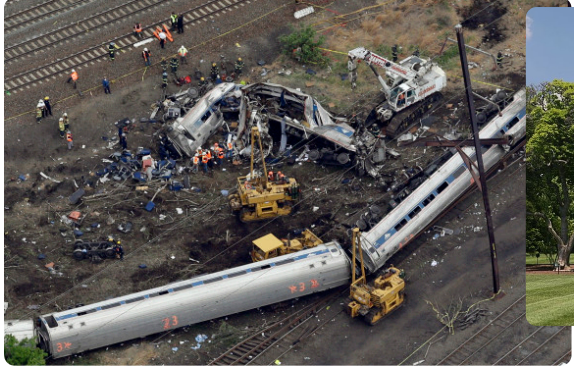
Towards Foundational Verification of Cyber-physical Systems

Gregory Malecha Daniel Ricketts
Mario M. Alvarez Sorin Lerner

SoSCYPS 2016

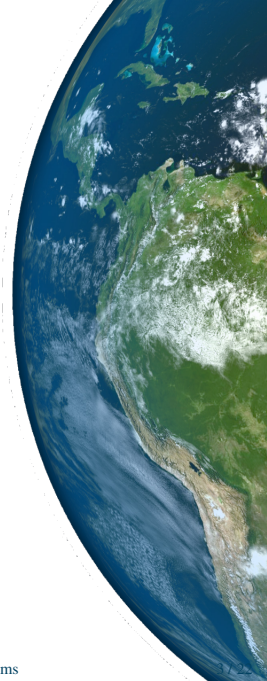



JOHNSON, STEVEN
117
77
87.9
42.2
22.3
86
58
108
88
97
95

System
(Hw+C)

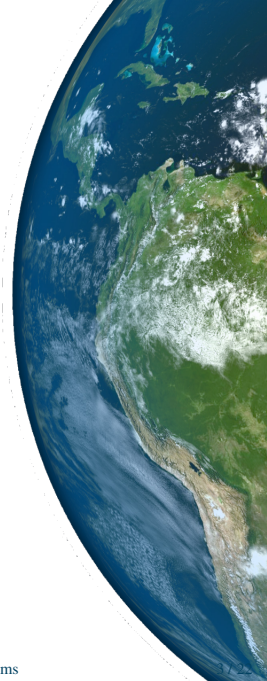
System
(Hw+C)

Specification
(Safety)



System
(Hw+C)

Verifier
(KeYmaera)



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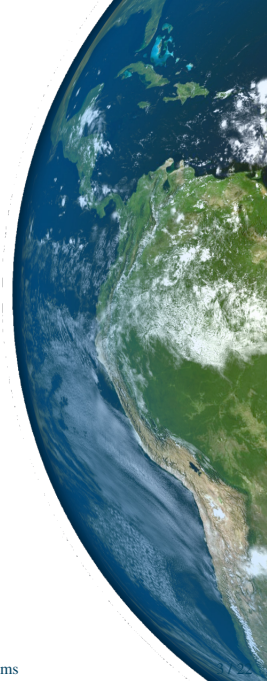

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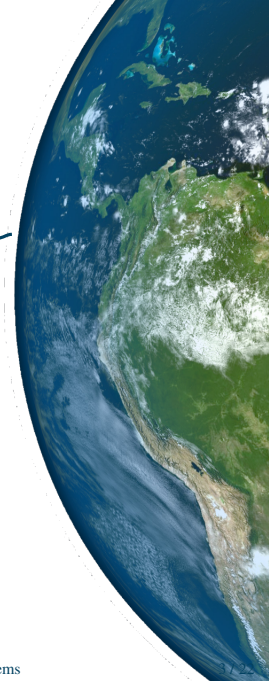

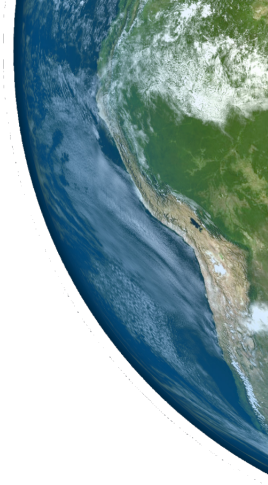

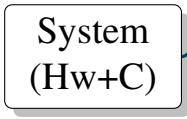
System
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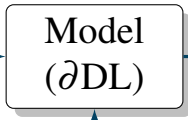
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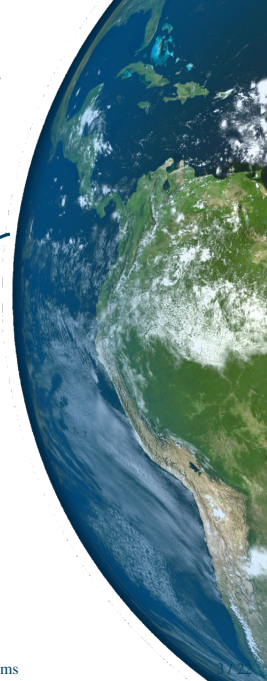

