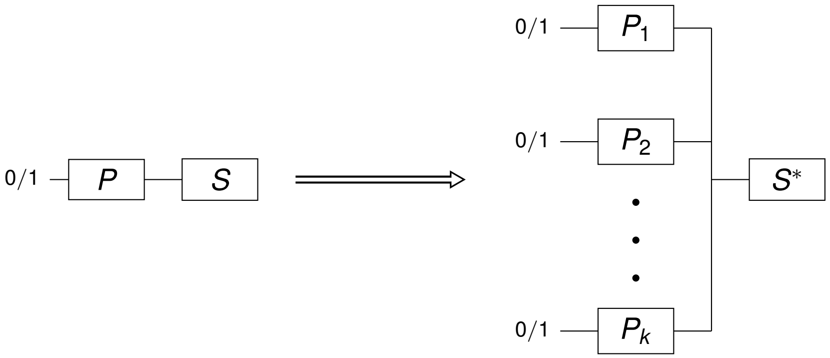
Hardness Amplification for Weakly Verifiable Cryptographic Primitives

Grzegorz Mąkosa

Advisors: Prof. Dr. Thomas Holenstein, Dr. Robin Künzler
Department of Computer Science, ETH Zürich

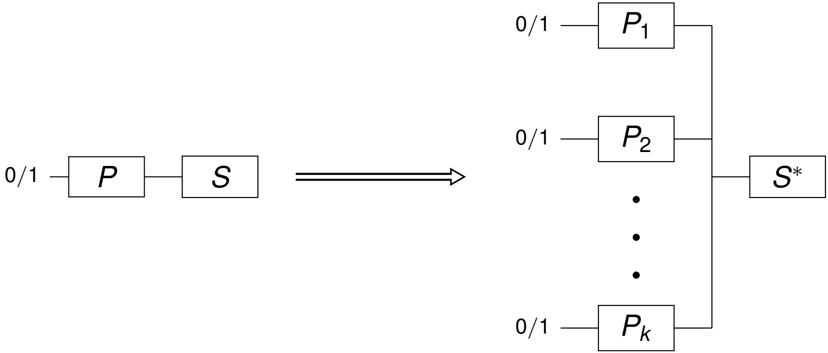
Hardness Amplification

- Is solving parallel repetition of problems substantially harder than a single instance of a problem?



Hardness Amplification

- Weak one-way function $\Longrightarrow$ strong one-way function
- What about MAC, signature schemes, CAPTCHAs?



Agenda

- Motivation and problem statement
- Background
  - Weakly Verifiable Puzzles
  - Threshold and monotone functions
  - Dynamic Puzzles
  - Interactive Puzzles
- Previous Works
- My Results
- Discussion and Questions

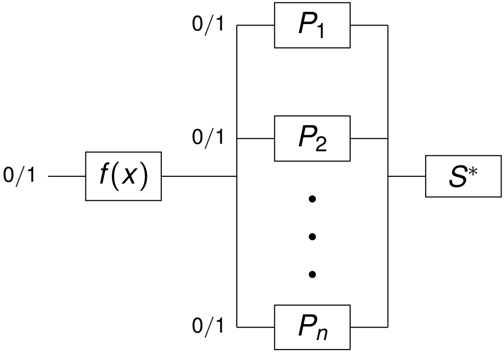
Threshold and Monotone Functions

Threshold function

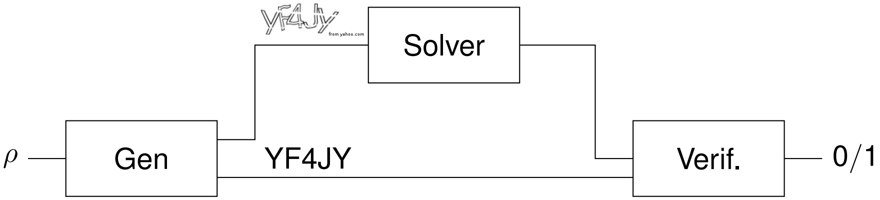
$$f_K(b_1, \ldots, b_n) = \begin{cases} 1 \text{ if } \sum_{i=1}^{n} b_i \geq K \\ 0 \text{ otherwise.} \end{cases}$$

Monotone function

$$f(b_0, \ldots, b_n) : \{0,1\}^n \rightarrow \{0,1\}$$



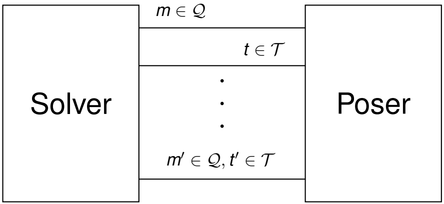
Weakly Verifiable Puzzles - CAPTCHA



- Small solutions space.
- Solver cannot efficiently verify correctness of solutions.

Dynamic Puzzles Example

- Game based security definition of MAC.



