Sharing in Ynot

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January 15, 2010

Outline

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Gaining Assurance

Observation

If there's one thing that we've learned in the past 20 years it's that all software has bugs.

- Tried and are trying a lot of approaches to mitigate this problem:
    - (Unit) Testing
    - Bug Finding Tools
    - Static Type Systems
    - Model Checking
    - Theorem Proving

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The Burden of Proofs

- Several projects have worked on verification.
  - Jahob - Verification in Java
  - *Spec#* - Verification in C#
  - *seL4* - Kernel verification in Agda
- Standard approach to verification:
  1. Write specifications.
  2. Write all of the code.
  3. Give specifications and code to VC Generator.
  4. Modify code/add annotations until and repeat until verification succeeds.

Type-Oriented Specifications: ML

- Most type systems don't express side-effects explicitly.

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(* swap : 'a ref -> 'a ref -> unit *)
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```
(* swap : 'a ref -> 'a ref -> unit *)
let swap a b =
  let t = !a in
  a := !b ;
  b := t
```

- Simplifies coding.
- But the types don't tell us whether a function is really a function!

Explicit IO: Haskell

- Haskell makes side-effects explicit using monads.

```
swap :: MVar a -> MVar a -> IO ()
```

Explicit IO: Haskell

- Haskell makes side-effects explicit using monads.

```
swap :: MVar a -> MVar a -> IO ()
swap p1 p2 =
  do { t1 <- takeMVar p1
     ; t2 <- takeMVar p2
     ; putMVar p1 t2
     ; putMVar p2 t1
     }
```

- Can now determine if a function doesn't have side effects.
- Only looking at the type, we know more, but not enough.

Specifications: Ynot

- Hoare logic-based specifications using dependent types.
- Index the IO monad by pre- and post-conditions.
  - Allows us to precisely specify the effects of a computation.

```
Definition swap : forall (p1 p2 : ptr) (v1 v2 : [nat]),
  Cmd (v1 ~~ v2 ~~ p1 ~~> v1 * p2 ~~> v2)
      (fun _ : unit => v1 ~~ v2 ~~ p1 ~~> v2 * p2 ~~> v1).
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DEMO

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  Cmd (v1 ~~ v2 ~~ p1 ~~> v1 * p2 ~~> v2)
      (fun _ : unit => v1 ~~ v2 ~~ p1 ~~> v2 * p2 ~~> v1).
  refine (fun p1 p2 v1 v2 =>
    t1 <- ! p1 ;
    t2 <- ! p2 ;
    p1 ::= t2 ;;
    {{ p2 ::= t1 }});
  sep fail auto. (** Proof **)
Qed.
```

Overview

1. Logic
   - Shallow embedding of separation logic.
   - Computational irrelevance.
2. Monad
   - `Cmd` monad indexed by pre- and post-conditions.
3. Tactics
   - *Ltac* automation for separation logic.

Separation Logic

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(** Predicates over heaps **)
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Definition hprop := heap -> Prop.
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Definition cell p v : hprop := fun h =>
  forall p', if p = p' then h p = Some v
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Definition hprop_sep (P Q : hprop) : hprop :=
  fun h => exists h1 h2, h ~> h1 * h2 /\ P h1 /\ Q h2.
```

Ynot Library : Command Monad

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Axiom Cmd : forall (pre : hprop) {A} (post : A -> hprop), Set.
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  pre2 T2 (post2 : T2 -> hprop)
  (st1 : Cmd pre1 post1)
  (_ : forall v, post1 v ==> pre2 v)
  (st2 : forall v : T1, Cmd (pre2 v) post2)
  : Cmd pre1 post2.
```

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  (_ : forall v, post1 v ==> pre2 v)
  (st2 : forall v : T1, Cmd (pre2 v) post2)
  : Cmd pre1 post2.

Axiom CmdRead : forall (T : Set) (p : ptr) (P : T -> hprop),
  Cmd (Exists v :@ T, p ~~> v * P v)
      (fun v => p ~~> v * P v).

(** ... and more ... **)
```

Outline

C-style Linked Lists

- Linked lists in ML.

```
module type LLIST =
struct
  type 'a t
  val new : unit -> 'a t
  (** ... **)
  val sub : 'a t -> int -> 'a option
end
```

- A type (`t`) and functions on it (`new`, `sub`).

