Hardness Amplification for Weakly Verifiable Cryptographic Primitives

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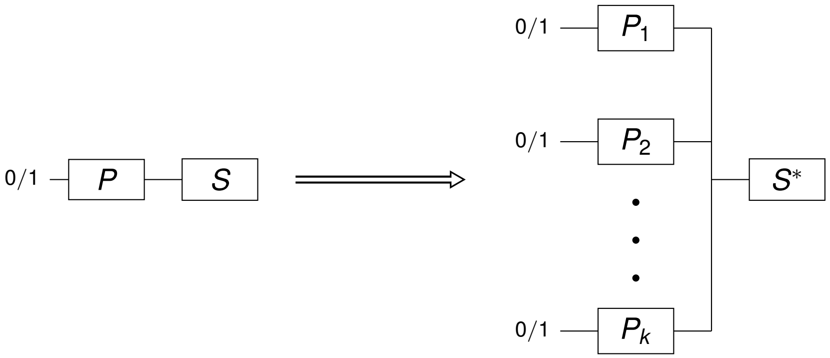
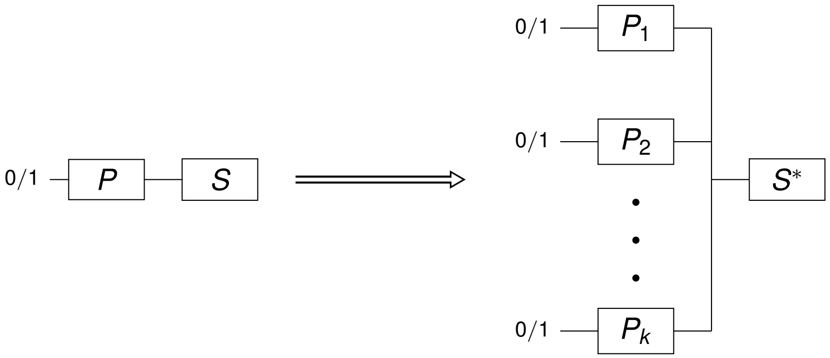
Hardness Amplification

- Is solving parallel repetition of problems substantially harder than a single instance?



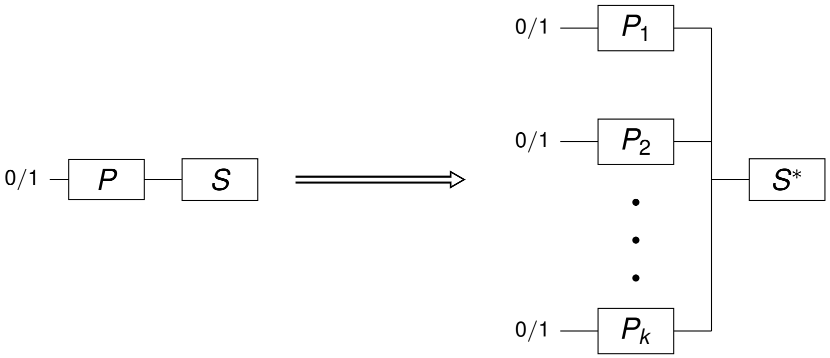
Hardness Amplification

- Weak one-way function $\Longrightarrow$ strong one-way function



Hardness Amplification

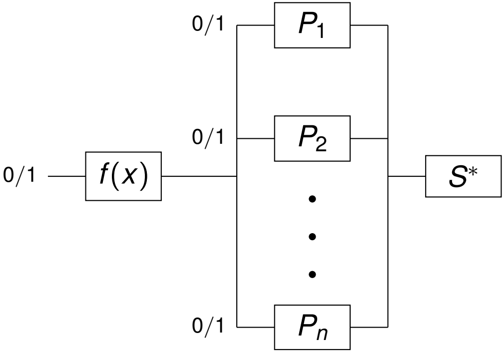
- Weak one-way function $\Longrightarrow$ strong one-way function
- What about MAC, signature schemes, CAPTCHAs?



