

## 2.8.- Plantillas

### 2.8.1.- Variables auxiliares

ln[\*]= (\*Cinemática directa\*)

$$\left\{ \begin{aligned} a1 \rightarrow & 1 + g^2, \quad b1 \rightarrow -2 f g + g 15 + 2 g 11 \cos[\theta 1] + 2 11 \sin[\theta 1], \quad c1 \rightarrow f^2 + 11^2 - 12^2 - f 15 + \frac{15^2}{4} + 11 (-2 f + 15) \cos[\theta 1] \end{aligned} \right\};$$

$$\left\{ f \rightarrow \frac{11^2 - 12^2 + 13^2 - 14^2 + 11 15 \cos[\theta 1] + 14 15 \cos[\theta 4]}{2 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])}, \quad g \rightarrow \frac{(-2 11 \sin[\theta 1] + 2 14 \sin[\theta 4])}{2 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])} \right\};$$

(\*Cinemática inversa\*)

(\*θ1\*)

$$\left\{ \begin{aligned} a1 \rightarrow & 11^2 - 12^2 - 11 15 + \frac{15^2}{4} + 2 11 X3 - 15 X3 + X3^2 + Y3^2, \\ b1 \rightarrow & -4 11 Y3, \quad c1 \rightarrow 11^2 - 12^2 + 11 15 + \frac{15^2}{4} - 2 11 X3 - 15 X3 + X3^2 + Y3^2 \end{aligned} \right\};$$

(\*θ4\*)

$$\left\{ \begin{aligned} a2 \rightarrow & -13^2 + 14^2 + 14 15 + \frac{15^2}{4} + 2 14 X3 + 15 X3 + X3^2 + Y3^2, \\ b2 \rightarrow & -4 14 Y3, \quad c2 \rightarrow -13^2 + 14^2 - 14 15 + \frac{15^2}{4} - 2 14 X3 + 15 X3 + X3^2 + Y3^2 \end{aligned} \right\};$$

### 2.8.2.- Cinemática directa

ln[\*]= (\*Brazos arriba\*)

$$(*g \left( \frac{b + \sqrt{-4 a c + b^2}}{2 a} \right) + f*)$$

$$\begin{aligned} Xmas = & \frac{11^2 - 12^2 + 13^2 - 14^2 + 11 15 \cos[\theta 1] + 14 15 \cos[\theta 4]}{2 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])} + \\ & \left( (-2 11 \sin[\theta 1] + 2 14 \sin[\theta 4]) \left( 2 11 \sin[\theta 1] + \frac{15 (-2 11 \sin[\theta 1] + 2 14 \sin[\theta 4])}{2 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])} + \right. \right. \\ & \quad \left. \frac{11 \cos[\theta 1] (-2 11 \sin[\theta 1] + 2 14 \sin[\theta 4])}{15 + 11 \cos[\theta 1] - 14 \cos[\theta 4]} - ((11^2 - 12^2 + 13^2 - 14^2 + 11 15 \cos[\theta 1] + 14 15 \cos[\theta 4]) \right. \\ & \quad \left. (-2 11 \sin[\theta 1] + 2 14 \sin[\theta 4])) / (2 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])^2) + \right. \\ & \quad \left. \sqrt{\left( \left( 2 11 \sin[\theta 1] + \frac{15 (-2 11 \sin[\theta 1] + 2 14 \sin[\theta 4])}{2 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])} + \frac{11 \cos[\theta 1] (-2 11 \sin[\theta 1] + 2 14 \sin[\theta 4])}{15 + 11 \cos[\theta 1] - 14 \cos[\theta 4]} - \right. \right. \right. \\ & \quad \left. \left. \left( (11^2 - 12^2 + 13^2 - 14^2 + 11 15 \cos[\theta 1] + 14 15 \cos[\theta 4]) (-2 11 \sin[\theta 1] + 2 14 \sin[\theta 4]) \right) / \right. \right. \\ & \quad \left. \left. (2 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])^2) \right)^2} - \right. \\ & \quad \left. 4 \left( 11^2 - 12^2 + \frac{15^2}{4} - \frac{15 (11^2 - 12^2 + 13^2 - 14^2 + 11 15 \cos[\theta 1] + 14 15 \cos[\theta 4])}{2 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])} + \right. \right. \\ & \quad \left. \frac{(11^2 - 12^2 + 13^2 - 14^2 + 11 15 \cos[\theta 1] + 14 15 \cos[\theta 4])^2}{4 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])^2} + 11 \cos[\theta 1] \right. \\ & \quad \left. \left( 15 - \frac{11^2 - 12^2 + 13^2 - 14^2 + 11 15 \cos[\theta 1] + 14 15 \cos[\theta 4]}{15 + 11 \cos[\theta 1] - 14 \cos[\theta 4]} \right) \right) \left( 1 + \frac{(-2 11 \sin[\theta 1] + 2 14 \sin[\theta 4])^2}{4 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])^2} \right) \right) \Bigg) \Bigg) / \\ & \left( 4 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4]) \left( 1 + \frac{(-2 11 \sin[\theta 1] + 2 14 \sin[\theta 4])^2}{4 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])^2} \right) \right); \end{aligned}$$

