Tutorial questions-3

1. Given the matrix A verify that the indicated matrix is in fact the inverse.

$$A=\begin{bmatrix}-4 & -2\\ 5 & 5\end{bmatrix} \qquad A^{-1}=\begin{bmatrix}-\frac{1}{2} & -\frac{1}{5}\\ \frac{1}{2} & \frac{2}{5}\end{bmatrix}$$

2. Find the inverse of the matrix

$$A=\begin{pmatrix}1 & 0 & 4\\ 1 & 1 & 6\\ -3 & 0 & -10\end{pmatrix}$$

3. Solve the system of equations by using inverse matrix

$$\begin{aligned} x+4z &= 2\\ x+y+6z &= 3\\ -3x-10z &= 4\end{aligned}$$

4. Let A be the matrix

$$\begin{bmatrix}2 & 0\\ 4 & 1\end{bmatrix}$$

In each part, compute the given quantity.

(a) A^3

(b) A^{-3}

(c) A^2-2A+I

5.Solve the following system of equations by using the inverse matrix

$$\begin{aligned} x+2y-z &= 11\\ 2x-y+3z &= 7\\ 7x-3y-2z &= 2\end{aligned}$$

6. For what value(s) of k is each of the matrices given below invertible?

a) $\begin{bmatrix}k & -1 & 4\\ 2 & 0 & 1\\ -1 & 0 & -1\end{bmatrix}$, b) $\begin{bmatrix}k & -1\\ -1 & 3\end{bmatrix}$

7. Matrix A is given by

$$A = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$$

Find a formula for the inverse of matrix A if none of the parameters a, b, c and d is equal to zero.

8. Use the inverse matrix to solve the system of equations

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ -1 & 1 & 2 & 0 \\ -2 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 2 \end{bmatrix}$$

9. The 3x 3 matrices A and B are defined in terms of a scalar constant k by

$$A = \begin{pmatrix} k & 9 & 2 \\ 1 & k & 0 \\ 5 & -1 & 1 \end{pmatrix} \quad and \quad B = \begin{pmatrix} 1 & -3 & 2 \\ k & 2 & -1 \\ 4 & 1 & 1 \end{pmatrix}$$

a) Find an expression for det (A) , in terms of k
b) Find the possible values of k given that AB is singular.

10. The 3x 3 matrices A and B are given below.

$$A = \begin{pmatrix} 3 & 4 & 2 \\ 1 & 1 & 4 \\ 4 & 5 & 7 \end{pmatrix} \quad and \quad B = \begin{pmatrix} -10 & -14 & 16 \\ 10 & 14 & -6 \\ 5 & 6 & 6 \end{pmatrix}$$

Show clearly that

$$A + A^{-1} + B = kA$$

stating the value of the scalar constant k.

11. Which of the following matrices is invertible?

(a) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 3 \\ 0 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 2 \\ 6 & 3 \end{bmatrix}$

12. Let $A = \begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix}$. Find A^{-1} and use it to solve the linear system of equations:

$$\begin{cases} 2x - 3y = 7 \\ -x + y = -3 \end{cases}$$

13. Calculate the determinant of the matrix:

а) $\begin{vmatrix} 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 5 & -1 & 0 \\ 2 & -3 & 7 & 2 & 3 \\ 1 & 0 & -4 & 2 & 0 \\ -1 & 2 & 0 & -1 & 2 \end{vmatrix}$;

б) $\begin{vmatrix} 3 & -2 & 1 & 1 \\ 0 & 1 & 2 & 0 \\ -1 & 2 & 4 & 2 \\ 1 & -3 & 0 & 0 \end{vmatrix}$;

14. Solve the equation:

$$\begin{vmatrix} x & x & x \\ 7 & 4 & 5 \\ 2 & -1 & 0 \end{vmatrix} = 0,$$

15. Find the matrix , if

$$AX + B = 2C, \qquad A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \\ -2 & 0 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 3 & 0 \\ 4 & -3 & 5 \\ 1 & -1 & 0 \end{bmatrix};$$