Mechanized Verification with Sharing

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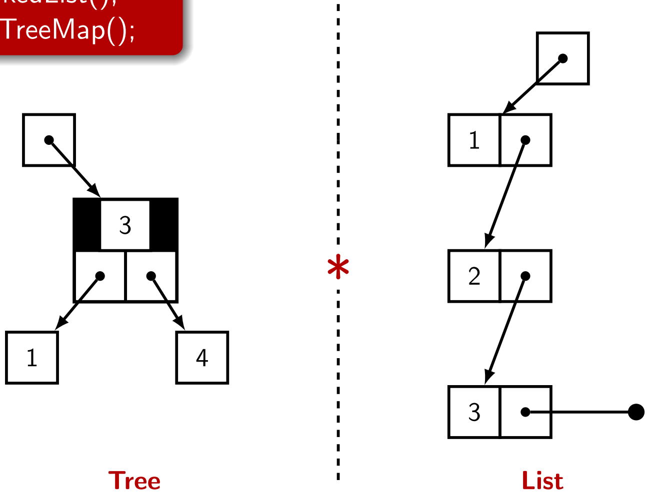
Separation Logic is Great

List lst = LinkedList();
Map map = TreeMap();



Aliasing Breaks Everything: External Sharing

```
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Iterator itr = Iterator(lst);
```
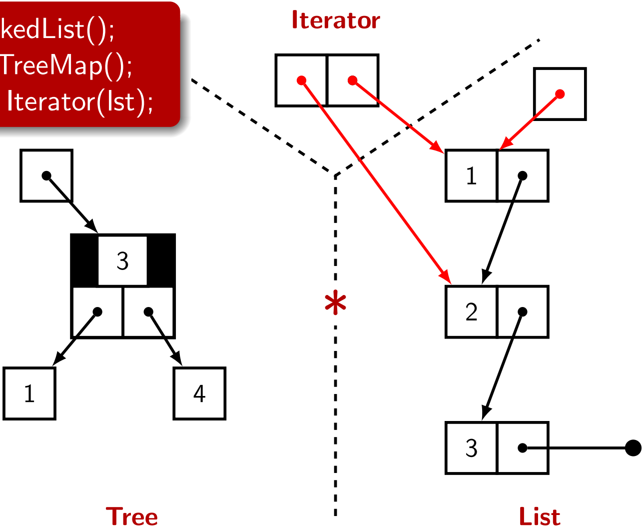
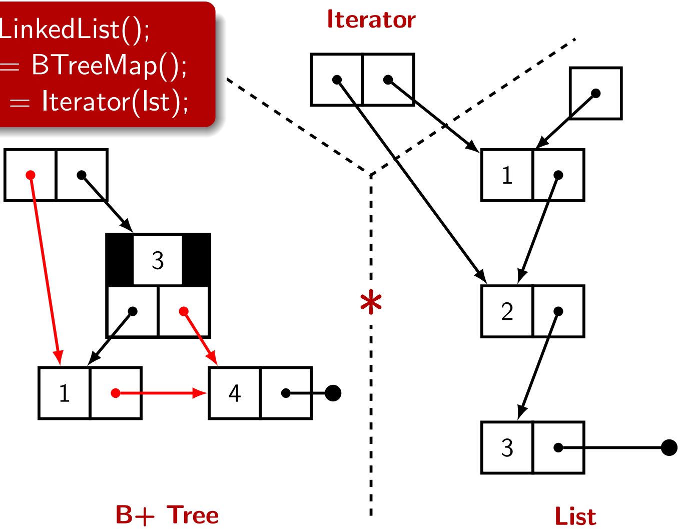



Aliasing Breaks Everything: **Internal** Sharing

List lst = LinkedList();
Map map = BTreeMap();
Iterator itr = Iterator(lst);



Outline

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Where have all the effects gone?

- Most type systems don't express effects.

$$\boxed{\text{swap} : \text{int } \textbf{ref} \rightarrow \text{int } \textbf{ref} \rightarrow \text{unit}}$$

- Don't know whether the function has effects.
- Must break encapsulation to reason about programs that call `swap`.

Black-box Effects

- Monads mitigate this problem to some extent.

$$\boxed{\text{swap} :: \text{MVar int} \rightarrow \text{MVar int} \rightarrow \textbf{IO}\ ()}$$

- Can make explicit the **absence** of effects.
- Does **not** explicitly express their presence...or exactly what they are.

Hoare Type Theory: Ynot

- We rely on *program logics* to express effects.

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    - Express side-effects using a dependent monad.

