
##
## data:  log_returns_2$ge
## t = 3.713, df = 860, p-value = 0.000218
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.003921037 0.012715061
## sample estimates:
##   mean of x
## 0.008318049
```

Conclusion:

Since p $<$ 0.05, we reject the null hypothesis. In other words, we have sufficient evidence to conclude that the mean of the log returns of GE stock differs significantly from zero.

Exercise III:

Consider the monthly stock returns of S&P composite index from January 1940 to September 2011 in Problem 2. Perform the following tests and draw conclusions using the 5% significance level.

3a) Test $H_0 : \mu = 0$ versus $H_a : \mu \neq 0$, where μ denotes the mean return.

Solution:

```
#Via Snedecor and Cochran, the t-ratio statistic of the sample
t.test(simple_return_series$sp, mean = 0, conf.level = 0.95) #use log returns or just regular returns?
```

```
##
##  One Sample t-test
##
## data:  simple_return_series$sp
## t = 4.2438, df = 859, p-value = 2.436e-05
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.003320588 0.009034840
## sample estimates:
##   mean of x
## 0.006177714
```

Conclusion:

Since p $<$ 0.05, we reject the null hypothesis. In other words, we have sufficient evidence to conclude that the mean returns of the GE stock differs significantly from zero.

3b) Test $H_0: m_3 = 0$ versus $H_a: m_3 \neq 0$, where m_3 denotes the skewness.

Solution:

```
#Via Snedecor and Cochran, the t-ratio statistic of the sample skewness is: S(r)/sqrt(6/T)
nrow(simple_return_series) #num entries in the data frame
```

```
## [1] 861
```

```
skew_ge <- skewness(simple_return_series$ge)
skew_stat <- skew_ge/(sqrt(6/nrow(simple_return_series)))
skew_stat
```

```
## [1] 0.6183421
## attr(,"method")
## [1] "moment"
```

Conclusion:

Since |t| $< Z_{1-0.05/2} =$ |t| $<$ 1.96, so we fail to reject the null hypothesis. In other words, we do not have sufficient evidence to conclude that the skewness measure of the returns for the GE stock is not zero.

3c) Test $H_0: K - 3 = 0$ versus $H_a: K - 3 \neq 0$, where K denotes the kurtosis.

Solution:

```
#Via Snedecor and Cochran, the t-ratio statistic of the sample excess kurtosis is: K(r)-3/sqrt(24/T)
nrow(simple_return_series) #num entries in the data frame
```

```
## [1] 861
```

```
kurtosis_ge <- kurtosis(simple_return_series$ge)
kurtosis_stat <- (kurtosis_ge - 3)/(sqrt(6/nrow(simple_return_series)))
kurtosis_stat
```

```
## [1] -21.08944
## attr(,"method")
## [1] "excess"
```

Conclusion:

Since |t| $> Z_{1-0.05/2} =$ |t| $>$ 1.96, so we reject the null hypothesis. In other words, we do have sufficient evidence to conclude that the excess kurtosis of the mean returns for the GE stock is not zero.

Exercise IV:

Consider the daily log returns of American Express stock from September 1, 2001 to September 30, 2011 as in Problem 1. Use the 5% significance level to perform the following tests:

4a) Test the null hypothesis that the skewness measure of the returns is zero

```
#Via Snedecor and Cochran, the t-ratio statistic of the sample skewness is: S(r)/sqrt(6/T)
nrow(log_returns_1) #num entries in the data frame
```

```
## [1] 2535
```

```
skew_axp <- skewness(log_returns_1$axp)
skew_stat <- skew_axp/(sqrt(6/nrow(log_returns_1)))
skew_stat
```

```
## [1] 0.4314821
## attr(,"method")
## [1] "moment"
```

Conclusion:

Since $|\text{t}| < Z_{1-0.05/2} = |\text{t}| < 1.96$, so we fail to reject the null hypothesis. In other words, we do not have sufficient evidence to conclude that the skewness measure of the log returns for American Express stock is not zero.

4b) Test the null hypothesis that the excess kurtosis measure of the returns is zero

```
#Via Snedecor and Cochran, the t-ratio statistic of the sample excess kurtosis is: K(r)-3/sqrt(24/T)
nrow(log_returns_1) #num entries in the data frame
```

```
## [1] 2535
```

```
kurtosis_axp <- kurtosis(log_returns_1$axp)
kurtosis_stat <- kurtosis_axp/(sqrt(6/nrow(log_returns_1)))
kurtosis_stat
```

```
## [1] 185.4146
## attr(,"method")
## [1] "excess"
```

Conclusion:

Since $|\text{t}| > Z_{1-0.05/2} = |\text{t}| > 1.96$, so we reject the null hypothesis. In other words, we do have sufficient evidence to conclude that the excess kurtosis of the log returns for the American Express stock is not zero.