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A Comparison of Four Spatial Regression Models for Yield Monitor Data: A Case Study from Argentina

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Abstract. The gap between data analysis and site-specific recommendations has been identified as one of the key constraints on widespread adoption of precision agriculture technology. This disparity is in part due to the fact that analytical techniques available to understand crop GIS layers have lagged behind development of data gathering and storage technologies. Yield monitor, sensor and other spatially dense agronomic data is often autocorrelated, and this dependence among neighboring observations violates the assumptions of classical statistical analysis. Consequently, reliability of estimates may be compromised. Spatial regression analysis is one way to more fully exploit the information contained in spatially dense data. Spatial regression techniques can also adjust for bias and inefficiency caused by spatial autocorrelation. The objective of this paper is to compare four spatial regression methods that explicitly incorporate spatial correlation in the economic analysis of variable rate technology: (1) a regression approach adopted from the spatial econometric literature; (2) a polynomial trend regression approach; (3) a classical nearest neighbor analysis; and (4) a geostatistical approach. The data used in the analysis is from a variable rate nitrogen trial in the Córdoba Province, Argentina, 1999. The spatial regression approaches offered stronger statistical evidence of spatial heterogeneity of corn yield response to nitrogen than ordinary least squares. The spatial econometric analysis can be implemented on relatively small data sets that do not have enough observations for estimation of the semivariogram required by geostatistics. The nearest neighbor and polynomial trend analyses can be implemented with ordinary least squares routines that are available in GIS software. The main result of this study is that conclusions drawn from marginal analyses of this variable rate nitrogen trial were similar for each of the spatial regression models, although the assumptions about spatial process in each model are quite different.

Keywords: yield monitor data, variable rate nitrogen, spatial regression models, negative spatial correlation

Introduction

Precision agriculture (PA) has captured the imagination of producers and agribusiness, but adoption has been relatively slow. For the 2001 harvest, about 34% of US corn area was harvested with a combine equipped with a yield monitor (Daberkow *et al.*, 2002), but only about one third of those combines were equipped with a GPS receiver that would allow them to make yield maps. About 11% of corn, 6% of soybean and 4% of the cotton areas were managed with variable rate fertilizer applications in 2000. Variable-rate seeding and pesticide application were used on 1%–3% of area depending on the crop. Bullock *et al.* (2002) identified the lack of site-specific crop response information as a constraint to adoption of spatial crop management practices. Most variable rate input application is still based on whole field (WF) crop response information. Bullock *et al.* argue that if producers could more easily gather and analyze the crop responses for specific soils, micro-climate, and management zones, then PA may be more profitable, and the social goals of using this technology to improve environmental performance and food safety could be more easily achieved.

Most of the whole field crop response information reported in the literature has been analyzed with ordinary least squares (OLS) and similar statistical tools. But reliability of yield response functions based on OLS estimates can be compromised by spatially autocorrelated data (Kessler *et al.*, 1998). Classical statistics applied to agronomic and on-farm experiments assume that observations are independent. But, in the case of PA data, this assumption of independence is untenable. For example, any yield monitor observation is clearly correlated with its neighboring observations. Consequently, field heterogeneity may be underestimated, and inferences about crop response to variable fertilizer rates may be misleading. Studies of on-farm trials have demonstrated that analysis of experiments comparing variable rate nitrogen (VRN) to conventional, uniform fertilization rates (Bongiovanni and Lowenberg-DeBoer, 2002; Hurley *et al.*, 2004., Lambert *et al.*, 2002) may lead to inaccurate conclusions about which input management strategies are best for fields with distinct, identifiable management zones. In these studies, more reliable estimates of variable rate technology (VRT) profitability were developed when spatial autocorrelation was taken into consideration. A key step in developing methods to determine whether precision farming is profitable and practical is the development of consistent and reliable estimation procedures that take into account the spatially autocorrelated nature of PA data. From an economic perspective, better estimates translate into more accurate assessments of site-specific management profitability.

Spatial dependence is the special case where the dependent variable or error term at each location is correlated with observations of the dependent variable or error terms at other locations (Anselin, 1992). Regression methods that model spatial correlation have been developed in a variety of contexts (for example, geography, agronomy, regional economics, and geology). The primary objective of this paper is to compare returns to VRN where rates are based on site-specific corn yield response to nitrogen estimated with different spatial regression techniques. Corn yield response heterogeneity was estimated using four spatial regression models, and ordinary least square (OLS). The spatial regression techniques compared were: (i) a restricted maximum likelihood (REML) geostatistical approach (Cressie, 1993; Schabenberger and Pierce, 2002); (ii) a spatial regression approach using polygons as discrete units of observation (or spatial autoregression, SAR) (Anselin, 1988); (iii) a polynomial trend (PTR) approach (Tamura *et al.*, 1988); and (iv) a classical nearest neighbor (NN) approach first suggested by Papadakis (1937). Each approach models spatial autocorrelation differently. The difference between estimation techniques for spatial data revolves around the assumption of whether relations between observations are best described as *discrete* or *continuous* relationships (for details on this distinction, see Anselin, 1988). The SAR and the NN approaches assume that spatial correlation is a discrete relationship between specific points or polygons. Reflecting their origin in methods for mapping and interpolation, the geostatistical and the PTR approaches assume that the spatial structure is continuous over space.

The empirical results and partial budgets presented in this analysis of the 1999–2000 crop season data are an example of how response estimates differ when alternative spatial regression methods are used. However, the main focus of this case study is the comparison of alternative regression methods that model spatial correlation differently, and on methodology for deriving site-specific results. The methodological results are intended to be an example of what happens when various methods are applied to the same data. Results are a step towards identifying key differences between methods.

Theory

Nearest-neighbor approach and spatial regression (NN)

The classical experimental design in agronomy is the randomized complete block (RCB). An RCB design is essentially a strategy to control experimental error. Developed by Fisher in the 1920s, the RCB was hailed as a correction for non-homogeneous experimental units in agronomic trials, particularly with respect to heterogeneous landscapes exhibiting different soil types or drainage characteristics. Papadakis (1937) responded to Fisher's blocking methodology by introducing a nearest-neighbor approach (NN). In this approach, experimental results of individual sub-blocks (y_{ij}) for sub-block i in treatment j within a treatment block are subtracted from the overall treatment mean of the parent block ($\hat{y}_j$). The difference between the sub- and whole-block values is the experimental error for y_{ij}. In the classical NN analysis, neighbors are arranged perpendicularly: every observation has four neighbors. Thus the error of y_{ij} is the average of the error terms of its four neighbors sharing the same boundary.

Stroup *et al.* (1994) also compared NN approaches to standard blocking methods. Using a lattice experimental design, Vollmann *et al.* (2000) used the classic NN approach to identify spatial patterns between experimental plots for soybean. They found that soybean yield, seed protein quantity, and seed size were affected by spatial heterogeneity between plots. An iterative NN approach used by Helms *et al.* (1999) compared block by treatment and pooled error means comparing soybean variety performance. Using analysis of variance (ANOVA), they found little difference between classical blocking and NN techniques in regards to reducing error caused by within-block spatial heterogeneity. Precision of between-plot variance estimates (pure error) was similar for NN and classical experimental designs.

