Boost Converter Project

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1 Analytical Calculations

1.1 Inductor Resistance

Since our physical inductor is not ideal, we need to measure the resistance of it for our analytical simulation.

$$R_L = 52\Omega \tag{1}$$

1.2 Duty cycle and R_3

Our target duty cycle was 90%. Using the duty cycle equation:

$$D = \frac{T_{on}}{T_{total}} = \frac{R_A + R_B}{R_A + 2R_B} \tag{2}$$

To find a value for R_3, we find the lower and upper bound by setting R_{pot} to 0Ω and $10\text{k}\Omega$. The lower bound value for R_3 is 0Ω To find the upper bound value, we solve for R_B, where $R_A = R_{pot} = 10k\Omega$ and $R_B = R_3$

$$0.9 = \frac{10\text{k} + R_3}{10\text{k} + 2R_3} \tag{3}$$

$$0.9(10\text{k} + 2R_3) = 10\text{k} + R_3 \tag{4}$$

$$9\text{k} + 1.8R_3 = 10\text{k} + R_3 \tag{5}$$

$$0.8R_3 = 1\text{k} \;\Rightarrow\; R_3 = 1.25\text{k}\Omega \tag{6}$$

Since the calculated upper bound value of R_3 is $1.25k\Omega$, the closest lab kit value is $1.2k\Omega$.

$$\frac{10\text{k} + 1.2\text{k}}{10\text{k} + 2(1.2\text{k})} = 0.903 \approx 90\% \text{ duty cycle} \tag{7}$$

Therefore, for our analytical simulation, we chose the $1.2\text{k}\Omega$ resistor.

1.3 Timing capacitor C_2

We use the frequency equation to find the upper and lower bound values of the timing capacitor, C_2.

$$f = \frac{1}{T_{\text{total}}}, \Rightarrow T_{\text{total}} = 0.693\,(R_{pot} + 2R_3)\,C_2 \Rightarrow C_2 = \frac{1}{f\,0.693\,(R_{pot} + 2R_3)} \tag{8}$$

1.3.1 Lower Bound

$$C_2 = \frac{1}{(500)(0.693)(0 + 2(1.2\text{k}))} \approx 1.2 \times 10^{-6} F = 1.2\mu F \tag{9}$$

1.3.2 Upper Bound

$$C_2 = \frac{1}{(500)(0.693)(10\text{k} + 2(1.2\text{k}))} \approx 2.33 \times 10^{-7} F = 2.33\mu F \tag{10}$$

For our analytical simulation, we chose the $0.47\mu F$ capacitor from the lab kit.

1.4 Potentiometer Resistance

We want to find the resistance that the potentiometer should be set at for 500 Hz PWM signal with our chosen values: $R_3 = 1.2\text{k}\Omega$, $C_2 = 0.47\mu F$. We use the frequency equation:

$$f = \frac{1}{(0.693)(R_{pot} + 2(R_3))(C_2)} \Rightarrow R_{pot} + 2R_3 = \frac{1}{(0.693)(f)(C_2)} \tag{11}$$

$$R_{pot} + 2R_3 \approx 6140.43\Omega \Rightarrow R_{pot} \approx 3740.43\Omega \tag{12}$$

The resistance of the potentiometer should be set to 3740Ω for a 500 Hz PWM signal with R_3= $1.2\text{k}\Omega$ and C_2=$0.47 \times 10^{-6}\mu$F.

1.5 Energy storage capacitor C_3

We found the value of C_3 by calculating the amount of current that load draws during each cycle of the PWM signal.

$$I_{load} = \frac{V_C - V_{LED}}{R_4} = \frac{12 - 2}{2.2\text{k}} \approx 4.55\text{mA} \tag{13}$$

$$T = (0.693)(R_{pot} + 2(R_3))(C_3) \approx 2\text{ms} \tag{14}$$

$$\Delta V = 12 \times 0.01 = 0.12\text{V} \tag{15}$$

$$\Delta V = \frac{\Delta Q}{C_3} = \frac{(I_{load})(T)}{C_3} \Rightarrow C_3 = \frac{(I_{load})(T)}{\Delta V} = \frac{(4.55\text{m})(2\text{m})}{0.12} \approx 7.58 \times 10^{-6} F = 75.8\mu F \tag{16}$$

For our analytical simulation, we chose the $100\mu F$ capacitor from our lab kit.

1.6 Calculation of Inductor Energy

To verify that the inductor stores enough energy to power the boost converter, we solve the inductor differential equation over each PWM interval (ON and OFF) and compute the steady-state current ripple and energy transfer per cycle.

