MXB326 Group Project - Reservoir Simulation

MXB326_23se1 Computational Methods 2
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1 Introduction

Oil is a natural resource which is used abundantly in modern society. Used in the creation of fuels which transport us around the world, to materials in products we use hundreds of times a day, and many places in between, it is no stretch to say that civilisation as we know it relies on oil. But the production of oil can be a slow and costly process. One method commonly used in the oil industry to improve production is called water-flooding. This processes uses the injection of water into the porous rock structure of an oil reservoir to increase the pressure, driving more oil out. Modelling this process mathematically is a key interest of a field of research known as reservoir engineering, and is of vital importance to ensure that the process of water-flooding runs smoothly.

This modelling, involving the simultaneous flow of two viscous fluids through a horizontal reservoir in three dimensions, is quite complex. Fortunately, the model can be substantially simplified when considering a homogeneous, one-dimensional reservoir. This simplification in turn reduces what would be a coupled system of partial differential equations into a single, one-dimensional PDE. To describe the mechanics of the system, the conservation of mass of oil and water assuming immiscibility and incompressibility, are given by the transport equations:

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial q_w}{\partial x} = 0 \tag{1}$$

$$\phi \frac{\partial S_o}{\partial t} + \frac{\partial q_o}{\partial x} = 0 \tag{2}$$

In this system, ϕ indicates how porous the medium of the reservoir is, and S_w and q_w, S_o and q_o indicate the saturation and flow rate of the water and oil respectively. We relate the pressure in each fluid as the capillary pressure: $P_c = p_o - p_w$, and assume that the capillary pressure is positive. Furthermore, the saturations of the oil and water can be related as $S_w + S_o = 1$, allowing a single saturation, $S = S_o$ to be the sole variable to be modelled. Also defined are the minimum, irreducible saturations S_{or} and S_{wr}.

For a comprehensive description of the mathematical system, it is also necessary to define the conditions at the boundaries of the domain. Figure 1 demonstrates the domain of the problem.

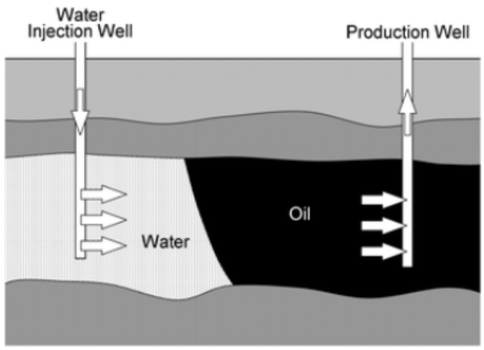

Figure 1: Schematic of the oil injection process.

Assuming a constant injection of water at $x = 0$ at rate q:

$$q_w(0, t) = q \tag{3}$$

$$q_o(0, t) = 0 \tag{4}$$

And assuming oil saturation begins at its maximum value:

$$S(x, 0) = 1 - S_{wr} \tag{5}$$

Finally, assuming a semi-infinite reservoir:

$$\lim_{x \to \infty} S(x, t) = 1 - S_{wr} \tag{6}$$

For an efficient and cost-effective water-flooding operation, it is critical to prevent the water from mixing with the oil in the well-bore (extraction point). We thus define the breakthrough time as $T_B = 1 - S_{or} - S_{wr}$, the time taken for the water to reach the well-bore during the water-flooding. Specification of a suitable injection rate q is capable of preventing this issue from arising.

This report contains a pilot study of the simplified system above, including a mathematical model with a semi-analytic solution and a numerical solution. These solutions will be analysed thoroughly, with the goal of demonstrating the capability of a fully detailed study in the future.

2 Mathematical Model

3 Semi-Analytical Solution

4 Numerical Solution

5 Analysis and Findings

6 Conclusions

7 References