Brownie *et al.* (1993) described the NN model:

$$Y_{ij} = \mu + \tau_{ij} + \theta z_{ij} + \varepsilon_{ij}, \tag{1}$$

where Y is yield, μ the overall mean yield, τ_{ij} the treatment effect, z_{ij} the set of nearest neighbor residuals perpendicular to y_{ij}, and θ is a slope coefficient of the covariance between the residual errors of yield y_{ij} and its z_{ij} neighbors. The residual error differences were expressed as $r_{ij} = y_{ij} - \hat{Y}_k$, where $\hat{Y}_k$ was the overall mean for treatment k. The average of the NN residuals for y_{ij} was determined as $z_{ij} = (r_{i,j-1} + r_{i,j+1} + r_{i-1j} + r_{i+1,j})/4$. The structure of the NN model as expressed in (1) is that of the familiar ANOVA model commonly used to test for treatment differences for on-farm trials. Equation (1) can be generalized into the familiar regression model by inserting the z_{ij} into an n x k matrix of explanatory variables, $\mathbf{X}$. This re-specification is important since the primary interest of this study is to estimate site-specific yield response to nitrogen (N). The NN model becomes $\mathbf{y} = \mathbf{X}\beta + \theta\mathbf{z} + \boldsymbol{\varepsilon}$, where the covariance parameter θ is an averaging parameter for the neighborhood of residual errors. Equation (1) is estimated with OLS.

Polynomial trend regression and spatial regression (PTR)

Tamura *et al.* (1988) proposed another alternative to modeling spatial dependence by inserting a polynomial trend variable (T_{ij}) into the familiar ANOVA model. This approach is somewhat related to the spatial expansion regression methodology that has received attention in urban and regional geography (Anselin, 1988). A trend surface is introduced into the model to capture spatial relationships between observations. This approach assumes that omission of spatial dependence is analogous to the omitted variable problem in the econometric literature. The omitted variable problem is handled by inclusion of trend variables in ANOVA models. Like the NN method proposed by Papadakis, Tamura *et al.*'s PTR model was developed to account for spatially structured error processes not dealt with by conventional blocking techniques. The simultaneous estimation of a polynomial response surface with the regression model separates systematic error components caused by spatial dependence from the unsystematic portion of $\boldsymbol{\varepsilon}_{ij}$ (Kirk *et al.*, 1980). Parameter estimates are derived only with respect to remaining random components, $\boldsymbol{\varepsilon}_{ij}$'s. In effect, addition of a system of coordinates relating observation i to j into the familiar regression model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ expressed in terms of polynomials eliminates the omitted variable problem, assuming the trend surface specified by the polynomial expression is of the correct specification. The omitted variable(s) in question would be those that explain spatial structure in error residuals.

The PTR model is specified as

$$Y_{ij} = \mu + \tau_{kij} + T_{ij} + \varepsilon_{ij}, \tag{2}$$

where Y the yield, μ the overall mean, τ_k the treatment effect, T is a polynomial trend, and $\boldsymbol{\varepsilon}$ is an independent and identically distributed (i.i.d.) random error component. The quadratic trend term is estimated as

$$T_{ij} = \varphi_1 x + \varphi_2 y + \varphi_3 x^2 + \varphi_4 y^2 + \varphi_5 xy, \tag{3}$$

where φ_i is a slope coefficient for the Cartesian (x,y) coordinate of observation y_{ij}. The (x,y) coordinates are expressed as row/column pairs. Like the NN approach, the

PTR method was developed for ANOVA of treatment effects controlling for spatial dependence. In this study, the objective is to identify differences in the marginal response of corn to nitrogen over a spatially variable terrain. To accomplish this goal, the PTR model is re-written as and then estimated with the familiar regression equation $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, including the (x,y) coordinates, their squares, and their interactions in $\mathbf{X}$. The model is estimated using OLS.

Geostatistical approach to spatial regression (REML)

Many agronomists have used geostatistical tools to model crop and soil spatial relationships. Perhaps this is because of the disciplinary links between soil science and geology. Originally, geostatistics was developed to produce maps by interpolation between observations. To facilitate mapping, geostatistics assumes that spatial variability is a continuous function of distances modeled by a semivariogram. Within the geostatistical framework, inferential testing of the relationships between variables (for example, layers in the crop GIS) at a given point has developed relatively recently. Cressie (1993) introduced the REML-geostatistical approach. Little *et al.* (1996) and Schabenberger and Pierce (2002) elaborated upon this approach, which entails estimating empirical semivariograms, and then using semivariogram parameter estimates as priors in a regression model to characterize spatial correlation between observations.

The REML-geostatistic approach outlined by Cressie (1993) and Schabenberger and Pierce (2002) has been used to analyze wheat hybrid trials (Stroup *et al.*, 1994), patterning of sudden infant death syndrome (SIDS) in North Carolina (Cressie, 1993), and heavy metals in soils (Schabenberger and Pierce, 2002). Lambert *et al.* (2002) used the REML-geostatistical approach to analyze yield monitor data. Hurley *et al.* (2004) took a similar approach combining geostatistics and regression techniques to analyze VRN corn trials in Minnesota, USA. They estimated the profitability of soil tests, topographical maps, and remote sensing information for corn using semivariogram priors to model spatial error processes. Lark and Wheeler (2003) used a variant to this model to estimate barley yield response in Europe.

The semivariogram is the backbone of the REML-geostatistical regression model. The semivariogram parameters (range, nugget, and sill) are estimated and then used as priors to model the regression covariance matrix. The regression model is estimated with the familiar model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$, but spatial covariance ($\mathbf{R}$) is modeled through $\mathbf{R} = \mathrm{Var}(\mathbf{e})$, where $\mathrm{Var}(\mathbf{e}) \equiv \mathbf{I}\,\sigma_n^2 + \sigma_s^2\mathbf{F}$, $\mathbf{F}$ is an $N \times N$ matrix whose i,j-th element is characterized by a distance decay function, and σ_n^2 and σ_s^2 are nugget and sill semivariogram estimates, and $\mathbf{I}$ is the identity matrix (Little *et al.*, 1996). The REML parameter estimates are estimated generalized least-squares (EGLS) estimates adjusted for spatial autocorrelation. If spatial dependence is present in OLS residuals, then the null hypothesis of the likelihood ratio (LR) test (a $\chi^2(2)$ variate) based on the difference between –2 times the log likelihood of the OLS and REML models is rejected (Little *et al.*, 1996). In this study, the robust semivariogram (Cressie, 1993) of the OLS residuals was estimated using the VARIOGRAM procedure in SAS (2000). Following Cressie (1993), weighted non-linear least squares (WNLS) is used to estimate the semivariogram of the OLS residuals. The REML regression was estimated using the MIXED procedure in SAS (2000).

Discrete spatial regression approach (SAR)

The discrete spatial regression approach assumes that spatial dependence is a relationship among discrete observations, or polygons. Spatial structure may be found in either the dependent variable (e.g., yield) or in regression residuals. Spatial structure is modeled assuming that the dependent variable or residuals are a function of a weighted average of neighboring observations. This approach has been used extensively in epidemiology, geography, and regional economics. In agriculture the structure of the data is similar, but the polygons are often soil types or management zones instead of states, counties, districts, or neighborhoods. This approach uses polygon data, enabling the simultaneous maximum likelihood estimation of the spatial structure and the relationships between GIS layers.