ON Interval (MOSFET Closed)

Including the MOSFET voltage drop v_{DS}, the inductor satisfies

$$L\frac{di}{dt} + R_{L1}\, i(t) = V_{in} - v_{DS}, \qquad 0 < t < T_{\text{on}}. \tag{17}$$

The steady-state current level for the ON interval is

$$K_{\text{on}} = \frac{V_{in} - v_{DS}}{R_{L1}}. \tag{18}$$

OFF Interval (MOSFET Open)

During the OFF interval, the diode conducts and

$$L\frac{di}{dt} + R_{L1}\, i(t) = V_{in} - V_{out}, \qquad T_{\text{on}} < t < T. \tag{19}$$

The corresponding steady-state current is

$$K_{\text{off}} = \frac{V_{in} - V_{out}}{R_{L1}}. \tag{20}$$

General Solution

For any interval of duration τ, the inductor current is

$$i(\tau) = i_0 e^{-a\tau} + K\left(1 - e^{-a\tau}\right), \tag{21}$$

where

$$a = \frac{R_{L1}}{L}. \tag{22}$$

Given Values

$$\begin{aligned} L &= 33 \text{ mH}, & R_{L1} &= 52\ \Omega, & (23)\\ V_{in} &= 5 \text{ V}, & V_{out} &= 12 \text{ V}, & (24)\\ f &= 500 \text{ Hz}, & D &= 0.9. & (25) \end{aligned}$$

$$\begin{aligned} T &= \frac{1}{f} = 0.002 \text{ s}, & (26)\\ T_{\text{on}} &= DT = 0.0018 \text{ s}, & (27)\\ T_{\text{off}} &= T - T_{\text{on}} = 0.0002 \text{ s}, & (28)\\ a &= \frac{52}{0.033} = 1575.758 \text{ s}^{-1}. & (29) \end{aligned}$$

MOSFET Voltage Calculation

The MOSFET drain–source voltage is computed using

$$v_{DS} = \left(v_{GS} - V_t + \frac{1}{R_D K_n}\right) \pm \sqrt{\left(v_{GS} - V_t + \frac{1}{R_D K_n}\right)^2 - \frac{2V_{DD}}{R_D K_n}}. \tag{30}$$

Given:

$$V_t = 1.824\ \text{V}, \qquad V_{DD} = 5\ \text{V}, \tag{31}$$
$$K_n = 0.1233\ \text{A/V}^2, \qquad R_D = 52\ \Omega, \tag{32}$$
$$v_{GS} = 5\ \text{V}, \tag{33}$$

the resulting on-state voltage drop is

$$v_{DS} \approx 0.243\ \text{V}. \tag{34}$$

Thus the corrected ON steady-state current is

$$K_{\text{on}} = \frac{5 - 0.243}{52} = 0.09148\ \text{A}. \tag{35}$$

The OFF steady-state current is unchanged:

$$K_{\text{off}} = -0.13462\ \text{A}. \tag{36}$$

Steady-State Inductor Currents

Let i_{min} be the current at the start of the ON interval and i_{max} the current at the end:

$$i_{\text{max}} = i_{\text{min}} e^{-aT_{\text{on}}} + K_{\text{on}}\left(1 - e^{-aT_{\text{on}}}\right), \tag{37}$$
$$i_{\text{min}} = i_{\text{max}} e^{-aT_{\text{off}}} + K_{\text{off}}\left(1 - e^{-aT_{\text{off}}}\right). \tag{38}$$

Solving the system yields:

$$i_{\text{min}} \approx 0.0310\ \text{A}, \tag{39}$$
$$i_{\text{max}} \approx 0.09233\ \text{A}. \tag{40}$$

Inductor Energy per Cycle

The inductor releases the energy associated with the current drop during OFF:

$$E_L = \frac{1}{2} L \left(i_{\text{max}}^2 - i_{\text{min}}^2\right) \approx 1.248 \times 10^{-4}\ \text{J/cycle}. \tag{41}$$

Load Energy Requirement

$$E_{\text{load}} = \frac{V_{out} I_{load}}{f} = \frac{12(4.545\ \text{mA})}{500} \approx 1.091 \times 10^{-4}\ \text{J/cycle}. \tag{42}$$

Conclusion

$$E_L > E_{\text{load}}, \tag{43}$$

so the inductor stores and delivers enough energy each PWM cycle to sustain the 12 V output even after including the MOSFET voltage drop. This satisfies the requirement to set up and solve the inductor differential equations for both intervals and compute the cycle-by-cycle energy transfer.

