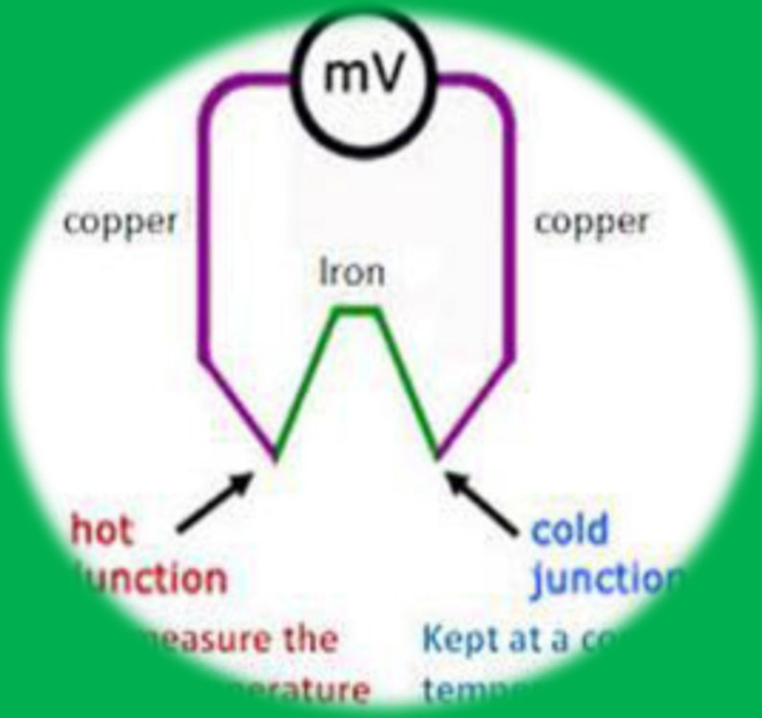
Physics for

ADVANCED LEVEL

Heat & Thermodynamics



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2nd Edition 2022

Competent Based Curriculum

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- Environmental Physics
- Alternating Current Theory (A.C)
- Fundamental Of Mechanics

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ABOUT THE BOOK.

This text extracted to cover the whole **heat and thermodynamics** concepts for advanced level with reference to the ***first re-print Advanced level Physics syllabus of 2010***. Everything is extracted from different sources but are within syllabus. The book designed in such a way that every advanced level physics students can grasp the required knowledge of solving different questions, it is simplified and non-complicated. It consists of almost 500 questions including NECTA question. This book written to reduce some complexity in heat and thermodynamics for advanced leaner.

NB:

Further Remarks, advice and any other additional idea/knowledge for the readers are completely considered for further modification.

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DEDICATION OF THE BOOK

To “Sir Isaac Newton” in contribution to Science

Typing, drawing and all other system type setting was done by mwl. Chambilo

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TABLE OF CONTENT

1 Thermometer

Introduction

Heat is the form of energy which flow from one point to another point due to temperature difference.

The device used to measure heat is called **calorimeter.** And the unit of heat is are Joules (J) or Caloric (cal).

$$1\text{cal} = 4.2\text{J}$$

Calorie is defined as amount of heat required to rise a temperature and its measurements of 1gm of water through 1℃

Thermometer

Thermometry is the study of temperature and its measurements Temperature is a degree of hotness or coldness of the body. The device used to measure temperature is called **THERMOMETER**

The most common units of temperature are known as Kelvin (k) or Centigrade (℃)

1.1 Establishment Of Thermometry.

The factors for the establishment of thermometer

i. Fixed point thermometer
ii. Thermometry property.