A spatial weights matrix is constructed to identify neighbors in a dataset. The matrices are designed to incorporate processes such as gravity, entropy, or decay into regression models (Anselin, 1988). Data arranged in regular rectangular lattices are defined using three criteria: 'bishop', 'rook', or 'queen'. These classes describe the level of contiguity, or common boundaries, between polygons. Following Bongiovanni and Lowenberg-DeBoer (2000), the SAR regression in this study uses the 'queen' criterion: individual grid cells have both a border and a corner in common with one or more other cells. In spatial terms, contiguity is defined as a function of the distance that separates one cell from another. Blocks belonging to the same neighborhood share the same weight, and the composite of neighborhoods covering the entire grid defines the spatial weights matrix. This matrix (**W**) is an $N \times N$, positive definite matrix with elements w_{ij}, and zeroes along the diagonal. Before spatial weights matrices are used to estimate spatial effects in regression models, they are row-standardized. This facilitates comparison of spatial characteristics across neighborhoods. Each element in a row is divided by the row sum.

Anselin (1988) identified two general patterns whereby spatial dependence may manifest itself in regression analysis: spatial *lag* and spatial *error*. If spatial error processes are ignored, OLS estimates are inefficient, but remain unbiased. If spatial lag processes are ignored, then OLS estimates are inconsistent and biased. The presence of these effects is determined when a regression model is estimated concomitantly with an associated spatial matrix.

The first step in determining the presence of spatial autocorrelation entails estimating the standard OLS model $\mathbf{y} = \mathbf{X\beta} + \boldsymbol{\varepsilon}$ combined with an associated spatial matrix. By incorporating the spatial weight matrix **W** into the regression model, relations between the dependent variable y_i (e_i) with neighboring y_j's (e_j's) are determined for lag (error) classifications. For lag processes, the modified regression model becomes $\mathbf{y} = \rho\mathbf{Wy} + \mathbf{X\beta} + \boldsymbol{\varepsilon}$; with ρ as the autoregressive moving average parameter for neighboring y_j's. The spatial error model is specified as $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ with $\boldsymbol{\varepsilon} = \lambda\mathbf{W}\boldsymbol{\varepsilon} + \mathbf{u}$, where $\mathbf{u}$ represents well-behaved, non-heteroskedastic, uncorrelated errors. The β_i's are EGLS estimates corrected for spatial autocorrelation.

Lagrange multiplier tests (LM, distributed as a $\chi^2(1)$ variate) can be used to detect the presence of spatial dependence in OLS residuals. The alternative of the LM_{error} test is that residuals follow a spatial pattern, while the alternative for the LM_{lag} test is that individual observations on explanatory and/or the dependant variables are correlated with the average of other values of the same variables in a given neighborhood of observations. Rejection of the null for the LM_{lag} test means that we face an omitted variable problem; OLS estimates are biased and inconsistent. If we reject the null of the LM_{error} test, we face an efficiency problem; OLS estimates are not biased, but they are inefficient.

Materials and methods

Corn nitrogen response data from the study by Bongiovanni and Lowenberg-DeBoer (2000) were used in this analysis. The data were collected from strip trials at the "Las Rosas" farm located near Río Cuarto in the southwestern corner of Córdoba Province, Argentina, in the 1998–1999 crop season (Figure 1). The strips were the width of the N applicator (9.8 m), with a zero N control and five other rates of elemental N: 29, 53, 66, 106, and 131.5 kg ha^{-1}. The N rate was constant for the whole strip, across the four topographies identified by National Institute for Agricultural Technology (INTA) agronomists. The highest N rate was higher than the expected yield maximizing level. The N source was urea. Data were collected with a standard AgLeader^{TM} (Agri-Tech Solutions, Onawa, IA, USA) yield monitor. Since the raw data includes data points that are closer within the same row than between rows, these yield points were averaged for a within-row distance equivalent to the between-rows distance. This was accomplished using the GIS software SSToolbox^{TM} (SST Development Group, Stillwater, OK, USA) by superimposing a grid of 9.8 × 9.8 m cells (the width of the N applicator) over the yield observations. Data points at the extreme left and at the extreme right were deleted, because they reflect an empty combine entering the row. Finally, and after averaging the data within each cell, 1738 cells were digitized as polygons. Centroid points generated by ArcView^{TM} (ESRI, Redlands, CA, USA) of each cell were used to estimate empirical semivariograms.

The base regression model was quadratic

$$Y = \beta_0 + \sum_i \delta_{i,o} + \left(\beta_1 + \sum_i \delta_{i,1}\right)N + \left(\beta_2 + \sum_i \delta_{i,2}\right)N^2 + \varepsilon, \tag{4}$$

where Y is the corn yield (t ha^{-1}), N is the (kg ha^{-1}) of elemental nitrogen fertilizer, $\delta_{i,k}$ a dummy variable constrained as $\sum_{i=1}^{4} \delta_{i,k} = 0$ indicating topographical variability (TOP1: lowland; TOP2: east slope; TOP3: hilltop; TOP4: west slope), and $\boldsymbol{\varepsilon}$ is a random disturbance term.