2 Numerical KiCad Verification

Using the calculated values where we chose lab kit values, we simulated our circuit in KiCad.

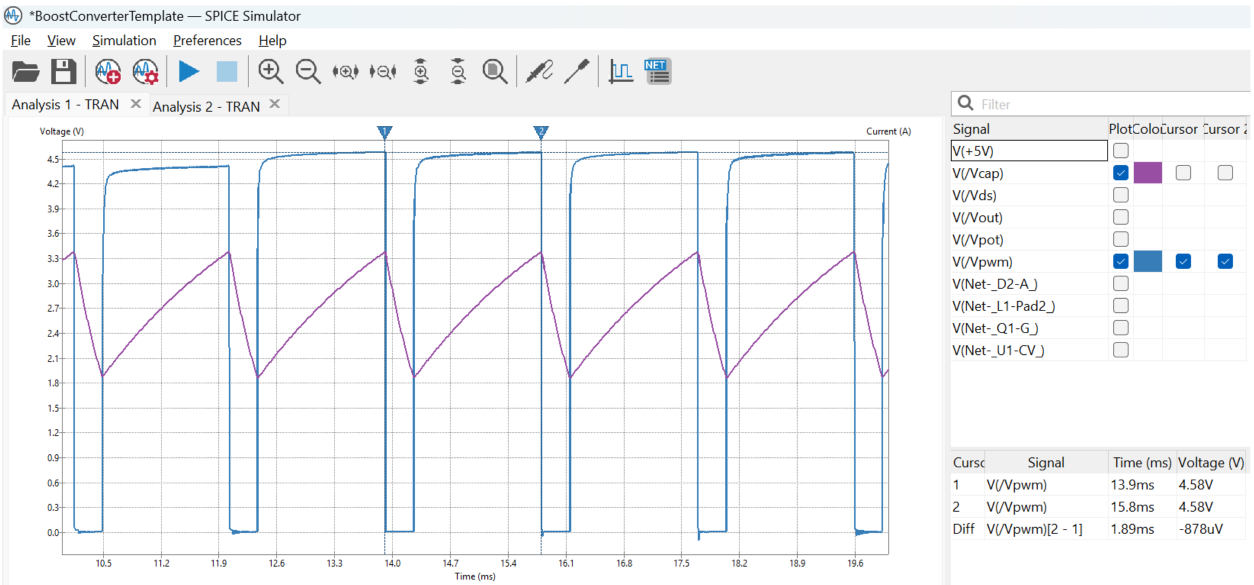

Figure 1: Plot of PWM Signal at 500 Hz

The total period was 1.89 ms. This gave us a PWM signal of $\approx$ 529 Hz. The period of one on period was 1.59 ms, giving us a duty cycle of $\approx 84\%$.