Agenda

- Setting and Type of Problems
  - Threshold and Monotone Functions
  - Weakly Verifiable Puzzles
  - Dynamic Weakly Verifiable Puzzles
  - Interactive Weakly Verifiable Puzzles
- Previous Works
- My Results
- Discussion and Questions

Threshold and Monotone Functions



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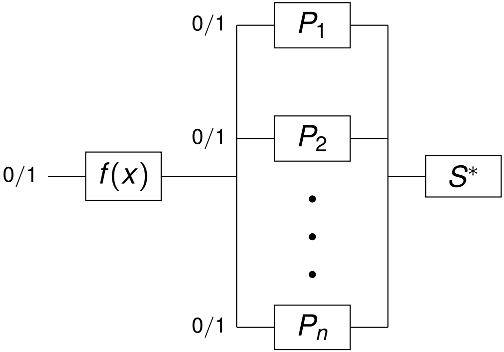
Threshold and Monotone Functions

Threshold function

$$f_K(b_1, \dots, b_n) = \begin{cases} 1 \text{ if } \sum_{i=1}^{n} b_i \geq K \\ 0 \text{ otherwise.} \end{cases}$$



Threshold and Monotone Functions

Threshold function

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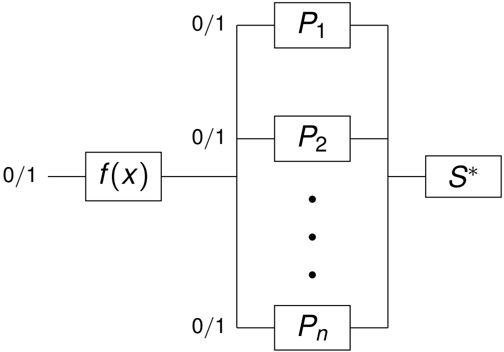
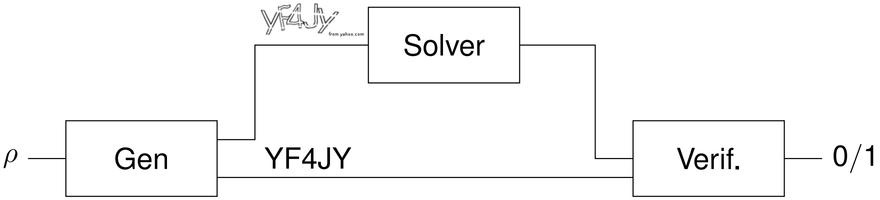
Monotone function

$$f(b_0, \ldots, b_n) : \{0,1\}^n \to \{0,1\}$$



Weakly Verifiable Puzzles - CAPTCHA



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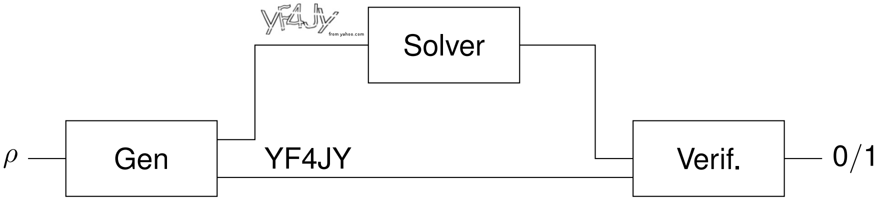

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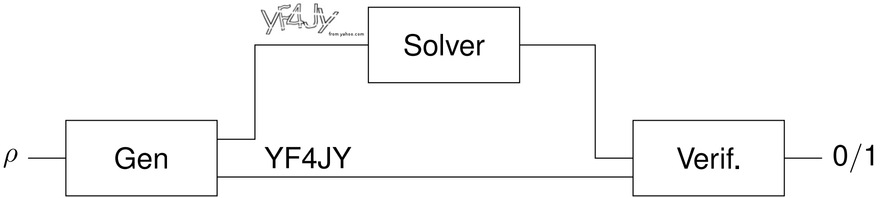
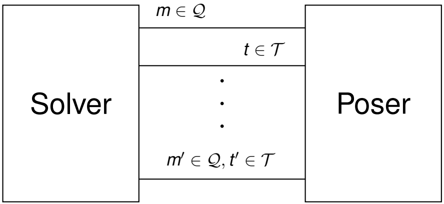
Weakly Verifiable Puzzles - CAPTCHA



- Small solutions space.
- Solver cannot efficiently verify correctness of solutions.

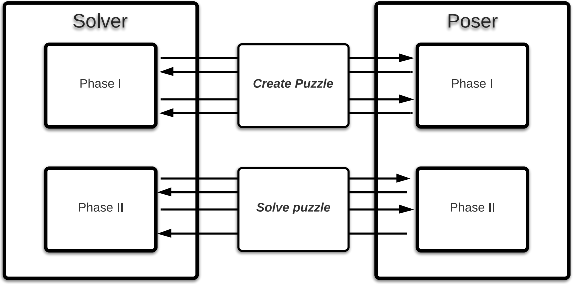
Dynamic Weakly Verifiable Puzzles

- Game-based security definition of MAC.