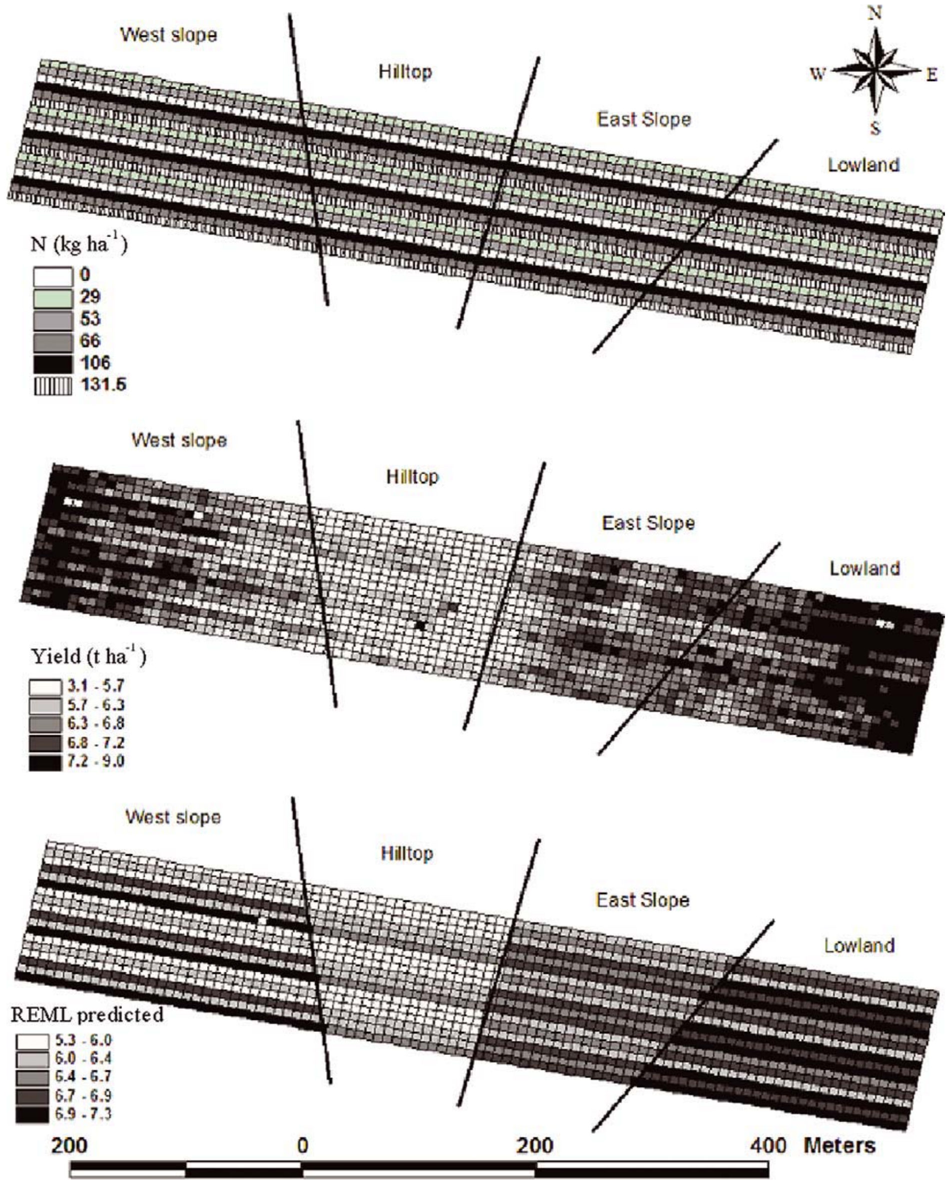

Figure 1. Experimental design (top panel), actual yields (middle panel), and predicted yields (bottom panel), t ha^{-1}. Predicted yields are estimated with the REML-geostatistic model.

INTA agronomists delineated topographical zones as areas with common landscape attributes (e.g. slope, aspect, soil color). The constraint on the dummy variables tested whether the intercept, and nitrogen by topography interactions terms for a given topography zone were different from the WF average.

Table 1. Sample moments for Las Rosas corn response to variable rate nitrogen trial

Moment	Top 1—Lowland ($t\ ha^{-1}$)	Top 2—East slope ($t\ ha^{-1}$)	Top 3—Hilltop ($t\ ha^{-1}$)	Top 4—West slope ($t\ ha^{-1}$)	Whole field ($t\ ha^{-1}$)
Mean	7.1	6.5	5.3	6.67	6.5
Std. error	0.03	0.03	0.032	0.034	0.021
Skewness	0.3415	−0.6597	0.0568	0.0135	−0.2382
Kurtosis	−0.1379	0.2673	0.7858	−0.1213	−0.1632
Min	5.6	4.7	3.1	4.8	3.1
Max	9.0	7.6	8.0	9.0	9.0

Results and discussion

Descriptive statistics for corn response to nitrogen

The unconditional, WF corn yield average was 6.5 ± 0.02 t ha^{-1}, (mean ± -standard deviation) with a minimum and maximum yield of 3.1 and 9.0 t ha^{-1}, respectively (Table 1). The hilltop area (TOP3) had the lowest yield mean (5.3 ± 0.03 t ha^{-1}). The average corn yield was highest for the lowland area (7.1 ± 0.03 t ha^{-1}). This is expected since this portion of the field is a sink for organic matter and run-off, while the hilltop area is more susceptible to wind erosion. These soils are loess soils originally deposited by the wind. When vegetation is removed, wind easily moves these soils again. The hilltop region is also susceptible to water erosion when the soil is tilled. Average corn yield was similar for the sloping portions of the field (east and west slope), and closely approximated the WF average (6.5 and 6.6 t ha^{-1}, respectively). The hilltop portion of the field exhibited the greatest yield variability compared to the other topographical zones (range = 4.9, CV = 11%). The relatively large kurtosis (K) of this portion of the field (0.79) suggests a fat-tailed distribution of yields (more variability) compared to topography zones ($-0.13 < K < 0.27$ for all other zones, Table 1).

REML-geostatistical approach

A spherical semivariogram model was used to fit the empirical semivariogam of the OLS residuals. The WNLS parameter estimates for the nugget (9), range (140 m), and sill (35) estimates were significant at the 1% level. The F-test for the fitted semivariogram was significant at $P < 0.0001$ ($F = 605$), and the coefficient of determination (estimated as 1—SSE/CSS, where SSE is the variance of the full model and CSS is the variance of the mean model) for the spherical model was 0.70. The proportion of spatial variation explained by the semivariogram was 80% (calculated as sill/[nugget + sill], Schabenberger and Pierce, 2002). The LR test was strongly

Table 2. Measures of fit for SAR, REML LRP, NN, PTR and OLS models

Model	Adj. R^2	AIC	LIK[a]	Rank[b]	LR test
OLS	0.60	10914	−5445	5	–
PTR	0.71	10392	−5179	3	532
NN	0.66	10636	−5304	4	282
REML-Spherical	–	9730	−4865	2	1160
SAR	–	9683	−4830	1	1231

[a]Log likelihood.
[b]Following Akaike's information criterion (smaller is better): AIC = −2 LIK + 2k, where k is the number of parameters and LIK is the log likelihood.

rejected for the REML-spherical regression model (LR = 1160, Table 2), indicating that the REML-geostatistical model significantly explained spatial dependence in the OLS yield residuals. Estimated generalized least squares parameters for the REML-spherical model carried the expected signs consistent with corn yield response to nitrogen (Table 3).

Discrete spatial regression approach (SAR)

A Lagrange multiplier (LM) test for spatial error dependence identified the presence of spatial error structure in the OLS residuals (LM = 705, df = 1). The $\mathrm{LM_{lag}}$ test was significant as well ($\mathrm{LM_{lag}} = 514$, df = 1). Since both LM tests were highly significant, the robust LM lag and error scores were used to determine which SAR model best described the spatial process. The robust tests correct for asymptotic interdependence between LM error and lag tests (Anselin, 1992). The robust LM test for spatial lag was not significant at the 5% level ($\mathrm{LM_{lag}} = 3.21$, df = 1, P = 0.07), whereas the $\mathrm{LM_{error}}$ test was strongly rejected at the 1% level ($\mathrm{LM_{error}} = 195$, df = 1). These diagnostics are in agreement with the REML-spherical results: the SAR error model also detects spatial error process in this data set. The null hypothesis of spatial independence was also strongly rejected by the LR for the SAR model (1231, respectively, df = 2, Table 2).

Parameter estimates for SAR model are presented in Table 3. The Z-tests associated with the autoregressive (AR) nuisance parameter λ were highly significant ($\lambda = 0.86$, $Z = 58$, $P < 0.0001$), indicating that the spatial AR term significantly explained the spatial structure in the residual error terms.