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Pre-condition

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Post-condition (depends on return value)

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swap : $\forall$ p q (v u : int),
Cmd (p $\mapsto$ v $*$ q $\mapsto$ u)
(_ : unit $\Rightarrow$ p $\mapsto$ u $*$ q $\mapsto$ v)

Basic Typing Rule: Read a Pointer

- Hoare Logic

$$\frac{P \Longrightarrow p \mapsto v * P'\ v}{\{P\}!p\{r \Rightarrow p \mapsto r * P'\ r\}}\ \textsc{Read}$$

- Hoare Type

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read : ∀ p P',
  Cmd (∃ v. p ↦ v ∗ P' v)
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C-style Linked Lists

```
module type LLIST =
struct
  type tlst (** = list int **)
  val empty : unit → tlst
  (** ... **)
  val sub   : tlst → int → int option
end
```

- An abstract type (`tlst`) and functions on it (`empty`, `sub`).

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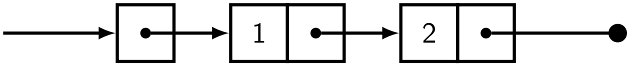
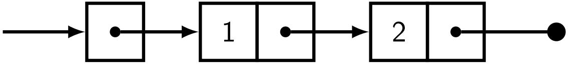
- An abstract type (`tlst`) and functions on it (`empty`, `sub`).
- To reason about correctness, we need specifications.
  1. Relate the type `tlst` to a functional model.
  2. Describe the heap in terms of the model.
  3. Provide specifications for functions.

(1&2) Representation Predicate



(1&2) Representation Predicate



Record llNode := mkNode { val : int ; next : optr }.

(1&2) Representation Predicate


```
Record llNode := mkNode { val : int ; next : optr }.

Equations llseg (pStart pEnd : optr) (ls : list int) :=
 llseg pStart pEnd nil :=
   pStart = pEnd
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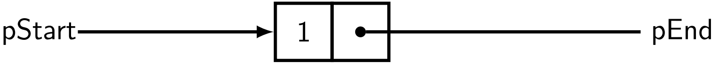


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Equations llseg (pStart pEnd : optr) (ls : list int) :=
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 llseg (Ptr st) pEnd (a :: b) :=
   ∃ nx : optr, st ↦ mkNode a nx ∗ llseg nx pEnd b
```
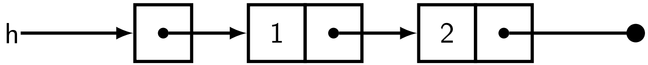

(1&2) Representation Predicate

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Definition tlst := ptr.
Definition llist (h : tlst) (m : list T) :=
   ∃ st : optr, h ↦ st ∗ llseg st Null m.
```

(3) Correctness Specifications

$$
\begin{array}{l}
\text{empty} : \textbf{Cmd}\ (\text{emp}) \\
\qquad\qquad\qquad (\text{r} : \text{tlst} \Rightarrow \text{llist r nil}) \\
\\
\text{sub} : \forall\ (\text{t} : \text{tlst})\ (\text{i} : \text{int})\ (\text{m} : \text{list int}), \\
\quad \textbf{Cmd}\ (\text{llist t m}) \\
\qquad\qquad (\text{r} : \text{option int} \Rightarrow \text{llist t m} * \text{r} = \text{nth m i})
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Expressing Iterators

```
Class ListIterable (titr : Type) :=
{ rep : titr → list int → nat → hprop
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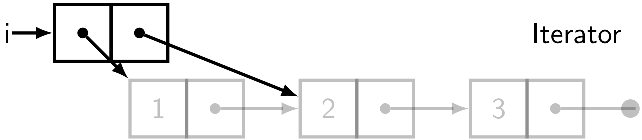
Class ListIterable (titr : Type) :=
{ rep : titr → list int → nat → hprop
; next : ∀ (t : titr) (m : list int) (idx : nat),
    Cmd (rep t m idx)
        (res : option int ⇒ res = nth m idx *
         rep t m (idx + 1))
}.
```

A Simple Iterator




```
Definition titr := ptr.

Definition liter (t : titr) (ls : list int) (n : nat) :=
  ∃ st : optr, ∃ cur : optr,
  t ↦ (st, cur) ∗
```

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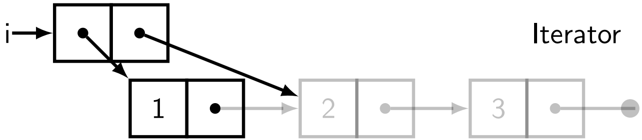


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  llseg st cur (firstn n ls) ∗
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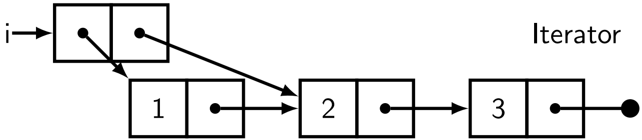


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   llseg cur Null (skipn n ls).
```
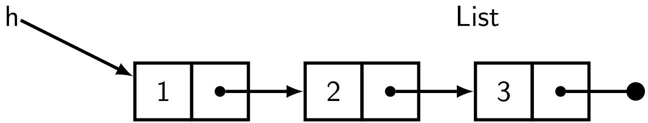

The Sharing Problem

- Requires access to the same memory as the underlying list.
  - Creating an iterator *consumes* the underlying list.
  - Can't have multiple iterators.



