MXB326 Group Project - Reservoir Simulation

MXB326_23se1 Computational Methods 2
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1 Introduction

Oil is a natural resource which is used abundantly in modern society. Necessary for the creation of fuels which transport us around the world, to materials in products we use hundreds of times a day, and many applications in between, it is no stretch to say that civilisation as we know it relies on oil. But the production of oil can be a slow and costly process. One method commonly used in the oil industry to improve production is called water-flooding. This processes uses the injection of water into the porous rock structure of an oil reservoir to increase the pressure, driving more oil out. Modelling this process mathematically is a key interest of a field of research known as reservoir engineering, and is of vital importance to ensure that the process of water-flooding runs smoothly.

This modelling, involving the simultaneous flow of two viscous fluids through a horizontal reservoir in three dimensions, is quite complex. Fortunately, the model can be substantially simplified when considering a homogeneous, one-dimensional reservoir. This simplification in turn reduces what would be a coupled system of partial differential equations into a single, one-dimensional PDE. To describe the mechanics of the system, the conservation of mass of oil and water assuming immiscibility and incompressibility, are given by the transport equations:

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial q_w}{\partial x} = 0 \tag{1}$$

$$\phi \frac{\partial S_o}{\partial t} + \frac{\partial q_o}{\partial x} = 0 \tag{2}$$

In this system, ϕ indicates how porous the medium of the reservoir is, and S_w and q_w, S_o and q_o indicate the saturation and flow rate of the water and oil respectively. We relate the pressure in each fluid as the capillary pressure: $P_c = p_o - p_w$, and assume that the capillary pressure is positive. Furthermore, the saturations of the oil and water can be related as $S_w + S_o = 1$, allowing a single saturation, $S = S_o$ to be the sole variable to be modelled. Also defined are the minimum, irreducible saturations S_{or} and S_{wr}.

For a comprehensive description of the mathematical system, it is also necessary to define the conditions at the boundaries of the domain. Figure 1 demonstrates the domain of the problem.

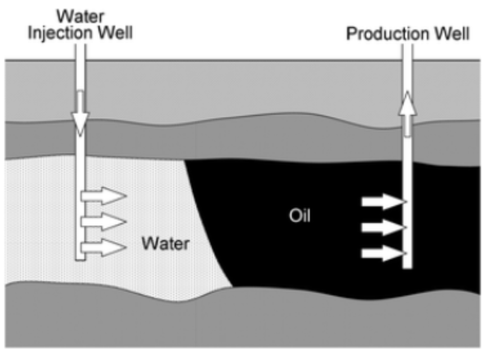

Figure 1: Schematic of the oil injection process.

Assuming a constant injection of water at $x = 0$ at rate q:

$$q_w(0,t) = q \quad \text{and} \quad q_o(0,t) = 0 \tag{3}$$

And assuming oil saturation begins at its maximum value:

$$S(x,0) = 1 - S_{wr} \tag{4}$$

Finally, assuming a semi-infinite reservoir:

$$\lim_{x \to \infty} S(x,t) = 1 - S_{wr} \tag{5}$$

For an efficient and cost-effective water-flooding operation, it is critical to prevent the water from mixing with the oil in the well-bore (extraction point). We thus define the breakthrough time as $T_B = 1 - S_{or} - S_{wr}$, the time taken for the water to reach the well-bore during the water-flooding. Specification of a suitable injection rate q is capable of preventing this issue from arising.

This report contains a pilot study of the simplified system above, including a mathematical model with a semi-analytic solution and a numerical solution. These solutions will be analysed thoroughly, with the goal of demonstrating the capability of a fully detailed study in the future.

2 Mathematical Model

It is possible to derive an exact solution for the system described above using methods from *Fokas and Yortsos*, in the case of constant injection rates when the water to oil viscosity ratio F has the form

$$F = \frac{1 - S_{wr} + \frac{\gamma}{\beta}}{S_{or} + \frac{\gamma}{\beta}} \tag{6}$$