C-style Linked Lists

- Linked lists in ML.

```
module type LLIST =
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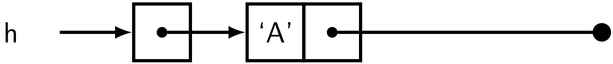
- A type (`t`) and functions on it (`new`, `sub`).
- To reason about correctness, we need specifications.
  1. Relate the type `t` to a computationally irrelevant model.
  2. Provide a predicate that describes the heap in terms of model.
  3. Provide specifications as stronger types for the functions.

Representation Predicate



- Describe the heap computationally using a functional model.

.

Representation Predicate

pStart ———————————— pEnd

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```
Fixpoint llseg (pStart pEnd : optr) (ls : list T) : hprop :=
  match ls with
    | nil     => [pStart = pEnd]
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Representation Predicate

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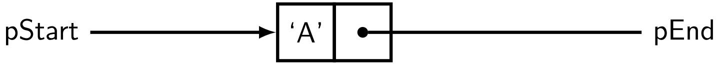

- Describe the heap computationally using a functional model.

```
Record llNode := mkNode { val : T ; next : optr }.

Fixpoint llseg (pStart pEnd : optr) (ls : list T) : hprop :=
  match ls with
    | nil    => [pStart = pEnd]
    | a :: b => match pStart with
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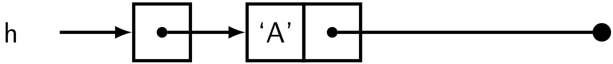
                  | Some p => Exists nx :@ option ptr,
                    p ~~> mkNode a nx * llseg nx pEnd b
  end end.
```

Representation Predicate



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  end end.

Definition tlst := ptr.
Definition llist (h : tlst) (m : list T) : hprop :=
  Exists st :@ option ptr, h ~~> st * llseg st None m.
```

Linked Lists: sub

```
Definition sub : forall (t : tlst) (i : nat) (m : [list T]),
  Cmd (m ~~ llist t m)
      (fun res : option T =>
         m ~~ llist t m * [res = nth_error m i]).
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      (fun res : option T =>
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  refine (fun t i m =>
    hd <- ! t ;
    {{ Fix3 (fun hd j m => m ~~ llseg hd None m)
            (fun hd j m (r : option T) =>
               m ~~ llseg hd None m * [r = nth_error m j])
            (fun self hd j m =>
              IfNull hd Then {{ Return None }}
              Else
                nde <- ! hd ;
                IfZero j Then
                  {{ Return (Some (val nde)) }}
                Else
                  {{ self (next nde) j (m ~~~ tail m) <@> _ }}
            ) hd i m <@> _ }});
  try clear self; sep' s tac.
Qed.
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  try clear self; sep' s tac.
Qed.
```

Linked Lists: mfold_left

- Can even write higher-order computations while maintaining abstraction.

```
Definition mfold_left : forall {U : Type} (t : tlst)
  (I : list T -> U -> hprop) (a : U) (m : [list T])
  (cmd : forall (c : T) (a : U) (m : [list T]),
     Cmd (m ~~ I m a)
         (fun a : U => m ~~ I (m ++ c :: nil) a)),
  Cmd (m ~~ llist t m * I nil a)
      (fun r : U => m ~~ llist t m * I m r).
```

- I is the invariant.
- a is the initial accumulator.
- cmd is the folded computation.

Outline

Adding Iterators

- Iterators and collections go hand-in-hand.

```
Class ListIterable (h : Set) (T : Type) : Type := {
  rep   : h -> list T -> nat -> hprop ;
```

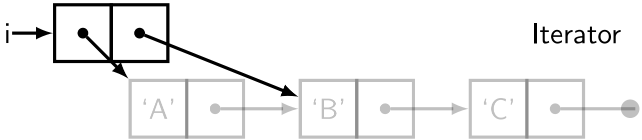
Adding Iterators

- Iterators and collections go hand-in-hand.

```
Class ListIterable (h : Set) (T : Type) : Type := {
  rep  : h -> list T -> nat -> hprop ;
  next : forall (t : h) (m : [list T]) (idx : [nat]),
    Cmd (m ~~ idx ~~ rep t m idx)
        (fun res : option T => m ~~ idx ~~
           rep t m (nextIndex idx (length m)) *
           [res = nth_error m idx])
}.
```

- `h` is the type of the iterator handle.
- `T` is the type of values being iterated over.
- Representation predicate (`rep`) and `next` command.

A Naïve Iterator




```
Definition titr := ptr.
(** Representation predicate **)
Definition liter (t : titr) (ls : list T) (idx : nat)
    : hprop :=
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  t ~~> (cur, st) *
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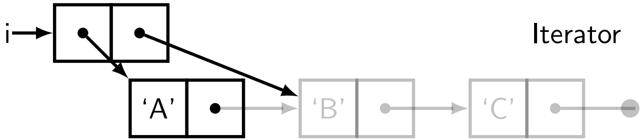


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  llseg st cur (firstn idx ls) *
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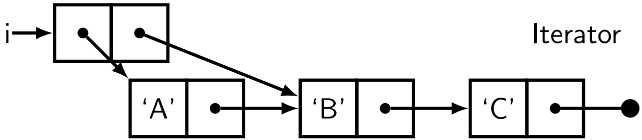


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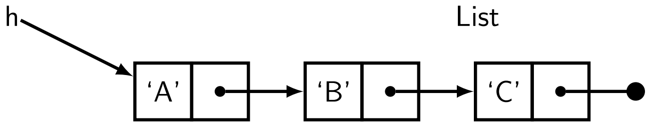
  llseg st cur (firstn idx ls) *
  llseg cur None (skipn idx ls).
```

The Sharing Problem

- Requires access to the same memory as the underlying list.
  - Creating an iterator *consumes* the underlying list.
  - Can't have multiple iterators.



