LAB – 5 : SOLUTION OF ODE OF FIRST ORDER AND FIRST DEGREE BY RUNGE-KUTTA 4TH ORDER METHOD AND MILNE'S PREDICTOR AND CORRECTOR METHOD

RUNGE-KUTTA 4TH ORDER METHOD

$$\frac{dy}{dx} = f(x, y) \ , \quad y(x_0) = y_0$$

$$k_1 = h\, f(x_0, y_0)$$

$$k_2 = h\, f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$$

$$k_3 = h\, f(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$$

$$k_4 = h\, f(x_0 + h, y_0 + k_3)$$

$$y = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

MILNE'S PREDICTOR AND CORRECTOR METHOD

$$\frac{dy}{dx} = f(x, y) \ , \quad y(x_0) = y_0$$

$$y_p = y_0 + \frac{4h}{3}(2f_1 - f_2 + f_3)$$

$$y_c = y_2 + \frac{h}{3}(f_2 + 4f_3 + f_4)$$

```python
# Runge-Kutta Method
from sympy import *
x,y=symbols('x,y')
f=(y**2-x**2)/(y**2+x**2)
x0=float(input('Enter the initial value of x : '))
y0=float(input('Enter the initial value of y : '))
h=float(input('Enter the value for step length h = '))

k1=h*f.subs({x:x0 , y:y0})
k2=h*f.subs({x:x0+(h/2) , y:y0+(k1/2)})
k3=h*f.subs({x:x0+(h/2) , y:y0+(k2/2)})
k4=h*f.subs({x:x0+h , y:y0+k3})
solution=y0+(1/6)*(k1+(2*k2)+(2*k3)+k4)
print('\nk1 = %0.4f'%k1,'\tk2 = %0.4f'%k2,'\tk3 = %0.4f'%k3,'\tk4 = %0.4f'%k4)
print(f'y({x0+h})=%0.4f'%solution)
```

OUTPUT:

```
Enter the initial value of x : 0
Enter the initial value of y : 1
Enter the value for step length h = 0.2

k1 = 0.2000     k2 = 0.1967     k3 = 0.1967     k4 = 0.1891
y(0.2)=1.1960
```

```python
# Milne's Predictor and corrector formula
from sympy import *
x,y=symbols('x,y')
f=x-(y**2)
h=float(input('Enter the value for step length h = '))
print('Enter the values for x and y ')
x0=float(input('x0 = '))
x1=x0+h
x2=x1+h
x3=x2+h
x4=x3+h
y0=float(input('y0 : '))
y1=float(input('y1 : '))
y2=float(input('y2 : '))
y3=float(input('y3 : '))
f1=f.subs({x:x1 , y:y1})
f2=f.subs({x:x2 , y:y2})
f3=f.subs({x:x3 , y:y3})
y4p=y0+((4*h)/3)*(2*f1-f2+2*f3)
print(f'\nFrom Milnes Predictor formula y({x4})=%0.4f'%y4p)
f4=f.subs({x:x4 , y:y4p})
yc=0
dif=1
i=1
print('\nFrom Milnes Corrector formula')
while dif>0.0009:
    y4c=y2+(h/3)*(f2+4*f3+f4)
    dif=abs(y4c-yc)
    yc=y4c
    print(f'{i} - Iteration : y({x4})=%0.4f'%y4c)
    f4=f.subs({x:x4 , y:y4c})
    i+=1
```

OUTPUT:

```
Enter the value for step length h = 0.2
Enter the values for x and y
x0 = 0
y0 : 0
y1 : 0.02
y2 : 0.0795
y3 : 0.1762

From Milnes Predictor formula y(0.8)=0.3049

From Milnes Corrector formula
1 - Iteration : y(0.8)=0.3046
2 - Iteration : y(0.8)=0.3046
```

Exercise: Write python program for the following

1. Using Runge-Kutta method , find $'y'$ at $x = 0.2$ of the initial value problem $\frac{dy}{dx} = 3x + \frac{y}{2}$ with $y(0) = 1$ by taking $h = 0.2$.
2. Using Runge-Kutta method, solve $\frac{dy}{dx} = 2y + 3e^x$ with $y(0) = 0$. Find $y(0.1)$ by taking $h = 0.1$.
3. Apply Milne's method to compute $y(1.4)$, given $\frac{dy}{dx} = x^2 + \frac{y}{2}$

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514

4. Apply Milne's method to compute $y(0.4)$, given $\frac{dy}{dx} = 2e^x y$

x	0	0.1	0.2	0.3
y	2.4	2.473	3.129	4.059