Nearest-neighbor approach

The NN model improved the coefficient of determination by 6%, compared to the OLS estimates (Adjusted $R^2 = 0.66$, Table 2). The appropriate measure of fit statistic is Akaike's information criterion (AIC) criterion since an additional parameter

Table 3. Regression coefficients, asymptotic T- and Z-tests for OLS and spatial regression models. The dependent variable is corn yield (t ha^{-1})

	OLS		SAR		REML-Spherical		PTR		NN	
Variable	Estimate	T	Estimate	Z	Estimate	T	Estimate	T	Estimate	T
Intercept	5.864	190.88	5.891	90.50	5.894	79.94	5.886	66.39	5.851	206.47
N	0.012	10.72	0.011	17.32	0.011	14.46	0.011	12.05	0.012	12.33
N^2	−3.579E−05	−4.64	−2.433E−05	−5.40	−2.400E−05	−4.36	−2.680E−05	−4.03	−4.009E−05	−5.63
TOP1 (lowland)	0.851	14.76	0.521	6.78	0.556	5.76	0.702	14.71	0.812	15.32
TOP2 (east slope)	0.200	3.59	0.227	3.08	0.220	2.50	0.428	8.42	0.220	4.28
TOP3 (hilltop)	−1.206	−21.51	−0.535	−7.23	−0.565	−6.38	−1.027	−21.02	−1.130	−21.78
TOP4 (west slope)	0.155	3.16	−0.213	−2.63	−0.211	−2.23	−0.103	−2.11	0.099	2.18
N X TOP1	−2.807E−03	−1.45	−4.151E−03	−3.87	−3.906E−03	−3.05	−2.346E−03	−1.33	−3.397E−03	−1.90
N X TOP2	−1.060E−03	−0.54	−1.236E−03	−1.08	−8.240E−04	−0.60	−4.441E−04	−0.26	−3.360E−04	−0.19
N X TOP3	3.353E−03	1.69	3.145E−03	2.72	3.465E−03	2.50	3.280E−03	1.92	3.126E−03	1.71
N X TOP4	5.140E−04	0.30	2.243E−03	2.24	1.265E−03	1.05	−4.904E−04	−0.33	6.070E−04	0.38
N^2 X TOP1	1.010E−05	0.68	2.133E−05	2.51	1.950E−05	2.20	3.800E−06	0.16	1.413E−05	1.01
N^2 X TOP2	−5.578E−06	−0.39	−9.953E−06	−1.20	−1.100E−05	−1.13	−1.370E−05	−1.13	−1.281E−05	−0.98
N^2 X TOP3	−7.092E−06	−0.50	−1.433E−05	−1.71	−1.600E−05	−1.64	−2.700E−06	−0.22	−6.754E−06	−0.51
N^2 X TOP4	2.569E−06	0.21	2.952E−06	0.41	7.500E−06	0.86	1.260E−05	1.19	5.433E−06	0.48
λ			0.86	58.04						
θ									0.60	17.44

was included in the regression model. The NN regression improved the AIC criterion over OLS by about 3%. However, the LR test for spatial dependence was significant (LR $= 282$, df $= 2$), indicating that the presence of spatial structure in the OLS model error terms was explained using the NN approach. Spatial dependence between the neighborhoods of error terms as identified by the NN approach was also significantly explained as indicated by the significant T-test for θ ($\theta = 0.60$, $T = 17$, $P < 0.0001$, Table 3).

Polynomial trend regression

The null hypothesis of no spatial structure in the regression error terms was strongly rejected when the model was estimated using the PTR specification (LR $= 984$, df $= 2$, Table 2). Compared to the original OLS model fit, the Adjusted R^2 for the PTR increased by 18% with the addition of the trend coefficients. Following the AIC criterion, the PTR model improved the overall fit of the data by 9%. Yield response parameter estimates from the PTR regression are in Table 3. Like the other regression models, the linear and quadratic response coefficients carried the expected signs.

Comparison of the spatial regression models

Overall, the base OLS model AIC fit criterion improved between 3% and 15% when error spatial dependence was included in the model (Table 2). All models produced the expected signs for the quadratic yield response to nitrogen, and all topography intercept terms were significant in each of the models. In general, the frequency of significant parameter estimates and the size of the T-tests increased with all models that modeled spatial dependence (Table 3).

Marginal response to nitrogen was different for the EGLS approaches (REML and SAR) in the lowland topography zone when compared to the OLS-based methods (NN, PTR, and OLS regressions): N recommendations for this zone were considerably less than the slope and hilltop zones based on the EGLS estimates (Table 4). The variability of the marginal effect of the first kilogram of N on corn growth decreased when estimated with the spatial regression models (Figure 2). A clear distinction between the marginal effects of N by topography zones 1 (lowland), versus 3 and 4 (hilltop and west slope, respectively) was evident with the SAR model. The REML model also detected site-specific differences in marginal response variability between zones 1 and 3 and the PTR model detected yield response differences from the WF average at the 10% level for the hilltop region. The distinction between site-specific responses is less clear with the NN and OLS regressions. A larger yield response to N was expected in the hilltop areas since the area is more susceptible to wind erosion and macronutrients are less abundant. Yield response to N was moderated in the lowland site since eroded soil nutrients are deposited in this zone, resulting in better soil for plant growing conditions. The SAR and REML (at the 5%

Table 4. Economically optimal nitrogen rates (EORN, kg ha^{-1}) and economically optimal yields (EOY, t ha^{-1}) for Lowland (TOP1), east slope (TOP2), hilltop (TOP3), and west slope (TOP4) management zones of Las Rosas

	TOP1	TOP2	TOP3	TOP4
OLS				
EORN	37	44	94	79
EOY	7.0	6.4	5.7	6.8
REML-spherical				
EORN	22	47	95	163
EOY	6.6	6.5	6.3	7.2
SAR				
EORN	0	41	93	147
EOY	6.4	6.5	6.3	7.1
PTR				
EORN	43	48	129	135
EOY	7.0	6.5	6.0	7.2
NN				
EORN	29	46	87	89
EOY	6.9	6.5	5.7	6.8

level), and PTR (at the 10% level) regressions pick up these physical processes in this particular data set, whereas the OLS and NN approaches do not.

The yield response curvature for all models diverged after the highest amount of nitrogen applied in the field experiment (Figure 3). After the highest N-rate

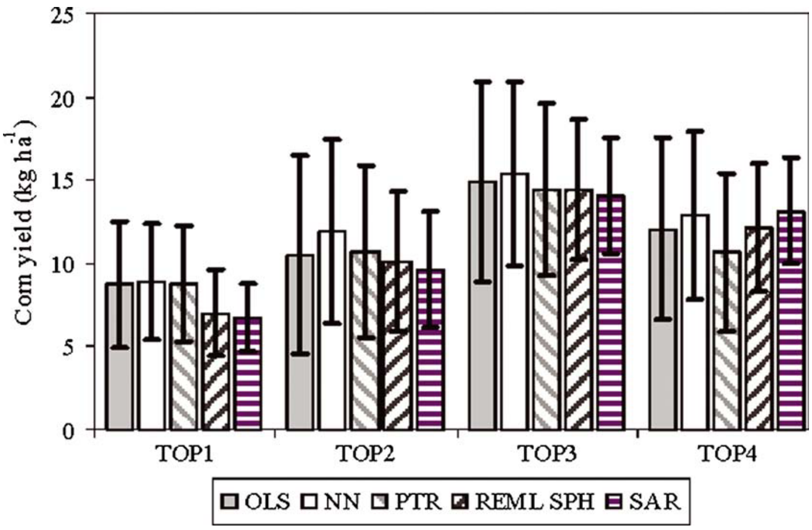

Figure 2. Marginal effect of elemental nitrogen on corn yield evaluated at N = 0 kg ha^{-1}. (bars are 95% confidence intervals). Key: TOP1 : low east; TOP2 : east slope; TOP3 : hilltop; TOP4 : west slope.

