LAB – 3 : COMPUTATION OF AREA UNDER THE CURVE USING TRAPEZOIDAL, SIMPSON'S $\left(\frac{1}{3}\right)^{\text{rd}}$ AND $\left(\frac{3}{8}\right)^{\text{th}}$ RULE

TRAPEZOIDAL RULE

$$\int_{x_0=a}^{x_n=b} y\,dx = \frac{h}{2}\left[\left(y_0 + y_n\right) + 2\left(y_1 + y_2 + y_3 + \cdots\cdots + y_{n-1}\right)\right] \text{ Where } h = \frac{b-a}{n}$$

SIMPSON'S $\left(\frac{1}{3}\right)^{\text{rd}}$ RULE

$$\int_{x_0=a}^{x_n=b} y\,dx = \frac{h}{3}\left[\left(y_0 + y_n\right) + 4\left(y_1 + y_3 + y_5 + \cdots + y_{n-1}\right) + 2\left(y_2 + y_4 + y_6 + \cdots + y_{n-2}\right)\right]$$

SIMPSON'S $\left(\frac{3}{8}\right)^{\text{th}}$ RULE

$$\int_{x_0=a}^{x_n=b} y\,dx = \frac{3h}{8}\left[\left(y_0 + y_n\right) + 3\left(y_1 + y_2 + y_4 + \cdots + y_{n-1}\right) + 2\left(y_3 + y_6 + y_9 + \cdots + y_{n-3}\right)\right]$$

Program 1: Program to evaluate $\int_0^1 \frac{1}{1+t^2}\,dt$ using Trapezoidal rule

```
# Trapezoidal rule
from sympy import *
t=Symbol('t')
f=1/(1+t**2)
a=float(input('Enter the value of lower limit : '))
b=float(input('Enter the value of upper limit : '))
n=int(input('Enter the number of division : '))
x=zeros(n+1)
y=zeros(n+1)
h=(b-a)/n
x[0]=a
y[0]=f.subs(t,x[0])
for i in range (0,n):
    x[i+1]=x[i]+h
    y[i+1]=f.subs(t,x[i+1])
for i in range(0,n+1):
    print(f'x{i} = %0.4f'%x[i],f'\t\t y{i}=%0.4f'%y[i])
I1=0
for i in range(1,n):
    I1=I1+y[i]
I=(h/2)*(y[0]+y[n]+2*I1)
print('Integral value from Trapezoidal rule = %0.4f'%I)
actualvalue=integrate(1/(1+t**2),(t,a,b))
print('Actual value of the integral = %0.4f'%actualvalue)
```

OUTPUT:

```
Enter the value of lower limit : 0
Enter the value of upper limit : 1
Enter the number of division : 10
x0 = 0.0000              y0=1.0000
x1 = 0.1000              y1=0.9901
x2 = 0.2000              y2=0.9615
x3 = 0.3000              y3=0.9174
x4 = 0.4000              y4=0.8621
x5 = 0.5000              y5=0.8000
x6 = 0.6000              y6=0.7353
x7 = 0.7000              y7=0.6711
x8 = 0.8000              y8=0.6098
x9 = 0.9000              y9=0.5525
x10 = 1.0000             y10=0.5000
Integral value from Trapezoidal rule = 0.7850
Actual value of the integral = 0.7854
```

Program 2: Program to evaluate $\int_0^4 e^{1/t}\, dt$ using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule

```python
# Simpson (1/3)rd rule
from sympy import *
t=Symbol('t')
f=exp(1/t)
a=float(input('Enter the value of lower limit : '))
b=float(input('Enter the value of upper limit : '))
n=int(input('Enter the number of division : '))
x=zeros(n+1)
y=zeros(n+1)
h=(b-a)/n
x[0]=a
y[0]=f.subs(t,x[0])
for i in range (0,n):
    x[i+1]=x[i]+h
    y[i+1]=f.subs(t,x[i+1])
for i in range(0,n+1):
    print(f'x{i} = %0.4f'%x[i],f'\t\t y{i}=%0.4f'%y[i])
I1=0
I2=0
for i in range(1,n):
    if i%2==0:
        I2=I2+y[i]
    else:
        I1=I1+y[i]
I=(h/3)*(y[0]+y[n]+4*I1+2*I2)
print('Integral value from Simpson one third rule = %0.4f'%I)
actualvalue=integrate(exp(1/t),(t,a,b))
print('Actual value of the integral = %0.4f'%actualvalue)
```