$$(* \frac{b + \sqrt{-4 a c + b^2}}{2 a} *)$$

$$\begin{aligned}
Y_{\text{mas}} = & \frac{1}{2 \left( 1 + \frac{(-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4])^2}{4 (15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4])^2} \right)} \\
& \left( \frac{2 \, 11 \, \text{Sin}[\theta 1] + \frac{15 (-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4])}{2 (15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4])} + \frac{11 \, \text{Cos}[\theta 1] (-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4])}{15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4]}}{((11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \, \text{Cos}[\theta 1] + 14 \, 15 \, \text{Cos}[\theta 4]) (-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4]))} \right) / \\
& (2 (15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4])^2) + \sqrt{\left( \left( 2 \, 11 \, \text{Sin}[\theta 1] + \frac{11 \, \text{Cos}[\theta 1] (-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4])}{15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4]} \right) + \right.} \\
& \left. \left( \left( 15 - \frac{11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \, \text{Cos}[\theta 1] + 14 \, 15 \, \text{Cos}[\theta 4]}{15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4]} \right) (-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4]) \right) \right) /} \\
& (2 (15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4]))^2 - \\
& \left( 4 \, 11^2 - 4 \, 12^2 + 15^2 - \frac{2 \, 15 (11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \, \text{Cos}[\theta 1] + 14 \, 15 \, \text{Cos}[\theta 4])}{15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4]} + \right. \\
& \left. \frac{(11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \, \text{Cos}[\theta 1] + 14 \, 15 \, \text{Cos}[\theta 4])^2}{(15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4])^2} + 4 \, 11 \, \text{Cos}[\theta 1] \right. \\
& \left. \left( 15 - \frac{11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \, \text{Cos}[\theta 1] + 14 \, 15 \, \text{Cos}[\theta 4]}{15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4]} \right) \right) \left( 1 + \frac{(-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4])^2}{4 (15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4])^2} \right) \right) \Bigg);
\end{aligned}$$

(\*Brazos abajo\*)

$$(*g \left( \frac{b - \sqrt{-4 \, a \, c + b^2}}{2 \, a} \right) + f*)$$

$$\begin{aligned}
X_{\text{menos}} = & \frac{11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \, \text{Cos}[\theta 1] + 14 \, 15 \, \text{Cos}[\theta 4]}{2 (15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4])} + \\
& \left( (-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4]) \left( 2 \, 11 \, \text{Sin}[\theta 1] + \frac{15 (-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4])}{2 (15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4])} + \right. \right. \\
& \left. \frac{11 \, \text{Cos}[\theta 1] (-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4])}{15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4]} - ((11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \, \text{Cos}[\theta 1] + 14 \, 15 \, \text{Cos}[\theta 4]) \right. \\
& \left. (-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4])) / (2 (15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4])^2) - \sqrt{\left( \left( 2 \, 11 \, \text{Sin}[\theta 1] + \right. \right.} \\
& \left. \frac{11 \, \text{Cos}[\theta 1] (-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4])}{15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4]} + \left( \left( 15 - \frac{11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \, \text{Cos}[\theta 1] + 14 \, 15 \, \text{Cos}[\theta 4]}{15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4]} \right) \right. \right. \\
& \left. \left. (-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4]) \right) \right) / (2 (15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4]))^2 -} \\
& \left( 4 \, 11^2 - 4 \, 12^2 + 15^2 - \frac{2 \, 15 (11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \, \text{Cos}[\theta 1] + 14 \, 15 \, \text{Cos}[\theta 4])}{15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4]} + \right. \\
& \left. \frac{(11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \, \text{Cos}[\theta 1] + 14 \, 15 \, \text{Cos}[\theta 4])^2}{(15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4])^2} + 4 \, 11 \, \text{Cos}[\theta 1] \right. \\
& \left. \left( 15 - \frac{11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \, \text{Cos}[\theta 1] + 14 \, 15 \, \text{Cos}[\theta 4]}{15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4]} \right) \right) \left( 1 + \frac{(-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4])^2}{4 (15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4])^2} \right) \right) \Bigg) / \\
& \left( 4 (15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4]) \left( 1 + \frac{(-2 \, 11 \, \text{Sin}[\theta 1] + 2 \, 14 \, \text{Sin}[\theta 4])^2}{4 (15 + 11 \, \text{Cos}[\theta 1] - 14 \, \text{Cos}[\theta 4])^2} \right) \right);
\end{aligned}$$