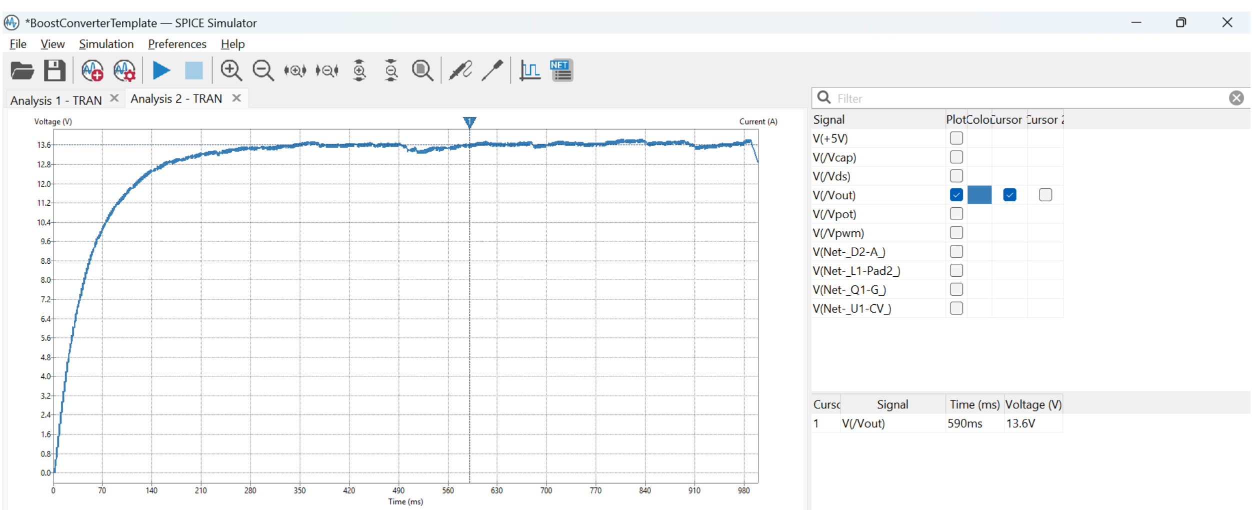

Figure 2: Plot of Output Voltage with PWM Signal at 500 Hz

3 Experimental Verification

3.1 Introduction

This section details the experimental verification of the assembled PCB to determine how well the final hardware meets the design requirements. Measurements of the 500 Hz PWM signal and the corresponding output voltage are presented, followed by a comparison to the expected design values and a brief discussion of any discrepancies.

