

Tutorial questions-5

1. Calculate the rank of the matrix by the Gaussian elimination method.

$$A = \begin{pmatrix} 1 & -4 & 2 & -1 \\ 3 & -12 & 6 & -3 \\ 2 & -1 & 0 & 1 \\ 0 & 1 & 3 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 & 0 & 7 \\ 1 & 0 & 1 & 3 \\ 3 & 2 & 7 & 7 \\ 3 & 2 & 7 & 7 \end{pmatrix}$$

2. If the matrix

$$A = \begin{bmatrix} 6 & -9 & 1 \\ k & 24 & 4 \\ 15 & 18 & -11 \end{bmatrix}$$

has rank 2, what is the value of k ?

3. Determine which values of k , if any, will give : a) a unique solution; b) no solution ; c) infinitely many solutions to the system

$$\begin{cases} x + y + kz = 2 \\ 3x + 4y + 2z = k \\ 2x + 3y - z = 1 \end{cases}$$

4. Solve the equation given by determinant:

$$\text{a) } \begin{vmatrix} x-1 & x & x+2 \\ 1 & 2 & 1 \\ 1 & x & 2 \end{vmatrix} = 0 \quad \text{b) } \begin{vmatrix} x & 1 & x+1 \\ 2 & x & 3 \\ x+1 & 4 & x \end{vmatrix} = -2x^2 + 11$$

5. Find the values of a that give : a) many solutions; b) no solutions ; c) unique solution

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2 - 14 & a + 2 \end{array} \right)$$

6. Determine all possible values of s and t for which the following system will have (i) no solution, (ii) a unique solution and (iii) infinitely many solutions.

In case of (ii) and (iii) find all the solutions.

$$3x - y + sz = 1$$

$$x + 3y + 2z = -t$$

$$x - 2y + 2z = 4$$

7. Find the inverse of the matrix

a) $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

b) $A = \begin{pmatrix} 1 & 1 & 1 \\ 6 & 5 & 4 \\ 13 & 10 & 8 \end{pmatrix}$