$$(\star \frac{b - \sqrt{-4 a h + o^2}}{2 a} \star)$$

$$\begin{aligned} Y_{\text{menos}} = & \frac{1}{2 \left( 1 + \frac{(-2 \, 11 \sin[\theta 1] + 2 \, 14 \sin[\theta 4])^2}{4 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])^2} \right)} \\ & \left( \frac{2 \, 11 \sin[\theta 1] + \frac{15 (-2 \, 11 \sin[\theta 1] + 2 \, 14 \sin[\theta 4])}{2 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])} + \frac{11 \cos[\theta 1] (-2 \, 11 \sin[\theta 1] + 2 \, 14 \sin[\theta 4])}{15 + 11 \cos[\theta 1] - 14 \cos[\theta 4]}}{((11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \cos[\theta 1] + 14 \, 15 \cos[\theta 4]) (-2 \, 11 \sin[\theta 1] + 2 \, 14 \sin[\theta 4]))} - \right. \\ & \left. (2 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])^2) - \sqrt{\left( \left( 2 \, 11 \sin[\theta 1] + \frac{11 \cos[\theta 1] (-2 \, 11 \sin[\theta 1] + 2 \, 14 \sin[\theta 4])}{15 + 11 \cos[\theta 1] - 14 \cos[\theta 4]} \right) + \right.} \right. \\ & \left. \left( \left( 15 - \frac{11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \cos[\theta 1] + 14 \, 15 \cos[\theta 4]}{15 + 11 \cos[\theta 1] - 14 \cos[\theta 4]} \right) (-2 \, 11 \sin[\theta 1] + 2 \, 14 \sin[\theta 4]) \right) \right) /} \\ & \left. (2 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4]))^2 \right) - \\ & \left( 4 \, 11^2 - 4 \, 12^2 + 15^2 - \frac{2 \, 15 (11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \cos[\theta 1] + 14 \, 15 \cos[\theta 4])}{15 + 11 \cos[\theta 1] - 14 \cos[\theta 4]} + \right. \\ & \left. \frac{(11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \cos[\theta 1] + 14 \, 15 \cos[\theta 4])^2}{(15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])^2} + 4 \, 11 \cos[\theta 1] \right. \\ & \left. \left( 15 - \frac{11^2 - 12^2 + 13^2 - 14^2 + 11 \, 15 \cos[\theta 1] + 14 \, 15 \cos[\theta 4]}{15 + 11 \cos[\theta 1] - 14 \cos[\theta 4]} \right) \right) \left( 1 + \frac{(-2 \, 11 \sin[\theta 1] + 2 \, 14 \sin[\theta 4])^2}{4 (15 + 11 \cos[\theta 1] - 14 \cos[\theta 4])^2} \right) \Bigg) \Bigg); \end{aligned}$$

### 2.8.3.- Cinemática inversa

$$In[\ast] = (\star \sigma 1 = 1 \star)$$

$$\theta 1_{\text{mas}} =$$

$$2 \operatorname{ArcTan} \left[ \left( 4 \, 11 \, Y3 + \sqrt{\left( 16 \, 11^2 \, Y3^2 - 4 \left( 11^2 - 12^2 + 11 \, 15 + \frac{15^2}{4} - 2 \, 11 \, X3 - 15 \, X3 + X3^2 + Y3^2 \right) \left( 11^2 - 12^2 - 11 \, 15 + \frac{15^2}{4} + 2 \, 11 \, X3 - 15 \, X3 + X3^2 + Y3^2 \right) \right)} \right) \right] / \left( 2 \left( 11^2 - 12^2 - 11 \, 15 + \frac{15^2}{4} + 2 \, 11 \, X3 - 15 \, X3 + X3^2 + Y3^2 \right) \right);$$