FIXED POINT

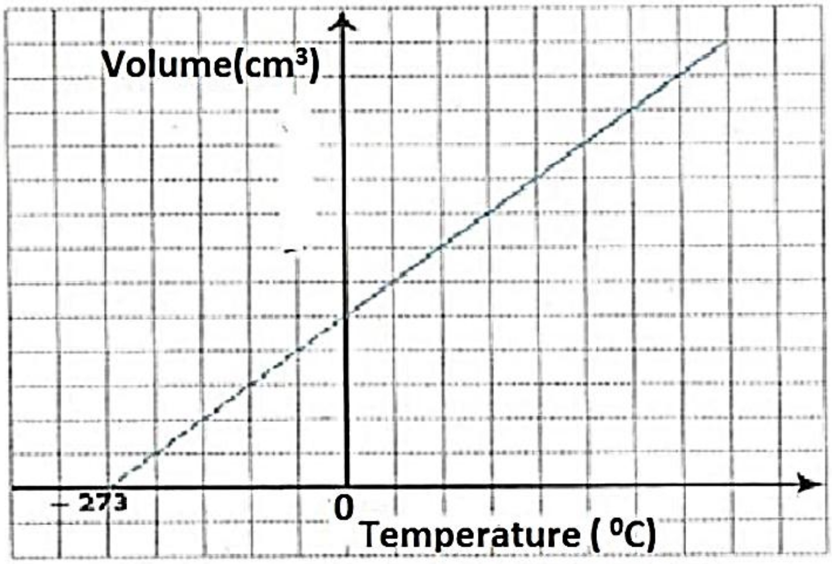
There are two fixed point which are lower fixed point and upper fixed point.

Lower fixed point is the temperature at which pure ice is in thermo equilibrium with pure water at standard atmospheric pressure (760mmHg).

NB: Kelvin or absolute temperature scale is standard temperature scale adopted from scientific measurements and its unit is Kelvin (k) It is denoted by "T"
$T = 273.15 + \theta℃$

1.2.Absolute Zero Temperature

Is the temperature at which the volume and pressure of the gas is theoretical,zero.i.e $(-273.15k)$



1.3. Tripple Point

Is the temperature at which pure water, pure ice and pure vapor together exist in equilibrium.

OR

Is the temperature at which all three states of water exist together in equilibrium. Triple point at water is $273.16k$

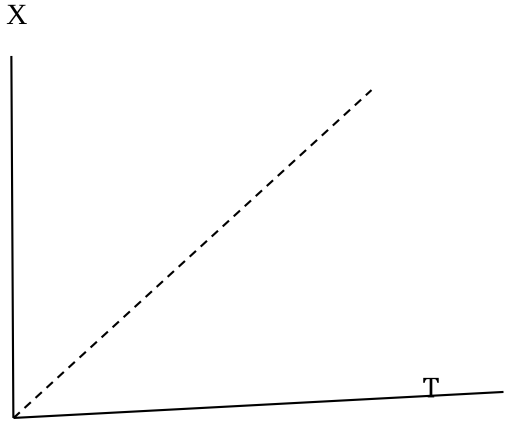
1.4. Thermodynamic Temperature Scale

Thermodynamic temperature scale is the standard temperature scale adopted for scientific measurement of temperature.

- It use a triple point of water as upper fixed point and absolute zero
- The triple point of water assigned the temperature 273.16K while the ice point is 273.15K and the steam point 373.15K
- Thermodynamic temperature is denoted by the symbol T and is expressed in Kelvin K

1.5. Thermometry Property

Is any property of body which vary linearly with temperature, and it is represented by letter "x"



Example of thermometry properties are

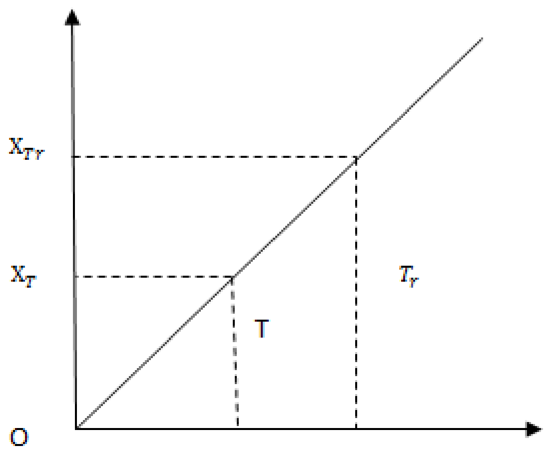
a) Pressure
b) Volume
c) Resistance
d) E. m. f
e) Length of the liquid in glass