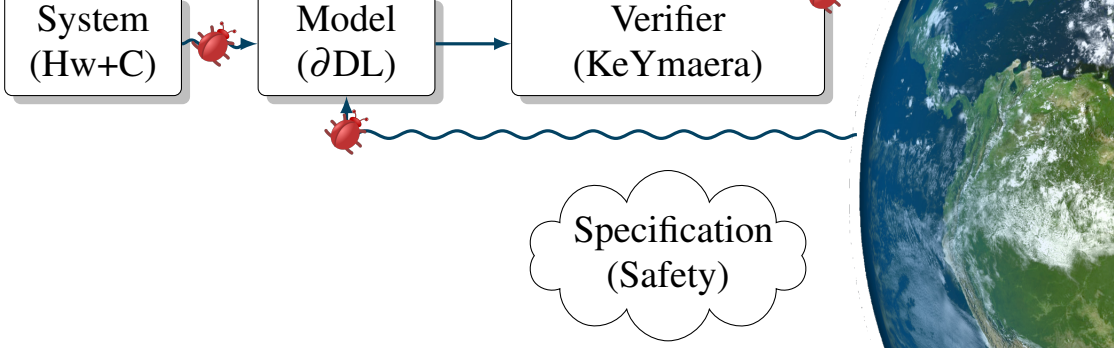
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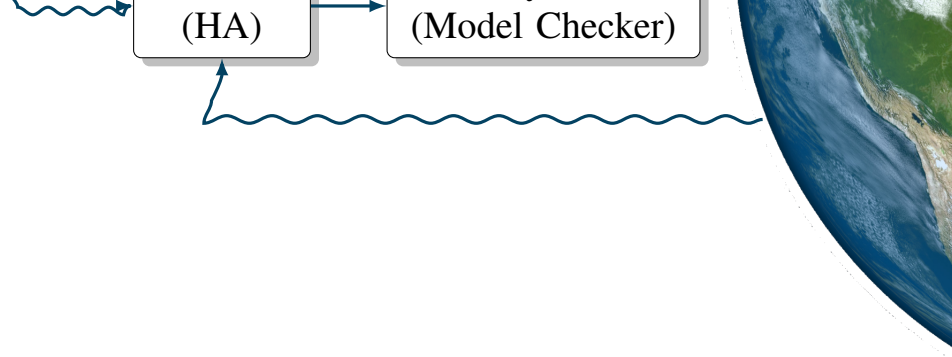
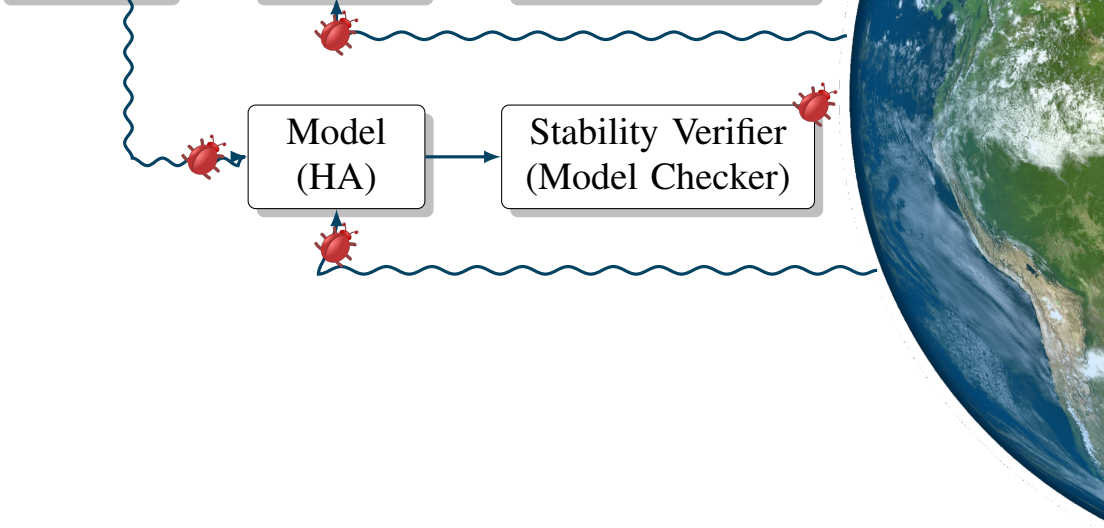
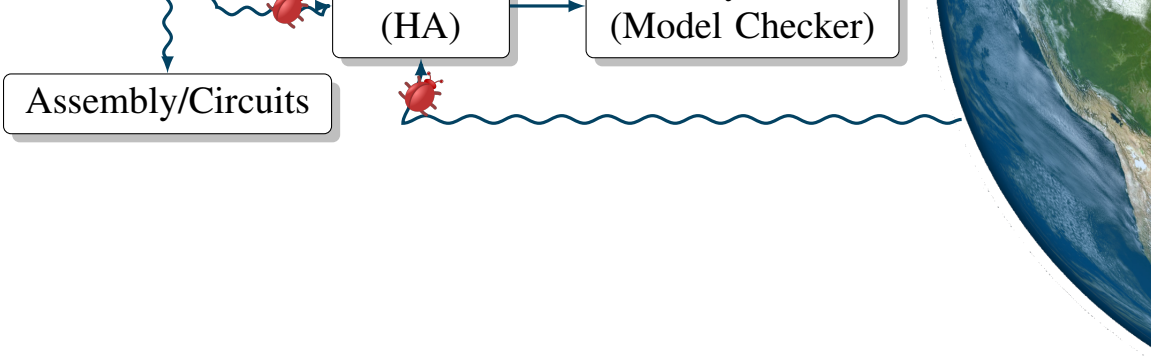
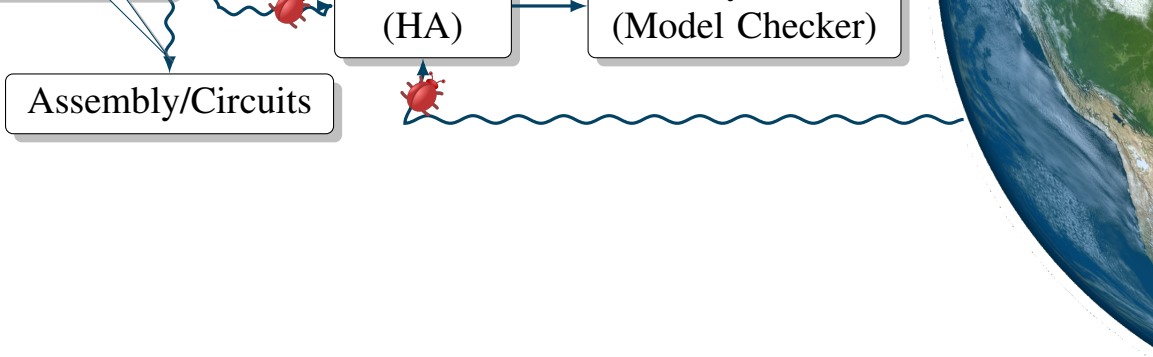

(HA)
(Model Checker)


Model
(HA)
Stability Verifier
(Model Checker)


(HA)
(Model Checker)
Assembly/Circuits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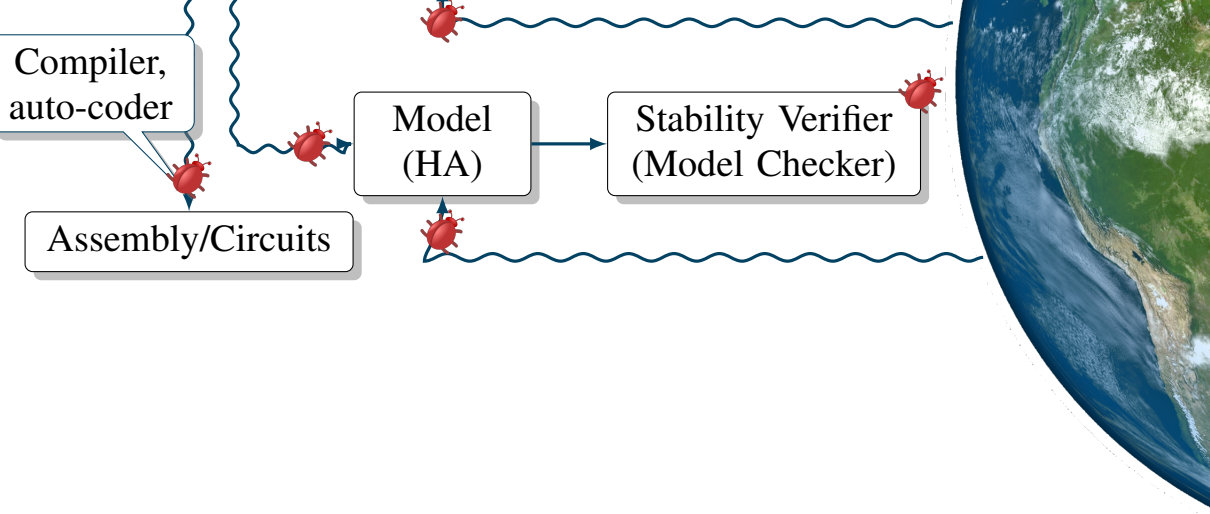
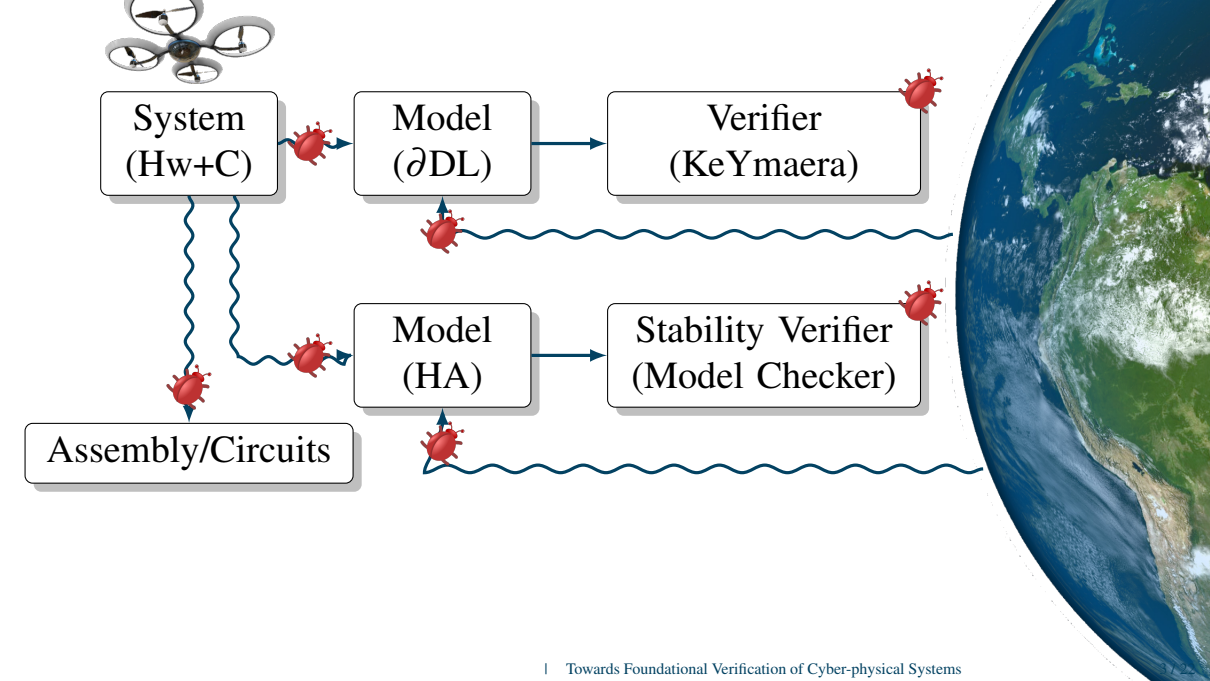