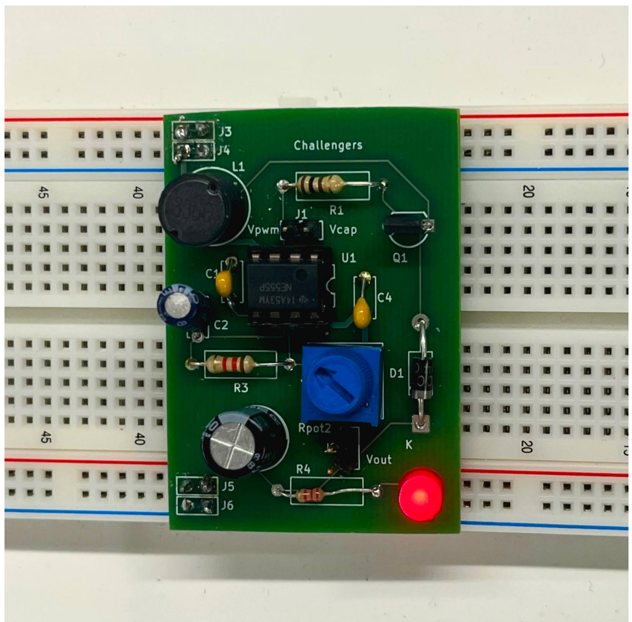

Figure 3: Finished PCB

3.2 Results

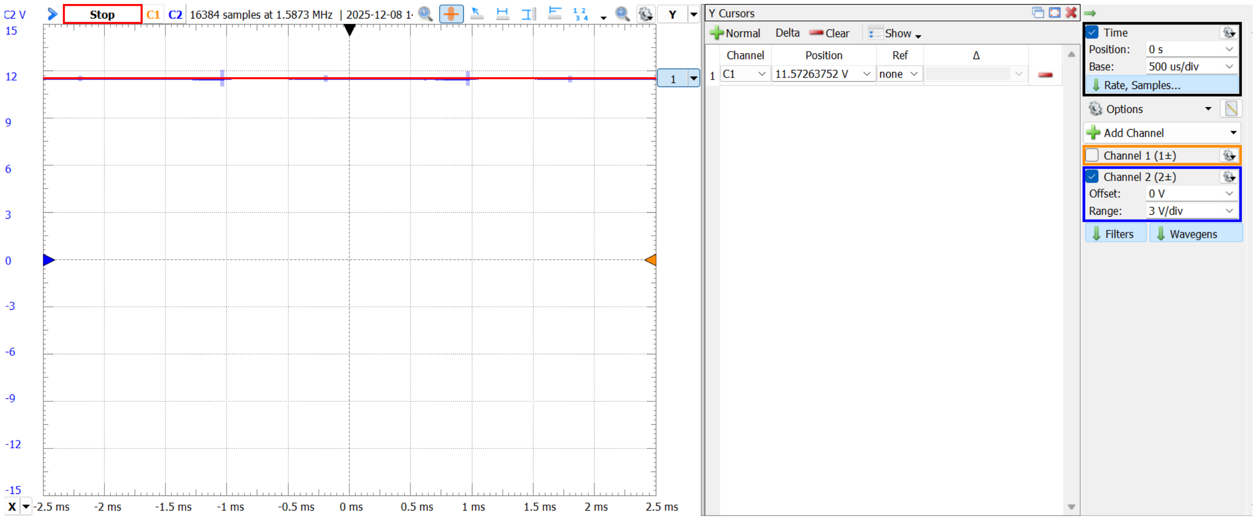

Figure 4: Experimental Vout

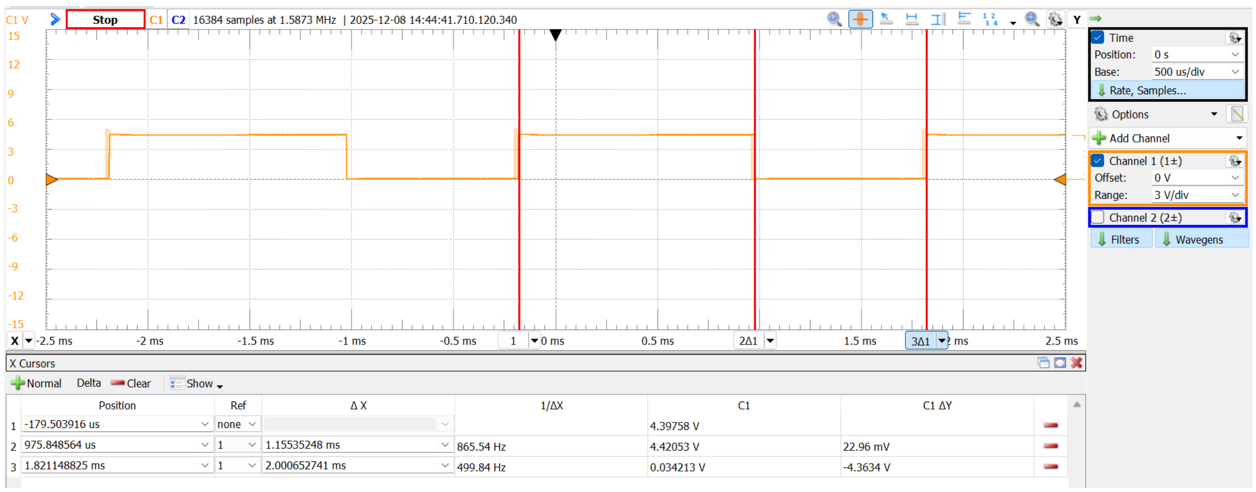

Figure 5: Experimental Vpwm

To achieve the results shown in the graphs, we adjusted the potentiometer until Vpwm produced a square wave with a frequency of 500.06 Hz, shown in orange. For our analytical design, we used $C_2 = 0.47\mu$F instead of the 1.0μF that we used on our PCB. The larger capacitor lowered the frequency of the 555 timer, but we compensated by readjusting the potentiometer. The duty cycle of this wave, calculated by dividing the pulse width (1.1554 ms) by the period (2.0007 ms), is 57.74%. Although this measurement falls within the design specifications of a duty cycle of 50 to 90%, it is significantly different from our calculated duty cycle of 90%. This difference can come from several sources: the real value of the potentiometer and resistors can be a few percent off from their labeled values, the 555 timer does not behave exactly like the ideal equations we used in our calculations, and the rising and falling edges of the PWM signal are not perfectly sharp, which can change how the oscilloscope measures the pulse width. The Vpwm pin is also very temperamental: it only read when pressure was applied to the side in a specific way. It is possible that the pin has a higher margin of error because of this tendency. Despite this technical difficulty, we still obtained precise, consistent readings from the pin. In addition, it is easy to slightly over- or under-adjust the potentiometer when doing this by hand, so the actual duty cycle we set on the board may not match the calculated ideal value. We then measured Vout at this frequency and found it to be 11.5726 V, as shown by the blue trace. This value is below the design specifications of a 12 V with 1% error, but only by a small margin. The error could also be caused by tolerances in the components, as their values might not exactly match their listed values. The MOSFET and diode each have a voltage drop, which reduces the maximum boost the circuit can provide. The output capacitor is also not ideal, so there is some loss, and the LED forward voltage and resistor tolerances in the load may not match the values we assumed in our design calculations. The oscilloscope used to read Vout has its own measurement error, which can slightly shift the reported voltage away from the ideal 12 V. The lower voltage might also have been caused by a faulty L1 inductor, as the LED initially would not turn on if the inductor was not held down. After some adjusting, the LED lit up without any force, but it is possible that the inductor is still not behaving as it should, reducing its output energy and therefore lowering the output voltage.

4 Collaboration Statement

Contributions of each team member for the team Challengers

Christine Han: Did analytical work (Milestone 1) and calculation of inductor energy, assisted with experimental verification

Kasie Nguyen: Designed PCB used, recalculated and verified analytical work, simulated design in KiCad, soldered components onto PCB

Eva Woodcock: Performed experimental verification and debugging on PCB, analyzed the experimental graphs of output voltage and PWM signal.