Algorithms Assignment 2

Asymptotic Analysis of Algorithms

Q1

a. $\sum_{i=3}^{n+1} 1$

$= \sum_{i=1}^{n+1} 1 - \sum_{i=1}^{2} 1$

$= (n+1) - 2$

$= n+1-2$

$= (n-1)$

$= n-1$

b. $\sum_{i=3}^{n+1} i$

$= \sum_{i=1}^{n+1} i - \sum_{i=1}^{2} i$

$= \frac{(n+1)(n+2)}{2} - (1+2)$

$= \frac{(n+1)(n+2)}{2} - 3$

$= \frac{(n+1)(n+2) - 6}{2}$

$= \frac{n^2+2n+n+2-6}{2} = \frac{n^2+3n-4}{2}$

Q1]

c]. $$\sum_{i=3}^{n+1} i(i+1)$$

$$\sum_{i=1}^{n+1} i^2+i \; - \; \sum_{i=1}^{\cancel{n+1}\,2} i^2+i \quad \text{——} \; 1$$

$$\sum_{i=1}^{n} i^2+i = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$\therefore \sum_{i=1}^{n+1} i^2+i = \frac{(n+1)(n+2)(2n+3)}{6} + \frac{(n+1)(n+2)}{2}$$

$$= \frac{(n^2+2n+n+2)(2n+3)}{6} + \frac{n^2+2n+n+2}{2}$$

$$= \frac{(n^2+3n+2)(2n+3)}{6} + \frac{n^2+3n+2}{2}$$

$$= \frac{2n^3+\cancel{6n^2}\,3n^2+6n^2+9n+4n+6}{6} + \frac{n^2+3n+2}{2}$$

$$= \frac{2n^3+9n^2+13n+6}{6} + \frac{3(n^2+3n+2)}{3(2)}$$

$$= \frac{2n^3+9n^2+13n+6+3n^2+9n+6}{6}$$

$$= \frac{2n^3+12n^2+22n+12}{6} \quad \text{——} \; 2$$

$$\sum_{i=1}^{2} i^2+i$$

$= (1^2+1) + 2^2+2$

$= (1+1) + 4 + 2$

$= 2 + 4 + 2$

$= 8$ ——— 3

from 1, 2, 3 we get

$$\sum_{i=3}^{n+1} = \frac{2n^3+12n^2+22n+12}{6} - 8$$

$$= \frac{2n^3+12n^2+22n+12-48}{6}$$

$$= \frac{2n^3+12n^2+22n-36}{6}$$

$$\sum_{i=3}^{n+1} = \frac{n^3+6n^2+11n-18}{3}$$

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d].

$$\sum_{i=3}^{n+1} \frac{1}{i(i+1)}$$

$$\frac{1}{i(i+1)} = \frac{1}{i} - \frac{1}{i+1}$$

$$\therefore \sum_{i=3}^{n+1} \left(\frac{1}{i} - \frac{1}{i+1}\right)$$

$$= \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \dots$$

$$\dots + \left(\frac{1}{n} - \frac{1}{n+1}\right) + \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

$$= \frac{1}{3} - \frac{1}{n+2}$$

$$= \frac{n+2-3}{3(n+2)} = \frac{n-1}{3(n+2)} = \frac{n-1}{3(n+2)}$$

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Q2]
a]. $T(N) = T(N-1) + 5$ for $n > 1$
$T(1) = 0$

$T(N) = T(N-1) + 5$

put $N = N-1$

$\therefore T(N-1) = T(N-1-1) + 5$
$T(N-1) = T(N-2) + 5$ —— (1)

$T(N-2) = T(N-2-1) + 5$
$T(N-2) = T(N-3) + 5$ —— (2)

$T(N) = T(N-1) + 5$

$\therefore T(N) = [T(N-2) + 5] + 5$ from 1
$T(N) = T(N-2) + 10$
$= [T(N-3) + 5] + 10$ from 2
$T(N) = T(N-3) + 15$

From the above pattern we can deduce that

$T(N) = T(N-K) + 3K$

When $K = N-1$ we get

$N - (N-1) = 1$

we have $T(1) = 0$

$\therefore \quad T(N) = T(N - N + 1) + 3(N-1)$
$= T(1) + 3N - 3$
$= 0 + 3N - 3$
$T(N) = 3N - 3$

$\therefore \quad T(N) = O(N)$

Q2] b). $T(N) = 3T(N-1)$ for $N > 1$ $\quad T(1) = 4$

$T(N) = 3T(N-1)$

put $N = N-1$

$\therefore \quad T(N-1) = 3T(N-1-1)$
$\therefore \quad T(N-1) = 3T(N-2) \quad \text{--- (1)}$

$T(N-2) = 3T(N-2-1)$
$T(N-2) = 3T(N-3) \quad \text{--- (2)}$

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$T(N) = 3T(N-1)$

$= 3T(N-1)$

$= 3 \times 3T(N-2)$ from 1

$= 9T(N-2)$

$= 9 \times 3T(N-3)$ from 2

$T(N) = 27T(N-3)$

From the above pattern we can deduce that

$T(N) = 3^k T(N-k)$

put $k = N-1$

$T(N) = 3^{N-1} T(N-N+1)$

$= 3^{N-1} T(1)$

$= 3^{N-1} 4$

$= 4 \times 3^N \times 3^{-1}$

$T(N) = \frac{4}{3} \times 3^N$

$\therefore T(N) = O(3^n)$

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Q2]

c].