Compiler,
auto-coder
Assembly/Circuits
Model
(HA)
Stability Verifier
(Model Checker)


System
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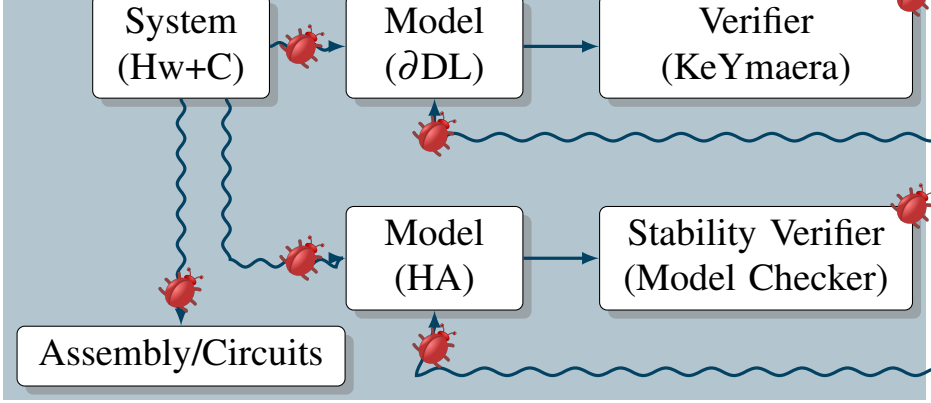
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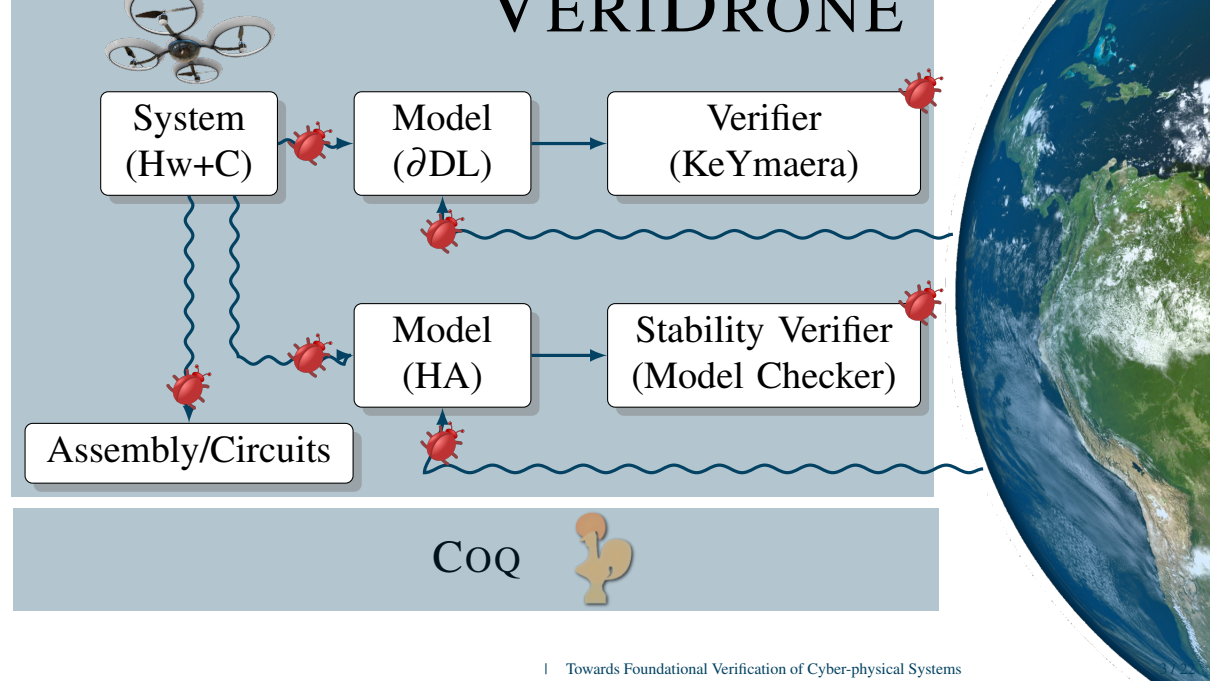
VERIDRONE
System
(Hw+C)
Model
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Verifier
(KeYmaera)
Model
(HA)
Stability Verifier
(Model Checker)
Assembly/Circuits
Coq

Benefits of Foundational Verification [YCER11]

Compiler	Bugs
GCC	122
Clang/LLVM	181
CompCert	

Benefits of Foundational Verification [YCER11]

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CompCert	0†

Verified in Coq

† In verified code

Benefits of Foundational Verification [YCER11]

Compiler	Bugs
GCC	122
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Verified in Coq

Foundational verification

- Strong guarantees, **and**
- Expressive logic

Verification in Coq

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Verification in Coq

Definitions

```
Def opt (c : c_prog) : c_prog :=
  ... c ...
```

Foundational verification

- Strong guarantees, **and**
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Verification in Coq

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Def opt (c : c_prog) : c_prog :=
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Thm opt_sou                prog,
  c ∼ opt c.
```

Definitions

Theorems

Foundational verification

- Strong guarantees, **and**
- Expressive logic

Verification in Coq

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Def opt (c : c_prog) : c_prog :=
  ... c ...

Thm opt_sound : ∀ c : c_prog,
  c ∼ opt c.
Proof.
  induction c.
  (* proof for each case *)
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Definitions

Theorems

Foundational verification

- Strong guarantees, and
- Expressive logic

Verification in Coq

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  ... c ...

Thm opt_sound : ∀ c : c_prog,
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Proof.
  induction c.
  (* proof for each case *)
Qed.
```

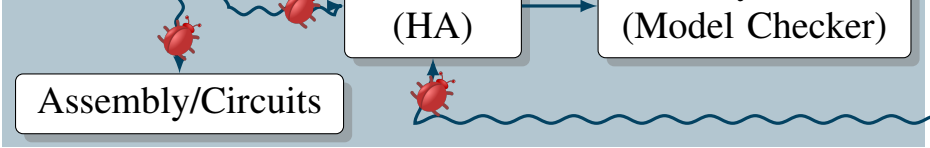
Definitions

Theorems

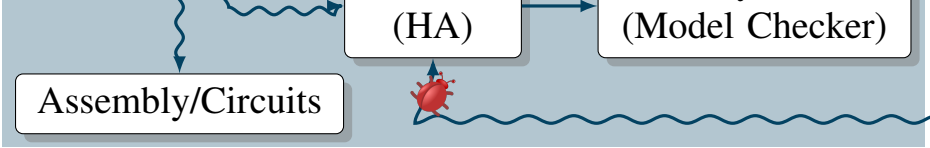
Interactive proof scripts

Foundational verification

- Strong guarantees, **and**
- Expressive logic

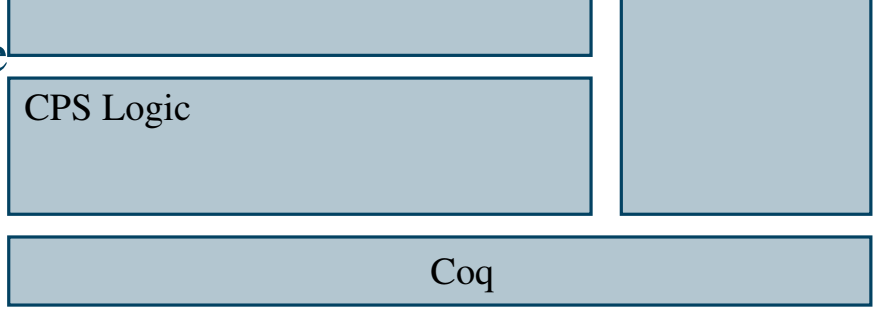


Coq

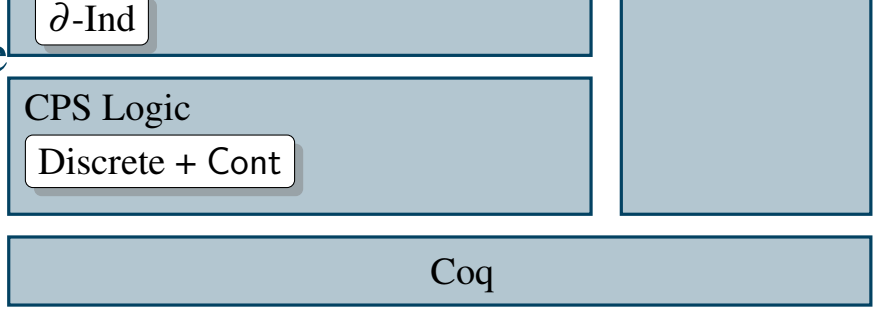


Coq

Outline



Outline



Case Study: Runtime Monitors [Dan15]

