Baroclinic instability with topography

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Abstract

We study the effect of bottom topography on the vertical mode structure in a two-layer quasigeostrophic model.

1 The 2-layer case

Consider a two layer fluid. The fluid's resting heights are H_1 and H_2. The total fluid depth is $H - h(x, y)$, where $H \stackrel{\text{def}}{=} H_1 + H_2$. The QGPV equations for each layer are:

$$q_{1t} + \mathsf{J}(\psi_1, q_1) + \beta\psi_{1x} = 0\,, \tag{1a}$$
eq:q1t

$$q_{2t} + \mathsf{J}(\psi_2, q_2 + \underbrace{\frac{f_0 h}{H_2}}_{\stackrel{\text{def}}{=}\eta}) + \beta\psi_{2x} = 0\,, \tag{1b}$$
eq:q2t

with

$$q_1 \stackrel{\text{def}}{=} \triangle\psi_1 + F_1(\psi_2 - \psi_1)\,, \tag{2a}$$
eq:q1def

$$q_2 \stackrel{\text{def}}{=} \triangle\psi_2 + F_2(\psi_1 - \psi_2)\,, \tag{2b}$$
eq:q2def

Now assume that there is a zonal mean flow $U_j(y)$ in each layer, i.e., $\psi_j \mapsto \psi_j - \int^y U_j(y')\,\mathrm{d}y'$. This implies:

$$q_1 \mapsto q_1 - U_{1y} + F_1 \int^y \left[U_2(y') - U_1(y')\right] \mathrm{d}y'\,, \tag{3a}$$

$$q_1 \mapsto q_2 - U_{2y} - F_2 \int^y \left[U_1(y') - U_2(y')\right] \mathrm{d}y'\,, \tag{3b}$$

and the EOM become:

$$q_{1t} + U_1 q_{1x} + \mathsf{J}(\psi_1, q_1) + \left[\beta - U_{1yy} - F_1(U_2 - U_1)\right]\psi_{1x} = 0\,, \tag{4a}$$
eq:Uq1t

$$q_{2t} + U_2(q_{2x} + \eta_x) + \mathsf{J}(\psi_2, q_2 + \eta) + \left[\beta - U_{2yy} - F_2(U_1 - U_2)\right]\psi_{2x} = 0\,. \tag{4b}$$
eq:Uq2t

After linearizing:

$$q_{1t} + U_1 q_{1x} + \left[\beta - U_{1yy} - F_1(U_2 - U_1)\right]\psi_{1x} = 0\,, \tag{5a}$$
eq:linUq1t

$$q_{2t} + U_2(q_{2x} + \eta_x) + \mathsf{J}(\psi_2, \eta) + \left[\beta - U_{2yy} - F_2(U_1 - U_2)\right]\psi_{2x} = 0\,. \tag{5b}$$
eq:linUq2t

eqs:lin2layer

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Parameters

Domain size: $L_x = L_y = 2$
$F_1 = 25$, $F_2 = 25/4$, $\beta = 0$
$U_1(y) = \operatorname{sech}(5y)^2$, $U_2(y) = 0$
$\eta = 10\cos(10\pi x)\cos(10\pi y)$

We want to study the stability of the imposed $U(y)$ with topography. The idea is that you start with some random, small-amplitude q_1, q_2 and then you either *(i)* integrate forward the linearized equation until the most unstable mode pops up or *(ii)* you integrate forward the nonlinear equations but every some prescribed Δt you rescale q_1, q_2 so that they remain small-amplitude and nonlinear terms won't matter much.

The two methods should give the same results.

You can also compute the growth rate by looking how much the energy of, e.g., q_1 grows for time Δt as:

$$\text{growth rate}_j = \frac{\log\left(\dfrac{\text{KE after } \Delta t}{\text{KE initially}}\right)}{2\Delta t} \tag{6}$$

As the structure of q_1, q_2 converges to the unstable mode the value of growth rate_j will converge to the corresponding growth rate.