**Tutorial questions- 11**

1. Let $\mathbf{u_1} = (3, -1, 2)$ and $\mathbf{u_2} = (3, 1, 5)$.

   (a) Express the vector $\mathbf{v} = (9, 11, 27)$ as a linear combination of $\mathbf{u_1}$ and $\mathbf{u_2}$ if possible.

   (b) Find $k$ such that the vector $\mathbf{w} = (-5, 4, k)$ is a linear combination of $\mathbf{u_1}$ and $\mathbf{u_2}$.

2. Let $\mathbf{a}_1 = \begin{bmatrix} 6 \\ 3 \\ 4 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 9 \\ -3 \\ 5 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 7 \\ 6 \\ h \end{bmatrix}$.

   (a) Find $h$ so that $\mathbf{b}$ is in $\mathrm{Span}\{\mathbf{a}_1, \mathbf{a}_2\}$.

   (b) For the $h$ that you found in the previous part, express $\mathbf{b}$ as a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$.

3. Let $\mathbf{b}_1 = (h, 5, 7)$, $\mathbf{b}_2 = (-1, 3, 7)$, and $\mathbf{b}_3 = (1, 1, 2)$.
   Find $h$ so that $\mathbf{b}_3 \in \mathrm{Span}\{\mathbf{b}_1, \mathbf{b}_2\}$.

4. Let $\mathbf{u_1} = (2, 0, 3, -1)$, $\mathbf{u_2} = (-4, 0, -6, 2)$, $\mathbf{u_3} = (5, 5, 0, 3)$, $\mathbf{u_4} = (1, 3, -6, 5)$,
   Determine whether each set is linearly independent or linearly dependent.

   (a) $\{\mathbf{u_1}, \mathbf{u_2}\}$

   (b) $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$

5. Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

(a) Express $\mathbf{a}_3$ as linear combinations of $\mathbf{a}_1$ and $\mathbf{a}_2$ if possible.

(b) Is $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ a basis for $\mathbb{R}^2$? Why or why not?

(c) Is $\{\mathbf{a}_2, \mathbf{a}_3\}$ a basis for $\mathbb{R}^2$? Why or why not?

6. Let $\mathbf{u_1} = (4, 2, 5)$, $\mathbf{u_2} = (3, -1, -2)$, and $\mathbf{u_3} = (6, 2, 0)$

(a) Is $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ a basis for $\mathbb{R}^3$? Justify.

(b) Is it possible to express $\mathbf{u_3}$ as a linear combination of $\mathbf{u_1}$ and $\mathbf{u_2}$? Justify without solving.

7. Let $\mathbf{a_1} = (2, 3, -1, 1)$, $\mathbf{a_2} = (-2, -3, 1, -1)$,
$\mathbf{a_3} = (2, 3, 1, 5)$, $\mathbf{a_4} = (2, 3, 2, 7)$, $\mathbf{a_5} = (4, 6, 3, 12)$.
Find a basis for $S = \operatorname{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5\}$.

8. Given that

$$S = \left\{ \begin{pmatrix} a & b \\ b & d \end{pmatrix} | a, b, d \in \mathbb{R} \right\}$$

Show that

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\} \text{ is a basis for } S$$