

## LAB - 8 : Finding gradient, divergence, curl .

### Gradient of a Scalar Function

$$\phi = \phi(x, y, z)$$

$$\text{grad } \phi = \frac{\partial \phi}{\partial x} \hat{i} + \frac{\partial \phi}{\partial y} \hat{j} + \frac{\partial \phi}{\partial z} \hat{k}$$

*#Finding the Gradient of a scalar function*

```
from sympy . physics . vector import *
```

```
from sympy import *
```

```
x,y,z=symbols('x,y,z')
```

```
v= ReferenceFrame ('v')
```

```
φ=log(x**2+y**2+z**2)
```

```
print ("\n Gradient of φ is")
```

```
display(Derivative(φ,x)*v.x+Derivative(φ,y)*v.y+Derivative(φ,z)*v.z)
```

```
gradφ=diff(φ,x)*v.x+diff(φ,y)*v.y+diff(φ,z)*v.z
```

```
display(gradφ)
```

### OUTPUT

Gradient of  $\phi$  is

$$\frac{\partial}{\partial x} \log (x^2 + y^2 + z^2) \hat{\mathbf{v}}_x + \frac{\partial}{\partial y} \log (x^2 + y^2 + z^2) \hat{\mathbf{v}}_y + \frac{\partial}{\partial z} \log (x^2 + y^2 + z^2) \hat{\mathbf{v}}_z$$

$$\frac{2x}{x^2 + y^2 + z^2} \hat{\mathbf{v}}_x + \frac{2y}{x^2 + y^2 + z^2} \hat{\mathbf{v}}_y + \frac{2z}{x^2 + y^2 + z^2} \hat{\mathbf{v}}_z$$

```

#Finding the Normal vector to the surface
from sympy . physics . vector import *
from sympy import *
x,y,z=symbols('x,y,z')
v= ReferenceFrame ('v')
φ=x**3+y**3+3*x*y*z-3
print ("\n Gradient of φ is")
display(Derivative(φ,x)*v.x+Derivative(φ,y)*v.y+Derivative(φ,z)*v.z)
gradφ=diff(φ,x)*v.x+diff(φ,y)*v.y+diff(φ,z)*v.z
display(gradφ)
NV=gradφ.subs({x:1,y:2,z:-1})
print('Normal vector to the surface is ')
display(NV)

```

## OUTPUT

Gradient of  $\phi$  is

$$\frac{\partial}{\partial x}(x^3 + 3xyz + y^3 - 3) \hat{\mathbf{x}} + \frac{\partial}{\partial y}(x^3 + 3xyz + y^3 - 3) \hat{\mathbf{y}} + \frac{\partial}{\partial z}(x^3 + 3xyz + y^3 -$$

$$(3x^2 + 3yz)\hat{\mathbf{x}} + (3xz + 3y^2)\hat{\mathbf{y}} + 3xy\hat{\mathbf{z}}$$

Normal vector to the surface is

$$-3\hat{\mathbf{x}} + 9\hat{\mathbf{y}} + 6\hat{\mathbf{z}}$$

## Divergence of a Vector point Function

$$\mathbf{f} = f_1(x, y, z)\hat{\mathbf{i}} + f_2(x, y, z)\hat{\mathbf{j}} + f_3(x, y, z)\hat{\mathbf{k}}$$

$$\text{div} \mathbf{f} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

```

#Finding the divergence of a vector field
from sympy . physics . vector import *
from sympy import *
x,y,z=symbols('x,y,z')
v= ReferenceFrame ('v')
f1=x*y**2
f2=2*x**2*y*z
f3=-3*y*z**2
f=f1*v.x+f2*v.y+f3*v.z
print ("\n Divergence of the vector field f is")
display(Derivative(f1,x)+Derivative(f2,y)+Derivative(f3,z))
divf=diff(f1,x)+diff(f2,y)+diff(f3,z)
display(divf)

