MXB326 Group Project - Reservoir Simulation

MXB326_23se1 Computational Methods 2
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Contents

1 Introduction

Oil is a natural resource which is used abundantly in modern society. Necessary for the creation of fuels which transport us around the world, to materials in products we use hundreds of times a day, and many applications in between, it is no stretch to say that civilisation as we know it relies on oil. But the production of oil can be a slow and costly process. One method commonly used in the oil industry to improve production is called water-flooding. This processes uses the injection of water into the porous rock structure of an oil reservoir to increase the pressure, driving more oil out. Modelling this process mathematically is a key interest of a field of research known as reservoir engineering, and is of vital importance to ensure that the process of water-flooding runs smoothly.

This modelling, involving the simultaneous flow of two viscous fluids through a horizontal reservoir in three dimensions, is quite complex. Fortunately, the model can be substantially simplified when considering a homogeneous, one-dimensional reservoir. This simplification in turn reduces what would be a coupled system of partial differential equations into a single, one-dimensional PDE. To describe the mechanics of the system, the conservation of mass of oil and water assuming immiscibility and incompressibility, are given by the transport equations:

$$\phi\frac{\partial S_w}{\partial t} + \frac{\partial q_w}{\partial x} = 0 \tag{1}$$

$$\phi\frac{\partial S_o}{\partial t} + \frac{\partial q_o}{\partial x} = 0 \tag{2}$$

In this system, ϕ indicates how porous the medium of the reservoir is, and S_w and q_w, S_o and q_o indicate the saturation and flow rate of the water and oil respectively. We relate the pressure in each fluid as the capillary pressure: $P_c = p_o - p_w$, and assume that the capillary pressure is positive. Furthermore, the saturations of the oil and water can be related as $S_w + S_o = 1$, allowing a single saturation, $S = S_o$ to be the sole variable to be modelled. Also defined are the minimum, irreducible saturations S_{or} and S_{wr}.

For a comprehensive description of the mathematical system, it is also necessary to define the conditions at the boundaries of the domain. Figure 1 demonstrates the domain of the problem.

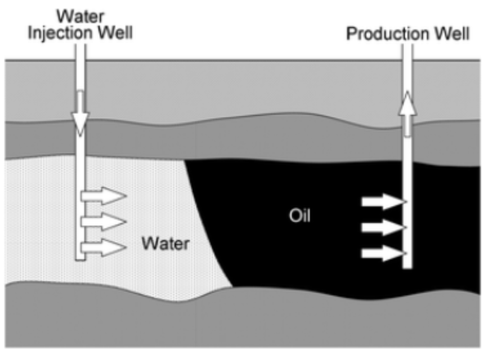

Figure 1: Schematic of the oil injection process.

Assuming a constant injection of water at $x = 0$ at rate q:

$$q_w(0,t) = q \quad \text{and} \quad q_o(0,t) = 0 \tag{3}$$

And assuming oil saturation begins at its maximum value:

$$S(x,0) = 1 - S_{wr} \tag{4}$$

Finally, assuming a semi-infinite reservoir:

$$\lim_{x\to\infty} S(x,t) = 1 - S_{wr} \tag{5}$$

For an efficient and cost-effective water-flooding operation, it is critical to prevent the water from mixing with the oil in the well-bore (extraction point). We thus define the breakthrough time as $T_B = 1 - S_{or} - S_{wr}$, the time taken for the water to reach the well-bore during the water-flooding. Specification of a suitable injection rate q is capable of preventing this issue from arising.

This report contains a pilot study of the simplified system above, including a mathematical model with a semi-analytic solution and a numerical solution. These solutions will be analysed thoroughly, with the goal of demonstrating the capability of a fully detailed study in the future.

2 Mathematical Model

It is possible to derive an exact solution for the system described above using methods from *Fokas and Yortsos*, in the case of constant injection rates when the water to oil viscosity ratio F has the form

$$F = \frac{1 - S_{wr} + \frac{\gamma}{\beta}}{S_{or} + \frac{\gamma}{\beta}} \tag{6}$$

Applying this, the following one-dimensional initial boundary value problem can be derived for the oil saturation:

$$\frac{\partial S}{\partial t} = \frac{\partial}{\partial x}\left[g(S)\frac{\partial S}{\partial x} + f(S)\right], \tag{7}$$

$$S(x,0) = 1 - S_{wr}, \tag{8}$$

$$\frac{\partial S}{\partial x}(0,t) = \frac{\alpha}{\beta}(\beta S(0,t) + \gamma) + \frac{\omega}{\beta}(\beta S(0,t) + \gamma)^2, \tag{9}$$

$$\lim_{x\to\infty} S(x,t) = 1 - S_{wr}. \tag{10}$$

By specifying $S_{wr}, S_{or}, F, \text{and} \beta$, the remaining parameters can be determined from (6) as well as the relations

$$\frac{\alpha}{\beta^2} = -\frac{(S_{or} + \frac{\gamma}{\beta})(1 - S_{wr} + \frac{\gamma}{\beta})}{1 - S_{wr} - S_{or}}, \tag{11}$$

$$\omega = \beta - \frac{\alpha}{\beta - \beta S_{wr} + \gamma}. \tag{12}$$

Furthermore, the functions $f(S)$ and $g(S)$, called the capillary-hydraulic properties of the fluid-porous system are given by

$$f(S) = \frac{\alpha}{\beta^2}\left[\frac{1}{1 - S_{wr} + \frac{\gamma}{\beta}} - \frac{1}{S + \frac{\gamma}{\beta}}\right], \quad g(S) = \frac{1}{(\beta S + \gamma)^2}.$$

These functions allow for an exactly solvable PDE (7), and also correspond to a physically meaningful model.

3 Semi-Analytical Solution

4 Numerical Solution

5 Analysis and Findings

6 Conclusions

7 References