# LAB 7: Evaluation of improper integrals, Beta and Gamma functions

## Gamma function:

$$\Gamma(n)=\int_0^\infty e^{-x}x^{n-1}dx,\ (n>0)$$

## Beta function:

$$\beta(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}\,dx,\ (m,n>0)$$

**<u>Program 1:</u>** Program for evaluate $\Gamma(n)$.

```
from numpy import *
from sympy import *
x=symbols('x')
n=float(input('Enter the value of n to find Γ(n):'))
f=exp(-x)*(x**(n-1))
I=integrate(f,(x,0,inf))
display(Eq(Integral(f,(x,0,inf)),I))
gn=gamma(n)
print(f'Γ({n})=%0.4f'%gn)
```

**<u>OUTPUT</u>**

```
Enter the value of n to find Γ(n):7
```

$$\int_0^\infty x^{6.0}e^{-x}\,dx = 720.0$$

```
I=720.0000
Γ(7.0)=720.0000
```

**Program 2:** Program for evaluate $\Gamma(n)$ & $\beta(m,n)$

```
from sympy import *
n=float(input('Enter the value of n to find Γ(n):'))
gn=gamma(n)
print(f'Γ({n})=%0.4f'%gn)
m=float(input('Enter the value of n to find β(m,n):'))
n=float(input('Enter the value of n to find β(m,n):'))
bmn=beta(m,n)
print(f'β({m},{n})=%0.4f'%bmn)
```

**OUTPUT**

```
Enter the value of n to find Γ(n):3.5
Γ(3.5)=3.3234
Enter the value of n to find β(m,n):2.5
Enter the value of n to find β(m,n):3.5
β(2.5,3.5)=0.0368
```

**Program 3:** Program to verify relation between $\beta(m,n)$ & $\Gamma(n)$

```
# Verification of Relation between Beta and Gamma functions
from sympy import *
n=float(input('Enter the value of n to find Γ(n):'))
m=float(input('Enter the value of n to find β(m,n):'))
gn=gamma(n)
gm=gamma(m)
gmn=gamma(m+n)
print(f'Γ({m})=%0.4f'%gm,f'\nΓ({n})=%0.4f'%gn,f'\nΓ({m}+{n})=%0.4f'%gmn)
bmn=beta(m,n)
print(f'β({m},{n})=%0.4f'%bmn)
rhs=(gn*gm)/gmn
print('R.H.S = %0.4f'%rhs)
if bmn==rhs:
    print('β(m,n)=(Γ(m)*Γ(n))/Γ(m+n)')
else:
    print('β(m,n)!=(Γ(m)*Γ(n))/Γ(m+n)')
```

**OUTPUT**

```
Enter the value of n to find Γ(n):2
Enter the value of n to find β(m,n):5
Γ(5.0)=24.0000
Γ(2.0)=1.0000
Γ(5.0+2.0)=720.0000
β(5.0,2.0)=0.0333
R.H.S = 0.0333
β(m,n)=(Γ(m)*Γ(n))/Γ(m+n)
```

**Exercise:** Write python program for the following

1. Evaluate

a) $\int_0^\infty e^{-x^2} dx$ b) $\int_0^\infty e^{-x} x^{7/2} dx$ c) $\int_0^\infty e^{-t} \cos 2t \, dt$

2. Find the value of the following

a) $\beta\left(\frac{3}{2}, \frac{9}{2}\right)$ b) $\beta(4,5)$ c) $\Gamma\left(\frac{7}{2}\right)$ d) $\Gamma(6)$

3. Verify $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ for $m = \frac{3}{2}$ and $n = \frac{1}{2}$.