LAB - 1 : Solution of algebraic and transcendental equation by Newton –Raphson and Regula – Falsi method

Newton Raphson Method

Newton Raphson method is a numerical method to solve algebraic and transcendental Equations.

Working Rule

To solve the equation $f(x) = 0$

1. Consider $f(x)$ from the given equation
2. Find $f'(x)$ or $f_1(x)$
3. Assume 'x_0'as the initial root.
4. $x_n = x_{n-1} - \dfrac{f(x_{n-1})}{f'(x_{n-1})}$

I – Iteration : $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

II – Iteration : $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

III – Iteration : $x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$

```python
# Newton Raphson Method with number of iterations
from sympy import *
x=Symbol('x')
f=x**2+3*x+1
df=diff(f,x)
x0=1
n=int(input('Enter the number of iterations: '))
i=1
print('  Iteration \t    x \t\t   f(x) ')
while i<=n:

    fx0=f.subs(x,x0)
    dfx0=df.subs(x,x0)
    xi=x0-(fx0/dfx0)
    fi=f.subs(x,xi)
    print(f'{i}-Iteration',f' \tx{i} = %0.4f'%xi,f'\tf(x{i})= %0.4f'%fi)
    x0=xi
    i+=1
```

```
Enter the number of iterations: 5
  Iteration         x              f(x)
1-Iteration     x1 = 0.0000     f(x1)= 1.0000
2-Iteration     x2 = -0.3333    f(x2)= 0.1111
3-Iteration     x3 = -0.3810    f(x3)= 0.0023
4-Iteration     x4 = -0.3820    f(x4)= 0.0000
5-Iteration     x5 = -0.3820    f(x5)= 0.0000
```

```python
# Newton Raphson Method (Root corrected to n decimal places)
from sympy import *
x=symbols('x')
f=x**2+3*x+1
df=diff (f,x)
x0=1
n=int(input('Enter the number of decimal places: '))
error=1/10**n
i=1
dif=1
print('  Iteration \t    x \t\t   f(x) ')
while dif>=error:
    fx0=f.subs(x,x0)
    dfx0=df.subs(x,x0)
    xi=x0-(fx0/dfx0)
    fi=f.subs(x,xi)
    print(f'{i}-Iteration',f' \tx{i} = %0.5f'%xi,f'\tf(x{i})= %0.5f'%fi)
    dif=abs(xi-x0)
    x0=xi
    i+=1
```

```
Enter the number of decimal places: 3
  Iteration          x              f(x)
1-Iteration     x1 = 0.00000    f(x1)= 1.00000
2-Iteration     x2 = -0.33333   f(x2)= 0.11111
3-Iteration     x3 = -0.38095   f(x3)= 0.00227
4-Iteration     x4 = -0.38197   f(x4)= 0.00000
5-Iteration     x5 = -0.38197   f(x5)= 0.00000
```

Regula Falsi Method

Regula Falsi method is a numerical method to solve algebraic and transcendental equations

Working Rule

To solve the equation $f(x) = 0$

1. Consider $f(x)$ from the given equation
2. Find 'a' and 'b' such that $f(a) < 0$ and $f(b) > 0$
3. $x_* = \dfrac{a\,f(b) - b\,f(a)}{f(b) - f(a)}$
4. If $f(x_*) > 0$ replace 'b' by 'x_*'

   If $f(x_*) < 0$ replace 'b' by 'x_*'

```
# Regula-Falsi Method with number of iterations
from sympy import *
x=Symbol('x')
f=x**3-4*x+9
a=-3
b=-2
n=int(input('Enter the number of iterations: '))
i=1
print('  a\t\t    b \t\t   f(a) \t  f(b)\t\t   x \t\t   f(x) \t    INTERVAL
while i<=n:
    fa=f.subs(x,a)
    fb=f.subs(x,b)
    xi=(a*fb-b*fa)/(fb-fa)
    fi=f.subs(x,xi)
    print('%0.4f\t\t'%a,'%0.4f\t'%b , '%0.4f\t'%fa ,'%0.4f\t'%fb , f' \tx{i
          f'\tf(x{i})= %0.4f\t'%fi,'(%0.4f,'%a,'%0.4f)'%b)
    if fi>0:
        b=xi
    elif fi<0:
        a=xi
    i+=1
```

```
Enter the number of iterations: 5
  a          b        f(a)     f(b)        x             f(x)          INTERVAL
-3.0000   -2.0000   -6.0000   9.0000   x1= -2.6000    f(x1)= 1.8240    (-3.0000,
-2.0000)
-3.0000   -2.6000   -6.0000   1.8240   x2 = -2.6933   f(x2)= 0.2372    (-3.0000,
-2.6000)
-3.0000   -2.6933   -6.0000   0.2372   x3 = -2.7049   f(x3)= 0.0289    (-3.0000,
-2.6933)
-3.0000   -2.7049   -6.0000   0.0289   x4 = -2.7063   f(x4)= 0.0035    (-3.0000,
-2.7049)
-3.0000   -2.7063   -6.0000   0.0035   x5 = -2.7065    f(x5)= 0.0004 (-3.0000,
-2.7063)
```

```python
# Regula-Falsi Method (Root corrected to n decimal places)
from sympy import *
x=Symbol('x')
f=x**3-4*x+9
a=-3
b=-2
n=float(input('Enter the number of decimal places: '))
error=1/10**n
i=1
diff=1
print('  a\t\t    b \t\t   f(a) \t  f(b)\t\t   x \t\t   f(x) \t    INTERVAL
while diff>=error:
    fa=f.subs(x,a)
    fb=f.subs(x,b)
    xi=(a*fb-b*fa)/(fb-fa)
    fi=f.subs(x,xi)
    print('%0.4f\t\t'%a,'%0.4f\t'%b , '%0.4f\t'%fa ,'%0.4f\t'%fb , f' \tx{i
          f'\tf(x{i})= %0.4f\t'%fi,'(%0.4f,'%a,'%0.4f)'%b)
    if fi>0:
        diff=abs(b-xi)
        b=xi
    elif fi<0:
        diff=abs(a-xi)
        a=xi
    i+=1
```

Exercise: Write python program for the following

1. Using Regula-Falsi method find the real root of the equation

   (a) $xe^x = 2$ (b) $xe^x - \cos x = 0$ (c) $2x - \log_{10} x = 7$

   (d) $x^3 - 2x - 5 = 0$, which lies between 2 and 3

   (e) $3x - \cos x = 1$, which lies between 0 and 1.

   (f) $x^4 + x^3 - 7x^2 - x + 5 = 0$, which lies between 2 and 3.

2. Using Newton-Raphson method find the real root of the equation

   (a) $xe^x = 2$, near 1 (b) $xe^x - \cos x = 0$, near 1 (c) $2x - \log_{10} x = 7$, near 3

   (d) $x^3 - 2x - 5 = 0$, near 2 (e) $3x - \cos x = 1$, near 1.

   (f) $x^4 + x^3 - 7x^2 - x + 5 = 0$, near 2.5.