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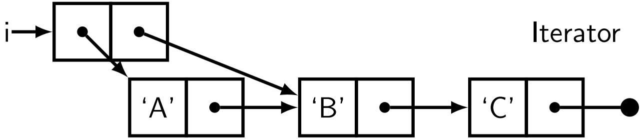

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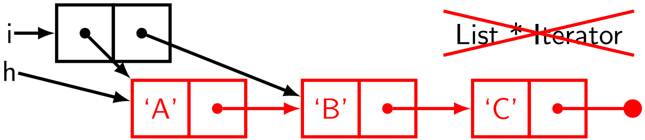


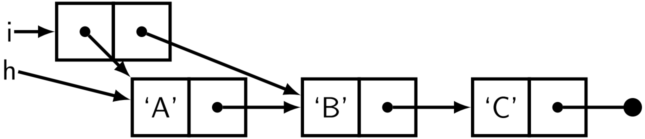
The Sharing Problem: Specifications

- Computations on iterators can't be called with the same underlying list.

```
Definition zip : forall (i1 i2 : titr)
    (l1 : [list T]) (l2 : [list U]),
  Cmd (l1 ~~ l2 ~~ liter i1 l1 0 * liter i2 l2 0 *
          [length l1 = length l2])
      (fun res : tlst => l1 ~~ l2 ~~
          liter i1 l1 (length l1) * liter i2 l2 (length l2) *
          llist res (zip l1 l2))
```

A Real Sharing Problem

- Who “owns” the list turns out to be a real problem.



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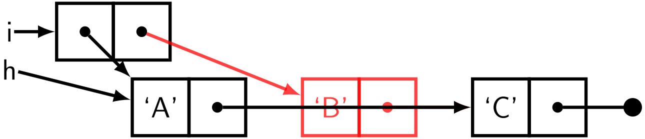

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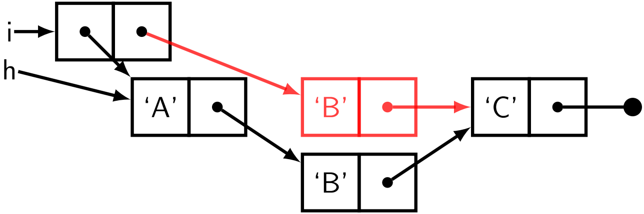

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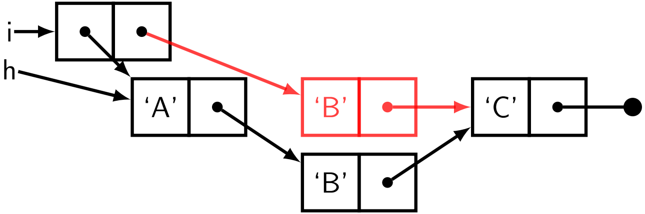
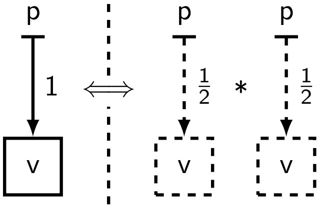


- Doesn't satisfy frame property!
  - Source of Java's `ConcurrentModificationException`.

Sharing with Fractional Permissions [1] (Boyland '03)

- Parameterize points-to by a fractional ownership.
  - `p ~[ q ]~> v`, `q` is the fraction.
- Ownership determines your capabilities:
  - Full permissions allows everything: read, write, free.
  - Partial permissions only allows reading.
  - Permissions can be split and joined.



[1] Ynot implementation by Avi Shinnar.

A Fractional Iterator

- Describe the iterator as owning a fraction of the whole list.

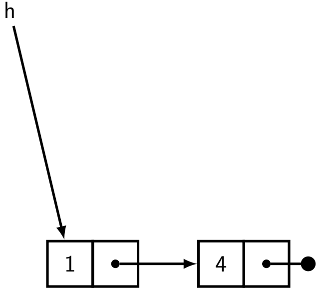
```
(** Representation predicate **)
Definition liter (owner : tlst) (q : Fp)
    (t : titr) (ls : list T) (idx : nat) : hprop :=
  Exists st :@ optr, Exists cur :@ optr,
  t ~~> (cur, st) *
  llseg st cur (firstn idx ls) q *
  llseg cur None (skipn idx ls) q.
```

- q is the fraction of the list that is owned.
- Allows multiple iterators over the same list.
  - As long as the fractions are compatible.