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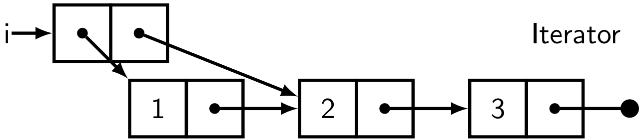

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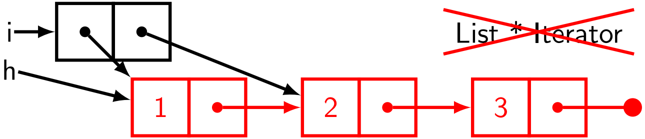
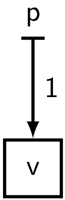


Sharing with Fractional Permissions (Boyland '03)

- Parameterize points-to by a fractional ownership.
  - p $\stackrel{q}{\mapsto}$ v, q is the fraction.



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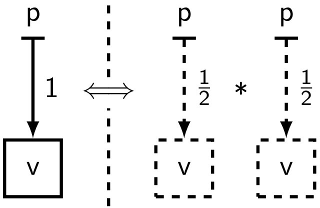

A Fractional Iterator

- Describe the iterator as owning a **fraction** of the list.

```
Definition liter (owner : tlst ) (q : Fp)
    (t : titr ) ( ls : list int ) (n : nat) : hprop :=
  ∃ st : optr , ∃ cur : optr ,
  t ↦ (st, cur) ∗
  llseg st cur ( firstn n ls ) q ∗
  llseg cur Null (skipn n ls ) q.
```

- During construction, only consume a fraction of the list.

```
Definition iterator : ∀ (t : tlst ) (m : list int ) (q : Fp),
  Cmd (llist t m q)
      (res : titr ⇒ liter t q res m 0).
```

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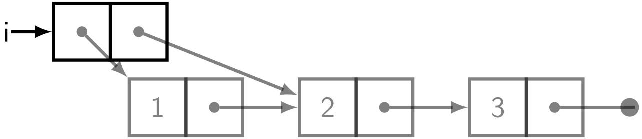

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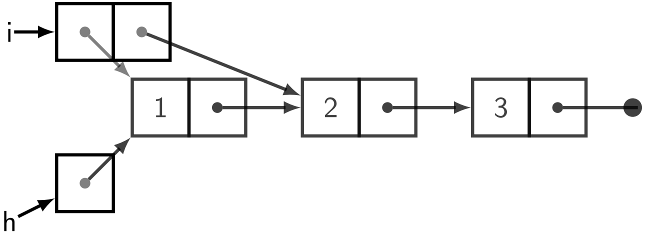
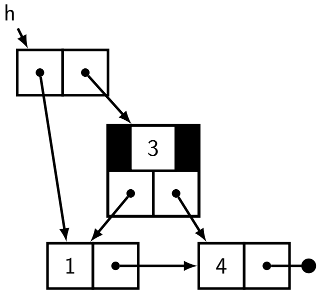


Outline

Aliasing

- Aliasing makes describing data structure more difficult.



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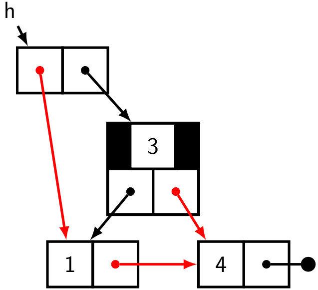

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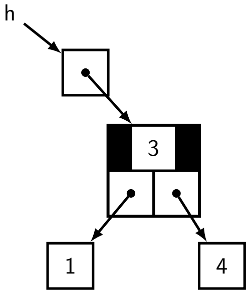

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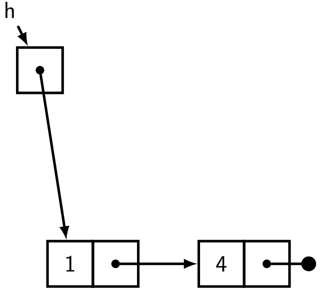

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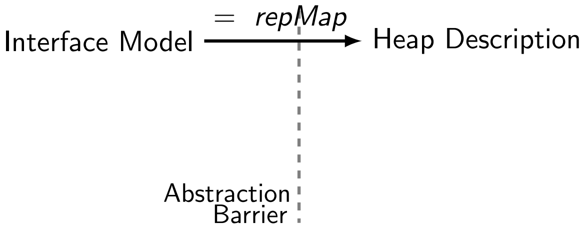

