

MXB326 Group Project - Reservoir Simulation

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1 Introduction

Oil is a natural resource which is used abundantly in modern society. Necessary for the creation of fuels which transport us around the world, to materials in products we use hundreds of times a day, and many applications in between, it is no stretch to say that civilisation as we know it relies on oil. But the production of oil can be a slow and costly process. One method commonly used in the oil industry to improve production is called water-flooding. This process uses the injection of water into the porous rock structure of an oil reservoir to increase the pressure, driving more oil out. Modelling this process mathematically is a key interest of a field of research known as reservoir engineering, and is of vital importance to ensure that the process of water-flooding runs smoothly.

This modelling, involving the simultaneous flow of two viscous fluids through a horizontal reservoir in three dimensions, is quite complex. Fortunately, the model can be substantially simplified when considering a homogeneous, one-dimensional reservoir. This simplification in turn reduces what would be a coupled system of partial differential equations into a single, one-dimensional PDE. To describe the mechanics of the system, the conservation of mass of oil and water assuming immiscibility and incompressibility, are given by the transport equations:

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial q_w}{\partial x} = 0 \quad (1)$$

$$\phi \frac{\partial S_o}{\partial t} + \frac{\partial q_o}{\partial x} = 0 \quad (2)$$

In this system, ϕ indicates how porous the medium of the reservoir is, and S_w and q_w , S_o and q_o indicate the saturation and flow rate of the water and oil respectively. We relate the pressure in each fluid as the capillary pressure: $P_c = p_o - p_w$, and assume that the capillary pressure is positive. Furthermore, the saturations of the oil and water can be related as $S_w + S_o = 1$, allowing a single saturation, $S = S_o$ to be the sole variable to be modelled. Also defined are the minimum, irreducible saturations S_{or} and S_{wr} .

For a comprehensive description of the mathematical system, it is also necessary to define the conditions at the boundaries of the domain. Figure 1 demonstrates the domain of the problem.

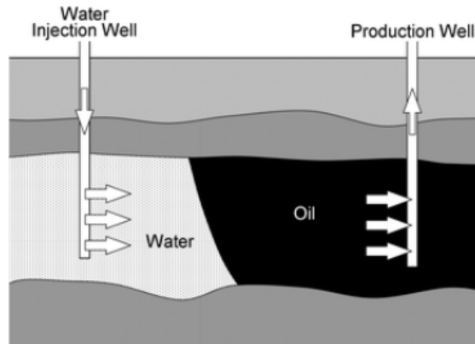


Figure 1: Schematic of the oil injection process.

Assuming a constant injection of water at $x = 0$ at rate q :

$$q_w(0, t) = q \quad \text{and} \quad q_o(0, t) = 0 \quad (3)$$

And assuming oil saturation begins at its maximum value:

$$S(x, 0) = 1 - S_{wr} \quad (4)$$

Finally, assuming a semi-infinite reservoir:

$$\lim_{x \rightarrow \infty} S(x, t) = 1 - S_{wr} \quad (5)$$

For an efficient and cost-effective water-flooding operation, it is critical to prevent the water from mixing with the oil in the well-bore (extraction point). We thus define the breakthrough time as $T_B = 1 - S_{or} - S_{wr}$, the time taken for the water to reach the well-bore during the water-flooding. Specification of a suitable injection rate q is capable of preventing this issue from arising.

This report contains a pilot study of the simplified system above, including a mathematical model with a semi-analytic solution and a numerical solution. These solutions will be analysed thoroughly, with the goal of demonstrating the capability of a fully detailed study in the future.

2 Mathematical Model

It is possible to derive an exact solution for the system described above using methods from *Fokas and Yortsos*, in the case of constant injection rates when the water to oil viscosity ratio F has the form

$$F = \frac{1 - S_{wr} + \frac{\gamma}{\beta}}{S_{or} + \frac{\gamma}{\beta}} \quad (6)$$

Applying this, the following one-dimensional initial boundary value problem can be derived for the oil saturation:

$$\frac{\partial S}{\partial t} = \frac{\partial}{\partial x} \left[g(S) \frac{\partial S}{\partial x} + f(S) \right], \quad (7)$$

$$S(x, 0) = 1 - S_{wr}, \quad (8)$$

$$\frac{\partial S}{\partial x}(0, t) = \frac{\alpha}{\beta}(\beta S(0, t) + \gamma) + \frac{\omega}{\beta}(\beta S(0, t) + \gamma)^2, \quad (9)$$

$$\lim_{x \rightarrow \infty} S(x, t) = 1 - S_{wr}. \quad (10)$$

By specifying S_{wr} , S_{or} , F , and β , the remaining parameters can be determined from (6) as well as the relations

$$\frac{\alpha}{\beta^2} = -\frac{(S_{or} + \frac{\gamma}{\beta})(1 - S_{wr} + \frac{\gamma}{\beta})}{1 - S_{wr} - S_{or}}, \quad (11)$$

$$\omega = \beta - \frac{\alpha}{\beta - \beta S_{wr} + \gamma}. \quad (12)$$

Using these equations, γ can be found with equation (6), α with γ and equation (11), and ω using the other two parameters with equation (12). This process has been implemented in the function `paramsFunc`.

