Definition 1.1 (Dynamic weakly verifiable puzzle.) *A dynamic weakly verifiable puzzle (DWVP) is defined by a probabilistic algorithms P called a problem poser. We denote the randomness P uses by π. A problem solver $S = (S_1, S_2)$ is a probabilistic two phase algorithm. The randomness used by S is denoted by ρ. In the first phase $P(\pi)$ interacts with $S_1(\rho)$. As the result of the interaction $P(\pi)$ outputs circuits Γ_V, Γ_H, a puzzle $x \in \{0,1\}^*$. The solver $S_1(\rho)$ produces no output. The circuit Γ_V takes as input $q \in Q$, an answer $y \in \{0,1\}^*$, and outputs 1 if y is a correct solution of x for q and 0 otherwise. The circuit Γ_H on input $q \in Q$ outputs a hint such that $\Gamma_V(q, \Gamma_H(q)) = 1$. In the second phase S_2 takes as input x, and has oracle access to Γ_V and Γ_H. The execution of S_2 with the input x and the randomness ρ is denoted by $S_2(x, \rho)$. The queries of S_2 to Γ_V are called verification queries, and to Γ_H hint queries. The algorithm S_2 can ask at most h hint queries, v verification queries, and successfully solves the puzzle if and only if it makes a verification query (q, y) such that $\Gamma_V(q, y) = 1$, when it has not previously asked for a hint query on q.*

Definition 1.2 (k-wise direct-product of DWVPs.) *Let $g : \{0,1\}^k \to \{0,1\}$ be a monotone function and $P^{(1)}$ a problem poser as in Definition 1.1. The k-wise direct product of $P^{(1)}$ is a DWVP defined by a probabilistic algorithm $P^{(g)}$. Let $\pi^{(k)} := (\pi_1, \ldots, \pi_k)$ be the randomness used by $P^{(g)}$, and $P^{(g)}(\pi^{(k)})$ denote the execution of $P^{(g)}$ with the randomness $\pi^{(k)}$. A solver $S := (S_!, S_2)$ is a probabilistic algorithm. In the first phase P interacts with S_1. As the result of the interaction $P^{(g)}$ outputs: a verification circuit*

$$\Gamma_V^{(g)}(q, y_1, \ldots, y_k) := g(\Gamma_V^1(q, y_1), \ldots, \Gamma_V^k(q, y_k)),$$

a hint circuit

$$\Gamma_H^{(k)}(q) := (\Gamma_H^1(q), \ldots, \Gamma_H^k(q)),$$

and a puzzle $x^{(k)} := (x_1, \ldots, x_k)$, where the i-th instance $(x_i, \Gamma_V^i, \Gamma_H^i) := \left\langle P^{(1)}(\pi_i), S_1(\rho) \right\rangle_{P^{(1)}}$.