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January 28, 2018

¹This document contains my notes on miscellaneous topics.

Part I

Mathematics

Chapter 1

Linear Algebra

1.1 Matrix Inverse Formulas

This chapter contains formulas for the inverses of various sums of matrices.

1.1.1 Inverse of $A + B$

Let $C + D$ be the inverse of $A + B$, then

$$\begin{aligned}(C + D)(A + B) &= I \\ \Rightarrow CA + CB + DA + DB &= I.\end{aligned}\tag{1.1}$$

Now let $C = A^{-1}$, then

$$\begin{aligned}A^{-1}A + A^{-1}B + DA + DB &= I \\ \Rightarrow A^{-1}B + DA + DB &= 0.\end{aligned}\tag{1.2}$$

Now let $D = A^{-1}MA^{-1}$, then

$$\begin{aligned}A^{-1}B + A^{-1}M + A^{-1}MA^{-1}B &= 0 \\ \Rightarrow A^{-1}(B + M + MA^{-1}B) &= 0 \\ \Rightarrow B + M + MA^{-1}B &= 0.\end{aligned}\tag{1.3}$$

Now let $M = BFB$, then

$$\begin{aligned}B + BFB + BFBA^{-1}B &= 0 \\ \Rightarrow B(I + FB + FBA^{-1}B) &= 0 \\ \Rightarrow I + FB + FBA^{-1}B &= 0 \\ \Rightarrow F(B + BA^{-1}B) &= -I \\ \Rightarrow F &= (B + BA^{-1}B)^{-1}.\end{aligned}\tag{1.4}$$

Therefore $D = A^{-1}B(B + BA^{-1}B)^{-1}BA^{-1}$. Substituting for C and D , the inverse of $A + B$ is then

$$(A + B)^{-1} = A^{-1} - A^{-1}B(B + BA^{-1}B)^{-1}BA^{-1} \quad (1.5)$$

Note that (1.5) only requires A to be invertible. If B is invertible (1.5) can be simplified to

$$(A + B)^{-1} = A^{-1} - A^{-1}B(I + A^{-1}B)^{-1}A^{-1} \quad (1.6)$$

or

$$(A + B)^{-1} = A^{-1} - A^{-1}(I + BA^{-1})BA^{-1} \quad (1.7)$$

1.2 Inverse of $A + BCD$

Using (1.5)

$$\begin{aligned} (A + BCD)^{-1} &= A^{-1} - A^{-1}BCD(BCD + BCDA^{-1}BCD)^{-1}BCDA^{-1} \\ &= A^{-1} + A^{-1}BCD(BC(D + DA^{-1}BCD))^{-1}BCDA^{-1} \\ &= A^{-1} + A^{-1}BCD(D + DA^{-1}BCD)DA^{-1} \\ &= A^{-1} + A^{-1}BCD((D(CD)^{-1} + DA^{-1}B)CD)^{-1}DA^{-1} \\ &= A^{-1} + A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1} \end{aligned} \quad (1.8)$$

1.3 Inverse of $AB + CD$

Using (1.5)

$$\begin{aligned} (AB + CD)^{-1} &= B^{-1}A^{-1} - (AB)^{-1}CD(CD + CD(AB)^{-1}CD)^{-1}CD(AB)^{-1} \\ &= B^{-1}A^{-1} - B^{-1}A^{-1}CD(CD(I + (AB)^{-1}CD))^{-1}CDB^{-1}A^{-1} \\ &= B^{-1}A^{-1} - B^{-1}A^{-1}CD(I + B^{-1}A^{-1}CD)^{-1}B^{-1}A^{-1} \end{aligned} \quad (1.9)$$

Bibliography