Furthermore, the functions $f(S)$ and $g(S)$, called the capillary-hydraulic properties of the fluid-porous system are given by

$$f(S) = \frac{\alpha}{\beta^2} \left[\frac{1}{1 - S_{wr} + \frac{\gamma}{\beta}} - \frac{1}{S + \frac{\gamma}{\beta}} \right], \quad g(S) = \frac{1}{(\beta S + \gamma)^2}.$$

These functions allow for an exactly solvable PDE (7), and also correspond to a physically meaningful model.

3 Semi-Analytical Solution

A semi-analytical solution for (7) can be derived at discrete space x_i and time t_n by applying the spacial transformation

$$\tilde{x} = \int_0^x (\beta S(\xi, t) + \gamma) d\xi + \omega t, \quad (13)$$

This linearises the equation and yields the solution for S :

$$S(x, t) = \frac{1}{\beta} \left[\frac{\alpha e^{\alpha x}}{\frac{\partial \phi}{\partial \tilde{x}}(\tilde{x}, t)} - \gamma \right], \quad (14)$$

where \tilde{x} is the solution of $\phi(\tilde{x}, t) = e^{\alpha x}$ for a given x and t . The solution of the nonlinear equation must be obtained using MATLAB's `fzero` function. The MATLAB documentation for this function indicates that it is used to find the roots of nonlinear function. Specifically, for some input $\mathbf{x} = \mathbf{fzero}(\mathbf{func}, \mathbf{x0})$, \mathbf{x} will return a point where $\mathbf{func}(\mathbf{x}) = 0$. The `fzero` function requires an initial guess x_0 such that $\mathbf{func}(x_0) \approx 0$. An appropriate guess for each $i = 1..N$ and $n = 1..M$ can be found by evaluating (13) at $x = x_i$ and $t = t_{n-1}$. Let \hat{x} indicate the approximate initial guess. Thus we have:

$$\hat{x}_{i,n} = (\beta S(x_i, t_{n-1}) + \gamma) x_i + \omega t_{n-1} \quad (15)$$

This results in an iterative process for the semi-analytical solution: for each t_n , solve for all x_i using $S(x_i, t_{n-1})$. For the initial guess at $t = t_1$, the initial condition (8) is applied. This provides \hat{x} for all x_i at $t = t_1$:

$$\hat{x}_{i,1} = [\beta(1 - S_{wr}) + \gamma] x_i \quad (16)$$

With an iterative solution defined and appropriate initial conditions, the solution can be straightforwardly determined using MATLAB. This solution is implemented in the MATLAB script `semianalytic.m`, and produces the following result:

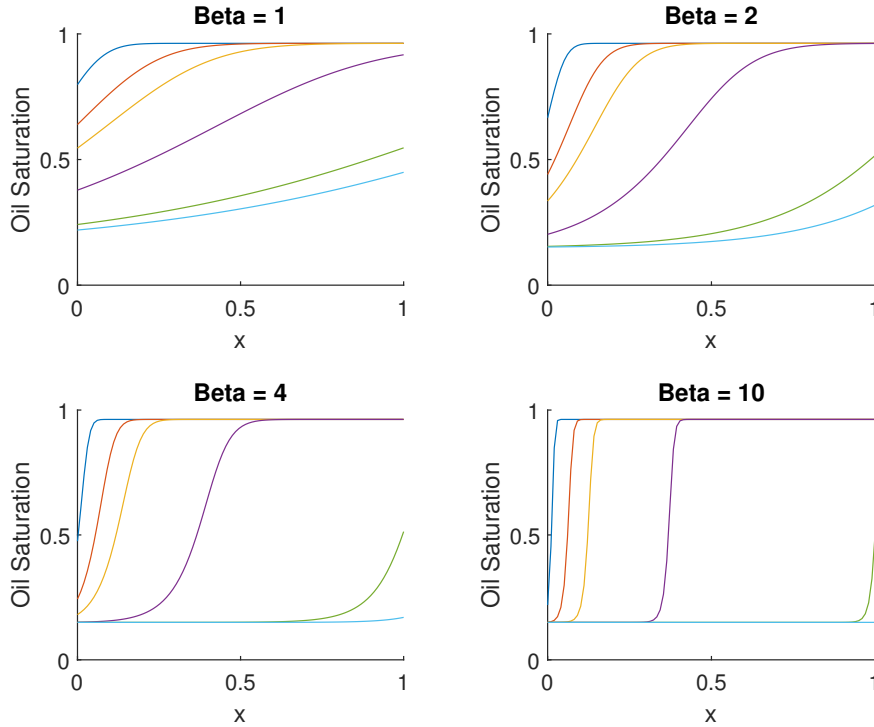


Figure 2: The semi analytical solution, plotted from $x = 0$ to $x = 1$ at $t = 0.01, 0.05, 0.1, 0.3, 0.8125, 1$, for $\beta = 1, 2, 4, 10$.

- 4 Numerical Solution
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