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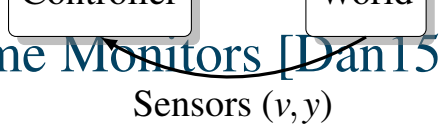

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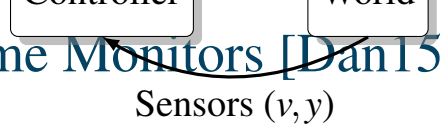

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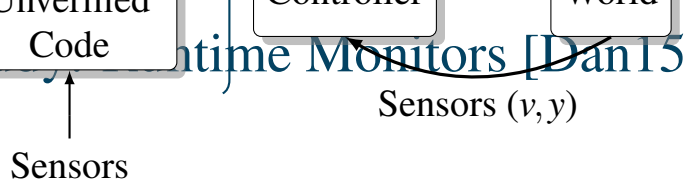

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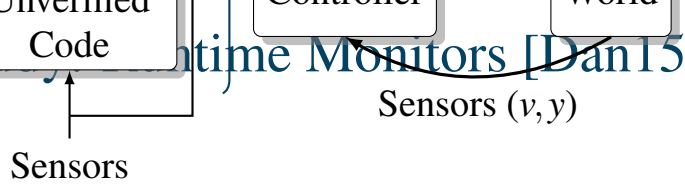

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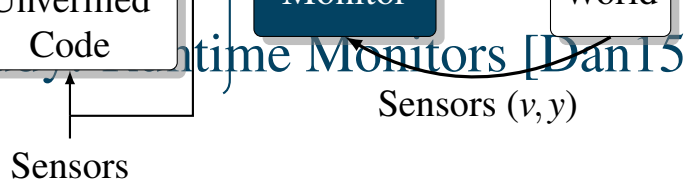

Case Study: Runtime Monitors [Dan15]

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"Sampled-Data System"

Case Study: Runtime Monitors [Dan15]

"Always"

Case Study: Runtime Monitors [Dan15]

Initial condition

Case Study: Runtime Monitors [Dan15]

$$\text{Init} \wedge \Box\, \text{Sys}_{\Delta}\Big(\underbrace{\text{Monitor}}_{\text{Monitor}}, \underbrace{\text{World}}_{\text{World}}\Big) \vdash \Box P$$

Boundness, stability, robustness, etc.

A Flavor of Customizable Verification

$$\frac{}{P \wedge \Box \mathsf{Sys}_{\Delta}(D, W) \vdash \Box P}\ \textsc{Sys-Ind}$$

Custom proof rules

A Flavor of Customizable Verification

′ = after the transition

(Discrete)

$$\frac{P \wedge D \wedge 0 \leq \tau' \leq \Delta \vdash P'}{P \wedge \Box \mathsf{Sys}_{\Delta}(D, W) \vdash \Box P}\ \textsc{Sys-Ind}$$

A Flavor of Customizable Verification

(Discrete)

$$\frac{P \wedge D \wedge 0 \leq \tau' \leq \Delta \vdash P'}{P \wedge \Box \mathsf{Sys}_{\Delta}(D, W) \vdash \Box P}\ \textsc{Sys-Ind}$$

A Flavor of Customizable Verification

(Continuous) $P \wedge \mathsf{Cont}(W \wedge \dot{\tau} = -1) \wedge 0 \leq \tau' \vdash P'$

(Discrete)

$$\frac{P \wedge D \wedge 0 \leq \tau' \leq \Delta \vdash P'}{P \wedge \Box \mathsf{Sys}_{\Delta}(D, W) \vdash \Box P}\ \textsc{Sys-Ind}$$

Continuous evolution

Derivative

A Flavor of Customizable Verification

(Continuous)

(Discrete)

$$\frac{P \wedge \mathsf{Cont}\,(W \wedge \dot{\tau} = -1) \wedge 0 \leq \tau' \vdash P' \qquad P \wedge D \wedge 0 \leq \tau' \leq \Delta \vdash P'}{P \wedge \Box \mathsf{Sys}_{\Delta}\,(D, W) \vdash \Box P}\ \textsc{Sys-Ind}$$

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A Flavor of Customizable Verification

Def

$$\frac{P \wedge \mathsf{Cont}(W \wedge \dot{\tau} = -1) \wedge 0 \leq \tau' \vdash P' \qquad P \wedge D \wedge 0 \leq \tau' \leq \Delta \vdash P'}{P \wedge \Box \mathsf{Sys}_{\Delta}(D, W) \vdash \Box P}\ \textsc{Sys-Ind}$$

Thm

Def

Aside: Simplified Quadcopter Dynamics

Linear dynamics

World dynamics

Angular thrus

$$W_{QC} \triangleq \left(\begin{array}{lll} C_{\theta\phi} & \rightarrow & \dot{\mathbf{x}} = \mathbf{v_x} \wedge \dot{\mathbf{y}} = \mathbf{v_y} \wedge \dot{\mathbf{z}} = \mathbf{v_z} \\ & \wedge & \dot{\mathbf{v_x}} = \mathbf{T} \cos\phi \sin\theta \\ & \wedge & \dot{\mathbf{v_y}} = -\mathbf{T} \sin\phi \\ & \wedge & \dot{\mathbf{v_z}} = \mathbf{T} \cos\phi \cos\theta - g \\ & \wedge & \dot{\phi} = 0 \wedge \dot{\theta} = 0 \wedge \dot{\mathbf{T}} = 0 \end{array} \right)$$

Pitch

Roll

$$C_{\theta\phi} \triangleq \quad |\theta| \leq 30° \wedge |\phi| \leq 30° \wedge 0 \leq \mathbf{T}$$

Simple dynamics if "mostly level" ($< 30°$)

Aside: Simplified Quadcopter Dynamics

$$W_{OC} \triangleq \left(\begin{array}{lll} C_{\theta\phi} & \rightarrow & \dot{\mathbf{x}} = \mathbf{v_x} \wedge \dot{\mathbf{y}} = \mathbf{v_y} \wedge \dot{\mathbf{z}} = \mathbf{v_z} \\ & \wedge & \dot{\mathbf{v_x}} = \mathbf{T}\cos\phi\sin\theta \\ & \wedge & \dot{\mathbf{v_y}} = -\mathbf{T}\sin\phi \\ & \wedge & \dot{\mathbf{v_z}} = \mathbf{T}\cos\phi\cos\theta - \\ & \wedge & \dot{\phi} = 0 \wedge \dot{\theta} = 0 \wedge \dot{\mathbf{T}} = 0 \end{array} \right)$$

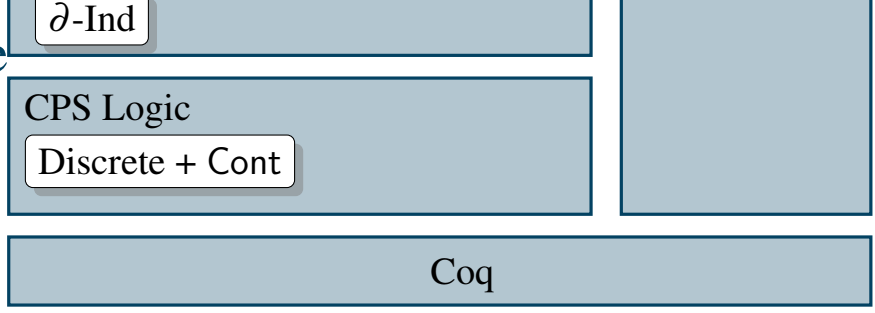
"Small-angle constraint"