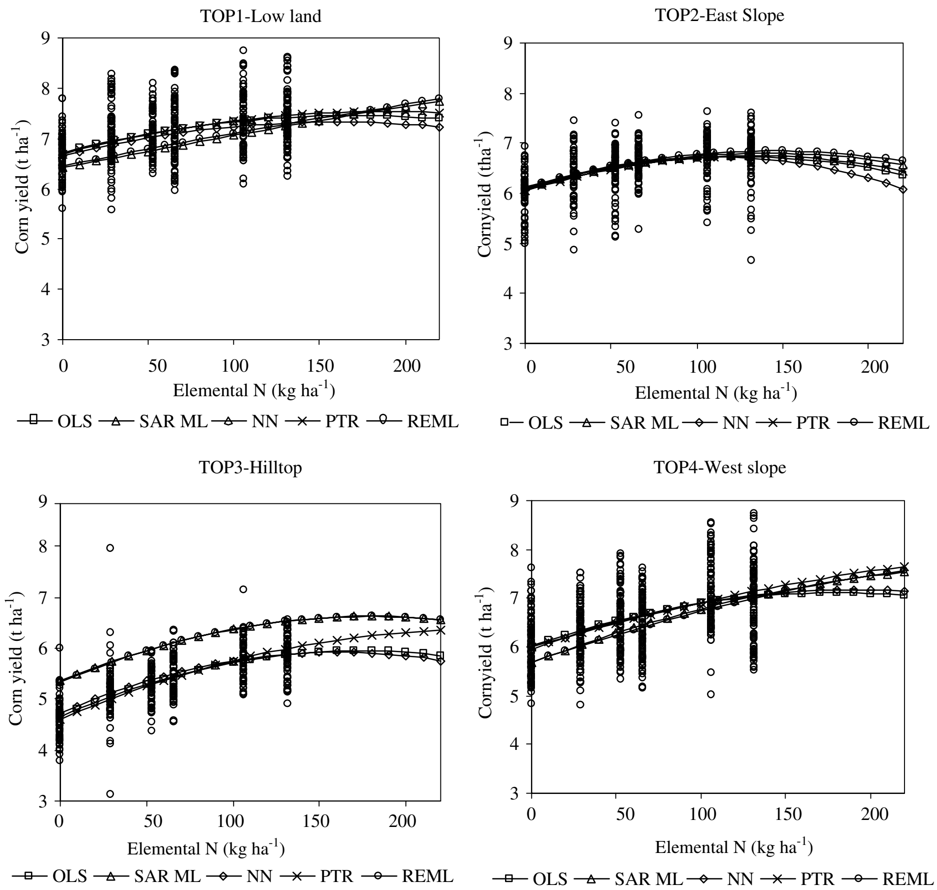

Figure 3. Yield response curves for the four yield response zones identified at the Las Rosas site estimated with OLS and SAR.

(136 kg ha^{-1}), the predicted yield response for NN, PTR, and OLS roughly maintained the same curvature. However, the EGLS response estimates diverged from the PTR, NN, and OLS estimates but stay parallel to each other. The biologically maximum N-rates were much higher when estimated with SAR and REML compared to OLS, NN, and PTR models in the lowland and west-slope management areas.

In many cases, economists do not consider intercept terms important; economic analysis focuses mainly on the marginal effects of explanatory variables on dependent variables. Likewise for this study, the intercept values do not affect the economics because the focus is on the marginal effects of N on corn yield. However, the intercept can be important in other agronomic situations. For instance, if the objective is to compare the profitability of alternative crops under different management schemes, overall revenue matters (not just the marginal revenue). In this case, the intercept would be important in estimating overall revenue.

With this in mind, major differences between the EGLS and OLS-based regression approaches are evident by inspection of the intercept terms for the hilltop region (Figure 3). The yield intercepts for corn estimated with the EGLS approaches were about 0.7 t ha^{-1} higher than the intercepts estimated with the NN, PTR, and OLS approaches in the hilltop area, but in the other topography zones, the EGLS intercepts were similar in magnitude to the NN, PTR, and OLS intercepts. The predicted regression lines for the EGLS estimators were substantially higher than most of the actual data in the hilltop zone. There are several explanations why this occurred with the EGLS estimators.

First, the average yield of the hilltop zone (5.3 t ha^{-1}) was 1.2 t ha^{-1} less than the WF average of 6.5 t ha^{-1}. The west and east slope areas only deviated from the WF mean by 0.1 and 0 t ha^{-1}, respectively and the high-yielding lowland area positively deviated from the WF average by 0.6 t ha^{-1}. The within-zone variability was also highest for the hilltop zone. The deviation of the hilltop unconditional yield average from the WF unconditional average, combined with the relatively large kurtosis and range values for this area, has a considerable impact on EGLS intercept estimates.

Second, the EGLS estimators used a data transform to re-weight observations. The WF intercept was uniformly re-scaled following transformation, but the transformation of the site-specific dummy variables was not constant. Due to dif-

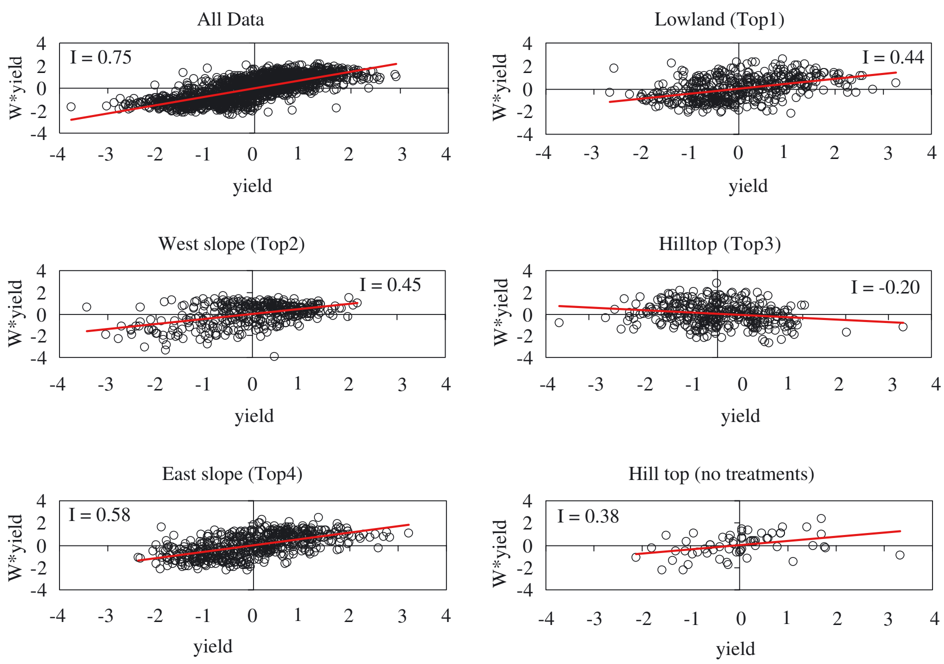

Figure 4. Moran's *I* for all of the Las Rosas corn-yield data ('All Data'), and the four topography zones. The yield data in the above panels are standardized to the normal distribution.

ferences in scaling, the EGLS site-specific intercept terms changed at different rates and the region-specific constant terms may vary significantly from the OLS-based intercept estimates, depending on the patterns characterizing yield response to N treatments in a given zone.

