Tutorial questions - 1

1. Given the matrices A, B, C and vector $\underline{x}$.

$$A = \begin{pmatrix} 4 & 0 & -1 \\ 3 & 5 & 2 \\ -3 & 1 & 7 \end{pmatrix}, \; B = \begin{pmatrix} 5 & -2 & 1 \\ 0 & 3 & 4 \end{pmatrix}, C = \begin{pmatrix} 2 & -1 \\ 6 & 3 \end{pmatrix}, \underline{x} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

Evaluate : $(a)\, A^T, (b)\, B - A, (c)\, AB, \; (d)\, CB, \; (e)\, B\underline{x}$.
If the result is not defined, explain why not.

2. Use Gaussian elimination to solve the system of equations

$$a) \begin{cases} x - 2y - z = 2 \\ x + y + z = 1 \\ 2x - y + 2z = 3 \end{cases} \qquad b) \begin{cases} x + 2y - z = 3 \\ 2x + 5y - 2z = 7 \\ -x + y + 5z = -12 \end{cases}$$

$$c) \begin{cases} 2x - 4y + 2z = 2 \\ x - 2y + z = 1 \\ x - 5y + 3z = 0 \end{cases}$$

3. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \; C = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix},$

$D = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$

a) Find the product of the sum of A and B and the difference between C and D.

b) Find the product of the difference between A and B and the sum of C and D.

4.For the matrices

$$A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} \quad and \quad B = \begin{pmatrix} -1 & 3 & 0 \\ 4 & 1 & -6 \end{pmatrix}$$

verify the equation $(AB)^T = B^T A^T$

5. Find the matrix B such that $A + B = C$, where

$$A = \begin{pmatrix} 2 & 0 \\ 1 & 4 \end{pmatrix} \quad and \quad C = \begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}$$

6. Given the matrix

$$A = \begin{pmatrix} 5 & -2 & 1 \\ 3 & 4 & 2 \end{pmatrix}$$

Calculate AA^T and A^TA.

7. Given the matrix

$$B = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}$$

Calculate B^3

8. Given that

$$3\begin{pmatrix} -1 & x & 2 \\ -3 & 1 & y \end{pmatrix} - \begin{pmatrix} 5 & 1 & z \\ -2 & 4 & 2 \end{pmatrix} = \begin{pmatrix} -8 & -1 & 3 \\ v & -1 & 10 \end{pmatrix}$$

Find the values x, y, z, v.

9. Calculate

$$(1 \quad 2 \quad 3) \cdot \begin{pmatrix} -1 & 3 \\ 4 & 2 \\ -2 & 5 \end{pmatrix} \cdot \begin{pmatrix} -15 \\ 1 \end{pmatrix}$$

10. Find the matrix X

$$a) \begin{pmatrix} -1 & 2 \\ -2 & 3 \\ 4 & 4 \end{pmatrix} + 2X = \begin{pmatrix} 5 & 2 \\ -2 & 5 \\ 2 & 2 \end{pmatrix}$$

$$b) \begin{pmatrix} 1 & -3 & -2 \\ 3 & 1 & -2 \\ -3 & 2 & 1 \end{pmatrix} + 3X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$