

CS 571 Homework 5

Due: Friday, May 3, 2024

1 Perform the Reductions

25 points

For each of the following λ -calculus expressions, execute the associated program by performing reductions in normal (i.e. call-by-name or lazy) order.

- $(\lambda x. (\lambda m. \lambda n. \lambda f. n (m f)) x (\lambda f. \lambda x. f f f x)) (\lambda f. \lambda x. f f f x)$
- $((\lambda x. \lambda y. x y (\lambda x. \lambda y. y)) (\lambda x. \lambda y. y) ((\lambda x. \lambda y. x (\lambda x. \lambda y. x) y) (\lambda x. \lambda y. y) (\lambda x. \lambda y. x))) (\lambda x. \lambda y. x) (\lambda x. \lambda y. y)$
- $(\lambda p. (\lambda p. p (\lambda x. \lambda y. y)) p) ((\lambda a. \lambda b. \lambda z. z a b) (\lambda x. \lambda y. x) (\lambda f. \lambda x. f f f x))$

Hint: To check your work, note that the above expressions correspond to the following syntactic sugar for λ -calculus programs:

- $(\lambda x. (x * 3)) 3$
- if* (*ff* and (*ff* or *tt*)) *then* *tt* *else* *ff*
- $(\lambda p. (\pi_2 p)) (\pi tt 3)$

2 Write the Program

10 points

Write λ -calculus expressions that implement the following programs.

- Write a function *sub* which performs subtraction. This function should take in two Church numerals n and m and return a Church numeral representing $n - m$.
- Write a function *fib* that computes Fibonacci numbers. Given an input Church numeral n , the function should return the n th Fibonacci number according to the recurrence:

$$fib\ 0 = 0$$

$$fib\ 1 = 1$$

$$fib\ n = (fib\ (n - 1)) + (fib\ (n - 2))$$

Hint: You will need to use a predecessor function, an example of which is presented in Lecture 20.