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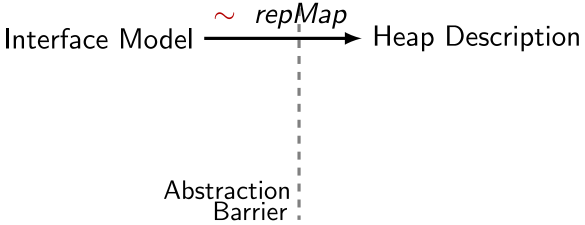

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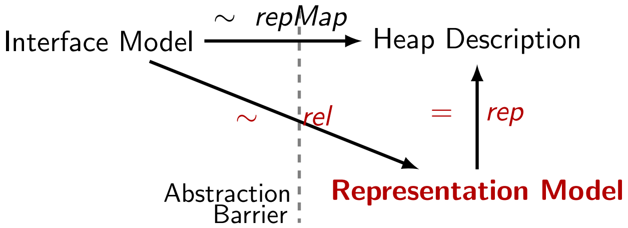

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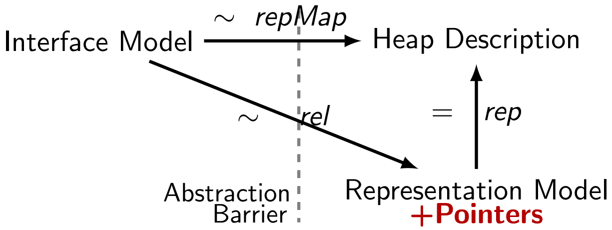

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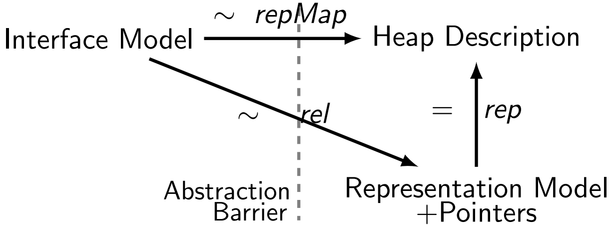

$\boxed{\begin{array}{c} p \textbf{ aliases } q \\ p = q \end{array}}$

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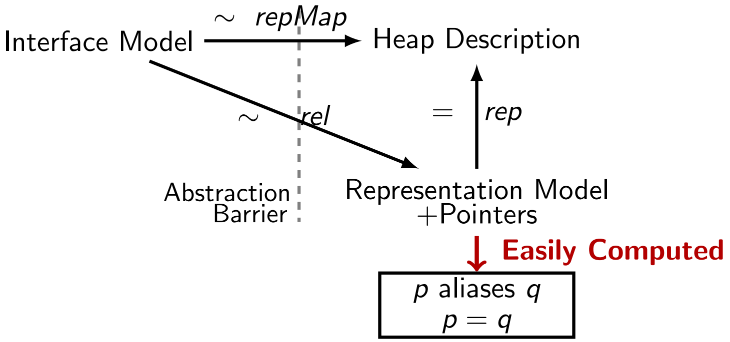
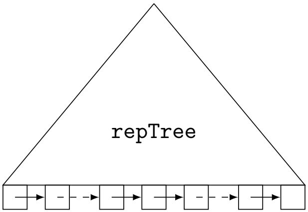


Implementing the Interface

- lookup, insert



Implementing the Interface

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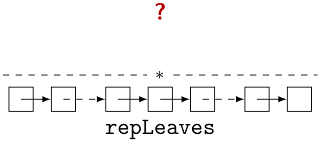

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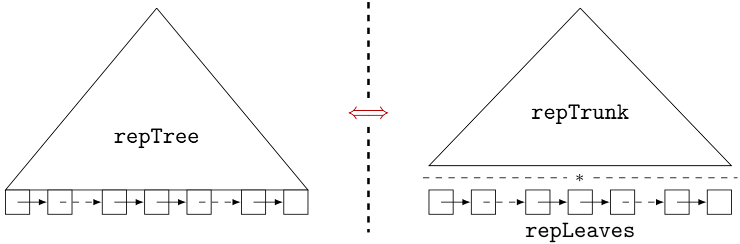

Lemma repTree_repTrunkLeaves : ∀ h p optr (m : ptree h),
repTree p optr m ⟺
repTrunk p optr m ∗ repLeaves (firstPtr m) optr (leaves m).

Outline

Take Away

1. **Sequential Sharing** — Even sequential code has sharing problems.
2. **Fractional permissions** — Expose sharing at interfaces.
3. **Internal Aliasing** – Hide details of efficient implementation.