Thirdly, closer examination of each site reveals several intriguing spatial patterns (Figure 4). Moran's I (Anselin, 1992) is the simplest tool by which to describe spatial dependence and it is particularly useful for understanding why the intercept shifts of the EGLS estimators appear to overestimate yield response in the hilltop region. A direct way of estimating I is accomplished by regressing $\mathbf{Wy} = \rho\mathbf{y} + \boldsymbol{\varepsilon}$, where $\mathbf{y}$ is the vector of dependant variables standardized as $\sim N(0,1)$, $\mathbf{W}$ is the $N \times N$ row-standardized weights matrix, and ρ is Moran's I. Closer inspection of the Las Rosas yield data reveals that there is significant *negative* spatial correlation between observations in the hilltop region, contrary to the east and west slope and lowland regions that exhibit positive spatial correlation between yield observations. Negative spatial correlation is the special case when the neighbors of an observation are opposite in magnitude to that observation. Conceptually, negative spatial correlation manifests as a 'checkerboard' pattern. This checkerboard pattern is also apparent in the grid plot of the actual yield data (Figure 1), and quantified by Moran's I for the hilltop area in Figure 4. The negative spatial correlation in the hilltop region (Figure 4, lower right) is an artifact of the interaction between the experimental design of the VRN study and the low-fertility characteristic of the hilltop region. Although the hilltop area is more susceptible to erosion, there are still some fertile pockets. In the check strips where no N was applied, yields were considerably less than the WF average because of the low fertility of the area. However, the marginal response to N in the hilltop area is also larger than the marginal response to N in the other management areas. Therefore, corn yield response to N in the hilltop region was greater than any of the other topography zones and corn yield in treatment strips that did receive N in this area were more likely to be higher relative to corn yield in check strips in the same area. When Moran's I was calculated for the hilltop region excluding treatment strips, moderate positive spatial correlation was detected (Figure 4).

Negative spatial autocorrelation in this data set has the effect of increasing the intercept term of the hilltop zone with the EGLS-SAR intercept estimates. As a simple example; in its most general form, the data transform for the SAR model is $y_i^* = y_i - \rho \sum_{j=1}^{n} w_{ij} y_j$, $i \neq j$. Thus, $\bar{y}^*$ will tend to be larger than $\bar{y}$ if there is negative spatial correlation. This *localized* mechanism appears to be driving the difference in the intercept location for the SAR intercept estimate.

A similar effect occurs with the REML-geostatistical approach, although the mechanism is slightly different. In this approach, it is likely that considerable anisotropy has resulted from the interaction of the treatment strips and the low fertility of the hilltop zone. It is worthwhile noting that the OLS-based approaches do not appear to be affected by the directional or highly localized variance differences in the hilltop area. This is not surprising since estimation does not involve a re-weighting transform of the variables.

On average, the standard errors of the linear and quadratic topographic by nitrogen interaction terms SAR model were 42% less than the OLS model. The standard errors for the estimated response coefficients (linear and quadratic) of the REML-spherical model were 30% less than the OLS standard error estimates. The estimated standard errors of the linear and quadratic response coefficients for the PTR and NN regression models were smaller than the OLS base model by 14% and 8%, respectively (both linear and quadratic terms). The AIC does not decrease substantially with the NN model, and only one nitrogen by topography interaction is significant at the 5% level. In general, the PTR approach performed less well in terms of identifying significant yield response by topography interactions.

Corn grown on the low-yielding hilltop zone significantly responded to nitrogen differently from the WF average at the 10% level for the NN, PTR, and OLS estimates. The REML-spherical SAR model estimated a significant interaction between nitrogen and the hilltop topography zone at the 5% level for the linear terms, and at the 10% level for the quadratic terms. The EGLS estimators for marginal response of corn to nitrogen grown in the lowland portion of the field (TOP1) were significant at the 5% level. The NN, PTR, and OLS models did not detect this site-specific response effect. Corn response to N was not different from the WF average for corn grown in the west and east slope portions of the field.

The issue of local variations in spatial process is worth pursuing in future VRT profitability studies that use regression techniques in economic analyses to determine profitability. In this study, careful analysis of the nature of local spatial dependencies suggested that a different model specification might be appropriate for this particular data set, or that the experimental design at the Las Rosas site might withstand improvement. The SAR and REML approaches link all observations to each other through the spatial weights matrix and the restricted covariance matrix, respectively. The dummy variable constraint forces the intercept terms of each management zone to center around a WF intercept or response term. When the dummy variable constraint $\sum_i \delta_i = 0$ is imposed, the linkages between observations are effectively doubled. Allowing each of the zones to have their own intercept and yield responses eliminates this strong assumption about the relation of observations over a relatively large surface. A model specification that would allow each zone to have its own intercept, linear, and response coefficients would eliminate the re-weighting effects of outlier observations observed with the EGLS estimators. If intercept terms are important for economic analysis, then allowing each zone to have its own intercept term avoids the effect of centering site-specific intercept terms on a WF average term. However, an extra step in estimating statistical differences between sites with respect to yield potential and response to inputs would be needed.

These steps do not eliminate the problem of anisotropy or negative spatial correlation that probably resulted from the experimental design. Wider treatment strips or blocks and thorough randomization of treatments might eliminate the treatment effects observed in this case study.

Practical applications of the results

If the discrete model of spatial dependence is a reasonable assumption, the SAR approach provides several advantages. SAR is a one step maximum likelihood estimation process, while the REML-geostatistical approach requires at least three steps. Second, SAR can work for a smaller number of observations than the REML-geostatistical approach. In some cases, the data has spatial structure, but the number of observations is too small to permit estimation of a semivariogram. A good example of this is the soil density research reported by Finck (2001). In that data, yields were reported by soil type polygon. There were 163 polygons in four separate fields. Because of spatial correlation within fields, the OLS estimates had inflated standard errors and few statistically significant coefficients. SAR provided a parsimonious model that allowed the analysts to identify statistically significant effect of the soil density treatment on heavy, lowland soils.