Instantaneous change

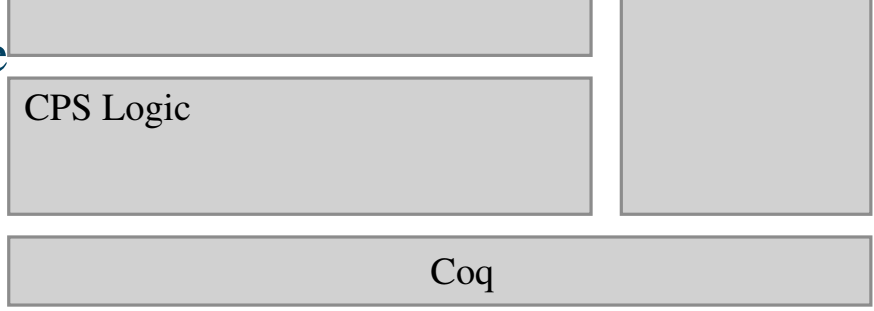
$$C_{\theta\phi} \triangleq \quad |\theta| \leq 30° \wedge |\phi| \leq 30° \wedge 0 \leq \mathbf{T}$$

† Approximation justified by fast angular dynamics and the small angle constraint [GHH+11].

Outline



Outline



Composing Monitors [Dan16a]

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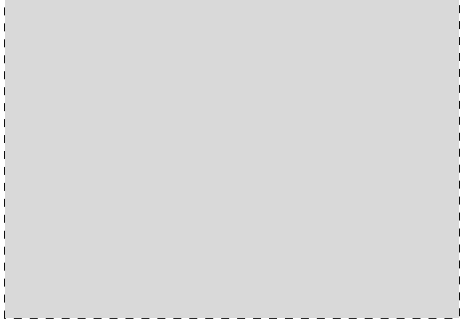
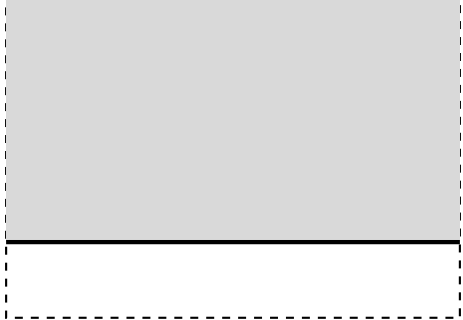


- Spatial transformation

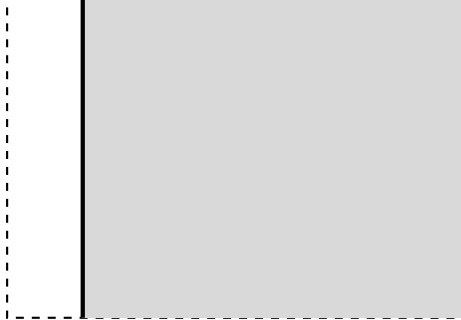
Composing Monitors [Dan16a]

- Spatial transformation



Composing Monitors [Dan16a]

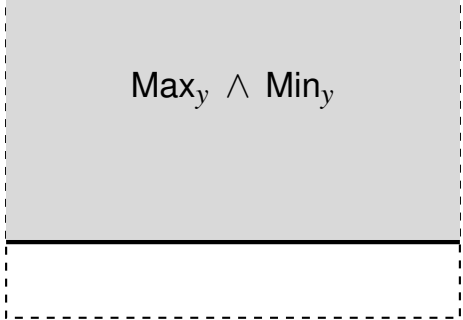
- Spatial transformation



Composing Monitors [Dan16a]

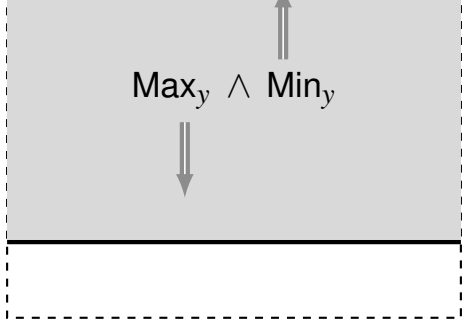
- Spatial transformation

Composing Monitors [Dan16a]



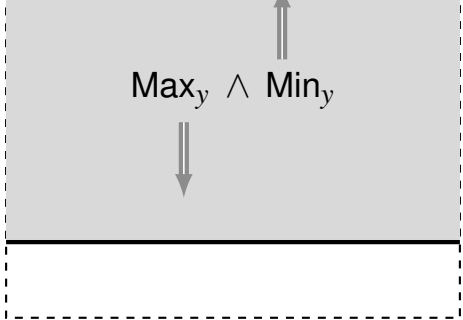
- Spatial transformation
- Conjunctive composition

Composing Monitors [Dan16a]



- Spatial transformation
- Conjunctive composition

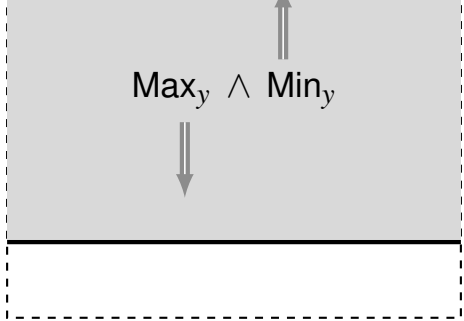
Composing Monitors [Dan16a]



Monitors might not be compatible!

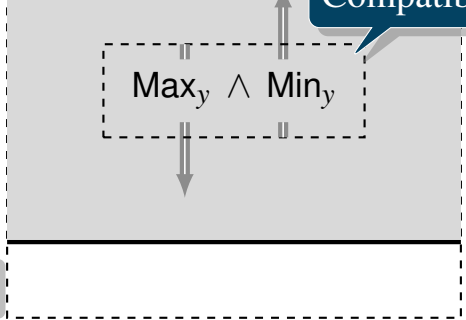
- Spatial transformation
- Conjunctive composition

Composing Monitors [Dan16a]



- Spatial transformation
- Conjunctive composition

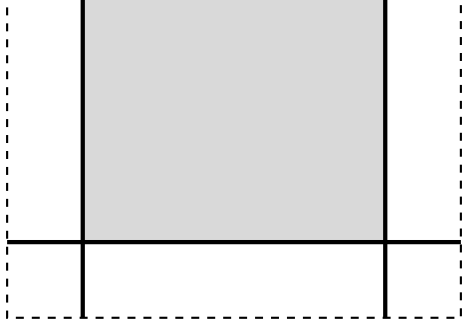
Composing Monitors [Dan16a]



- Spati
- Conjunctive composition

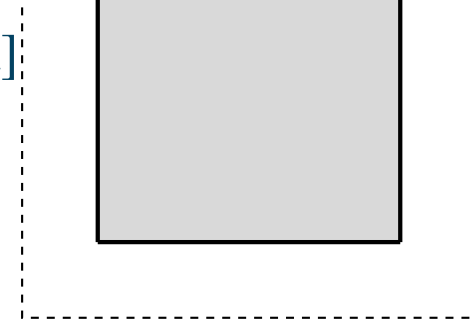
Formalize and build proof rules

Composing Monitors [Dan16a]



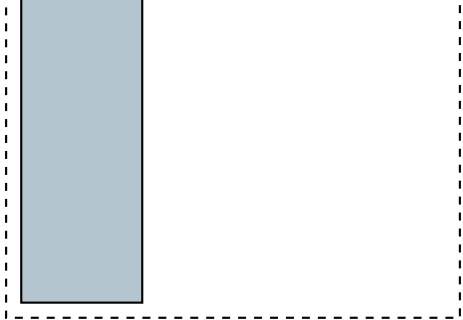
- Spatial transformation
- Conjunctive composition

Composing Monitors [Dan16a]



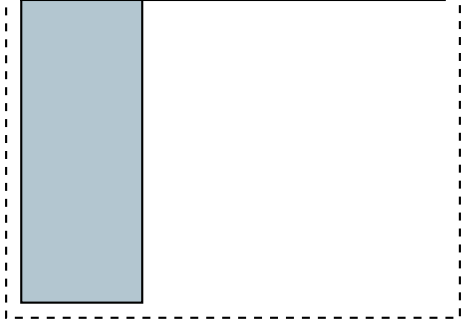
- Spatial transformation
- Conjunctive composition

