Tutorial questions- 6

Given the matrices A, B, C and vector $\underline{x}$.

$$A = \begin{pmatrix} 4 & 0 & -1 \\ 3 & 5 & 2 \\ -3 & 1 & 7 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & -2 & 1 \\ 0 & 3 & 4 \end{pmatrix}, C = \begin{pmatrix} 2 & -1 \\ 6 & 3 \end{pmatrix}, \underline{x} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

Evaluate : $(a)\ A^T$, $(b)\ B - A$, $(c)\ AB$, $(d)\ CB$, $(e)\ B\underline{x}$.

Find the matrix X

$$a) \begin{pmatrix} -1 & 2 \\ -2 & 3 \\ 4 & 4 \end{pmatrix} + 2X = \begin{pmatrix} 5 & 2 \\ -2 & 5 \\ 2 & 2 \end{pmatrix}$$

$$b) \begin{pmatrix} 1 & -3 & -2 \\ 3 & 1 & -2 \\ -3 & 2 & 1 \end{pmatrix} + 3X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Consider the equation

$$\begin{vmatrix} 1 & 0 & 0 \\ 5 & 2\sin x + \sqrt{2} & 0 \\ 2 & m & 1 + \cos 2x \end{vmatrix} = 0.$$

Determine all the possible values of x given that $0° \le x \le 360°$.

Find the value of x, if the matrix below is singular

$$A = \begin{bmatrix} 3 - x & 2 & 2 \\ 2 & 4 - x & 1 \\ -2 & -4 & -1 - x \end{bmatrix}$$

Let $f(t)=\begin{vmatrix}\cos t & t & 1\\ 2\sin t & t & 2t\\ \sin t & t & t\end{vmatrix}$, then $\lim_{t\to 0}\frac{f(t)}{t^2}$ is equal to

(a) 0
(b) -1
(c) 2
(d) 3

Let A be the matrix

$$\begin{bmatrix}2 & 0\\ 4 & 1\end{bmatrix}$$

In each part, compute the given quantity.

(a) A^3
(b) A^{-3}
(c) A^2-2A+I

For what value(s) of k is each of the matrices given below invertible?

a) $\begin{bmatrix}k & -1 & 4\\ 2 & 0 & 1\\ -1 & 0 & -1\end{bmatrix}$, b) $\begin{bmatrix}k & -1\\ -1 & 3\end{bmatrix}$

Which of the following matrices is invertible?

(a) $\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}$

(c) $\begin{bmatrix}0 & 3\\ 0 & 5\end{bmatrix}$

(b) $\begin{bmatrix}2 & 3\\ 3 & 5\end{bmatrix}$

(d) $\begin{bmatrix}4 & 2\\ 6 & 3\end{bmatrix}$

Solve the equation:

$$\begin{vmatrix}x & x & x\\ 7 & 4 & 5\\ 2 & -1 & 0\end{vmatrix}=0,$$

Consider a linear system whose augmented matrix is of the form

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ -1 & 4 & 3 & 2 \\ 2 & -2 & a & 3 \end{array}\right]$$

For what values of a will the system have a unique solution?

Consider a linear system whose augmented matrix is of the form

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 3 & 0 \\ -1 & 1 & \beta & 0 \end{array}\right]$$

(a) Is it possible for the system to be inconsistent? Explain.

(b) For what values of β will the system have infinitely many solutions?