```

## OUTPUT

Divergence of the vector field f

$$\frac{\partial}{\partial x}xy^2 + \frac{\partial}{\partial z}(-3yz^2) + \frac{\partial}{\partial y}2x^2yz$$

$$2x^2z + y^2 - 6yz$$

```

# To verify the vector is a solenoidal
from sympy . physics . vector import *
from sympy import *
x,y,z=symbols('x,y,z')
v= ReferenceFrame ('v')
f1=3*y**4*z**2
f2=4*x**3*z**2
f3=3*x**2*y**2
f=f1*v.x+f2*v.y+f3*v.z
print ("\n Divergence of the vector f is ")
display(Derivative(f1,x)+Derivative(f2,y)+Derivative(f3,z))
divf=diff(f1,x)+diff(f2,y)+diff(f3,z)
display(divf)
if divf==0:
    print('The vector f is solenoidal')
else:
    print('The vector f is not solenoidal')

```

## OUTPUT

Divergence of the vector f

$$\frac{\partial}{\partial z} 3x^2 y^2 + \frac{\partial}{\partial y} 4x^3 z^2 + \frac{\partial}{\partial x} 3y^4 z^2$$

0

The vector f is solenoidal

## Curl of a Vector Point Function

$$f = f_1(x, y, z)\hat{i} + f_2(x, y, z)\hat{j} + f_3(x, y, z)\hat{k}$$

$$\text{curl}f = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix} = \left( \frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z} \right) \hat{i} - \left( \frac{\partial f_3}{\partial x} - \frac{\partial f_1}{\partial z} \right) \hat{j} + \left( \frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} \right) \hat{k}$$

*#Program to find curl of  $F=xy^2\hat{i}+2x^2yz\hat{j}-3yz^2\hat{k}$*

```
from sympy . physics . vector import *
from sympy import *
x,y,z=symbols('x,y,z')
v= ReferenceFrame ('v')
f1=x*y**2
f2=2*x**2*y*z
f3=-3*y*z**2
f=f1*v.x+f2*v.y+f3*v.z
print (" curl of f is ")
display((Derivative(f3,y)-Derivative(f2,z))*v.x-(Derivative(f3,x)-Derivative(f2,z))*v.y+(Derivative(f2,x)-Derivative(f3,y))*v.z)
curlf=(diff(f3,y)-diff(f2,z))*v.x-(diff(f3,x)-diff(f1,z))*v.y+(diff(f2,x)-diff(f3,y))*v.z
display (curlf)
```

## OUTPUT

curl of f is

$$\left( \frac{\partial}{\partial y}(-3yz^2) - \frac{\partial}{\partial z}2x^2yz \right) \hat{\mathbf{v}}_x + \left( \frac{\partial}{\partial z}xy^2 - \frac{\partial}{\partial x}(-3yz^2) \right) \hat{\mathbf{v}}_y + \left( -\frac{\partial}{\partial y}xy^2 + \frac{\partial}{\partial x}2x^2yz \right) \hat{\mathbf{v}}_z$$

$$(-2x^2y - 3z^2) \hat{\mathbf{v}}_x + (4xyz - 2xy) \hat{\mathbf{v}}_z$$

**Exercise :** Write the python program for the following

1. If  $\phi = x^2y + y^2z + z^2x$  find  $\nabla\phi$  at  $(1,1,1)$
2. Find  $\nabla\phi$  if  $\phi = \log(x^2 + y^2 + z^2)$
3. Find the unit normal vector to the surface  $x^2y + 2xz = 4$  at  $(2, -2, 3)$

4. Find the unit normal vector to the surface  $xy^3z^2=4$  at  $(-1,-1,2)$
5. Find  $\nabla \cdot F$  and  $\nabla \times F$  if  $F = x^2yz\hat{i} + xy^2z\hat{j} + xyz^2\hat{k}$
6. Show that the vector  $(-x^2 + yz)\hat{i} + (-z^2x + 4y)\hat{j} + (2xz - 4z)\hat{k}$  is solenoidal.