Outline

Aliasing

- Data structures with aliasing are more difficult to describe.



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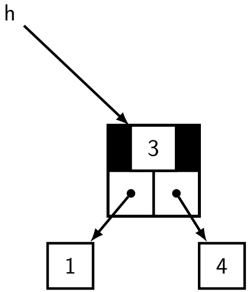

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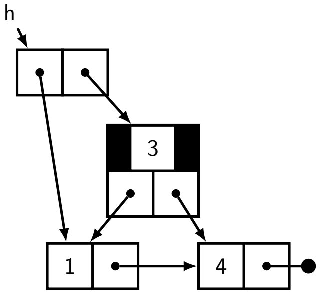

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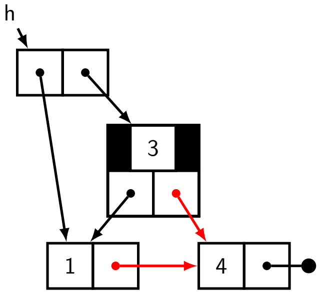

B+ Trees

- B+ trees are *n*-ary trees where the leaves are connected by a linked list.
  - Support fast lookup and in-order iteration.
  - Commonly used for database indices. (Malecha '10)
- Previous formalizations exist, but neither is mechanically verified:
  - Classical conjunction, $(\texttt{list} * \texttt{any}) \wedge (\texttt{tree})$. (Bornat '04)
  - B+ tree language. (Sexton '08)
- Both of these approaches seemed difficult to automate.

Difficulties of the Invariant

- Several difficulties describing this:
  - Have to encode pointer aliasing explicitly.
  - Many different B+ trees can describe the same finite map.
  - Enforce the tree balancedness.
  - Enforce the ordering of keys.
  - Invariants on the size of branches and leaves.

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Representation Invariant

- Existentially quantify an irrelevant model (`tr`) of the tree which contains the pointers.
  - Avoids existentials in the representation invariant, simplifies automation.
  - Makes the heap predicate (`repTree`) very computational.

```
Definition rep (p : BptMap) (m : Model) : hprop :=
  Exists pRoot :@ ptr, Exists h :@ nat, Exists tr :@ ptree h,
  p ~~> (pRoot, existT (fun h:nat => [ptree h]) h [tr]) *
  repTree pRoot None tr *
```

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  - Connect the logical model (`m`) to the physical model (`tr`).

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  [eqlistA entry_eq m (as_map tr)] *
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Representation Invariant

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  - Avoids existentials in the representation invariant, simplifies automation.
  - Makes the heap predicate (`repTree`) very computational.
  - Connect the logical model (`m`) to the physical model (`tr`).
  - Consolidate pure facts about the model in `inv`.

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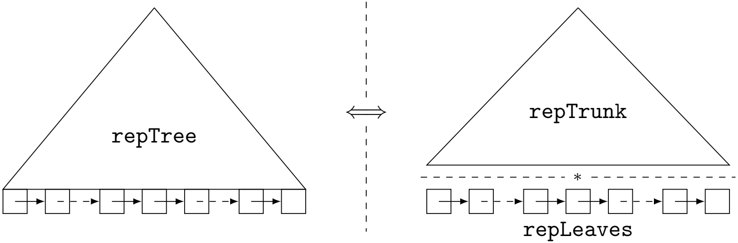
  [eqlistA entry_eq m (as_map tr)] *
  [inv _ tr MinK MaxK].
```

1 Model, 2 Views



- Can switch between views by proving and applying a lemma:

```
Lemma repTree_repTrunkLeaves : forall (h : nat)
    (p : ptr) (optr : optr) (m : ptree h),
  repTree p optr m
  <==>
  repTrunk p optr m *
  repLeaves (Some (firstPtr m)) (leaves m) optr.
```

Outline

Some Lessons

- Fractional permissions are necessary even for sequential code.
- Separation logic makes trees much easier than DAGs/graphs.
  - Can simplify things by re-ifying an irrelevant model.
  - Big win for automation.
- Higher-order ADT functions: `fold`
- Automation pays off when reasoning about separation logic.
- Higher-order abstraction simplifies specifications and proofs.

Other Projects & Outlook

Previous Projects

- Verified web application — trace-based I/O. (Wisnesky '09)
- Verified relational database. (Malecha '10)

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Future?

- Still a fair amount of work for a more realistic system.
  - Reasoning about concurrency.
    - Brookes '07, Appel '08, Nanevski '09
  - Reasoning about failures.
  - Proofs can still be tedious & long.
    - Domain specific external provers.

`http://ynot.cs.harvard.edu/`