Tutorial questions-4

1. Which of the matrices that follow are in row echelon form? Which are in reduced row echelon form?

(a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ **(d)** $\begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ **(f)** $\begin{bmatrix} 1 & 4 & 6 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$

(g) $\begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 & 4 \\ 0 & 0 & 1 & 3 & 6 \end{bmatrix}$ **(h)** $\begin{bmatrix} 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2. The augmented matrices that follow are in row echelon form. For each case, indicate whether the corresponding linear system is consistent. If the system has a unique solution, find it.

(a) $\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{array}\right]$ **(b)** $\left[\begin{array}{cc|r} 1 & 3 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$

(c) $\left[\begin{array}{crr|c} 1 & -2 & 4 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(d) $\left[\begin{array}{crr|r} 1 & -2 & 2 & -2 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$

(e) $\left[\begin{array}{ccc|r} 1 & 3 & 2 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{array}\right]$

3. The augmented matrices that follow are in reduced row echelon form. In each case, find the solution set to the corresponding linear system.

(a) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right)$ **(b)** $\left(\begin{array}{ccc|c} 1 & 4 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right)$

(c) $\left(\begin{array}{ccc|c} 1 & -3 & 0 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right)$

(d) $\left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 5 \\ 0 & 0 & 1 & 3 & 4 \end{array}\right)$

(e) $\left(\begin{array}{cccc|c} 1 & 5 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

(f) $\left(\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$

4. For each of the systems in Exercise 3, make a list of the lead variables and a second list of the free variables.

5. For each of the systems of equations that follow, use Gaussian elimination to obtain an equivalent system whose coefficient matrix is in row echelon form. Indicate whether the system is consistent. If the system is consistent and involves no free variables, use back substitution to find the unique solution. If the system is consistent and there are free variables, transform it to reduced row echelon form and find all solutions.

(a) $\begin{aligned} x_1 - 2x_2 &= 3 \\ 2x_1 - x_2 &= 9 \end{aligned}$ **(b)** $\begin{aligned} 2x_1 - 3x_2 &= 5 \\ -4x_1 + 6x_2 &= 8 \end{aligned}$

(c) $\begin{aligned} x_1 + x_2 &= 0 \\ 2x_1 + 3x_2 &= 0 \\ 3x_1 - 2x_2 &= 0 \end{aligned}$ **(d)** $\begin{aligned} 3x_1 + 2x_2 - x_3 &= 4 \\ x_1 - 2x_2 + 2x_3 &= 1 \\ 11x_1 + 2x_2 + x_3 &= 14 \end{aligned}$

(e) $\begin{aligned} 2x_1 + 3x_2 + x_3 &= 1 \\ x_1 + x_2 + x_3 &= 3 \\ 3x_1 + 4x_2 + 2x_3 &= 4 \end{aligned}$ **(f)** $\begin{aligned} x_1 - x_2 + 2x_3 &= 4 \\ 2x_1 + 3x_2 - x_3 &= 1 \\ 7x_1 + 3x_2 + 4x_3 &= 7 \end{aligned}$

6. Use Gauss–Jordan reduction to solve each of the following systems.

(a)
$$\begin{aligned} x_1 + x_2 &= -1 \\ 4x_1 - 3x_2 &= 3 \end{aligned}$$

(b)
$$\begin{aligned} x_1 + 3x_2 + x_3 + x_4 &= 3 \\ 2x_1 - 2x_2 + x_3 + 2x_4 &= 8 \\ 3x_1 + x_2 + 2x_3 - x_4 &= -1 \end{aligned}$$

(c)
$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ x_1 - x_2 - x_3 &= 0 \end{aligned}$$

(d)
$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ 2x_1 + x_2 - x_3 + 3x_4 &= 0 \\ x_1 - 2x_2 + x_3 + x_4 &= 0 \end{aligned}$$

7. Consider a linear system whose augmented matrix is of the form

$$\left(\begin{array}{rrr|r} 1 & 2 & 1 & 1 \\ -1 & 4 & 3 & 2 \\ 2 & -2 & a & 3 \end{array}\right)$$

8. For what values of a will the system have a unique solution?

Consider a linear system whose augmented matrix is of the form

$$\left(\begin{array}{rrr|r} 1 & 2 & 1 & 0 \\ 2 & 5 & 3 & 0 \\ -1 & 1 & \beta & 0 \end{array}\right)$$

(a) Is it possible for the system to be inconsistent? Explain.

(b) For what values of β will the system have infinitely many solutions?

9. Consider a linear system whose augmented matrix is of the form

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & a & b \end{array}\right]$$

(a) For what values of a and b will the system have infinitely many solutions?

(b) For what values of a and b will the system be inconsistent?