Composing Monitors [Dan16a]



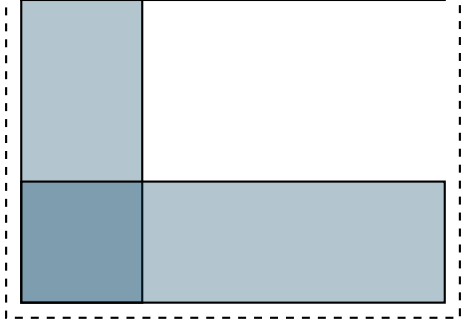
- Spatial transformation
- Conjunctive composition
- Disjunctive composition

Composing Monitors [Dan16a]



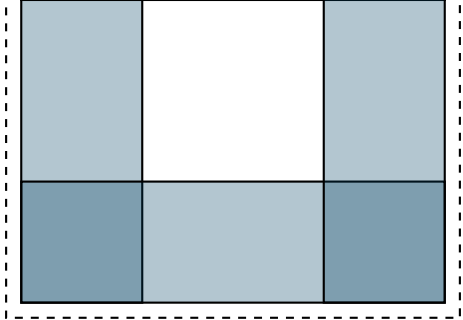
- Spatial transformation
- Conjunctive composition
- Disjunctive composition

Composing Monitors [Dan16a]



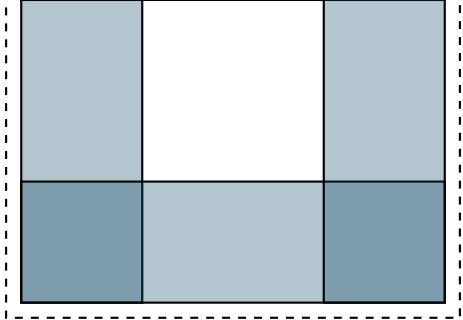
- Spatial transformation
- Conjunctive composition
- Disjunctive composition

Composing Monitors [Dan16a]



- Spatial transformation
- Conjunctive composition
- Disjunctive composition

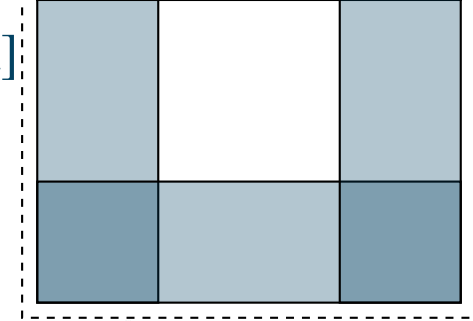
Composing Monitors [Dan16a]



- Spatial transformation
- Conjunctive composition
- Disjunctive composition

Composing Monitors [Dan16a]

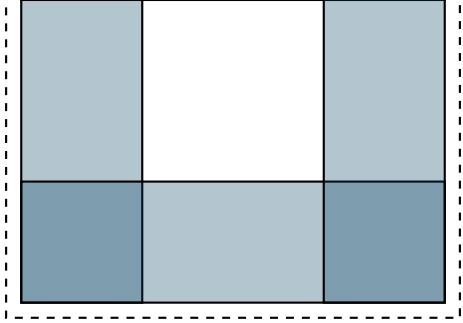
- Spatial transformation
- Conjunctive composition
- Disjunctive composition



Composing Monitors [Dan16a]

Thm

Thm

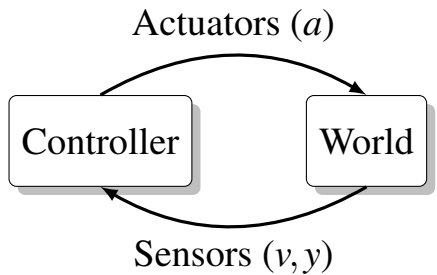


- Spatial transformation
- Conjunctive composition
- Disjunctive composition

Extending the Model: Robustness [Dan16b]

Tolerance to disturbances

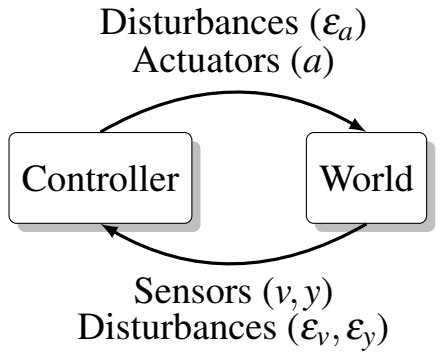
- Add disturbances
- Bound violations



Extending the Model: Robustness [Dan16b]

Tolerance to disturbances

- Add disturbances
- Bound violations

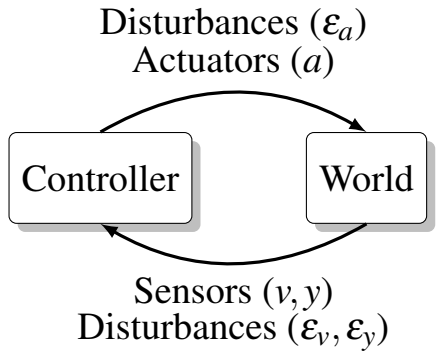


Extending the Model: Robustness [Dan16b]

Def

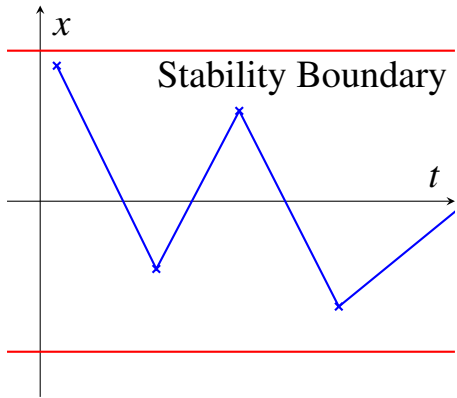
Tolerance to disturbances

- Add disturbances
- Bound violations



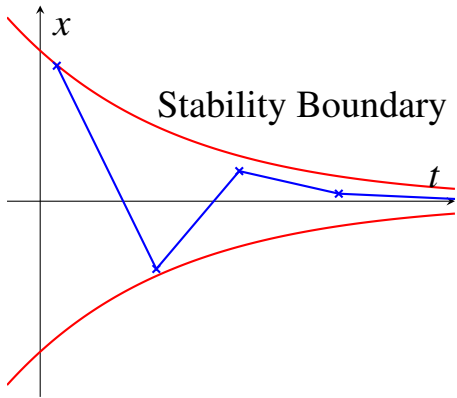
More Properties: Stability [Mat16]

- Boundedness over time



More Properties: Stability [Mat16]

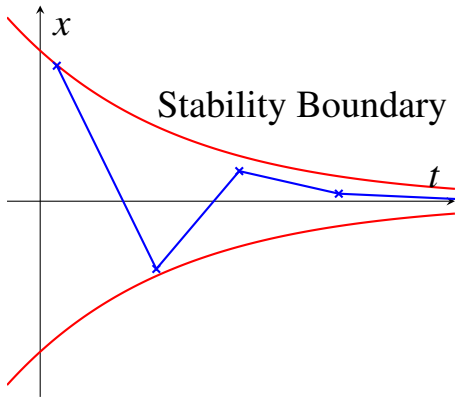
- Boundedness over time
- Convergence to a goal



More Properties: Stability [Mat16]

- Boundedness over time
- Convergence to a goal

- 100 years of control theory!



More Properties: Stability [Mat16]

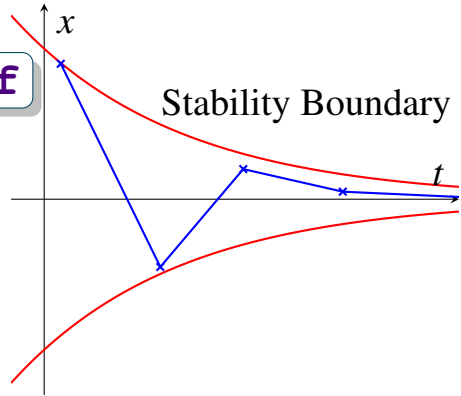
- Boundedness over time
- Convergence to a goal
- 100 years of control theory!