- Set of messages $\mathcal{Q}$
- Hint - solution for $q \in \mathcal{Q}$
- Set of hint indices $\mathcal{H} \subseteq \mathcal{Q}$
- Verification query solution for $q \in \mathcal{Q} \setminus \mathcal{H}$.
- Number of hint and verification queries limited.

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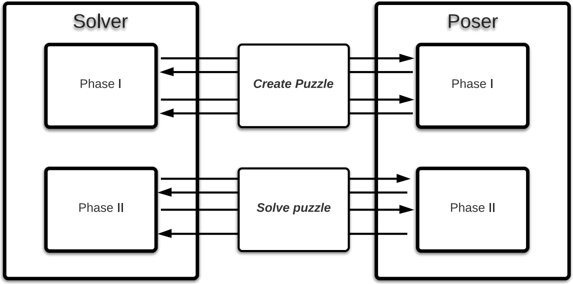
Interactive puzzle - commitment protocols



Hardness amplification results

- Weakly verifiable puzzles - e.g. CAPTCHA [CHS05]
- Dynamic weakly verifiable puzzles $+$ threshold functions
  e.g. MAC [DIJK09]
- Interactive weakly verifiable puzzles $+$ monotone function
  e.g. commitment protocols [HS11]

Goal

- Define a type of puzzles that generalize MAC, CAPTCHA, bit commitments.
- Hardness amplification result for this type of puzzles.

Monotone functions	+	Dynamic weakly verifiable puzzles	+	Interactive weakly verifi-able puzzles

Reduction

- A - solving a single puzzle is hard
- B - solving parallel repetition is hard

$$A \implies B$$

$$\neg B \implies \neg A$$

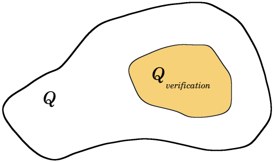
- Given a good solver C for parallel repetition
- Reduce C to a solver for single puzzle

Problems

- Fix a position for the input puzzle
- Generate $n - 1$ puzzles
- Run C multiple times
- If the solution is correct output it
- One has to run C multiple times
- Hint query may prevent block a solution that would be correct
- Not possible to check correctness of the solution for the input puzzle

Problem: conflicting hint queries

- The solver asks hint queries.
- Hint queries can prevent verification queries from succeeding.
- Use hash function to partition query domain [DIJK09].
- Can ask hints only on $\mathcal{Q} \setminus \mathcal{Q}_{\mathit{verification}}$.
- Substantial success probability for partitioned domain.

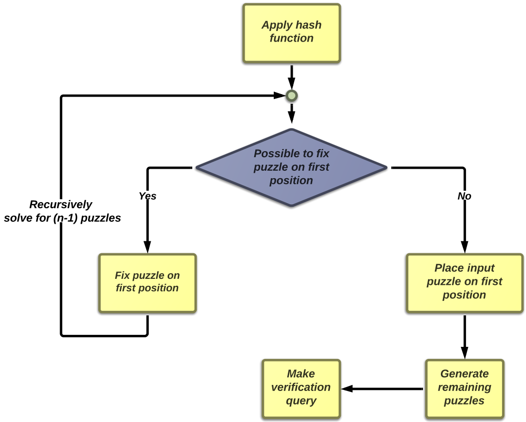


$$hash \leftarrow \mathcal{H}$$

$$hash : \mathcal{Q} \rightarrow \{0, 1, \ldots, 2(h+v) - 1\}$$

$$\mathcal{Q}_{\mathit{verification}} := q \in \mathcal{Q} : hash(q) = 0$$

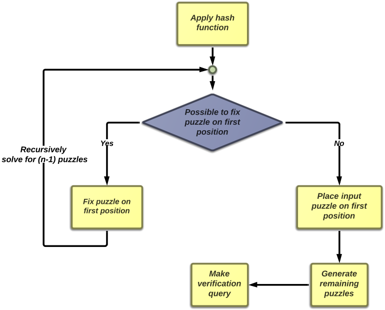
Approach overview



Problem: verifying the solution

- Cannot check whether the solution is correct.
- For a special case where all puzzles have to be solved.
- Look at the remaining $n-1$ puzzles that are generated.

$$\underbrace{\boxed{?}\,\overbrace{\boxed{1}\,\boxed{1}\,\cdots\cdots\,\boxed{1}}^{<\,\delta^{k-1}}}_{\geq\,\delta^{k}}$$



Result

Given a solver for parallel repetition of puzzles that satisfies

$$\geq \delta^k + \varepsilon \qquad\qquad \geq \Pr[g(u_1, \ldots, u_k) = 1] + \varepsilon,$$

where $\Pr[u_i = 1] = \delta$.

We devise a solver for a single puzzle that satisfies (almost surely)

$$\geq \frac{1}{16(h+v)}\left(\delta + \frac{\varepsilon}{6k}\right).$$

Discussion

- Not clear whether it is possible to improve the result

$$\geq \frac{1}{16(h+v)}\left(\delta + \frac{\varepsilon}{6k}\right).$$

- Tried to improve it. ✗
- Tried to show it is optimal. ✗

Questions

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