ASSIGNMENT: 1

Q1: $n = 20$

After ordering

4, 4, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 10

(A) (B)

Total sum = 137

$\text{Mean} = \frac{137}{20} = 6.85$ $\quad \text{Median} = \frac{A+B}{2} = 7$

Mode = 7 (most frequent)

Q2: $n = 35$

After ordering:

28, 40, 68, 70, 75, 75, 75, 75, 80, 86, 89, 90, 90, 97, 97,
100, 100, 100, 104, 104, 109, 113, 120, 120, 120, 122, 123, 123,
130, 140, 145, 170, 174, 194, 217

Total sum = 3763

$\text{Mean} = \frac{3763}{35} = 107.5$, Median = A = 100

Mode = 75 (most frequent)

Q3.

x	$p(x)$	$x_i - \mu$
0	0.09	-2.15
1	0.15	-1.15
2	0.40	-0.15
3	0.25	0.85
4	0.10	1.85
5	0.01	2.85

$n = 6$

Mean/expectation, $E(x) = \sum_{i=1}^{6} x_i p(x_i)$

$= 2.15 = \mu$

variance, $E(x-\mu)^2 = \sum_{i=1}^{6} (x_i - \mu)^2 p(x_i)$

$= 1.2275$

Q4: Probability Density Function,

$$\text{pdf}, f(x) = 20 e^{-20(x-12.5)} \quad \forall d \geq 12.5$$

$$= 0 \quad \forall d < 12.5$$

$$P(d > 12.6) = \int_{12.6}^{\infty} f(x)\,dx = \int_{12.6}^{\infty} 20 e^{-20(x-12.5)}\,dx$$

$$= -e^{-20(x-12.5)} \Big|_{12.6}^{\infty} = 0.135$$

Cumulative distribution function while $d < 11$:

$$P(d < 11) = \int_{-\infty}^{11} f(x)\,dx = 0$$

ASSIGNMENT: 1

Q5 : $P(\text{faulty}) = 0.3$

$P(\text{No faulty}) = 0.7$

$n = 6, \; x = 2$

$$P(x=2) = \frac{n!}{x!\,(n-x)!} \; p^x (1-p)^{n-x}$$

where $P = P(\text{faulty})$

$$\Rightarrow P(x=2) = \frac{6!}{2!\,4!} \; 0.3^2 \; 0.7^4$$

$$= \frac{6 \times 5}{2} \; 0.3^2 \; 0.7^4$$

$$= 0.32$$

Average faulty, $= n \times p = 6 \times 0.3$

$= 1.8$

Variance $= n \times p \times (1-p)$

$= 6 \times 0.3 \times 0.7 = 1.26$

Std. deviation $= \sqrt{\text{variance}} = 1.122$

Q6:

	Gaurav	Barkha
	$n_1 = 8$	$n_2 = 12$
correction →	$P(C_1) = 0.75$	$P(C_2) = 0.45$
	$P'(C_1) = 0.25$	$P'(C_2) = 0.55$
	$x_1 = 5$	$x_2 = 5$
	$P(x_1) = \frac{n_1!}{x_1!\,(n_1-x_1)!} \times P(C_1)^{x_1} \; P'(C_1)^{n_1-x_1}$	$P(x_2) = \frac{12!}{5!\,7!} \; 0.55^5 \; 0.45^7$
	$= \frac{8!}{5!\,3!} \; 0.25^5 \; 0.75^3$	
	$= 0.023$	$= 0.1489$
	$x_1 = 4$	$x_2 = 6$
	$P(x_1) = \frac{8!}{4!\,4!} \; 0.25^4 \; 0.75^4$	$P(x_2) = \frac{12!}{6!\,6!} \; 0.55^6 \; 0.45^6$
	$= 8.65\%$	$= 21.2\%$