Def

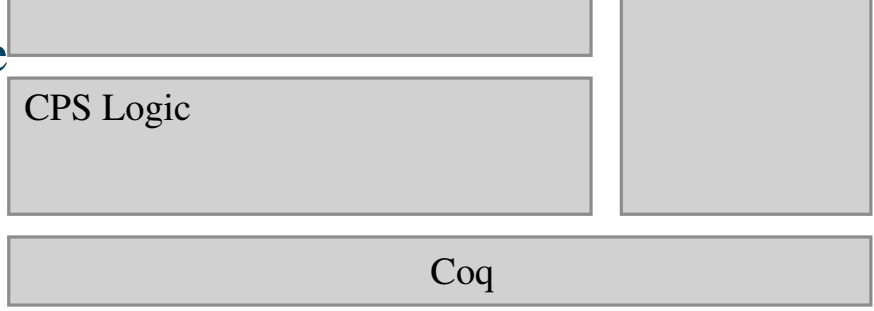
Def

Thm

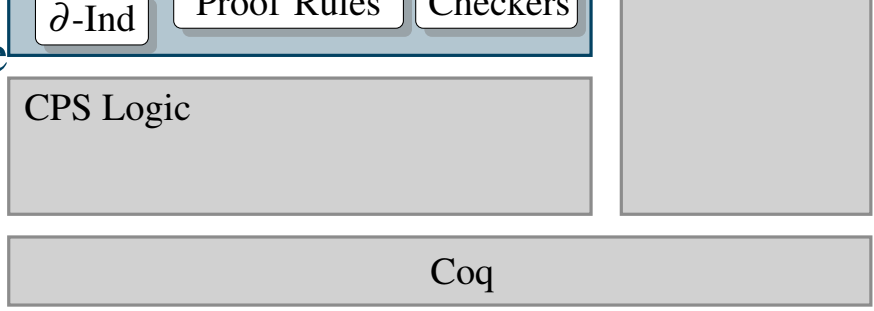
Thm



Outline



Outline



External Automation

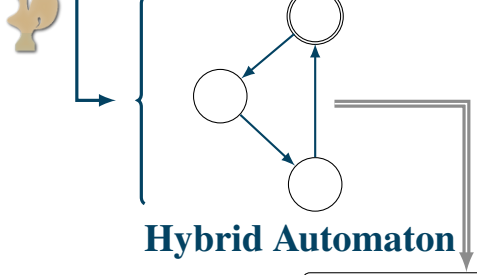
Connect to other tools

External Automation

Connect to other tools

- Formalize in Coq

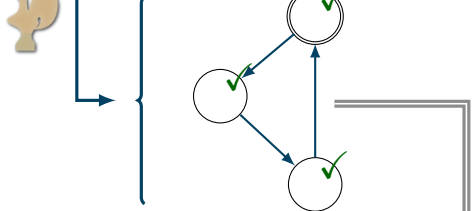
External Automation



Connect to other tools

- Formalize in Coq
- Leverage automation

External Automation

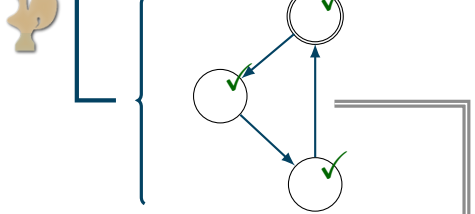


Connect to other tools

- Formalize in Coq
- Leverage automation

Model Checker
(e.g. SpaceEx)

External Automation



Connect to other tools

- Formalize in Coq
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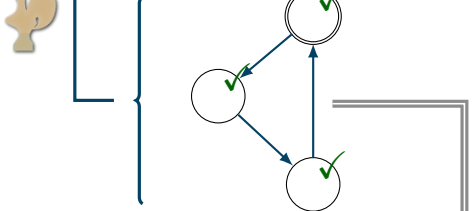
External Automation



Connect to other tools

- Formalize in Coq
- Leverage automation

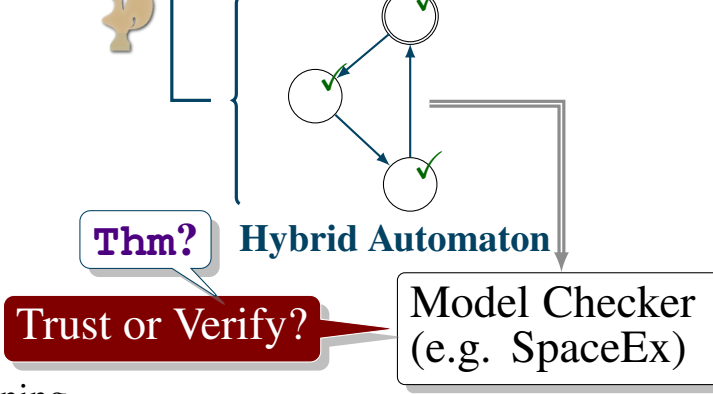
Trust or Verify?

Model Checker
(e.g. SpaceEx)

External Automation

Connect to other tools

- Formalize in Coq
- Leverage automation
- Combine with other reasoning



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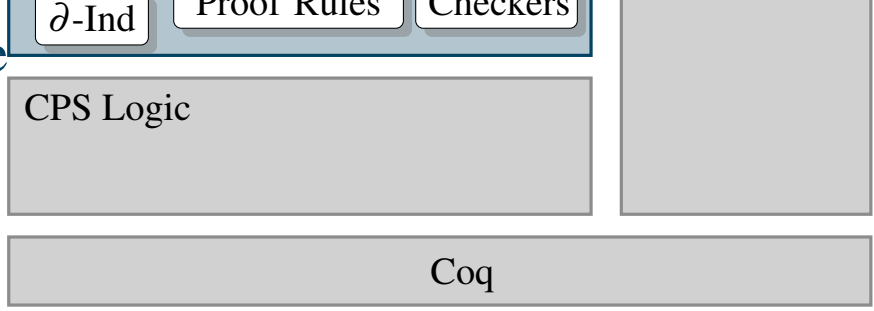

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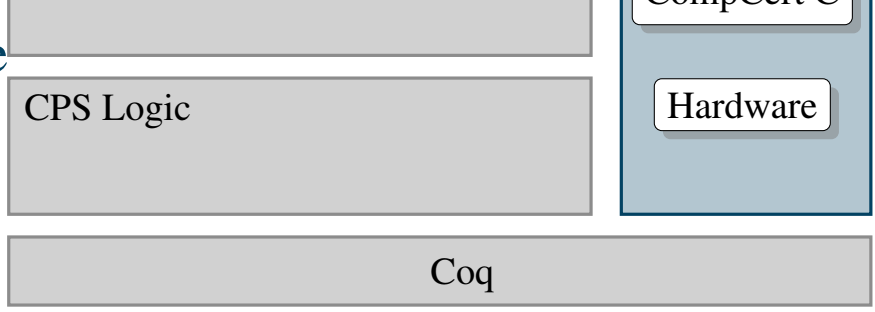