Applying this, the following one-dimensional initial boundary value problem can be derived for the oil saturation:

$$\frac{\partial S}{\partial t} = \frac{\partial}{\partial x}\big[g(S)\frac{\partial S}{\partial x} + f(S)\big], \tag{7}$$

$$S(x,0) = 1 - S_{wr}, \tag{8}$$

$$\frac{\partial S}{\partial x}(0,t) = \frac{\alpha}{\beta}(\beta S(0,t) + \gamma) + \frac{\omega}{\beta}(\beta S(0,t) + \gamma)^2, \tag{9}$$

$$\lim_{x\to\infty} S(x,t) = 1 - S_{wr}. \tag{10}$$

By specifying $S_{wr}, S_{or}, F, \text{and}\beta$, the remaining parameters can be determined from (6) as well as the relations

$$\frac{\alpha}{\beta^2} = -\frac{(S_{or} + \frac{\gamma}{\beta})(1 - S_{wr} + \frac{\gamma}{\beta})}{1 - S_{wr} - S_{or}}, \tag{11}$$

$$\omega = \beta - \frac{\alpha}{\beta - \beta S_{wr} + \gamma}. \tag{12}$$

Using these equations, γ can be found with equation (6), α with γ and equation (11), and ω using the other two parameters with equation (12). This process has been implemented in the function `paramsFunc`.

Furthermore, the functions $f(S)$ and $g(S)$, called the capillary-hydraulic properties of the fluid-porous system are given by

$$f(S) = \frac{\alpha}{\beta^2}\left[\frac{1}{1 - S_{wr} + \frac{\gamma}{\beta}} - \frac{1}{S + \frac{\gamma}{\beta}}\right], \quad g(S) = \frac{1}{(\beta S + \gamma)^2}.$$

These functions allow for an exactly solvable PDE (7), and also correspond to a physically meaningful model.

3 Semi-Analytical Solution

A semi-analytical solution for (7) can be derived at discrete space x_i and time t_n by applying the spacial transformation

$$\tilde{x} = \int_0^x (\beta S(\xi,t) + \gamma) d\xi + \omega t, \tag{13}$$

This linearises the equation and yields the solution for S:

$$S(x,t) = \frac{1}{\beta}\left[\frac{\alpha e^{\alpha x}}{\frac{\partial \phi}{\partial \tilde{x}}(\tilde{x},t)} - \gamma\right], \tag{14}$$

where $\tilde{x}$ is the solution of $\phi(\tilde{x},t) = e^{\alpha x}$ for a given x and t. The solution of the nonlinear equation must be obtained using MATLAB's `fzero` function. The MATLAB documentation for this function indicates that it is used to find the roots of nonlinear function. Specifically, for some input `x = fzero(func, x0)`, `x` will return a point where $\text{func}(x) = 0$. The `fzero` function requires an initial guess x_0 such that $\text{func}(x_0) \approx 0$. An appropriate guess for each $i = 1..N$ and $n = 1..M$ can be found by evaluating (13) at $x = x_i$ and $t = t_{n-1}$. Let $\hat{x}$ indicate the approximate initial guess. Thus we have:

$$\hat{x}_{i,n} = (\beta S(x_i, t_{n-1}) + \gamma)\, x_i + \omega t_{n-1} \tag{15}$$

This results in an iterative process for the semi-analytical solution: for each t_n, solve for all x_i using $S(x_i, t_{n-1})$. For the initial guess at $t = t_1$, the initial condition (8) is applied. This provides $\hat{x}$ for all x_i at $t = t_1$:

$$\hat{x}_{i,1} = [\beta(1 - S_{wr}) + \gamma]\, x_i \tag{16}$$

With an iterative solution defined and appropriate initial conditions, the solution can be straightforwardly determined using MATLAB. This solution is implemented in the MATLAB script `semianalytic.m`, and produces the following result:

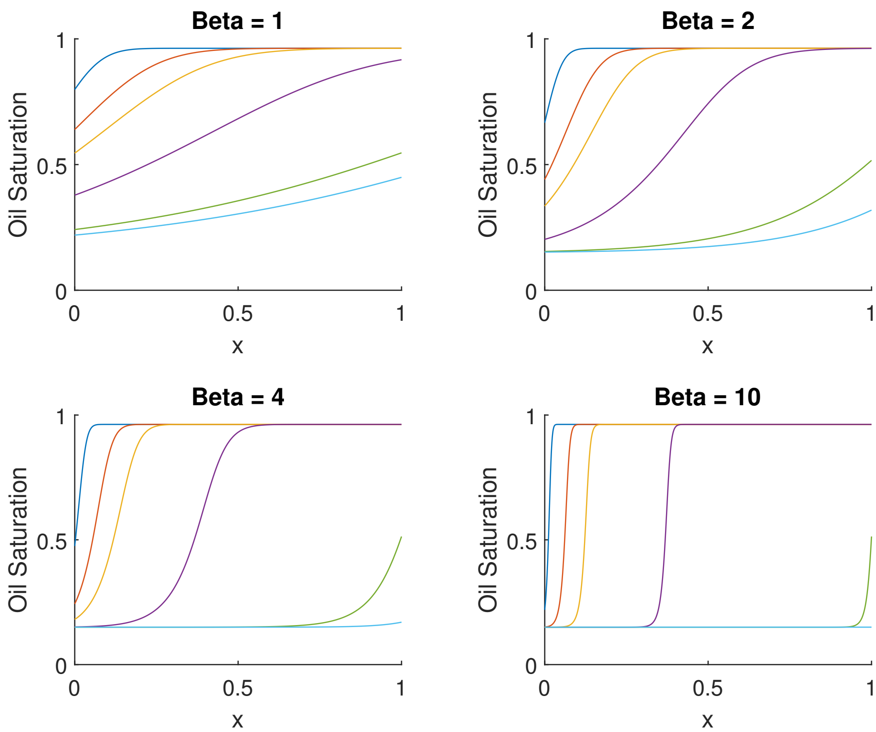

Figure 2: The semi analytical solution, plotted from $x = 0$ to $x = 1$ at $t = 0.01, 0.05, 0.1, 0.3, 0.8125, 1$, for $\beta = 1, 2, 4, 10$.

4 Numerical Solution

For a numerical approach, the Finite Volume Method and Newton's method were used to formulate a problem which can be solved numerically in order to approximate the real values of the PDE at discrete points in space and time.

4.1 Equivalent Boundary Conditions

5 Analysis and Findings

5.1 Comparison of semi-analytical solution and numerical solution

5.2 Average reservoir oil saturation

5.3 Computational efficiency analysis

5.4 Buckley-Leverett solution analysis

For large β, the capillary forces are negligible when compared to the viscous forces. As a result, the method of characteristics can be used to obtain a simplified solution. The solution exhibits Buckley-Leverett shock front behaviour given by

$$S(x,t) = \begin{cases} S_{or}, & x < vt, \\ 1 - S_{wr}, & x > vt. \end{cases}, \quad \text{where} \quad v = \frac{1}{1 - S_{wr} - S_{or}} \tag{17}$$

This is implemented in MATLAB as the function `BuckleyLeverett.m`. Plotting the Buckley-Leverett shock front against the semi-analytical solution with $\beta = 10$, it can be seen that the semi-analytical solution does exhibit the shock front behaviour:

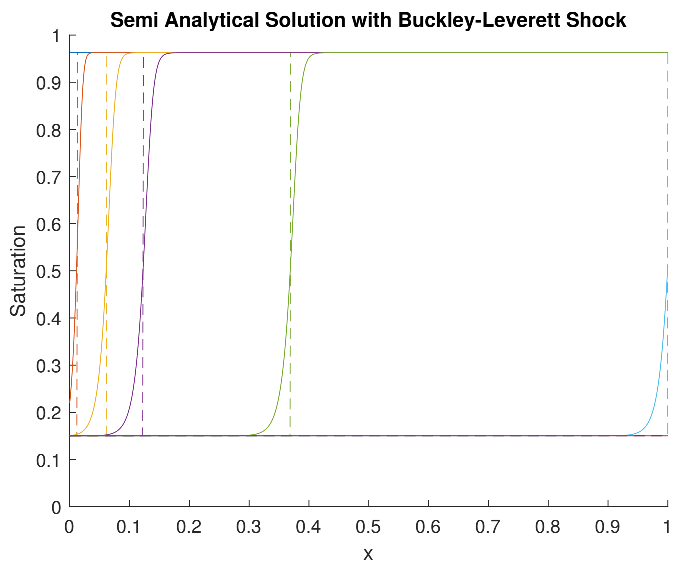

Figure 3: The semi-analytical solution with $\beta = 10$ plotted against the Buckley-Leverett shock front.

Similarly for the numerical solution:

As has been observed in the plots previously, increases of β correspond to increases of the slope of the front between the phases. As such, the shock front behaviour of becoming perfectly vertical as $\beta \to \infty$ is reasonable.

5.5 Impact of different injection rates

6 Conclusions

7 References