$$(\star \sigma 1 = -1 \star)$$

$$\theta 1_{\text{menos}} =$$

$$2 \operatorname{ArcTan} \left[ \left( 4 \, 11 \, Y3 - \sqrt{\left( 16 \, 11^2 \, Y3^2 - 4 \left( 11^2 - 12^2 + 11 \, 15 + \frac{15^2}{4} - 2 \, 11 \, X3 - 15 \, X3 + X3^2 + Y3^2 \right) \left( 11^2 - 12^2 - 11 \, 15 + \frac{15^2}{4} + 2 \, 11 \, X3 - 15 \, X3 + X3^2 + Y3^2 \right) \right)} \right) \right] / \left( 2 \left( 11^2 - 12^2 - 11 \, 15 + \frac{15^2}{4} + 2 \, 11 \, X3 - 15 \, X3 + X3^2 + Y3^2 \right) \right);$$

$$(\star \sigma 2 = 1 \star)$$

$$\theta 4_{\text{mas}} =$$

$$2 \operatorname{ArcTan} \left[ \left( 4 \, 14 \, Y3 + \sqrt{\left( 16 \, 14^2 \, Y3^2 - 4 \left( -13^2 + 14^2 - 14 \, 15 + \frac{15^2}{4} - 2 \, 14 \, X3 + 15 \, X3 + X3^2 + Y3^2 \right) \left( -13^2 + 14^2 + 14 \, 15 + \frac{15^2}{4} + 2 \, 14 \, X3 + 15 \, X3 + X3^2 + Y3^2 \right) \right)} \right) \right] / \left( 2 \left( -13^2 + 14^2 + 14 \, 15 + \frac{15^2}{4} + 2 \, 14 \, X3 + 15 \, X3 + X3^2 + Y3^2 \right) \right);$$

$$(\star \sigma 2 = -1 \star)$$

$$\theta 4_{\text{menos}} =$$

$$2 \operatorname{ArcTan} \left[ \left( 4 \, 14 \, Y3 - \sqrt{\left( 16 \, 14^2 \, Y3^2 - 4 \left( -13^2 + 14^2 - 14 \, 15 + \frac{15^2}{4} - 2 \, 14 \, X3 + 15 \, X3 + X3^2 + Y3^2 \right) \left( -13^2 + 14^2 + 14 \, 15 + \frac{15^2}{4} + 2 \, 14 \, X3 + 15 \, X3 + X3^2 + Y3^2 \right) \right)} \right) \right] / \left( 2 \left( -13^2 + 14^2 + 14 \, 15 + \frac{15^2}{4} + 2 \, 14 \, X3 + 15 \, X3 + X3^2 + Y3^2 \right) \right);$$

## 2.8.4 .- Dibujar Barras

```
In[*]:= (*Puntos de las barras*)
```

$$p2S2D = \left\{ 11 \cos[\theta 1] + \frac{15}{2}, 11 \sin[\theta 1] \right\};$$

$$p4S2D = \left\{ 14 \cos[\theta 4] - \frac{15}{2}, 14 \sin[\theta 4] \right\};$$

$$p1S2D = \left\{ \frac{15}{2}, 0 \right\};$$

$$p5S2D = \left\{ -\frac{15}{2}, 0 \right\};$$

$$p3S2D = \{Xmas, Ymas\};$$

```
DibujarBarrasFunction[P_, c_, text_] := Module[
  {elementos},
```

```
  (*Union de los elementos*)
```

```
  elementos = {{P[[1]], P[[2]]}, {P[[2]], P[[3]]}, {P[[3]], P[[4]]}, {P[[4]], P[[5]]}, {P[[5]], P[[1]]}} /.
    {11 -> c[[1]], 12 -> c[[2]], 13 -> c[[3]], 14 -> c[[4]], 15 -> c[[5]]} /. {θ1 -> c[[6]], θ4 -> c[[7]]};
```

```
  Show[
```

```
    Table[Graphics[Line[elementos[[k]]]], {k, 1, Length[elementos]}],
```

```
    Table[Graphics[Text[Style[StringForm["l`", k], Bold, Red, 15], Mean[elementos[[k]]]], {k, 1, 5}],
```

```
    Table[Graphics[Text[Style[ToString[k], Bold, Black, 15], elementos[[k, 1]]], {k, Length[elementos]}],
```

```
    Axes -> True, AxesLabel -> {x, y}, PlotLabel -> text,
```

```
    GridLines -> Automatic, GridLinesStyle -> Directive[Gray, Dashed]]
```

```
]
```