- Set of messages $\mathcal{Q}$
- Hint - solution for $q \in \mathcal{Q}$
- Set of hint indices $\mathcal{H} \subseteq \mathcal{Q}$
- Verification query solution for $q \in \mathcal{Q} \setminus \mathcal{H}$.
- Number of hint and verification queries limited.

Interactive puzzle - commitment protocols



Hardness amplification results

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- Dynamic weakly verifiable puzzles $+$ threshold functions e.g. MAC,[DIJK09]
- Interactive weakly verifiable puzzles $+$ monotone function e.g. commitment protocols, [HS11]

Goal

- Define puzzle that generalize MAC, CAPTCHA, bit commitments.
- Amplify hardness by parallel repetition.

Monotone functions	+	Dynamic weakly verifiable puzzles	+	Interactive weakly verifi-able puzzles

Reduction

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- Reduce C to a solver for single puzzle
- A - solving a single puzzle is hard
- B - solving parallel repetition is hard

$$\neg B \implies \neg A$$

$$A \implies B$$

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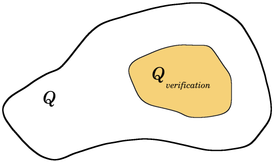


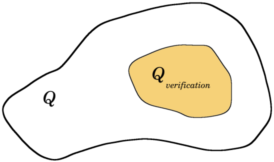
$$hash \leftarrow \mathcal{H}$$

$$hash : \mathcal{Q} \rightarrow \{0, 1, \ldots, 2(h+v)-1\}$$

$$\mathcal{Q}_{verification} := \{q \in \mathcal{Q} : hash(q) = 0\}$$

Conflicting hint queries

- The solver C can be run multiple times.
- Hint queries prevent verification queries from succeeding.
- Use hash function to partition query domain [DIJK09].
- Substantial success probability for partitioned domain.



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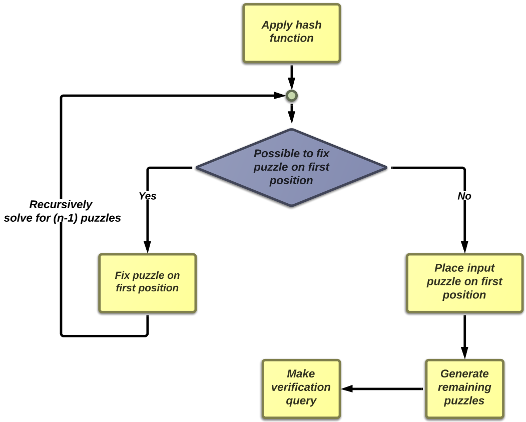
Approach overview



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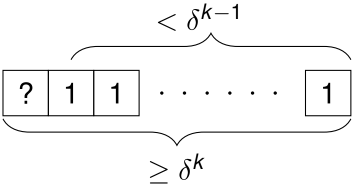

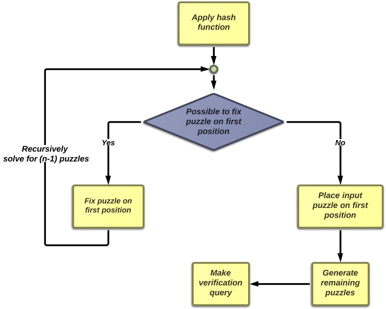

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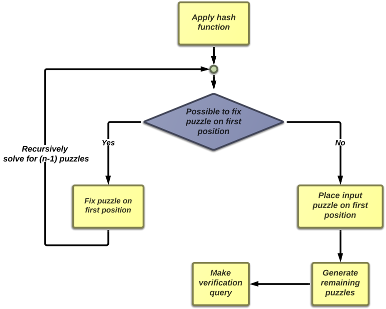
$$\underbrace{\boxed{?}\,\overbrace{\boxed{1}\boxed{1}\;\cdots\cdots\;\boxed{1}}^{<\,\delta^{k-1}}}_{\geq\,\delta^{k}}$$

- Possible to generalize for monotone functions [HS11].



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Given a solver for parallel repetition of puzzles that satisfies

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More generally using a monotone function

$$\geq \Pr[g(u_1, \ldots, u_k) = 1] + \varepsilon$$

where $\Pr[u_i = 1] = \delta$.
We devise a solver for a single puzzle that satisfies (with high probability)

$$\geq \frac{1}{16(h+v)}\left(\delta + \frac{\varepsilon}{6k}\right).$$

Discussion

- Not clear whether it is possible to improve the result

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Questions

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