1.6. Qualities Of Good Thermometric Property

- It should have unique value at a particular temperature
- It should be repeatable
- It should vary continuously and linearly with temperature
- It should change considerably for small change of temperature
- It should be easily produced in the laboratory

Let X be the value of thermometric property also, let x_{T_r} = thermometry property at tripple point, T_r = Triplle point

$$x_T = \text{thermometry property at certain temperature}$$



$$T = \text{certain temeprature}$$

$$\text{from } x \propto T$$

$$x_T \propto T$$

$$x_{T_r} \propto T$$

$$x_T = kT ----(i)$$

$$x_{T_r} = kT ----(ii)$$

dividing the two equation

$$\frac{x_T}{x_{T_r}} = \frac{kT}{kT_r}$$

$$T = \left(\frac{x_T}{x_{T_r}}\right)T_r$$

$$\text{but } T_r = 273.16k$$

$$T = \left(\frac{x_T}{x_{T_r}}\right) \times 273.16k$$

Example 01:

The pressure recorded by a constant volume gas thermometer at temperature T is $4.88 \times 10^4 Pa$. Calculate T if the pressure at triple point (273.16k) is $4.2 \times 10^4 Pa$.

solution

$$P_T = 4.88 \times 10^4 \text{Pa}$$

$$P_{T_r} = 4.2 \times 10^4 \text{Pa}$$

$$T_r = 273.16\text{K}$$

Required T

$$T = \left(\frac{P_T}{P_{T_r}}\right)T_r$$

$$T = \left(\frac{4.88 \times 10^4}{4.2 \times 10^4}\right) \times 273.16$$

$$T = 317.39\text{K}$$

Example 02: In a particular constant volume gas thermometer register a pressure of $1.937 \times 10^4 Pa$ at triple point of water and $2.168 \times 10^4 Pa$ at boiling point of liquid. Compute boiling point of liquid.

solution

$$P_{T_r} = 1.937 \times 10^4 \text{Pa}$$

$$P_T = 2.168 \times 10^4 \text{Pa}$$

T at boiling point

$$B.P = \left(\frac{P_T}{P_{T_r}}\right) 273.16$$

$$B.P = \left(\frac{2.168 \times 10^4}{1.937 \times 10^4}\right) 273.16$$

$$\text{Boiling point } (B.P) = 305.73\text{K}$$

Example 03:

The resistance thermometer reads 1.2Ω when measuring Kelvin temperature (T) of a body and 1.00Ω at triple point of water. Find T and its centigrade equivalent.

solution

$$R_T = 1.2\Omega$$

$$R_{T_r} = 1.00\Omega$$

$$T = ?$$

$$\theta = ?$$

$$T = \left(\frac{R_T}{R_{T_r}}\right) 273.16$$

$$T = \left(\frac{1.2}{1}\right) 273.16$$

$$T = 327.79\text{K}$$

$$\text{but } T = 273 + \theta$$

$$327.79 = 273 + \theta$$

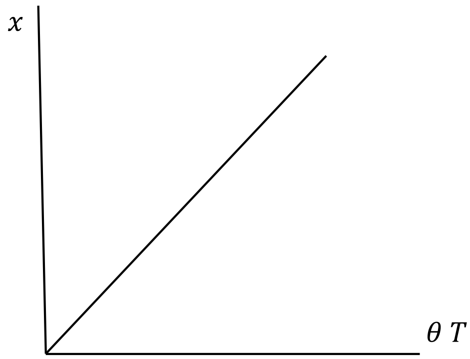
$$\theta = 327.79 - 273$$

$$\theta = 54.79°\text{C}$$

1.7.Fundamental Interval (Celcius Scale)

Is the temperature scale in which the lower and upper fixed point are $0°\text{C}$ and $100°\text{C}$ respectively.

