Tutorial questions-8

1. Calculate the determinant of the matrix:

а) $\begin{vmatrix} 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 5 & -1 & 0 \\ 2 & -3 & 7 & 2 & 3 \\ 1 & 0 & -4 & 2 & 0 \\ -1 & 2 & 0 & -1 & 2 \end{vmatrix}$;

б) $\begin{vmatrix} 3 & -2 & 1 & 1 \\ 0 & 1 & 2 & 0 \\ -1 & 2 & 4 & 2 \\ 1 & -3 & 0 & 0 \end{vmatrix}$;

2. Find the matrix X

$$a) \begin{pmatrix} -1 & 2 \\ -2 & 3 \\ 4 & 4 \end{pmatrix} + 2X = \begin{pmatrix} 5 & 2 \\ -2 & 5 \\ 2 & 2 \end{pmatrix}$$

$$b) \begin{pmatrix} 1 & -3 & -2 \\ 3 & 1 & -2 \\ -3 & 2 & 1 \end{pmatrix} + 3X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3. Consider a linear system whose augmented matrix is of the form

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 3 & 0 \\ -1 & 1 & \beta & 0 \end{array}\right]$$

For what values of β will the system have infinitely many solutions?

4. Let A be the matrix

$$\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

In each part, compute the given quantity.

(a) A^3

(b) A^{-3}

(c) $A^2 - 2A + I$

5. If the matrix

$$A = \begin{bmatrix} 6 & -9 & 1 \\ k & 24 & 4 \\ 15 & 18 & -11 \end{bmatrix}$$

has rank 2, what is the value of k?

6. Determine which values of k, if any, will give : a) a unique solution; b) no solution ; c) infinitely many solutions to the system

$$\begin{cases} x + y + kz = 2 \\ 3x + 4y + 2z = k \\ 2x + 3y - z = 1 \end{cases}$$

7. Solve the equation:

$$\begin{vmatrix} x & x & x \\ 7 & 4 & 5 \\ 2 & -1 & 0 \end{vmatrix} = 0,$$

8. **Solve the matrix equation.**

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} X = \begin{bmatrix} 6 & 6 \\ 2 & 4 \end{bmatrix}$$

9. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

and $(A + B)^2 = A^2 + B^2$ then the values of a and b are

10 If a ≠ b, b, c satisfy $\begin{vmatrix} a & 2b & 2c \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0$, then abc =