End-to-End Guarantees

$$\mathsf{Sys}_{\Delta}(\qquad D,\qquad W) \vdash P$$

Connect models & code

- Existing developments

End-to-End Guarantees

Floating point & execution

$$\mathsf{Sys}_\Delta(\ D,\ W) \vdash P$$

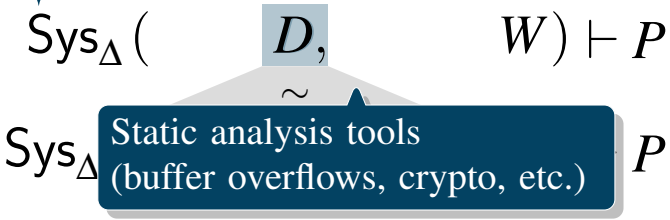
$$\sim$$

$$\mathsf{Sys}_\Delta(\ \texttt{C\{C-code\}},\ W) \vdash P$$

Connect models & code

- Existing developments

End-to-End Guarantees



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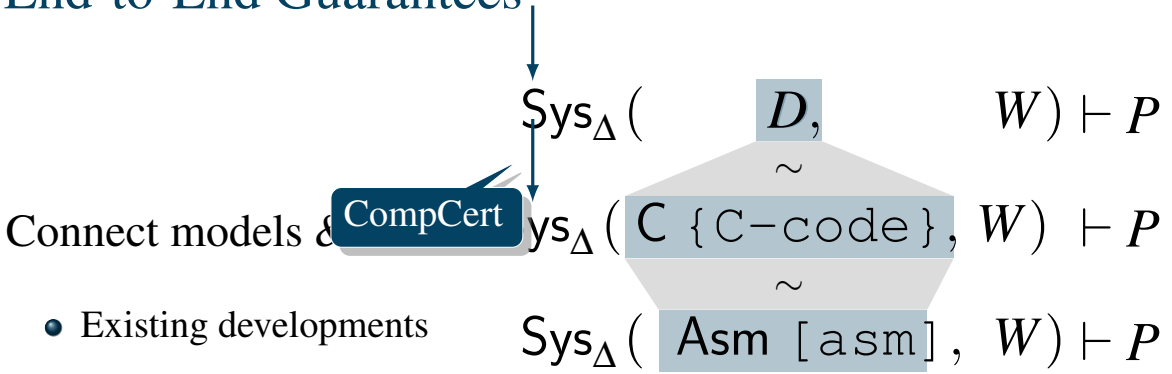
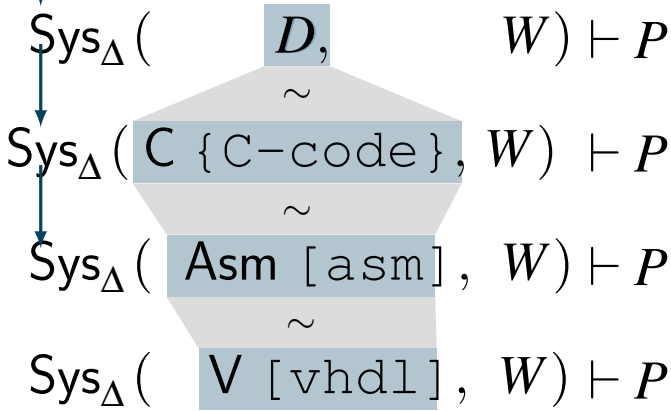
End-to-End Guarantees



End-to-End Guarantees

Connect models & code

- Existing developments



End-to-End Guarantees

Thm

Def

$$\mathsf{Sys}_\Delta(\ D,\ W) \vdash P$$

Def

$$\sim$$

$$\mathsf{Sys}_\Delta(\ \mathsf{C}\ \texttt{\{C-code\}},\ W) \vdash P$$

Connect models & code

Thm

Def

$$\sim$$

$$\mathsf{Sys}_\Delta(\ \mathsf{Asm}\ \texttt{[asm]},\ W) \vdash P$$

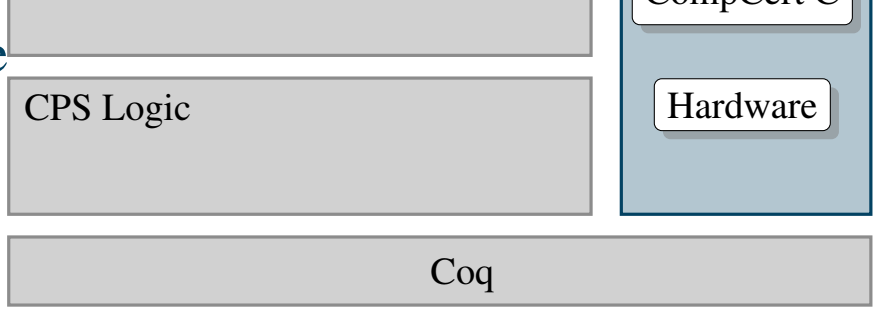
- Existing developments

Foundational proofs of *actual code*

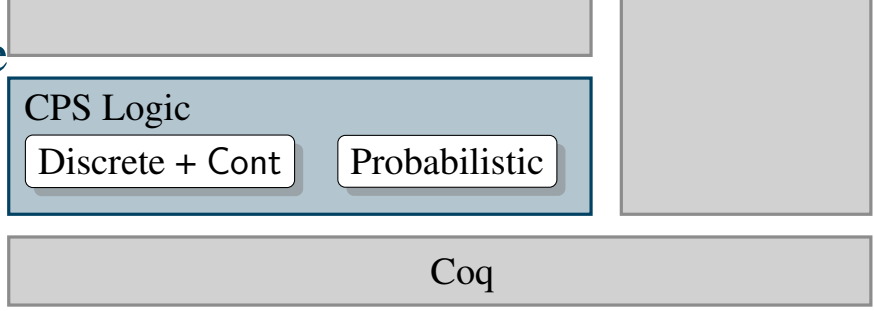
$$\sim$$

$$\mathsf{Sys}_\Delta(\ \mathsf{V}\ \texttt{[vhdl]},\ W) \vdash P$$

Outline

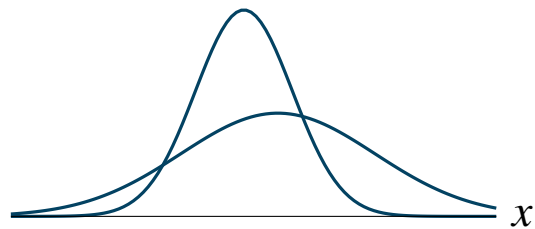


Outline



Uncertainty

- Probabilistic processes
- Minimize uncertainty



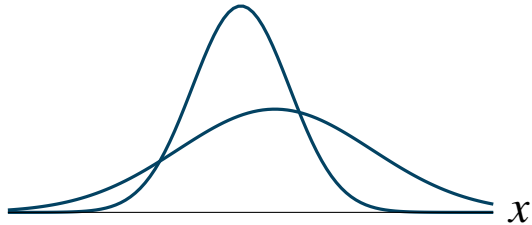
Uncertainty

fused

gps

barometer

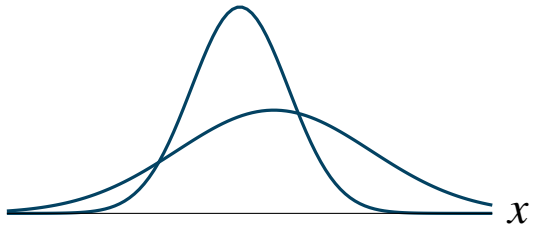
- Probabilistic processes
- Minimize uncertainty



Uncertainty

Safe (80%) Unsafe (20%)

- Probabilistic processes
- Minimize uncertainty
- Decide with uncertainty



Uncertainty

Safe (80%) Unsafe (20%)

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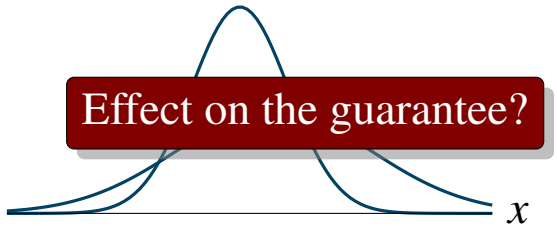
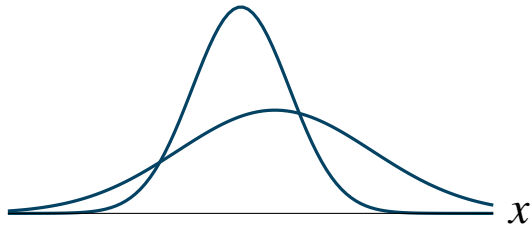
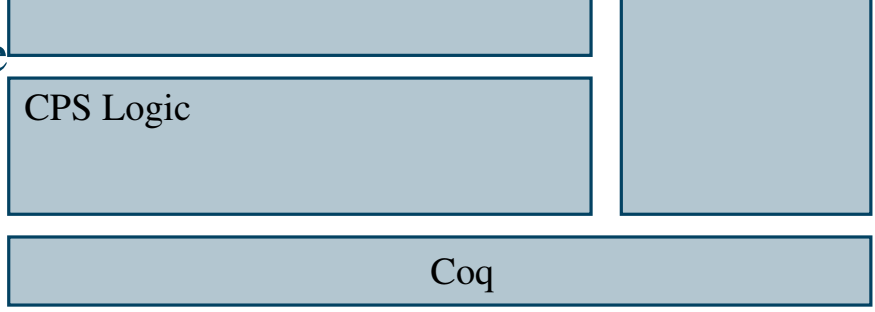


Uncertainty

- Probabilistic processes
- Minimize uncertainty
- Decide with uncertainty



Outline



Out

http://verifydone.ucsd.edu/

CPS Logic

Coq

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