$T(N) = T(N/3) + 1$ for $n > 1$

$T(1) = 1$

Solve $(N = 3^k)$

$$T(N) = T\left(\frac{N}{3}\right) + 1$$

$$T(N) = \left[T\left(\frac{N}{9}\right) + 1\right] + 1$$

$$T(N) = \left[T\left(\frac{N}{27}\right) + 1\right] + 1 + 1$$

From the above Equatⁿ we can generalize

$$T(N) = T\left(\frac{N}{3^k}\right) + k \times 1$$

Solving for $N = 3^k$

$$\therefore T(N) = T\left(\frac{3^k}{3^k}\right) + k \times 1$$

$$\therefore T(N) = T(1) + k$$

$T(1) = 1$ and $N = 3^k$ $\therefore$ $k = \log_3 N$

$\therefore\ T(N) = \log_3 N + 1$

$T(N) = 1 + \log_3 N$

i.e $T(N) = O(\log N)$.

Q3)

a). $T(N) = 2T(N/2) + N^4$

Using Masters Theorem, we know that

$T(N) = AT(N/B) + N^D$

Comparing given Equatⁿ with above Equatⁿ we get

$A = 2 \quad B = 2 \quad D = 4$

$\log_B A = \log_2 2 = 1$

$\log_B A < D$

Hence

$T(N) = \Theta(N^D)$

$\therefore\ T(N) = \Theta(N^4)$.

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b]. $T(N) = T\left(\frac{9N}{10}\right) + N$

Using Masters Thm we know that

$T(N) = AT(N/B) + N^D$

$A = 1 \quad B = \frac{10}{9} \quad D = 1$

$\log_B A = \log_{10/9} 1 = \underline{\underline{0}}$

$0 < 1$

$\therefore \log_B A < \underline{\underline{D}}.$

Hence $T(N) = \Theta(N^D)$

$\therefore T(N) = \Theta\underline{\underline{(N)}}.$

c]. $T(N) = 16T(N/4) + N^2$

Using Masters Th^m we know

$T(N) = AT(N/B) + N^D$

$\therefore$ $A = 16$ $B = 4$ $D = 2$

$\log_B A = \log_4 16 = 2$

$2 = 2$

$\therefore$ $\log_B A = D$

$\therefore$ $T(N) = \theta(N^D \log N)$

Hence

$\therefore$ $T(N) = \theta(N^2 \log N)$

d]. $T(N) = 2T(N/4) + \sqrt{N}$

Using Masters Th^m we know that

$T(N) = AT(N/B) + N^D$

$\therefore$ $A = 2$ $B = 4$ $D = 1/2$

$\log_B A = \log_4 2 = 1/2$

$\therefore 1/2 = 1/2$

$\therefore \log_B A = D$

$T(N) = \Theta(N^D \log N)$

Hence

$T(N) = \Theta(N^{1/2} \log N)$

$= \Theta(\sqrt{N} \log N)$

e). $T(N) = T(N-1) + N$

Solving by Substitut^n Method

$T(N) = T(N-1) + N$

Assume $T(1) = 1$

$\therefore T(1) = 1$

$\therefore T(2) = 1+2 = 3$

$\therefore T(3) = 1+2+3 = 6$

from above Equat^n we can see the pattern as follows

$T(N) = \frac{N(N+1)}{2}$

$$f(N) = \frac{N(N+1)}{2}$$

$$= \frac{N^2}{2} + \frac{N}{2}$$

By InducTn

$$f(N) \leq C \cdot g(N)$$

$$\frac{N(N+1)}{2} \leq C \cdot g(N)$$

Assume $C = \frac{1}{2}$

$$\therefore \frac{N^2}{2} + \frac{N}{2} \leq \frac{1}{2} g(N)$$

$$\therefore g(N) \geq N^2 + N$$

$$\therefore T(N) = O(N^2)$$

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8. $T(N) = T(\sqrt{N}) + 1$

By Substitut Method

$$T(N) = T(N^{1/2}) + 1$$
$$= T(N^{1/4}) + 2$$
$$= T(N^{1/8}) + 3$$

$\therefore \quad T(N) = T(N^{1/2K}) + K$

Solving the above Eqn we get

$\cancel{2^{2K}} \quad N = 2^{2K}$

i.e. $K = (\log(\log N))$

$\therefore \quad T(N) = \log(\log N)$.