The REML-geostatistical approach is a good alternative to SAR when: (1) enough data is available to estimate semivariograms, and (2) the discrete model of spatial variance structure is untenable. The REML-geostatistical approach may also facilitate interdisciplinary communication. For most economists, both the discrete spatial regression and the REML- geostatistical approach are modest extensions of familiar regression models. Many agronomists and soil scientists are familiar with geostatistics, but many do not regularly use regression analysis to estimate yield response to inputs over space. The SAR regression approach may appear very foreign to many agronomists and soil scientists, while the use of geostatistical concepts in the geostatistic REML approach may help create confidence. If the coefficient estimates are similar and the REML-geostatistical fit as close to that of the SAR estimates as in the Las Rosas 1999 case, the cost of using the REML-geostatistical seems to be relatively small. Another advantage of the REML-geostatistical approach is that it can be implemented in the widely available SAS™ software.

Illustrative example: nitrogen budgeting and VRN profitability

Accounting for spatial dependence in yield monitor data has an effect on the inferences drawn about VRN profitability in this case study (Table 5). Though the estimated N responses are site-specific and cannot be generalized to other fields, they demonstrate how incorrect management decisions might be made if spatial dependence in data is not considered. Whole-field and SSM profitability is determined using a partial budget analysis (Boehlje and Eidman, 1984). Marginal analysis is used to estimate net returns from applied nitrogen (Beattie and Taylor, 1985). This budget method states that profit is maximized when the value of the increased yield from added nitrogen equals the cost of applying an additional kilogram or when the marginal value product equals the marginal factor cost.

Returns from N above fertilizer cost were estimated for uniform application rates and for VRN by landscape position. The uniform N rate was 36.8 kg ha^{-1} recommended by Castillo *et al.* (1998). Estimated VRN applications assumed that N varied by landscape position according to the profit maximizing levels. All estimates use the landscape response curves to estimate yield, which is weighted by the corresponding topography areas (Lowland = 27%, east slope = 21%, hill-

Table 5. Expected net returns to N use [a] with OLS specification, PTR, NN, REML and SAR models. The net return base is N = 0 kg ha^{-1} (standard errors in parentheses)

Regression method	Uniform rate (\$ ha^{-1})	Variable rate (\$ ha^{-1})	Variable rate −6 (\$ ha^{-1}) application fee
OLS	8.35	11.63	5.63
	(3.32)[b]	(3.87)	(2.99)
SAR ML	8.11	15.68	9.68
	(3.82)	(4.16)	(3.36)
REML—Spherical	8.06	15.72	9.72
	(4.59)	(4.74)	(3.89)
Trend regression (PTR)	8.05	14.63	8.63
	(3.84)	(4.55)	(3.84)
Nearest-neighbor (NN)	9.76	13.73	7.73
	(3.19)	(3.76)	(2.85)

[a] The net return to N use is estimated as the difference between returns with the recommended uniform or VRT rate and N = 0 kg ha^{-1}.
[b] Standard errors of the point estimates for returns to VRN and uniform rates were approximated using the delta method (Cassella and Berger, 1990).

top = 20%, and west slope = 32%). Returns above fertilizer cost were estimated as follows:

$$E[\pi] = \sum_{i=1}^{4} \omega_i (P_{\mathrm{c}}[(\beta_0 + \delta_{o,i}) + (\beta_1 + \delta_{1,i})N + (\beta_2 + \delta_{2,i})N^2] - P_{\mathrm{N}}N - F), \qquad (5)$$

where $E[\pi]$ is expected profit (\$ ha^{-1}), *P*c the price of corn (\$0.685 t^{-1}), *i* the topographic zone (1: lowland; 2: east slope E; 3: hilltop; 4: west slope); *N* the N rate, -kg ha^{-1} (profit max N* rate for VRT computations), P_N the Price of N fertilizer (\$ 0.435 kg^{-1}), plus interest for 6 months at 15% annual interest rate, *F* the cost of VRN application ha^{-1}, and ω_i is the proportion of landscape represented by topography zone *i*.

The partial budget omits some costs of developing site-specific recommendations. The major costs in developing site-specific recommendations are: (1) the time involved in implementing the trial; (2) yield loss on strips with non-optimal inputs; and (3) analysis time. On the Las Rosas farm, as on many US farms, the farmer already has a yield monitor; the marginal cost of using that yield monitor for this field is negligible. Soil tests were not used at Las Rosas, so these information costs are not relevant. Analysis software cost is probably not a long-run cost issue either; either free software can be used, or spatial algorithms will be made part of standard mapping software used by growers and crop consultants.

The net return to N use is \$7 – \$8 ha^{-1} greater when using the spatial regression estimates. This would allow the producer to pay the estimated \$6 ha^{-1} fee for custom VRN application and retain a modest profit. As pointed out by Bongiovanni and Lowenberg-DeBoer (2000), marginal analysis using spatial regression estimates results in a different VRN management decision compared to using OLS estimates. A recommendation based on REML, PTR, NN or the SAR approaches would find statistical support for differences between N response by landscape area and economic evidence for VRN profitability. In the present case, a recommendation based on OLS would conclude that the N response is the same in all landscape areas and that VRN is unprofitable at the estimated \$6 ha^{-1} custom application fee.

The results of this case study should be put into perspective. First, a one-year, one-site data set is not a strong basis for drawing empirical conclusions in rain-fed agricultural research. Bongiovanni and Lowenberg-DeBoer (2002) demonstrate this in their examination of yield response coefficient stability over time in another on-farm VRN field trial in Argentina.

Likewise, another shortcoming with the economic analysis is that the partial budget is based on *ex post* yields. Ideally, the economic assessment of these approaches should be based on *ex ante* decision-making. However, the *ex post* analysis provides a reasonable starting point of comparison. If the *ex post* results were not in favor of VRN for this site, then the *ex ante* results would most likely not yield favorably either. However, the converse is uncertain. Second, if the response coefficients were stable from year to year, then the *ex post* results would be generalizable over all years. On the other hand, if the response coefficients are not stable, then the *ex post* results are relevant for the single production year and particular site. Multi-year data sets are a step towards addressing both of the above problems.

Conclusion

The spatial regression approaches compared in this study showed statistically significant coefficients for N response by topography. The explicit incorporation of a spatial component in the yield response model revealed site-specific interaction patterns that were not detected by the conventional OLS model. In this case study, OLS analysis of VRN profitability would have rejected the hypothesis of spatial variation in N response by landscape zone.

The REML-spherical model had a fit that was intermediate between PTR and SAR models. The SAR regression approach was not unambiguously superior to other spatial regression approaches. Parameter estimates are very similar for the REML-spherical and SAR estimates, but the SAR model had the most coefficient estimates that are statistically significant at the 5% level[1]. The NN and PTR models corrected for spatial structure in residual errors but were not as efficient as the EGLS approaches.

With increased information available to the producer, better, more precise statistical regression methodologies should be developed to take advantage of that information. This brings up the issue of which statistical methodologies are most appropriate when gauging profitability of precision technologies. In many cases, the information provided by these new tools is highly spatially correlated, and the only appropriate and unbiased way to properly estimate returns to these technologies is with statistical methods that explicitly model spatial correlation.

Notes

1. As a reviewer pointed out, it is difficult to make inferences about the properties of these estimators since this is a single data set. However, the properties of these estimators are well established in the regression literature. A more robust comparison of estimators for this single data set would include a Monte Carlo analysis.

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