Tutorial questions-2

1. Evaluate the determinant of a matrix:

$$a)\ A = \begin{pmatrix} 3 & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 3 & 4 \end{pmatrix} \qquad b)\ B = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 2 & 0 \\ 2 & 5 & -1 \end{pmatrix}$$

2. Without expansion, show that

$$\begin{vmatrix} 6 & 1 & 3 & 2 \\ -2 & 0 & 1 & 4 \\ 3 & 6 & 1 & 2 \\ -4 & 0 & 2 & 8 \end{vmatrix} = 0$$

3. Without expanding, find the value of the determinant

$$\begin{vmatrix} 8 & -3 & -2 \\ 7 & 1 & -8 \\ 24 & -9 & -6 \end{vmatrix}.$$

4. Consider the equation

$$\begin{vmatrix} 1 & 0 & 0 \\ 5 & 2\sin x + \sqrt{2} & 0 \\ 2 & m & 1 + \cos 2x \end{vmatrix} = 0.$$

Determine all the possible values of x given that $0° \leq x \leq 360°$.

5. Given that $n = \begin{vmatrix} 6 & -8 & 9 \\ 15 & -9 & -11 \\ -7 & 2 & -4 \end{vmatrix}$ and $m = \begin{vmatrix} 18 & -24 & 27 \\ 90 & -54 & -66 \\ -35 & 10 & -20 \end{vmatrix}$,

find a relation between m and n without expanding either determinant.

6. Evaluate the determinant of a matrix

$$\begin{bmatrix} 0 & 6 & -2 & -1 & 5 \\ 0 & 0 & 0 & -9 & -7 \\ 0 & 15 & 35 & 0 & 0 \\ 0 & -1 & -11 & -2 & 1 \\ -2 & -2 & 3 & 0 & -2 \end{bmatrix}$$

7. Find the value of x, if the matrix below is singular

$$\begin{bmatrix} 1 & 2 & x \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$$

8. Find the value of x, if the matrix below is singular

$$A = \begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$$

7. Use Cramer's Rule to solve the system of equations:

1) $-2x - 5y + 4z = 21$
$-5x - 5y + z = 21$
$-4y - 4z = 8$

2) $5x + y - 4z = -4$
$-3y - 6z = -21$
$-x - y - z = -6$

3) $-4x - 6z = -12$
$-6x - 4y - 2z = 6$
$-x + 2y + z = 9$

4) $4x - 4y + 2z = -14$
$4x + 2y = 14$
$-3y + z = -10$

Let $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$, **then** $\lim_{t \to 0} \frac{f(t)}{t^2}$ **is equal to**

(a) 0
(b) -1
(c) 2
(d) 3