BARKHA has a higher probability of answering correctly

Two main governing factor

$P(C)$ → lesser the good

n → No. of questions

ASSIGNMENT: 1

7. $\mu = 72$, $\mu_{desired} = \mu_{4min} = \frac{4 \times 72}{60} = 4.8$

a. $k = 5$

$$P(k) = \frac{e^{-4.8}\, 4.8^5}{5!} = A$$

b. not more than 3 customer

$$P(k \leq 3) = P(1) + P(2) + P(3)$$

$$= \frac{e^{-4.8}\, 4.8^1}{1!} + \frac{e^{-4.8}\, 4.8^2}{2!} + \frac{e^{-4.8}\, 4.8^3}{3!}$$

$$= B$$

c. more than 3

$$P(k > 3) = 1 - P(k \leq 3)$$

$$= 1 - B$$

8. Errors/hr = 6 no. of words/min = 77

With 455 words, the same can be entered in

$$\frac{455}{77} = 5.909 \text{ mins.}$$

Expected error in 5.909 min $= \frac{6 \times 5.909}{60} = 0.5909$

Probability of 2 errors in 455 words

$$P(2) = \frac{e^{-.5909}\, 0.5909^2}{2!}$$

$n_1 = 1000$

$t_1 = 1000/77$

$E_1 = \frac{6}{60} \times t_1 = 1.2987$

$$P(2) = \frac{e^{-1.2987}\, 1.2987^2}{2!}$$

$= 0.23$

$n_2 = 255$

$t_2 = 255/77$

$E_2 = \frac{6}{60} \times t_2 = 0.3312$

$$P(2) = \frac{e^{-.3312}\, 0.3312^2}{2!}$$

$= 0.04$

(9) SAME AS ASSIGNMENT: 1 Question 4

(10) a)

$P(Z > 1.26) = 1 - P(Z \leq 1.26) = 1 - 0.896 = 0.104$

$P(Z < -0.86) = 0.19489$

$P(Z > -1.37) = 1 - P(Z \leq -1.37) = 1 - 0.08534 = 0.91466$

$P(-1.25 < Z < 0.37) = P(Z > -1.25) - P(Z < 0.37)$

$= 0.89435 - 0.64431$

$P(Z \leq -4.6) \approx 0$

b. find Z_1 such that $P(Z > Z_1) = 0.05$

$P(Z > Z_1) = 1 - P(Z \leq Z_1) = 0.05$

$\Rightarrow P(Z \leq Z_1) = 0.95$

from z-table, $Z_1 = 1.65$

c. find value of Z_1 s.t. $P(-Z_1 < Z < Z_1) = 0.99$

$P(Z > -Z_1) - P(Z < Z_1) = 0.99$

$\Rightarrow Z_1 = 2.58$

ASSIGNMENT: 1

11. $\mu = 10\,mA$, $\sigma^2 = 4 \Rightarrow \sigma = 2$

a. $x = 13\,mA$

$Z_1 = \frac{x-\mu}{\sigma} = 1.5$

$P(Z > Z_1) = P(Z > 1.5) = 1 - P(Z \leq 1.5)$

$= 0.066807$

b. $x_1 = 9$, $x_2 = 11$

$Z_1 = -0.5$ $Z_2 = 0.5$

$P(Z_1 < Z < Z_2) = P(Z_2) - P(Z_1)$

$= 0.691 - 0.309$

$= 0.382$

c. $P(Z) = 0.98 \Rightarrow Z = 2.054$ (from z table)

$\Rightarrow \frac{x-\mu}{\sigma} = 2.054 \Rightarrow x = 2.054 \times \sigma + \mu$

$= 14.108$

12. $\mu_{spec} = 0.25$

$\sigma_{spec} = 0.0015$

$\Rightarrow$ LSL $= 0.25 - 0.0015 = 0.2485$

$\Rightarrow$ USL $= 0.25 + 0.0015 = 0.2515$

$\mu_a = 0.2508$

$\sigma_a = 0.0005$

$Z_{lower} = \frac{0.2485 - 0.2508}{0.0005} = -4.6$

$Z_{upper} = \frac{0.2515 - 0.2508}{0.0005} = 1.4$

$P(-4.6 < Z < 1.4) = P(Z < 1.4) - P(Z > -4.6)$

$= 0.9192 - 0$

$= 0.9192$