LAB – 4 : SOLUTION OF ODE OF FIRST ORDER AND FIRST DEGREE BY TAYLOR'S SERIES AND MODIFIED EULER'S METHOD

TAYLOR'S SERIES

$$\frac{dy}{dx} = f(x,y) \ , \qquad y(x_0) = y_0$$

$$y = y(x_0) + (x - x_0)\, y_1(x_0) + \frac{(x-x_0)^2}{2!} y_2(x_0) + \frac{(x-x_0)^3}{3!} y_3(x_0) + \cdots\cdots$$

MODIFIED EULER'S METHOD

$$\frac{dy}{dx} = f(x,y) \ , \quad y(x_0) = y_0$$

$$y^{(E)} = y_{n-1} + h\, f(x_{n-1}, y_{n-1})$$

$$y^{(ME)} = y_{n-1} + \frac{h}{2}\Big[f(x_{n-1}, y_{n-1}) + f\big(x_n, {y^{(E)}}_n\big)\Big]$$

```python
#Taylor's sreies
from sympy import *
x,y=symbols('x,y')
f=x**2+y**2
x0=float(input('Enter the initial value for x : '))
y0=float(input('Enter the initial value for y : '))
n=int(input('Enter the number of terms required in the series : '))
print('y1 = %0.4f'%f.subs({x:x0 , y:y0}))
series=y0+(x-x0)*(f.subs({x:x0 , y:y0}))
dy=f
for i in range (2,n):
    dy=diff(dy,x)+diff(dy,y)*f
    dy0=dy.subs({x:x0,y:y0})
    print(f'y{i} = %0.4f'%dy0)
    series=series+((((x-x0)**i)*dy0)/factorial(i))
display(series)
xvalue=float(input('Enter the value of x at which we have to find y : '))
print(f'y({xvalue}) = %0.4f'%series.subs({x:xvalue , y:y0}))
```

OUTPUT:

```
Enter the initial value for x : 0
Enter the initial value for y : 1
Enter the number of terms required in the series : 5
y1 = 1.0000
y2 = 2.0000
y3 = 8.0000
y4 = 28.0000
```

$$1.16666666666667x^4 + 1.33333333333333x^3 + 1.0x^2 + 1.0x + 1.0$$

```
Enter the value of x at which we have to find y : 1.4
y(1.4) = 12.5005
```

```python
#Modified Euler's method to solve ODE
from sympy import *
x,y=symbols('x,y')
f=x+sqrt(y)
x0=float(input('Enter the initial value for x : '))
y0=float(input('Enter the initial value for y : '))
h=float(input('Enter the step lenght h : '))
n=int(input('Enter the number of iteration required in Modified Eulers Metl
yE=y0+h*f.subs({x:x0 , y:y0})
x1=x0+h
yME=yE
print('\nFrom Eulers Method : y = %0.4f'%yE)
print('\nFrom Modified Eulers Method')
for j in range (1,n+1):
    yME=y0+(h/2)*(f.subs({x:x0 , y:y0})+f.subs({x:x1 , y:yME}))
    print(f'{j} - Iteration : y = %0.4f'%yME)
```

OUTPUT:

```
Enter the initial value for x : 0
Enter the initial value for y : 1
Enter the step lenght h : 0.2
Enter the number of iteration required in Modified Eulers Method : 2
Enter the total number of values of x at which y should be determained :
2

From Eulers Method : y = 1.2000

From Modified Eulers Method
1 - Iteration : y = 1.2295
2 - Iteration : y = 1.2309
```

```python
#Modified Euler's method to solve ODE (for mulitiple values of 'x')
from sympy import *
x,y=symbols('x,y')
f=x+sqrt(y)
x0=float(input('Enter the initial value for x : '))
y0=float(input('Enter the initial value for y : '))
h=float(input('Enter the step lenght h : '))
n=int(input('Enter the number of iteration required in Modified Eulers Meth
m=int(input('Enter the total number of values of x at which y should be det
for i in range (1,m+1):
    yE=y0+h*f.subs({x:x0 , y:y0})
    x1=x0+h
    yME=yE
    print('\nFrom Eulers Method : y = %0.4f'%yE)
    print('\nFrom Modified Eulers Method')
    for j in range (1,n+1):
        yME=y0+(h/2)*(f.subs({x:x0 , y:y0})+f.subs({x:x1 , y:yME}))
        print(f'{j} - Iteration : y = %0.4f'%yME)
    x0=x1
    y0=yME
```

OUTPUT:

```
Enter the initial value for x : 0
Enter the initial value for y : 1
Enter the step lenght h : 0.2
Enter the number of iteration required in Modified Eulers Method : 2
Enter the total number of values of x at which y should be determained :
2

From Eulers Method : y = 1.2000

From Modified Eulers Method
1 - Iteration : y = 1.2295
2 - Iteration : y = 1.2309

From Eulers Method : y = 1.4928

From Modified Eulers Method
1 - Iteration : y = 1.5240
2 - Iteration : y = 1.5253
```

Exercise: Write python program for the following

1. Using Taylor's series method , find $'y'$ at $x = 0.4$ of the initial value problem $\frac{dy}{dx} = x^2 y + 1$ with $y(0) = 0$ considering up to 4^{th} degree term.
2. Using Taylor's series method , find $'y'$ at $x = 0.1$ of the initial value problem $\frac{dy}{dx} = 2y + 3e^x$ with $y(0) = 0$ considering up to 4^{th} degree term.
3. Solve $\frac{dy}{dx} = 3x + \frac{y}{2}$ with $y(0) = 1$ then find $'y'$ at $x = 0.2$ with $h = 0.2$ using Modified Euler's method.
4. Solve $\frac{dy}{dx} = x + \sin(y)$ with $y(0) = 1$ then find $'y'$ at $x = 0.4$ with $h = 0.2$ using Modified Euler's method.