OUTPUT:

```
Enter the value of lower limit : 1
Enter the value of upper limit : 4
Enter the number of division : 6
x0 = 1.0000              y0=2.7183
x1 = 1.5000              y1=1.9477
x2 = 2.0000              y2=1.6487
x3 = 2.5000              y3=1.4918
x4 = 3.0000              y4=1.3956
x5 = 3.5000              y5=1.3307
x6 = 4.0000              y6=1.2840
Integral value from Simpson one third rule = 4.8620
Actual value of the integral = 4.8555
```

Program 3 : Program to evaluate $\int_{2}^{8} \log_{10}(t)\, dt$ using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule

```
# Simpson (3/8)th rule
from math import *
from sympy import *
t=Symbol('t')
f=1/log(t,10)
a=float(input('Enter the value of lower limit : '))
b=float(input('Enter the value of upper limit : '))
n=int(input('Enter the number of division : '))
x=zeros(n+1)
y=zeros(n+1)
h=(b-a)/n
x[0]=a
y[0]=f.subs(t,x[0])
for i in range (0,n):
    x[i+1]=x[i]+h
    y[i+1]=f.subs(t,x[i+1])
for i in range(0,n+1):
    print(f'x{i} = %0.4f'%x[i],f'\t\t y{i}=%0.4f'%y[i])
I1=0
I2=0
for i in range(1,n):
    if i%3==0:
        I2=I2+y[i]
    else:
        I1=I1+y[i]
I=((3*h)/8)*(y[0]+y[n]+3*I1+2*I2)
print('Integral value from Simpson three eighth rule = %0.4f'%I)
actualvalue=integrate((1/log(t,10)),(t,a,b))
print('Actual value of the integral = %0.4f'%actualvalue)
```

OUTPUT:

```
Enter the value of lower limit : 2
Enter the value of upper limit : 8
Enter the number of division : 6
x0 = 2.0000              y0=3.3219
x1 = 3.0000              y1=2.0959
x2 = 4.0000              y2=1.6610
x3 = 5.0000              y3=1.4307
x4 = 6.0000              y4=1.2851
x5 = 7.0000              y5=1.1833
x6 = 8.0000              y6=1.1073
Integral value from Simpson three eighth rule = 9.7374
Actual value of the integral = 9.6906
```

Exercise: Write python program for the following

1. Program to evaluate the following definite integrals using the trapezoidal rule

(a) $\int_{4}^{5.2} \log x \, dx$ with $n=6$ (b) $\int_{0}^{\pi} \sin\theta \, d\theta$ with $n=10$

(c) $\int_{0}^{\pi/2} \sqrt{\cos\theta} \, d\theta$ with $n=8$ (d) $\int_{0}^{1} e^{-x^2} \, dx$ with $n=10$

2. Program to evaluate the following definite integrals using Simpson's (1/3)rd rule.

(a) $\int_{4}^{5.2} \log x \, dx$ with $n=6$ (b) $\int_{0}^{\pi} \sin\theta \, d\theta$ with $n=10$

(c) $\int_{0}^{\pi/2} \sqrt{\cos\theta} \, d\theta$ with $n=8$ (d) $\int_{0}^{1} e^{-x^2} \, dx$ with $n=12$

3. Program to evaluate the following definite integrals using Simpson's (3/8)th rule.

(a) $\int_{4}^{5.2} \log x \, dx$ with $n=6$ (b) $\int_{0}^{\pi} \sin\theta \, d\theta$ with $n=9$

(c) $\int_{0}^{\pi/2} \sqrt{\cos\theta} \, d\theta$ with $n=9$ (d) $\int_{0}^{1} e^{-x^2} \, dx$ with $n=12$