Considering the graph below



Thermometry property is directly proportional to the change in temperature

$$x \propto \Delta\theta$$

$$\text{let } 0°C = \text{ lower point}$$

$$x°C = \text{Thermometry property at ice point}$$

$$100°C = \text{steam point}$$

$$x_{100°C} = \text{thermometry property at steam point}$$

$$\theta°C = \text{be any temperature}$$

$$x_{0°C} = \text{Thermometry property at any certain temperature}$$

$$\Delta x \propto \Delta\theta$$

$$\Delta x \propto \Delta T$$

$$x_{100°C} - x_{0°C} \propto 100°C - 0°C$$

$$x_{100°C} - x_{0°C} \propto 100°C$$

$$x_{100°C} - x_{0°C} = k100°C ----(i)$$

$$x_{\theta} - x_{o} = k\theta ----(ii)$$

$$\frac{x_{100°C} - x_{0°C}}{x_{\theta} - x_{o}} = \frac{k100°C}{k\theta}$$

$$\frac{x_{100°C} - x_{0°C}}{x_{\theta} - x_{o}} = \frac{100°C}{\theta}$$

$$\theta = \left(\frac{x_{\theta} - x_{o}}{x_{100°C} - x_{0°C}}\right) 100°C$$

Example 04:

(i) Write down an expression which calibrates a thermometry property "x" to read temperature in celcius scale.

(ii) Find the temperature of the system which its pressure is 4.6Pa. Given that the lower and upper fixed points of pressure are 1.5Pa and 3.0Pa

solution

$$\text{(i)}\ \theta = \left(\frac{x_\theta - x_o}{x_{100°C} - x_{0°C}}\right)100°C$$

$$\text{(ii) data given } P_\theta = 4.6\ Pa$$

$$P_{0°C} = 1.5Pa$$

$$P_{100°C} = 3.0Pa$$

$$\theta = \left(\frac{P_\theta - P_{0°C}}{P_{100°C} - P_{0°C}}\right)100°C$$

$$\theta = \frac{4.6 - 1.5}{3.0 - 1.5} \times 100°C$$

$$\theta = \frac{3.1}{1.5} \times 100°C$$

$$\theta = 206.7°C$$

Example 05:

A thermometer used liquid in glass thermometer show that the length of mercury at 0°C and at 100°C are 5cm and 7cm respectively at a certain temperature the length is 6.5cm. Find its certain temperature

solution

$$l_{0°C} = 5cm$$

$$l_{100°C} = 7cm$$

$$l_{\theta°C} = 6.5cm$$

$$\text{required}\ \ \theta = ?$$

from the formula

$$\theta = \left(\frac{l_\theta - l_{0°C}}{l_{100°C} - l_{0°C}}\right)100°C$$

$$\theta = \left(\frac{6.5 - 5}{7 - 5}\right)100°C$$

$$\theta = \left(\frac{1.5}{2}\right)100$$

$$\theta = 75°C$$

Example 06:

The pressure of air in constant volume gas thermometer is $80 atm\ and\ 109.3 atm$ at $0°C\ and\ 100°C\ respectively$.when a bulb is placed in hot water the pressure is $100 atm$. Calculate the temperature of the hot water.

solution

$$P_\theta = 100\text{atm}$$

$$P_{0°C} = 80\text{atm}$$

$$P_{100°C} = 109.3\text{atm}$$

required temperature θ

$$\theta = \left(\frac{P_\theta - P_{0°C}}{P_{100°C} - P_{0°C}}\right)100$$

$$\theta = \left(\frac{100 - 80}{109.3 - 80}\right)100$$

$$\theta = \left(\frac{20}{29.3}\right)100$$

$$\theta = 68.259°C$$

The temperature of the hot water 68.26°C

Example 07:

A platinum resistance wire of 2.0Ω, 2.77Ω and 5.28Ω at melting ice, steam point and boiling point of sulphur respectively. Calculate boiling point of sulphur.

solution

$$R_\theta = 5.2\Omega$$

$$R_{0°C} = 2\Omega$$

$$R_{100°C} = 2.77\Omega$$

$$\theta = \left(\frac{R_\theta - R_{0°C}}{R_{100°C} - R_{0°C}}\right)100°C$$

$$\theta = \left(\frac{5.2 - 2}{2.77 - 2}\right)100$$

$$\theta = \left(\frac{3.2}{0.77}\right)100$$

$$\theta = 415.58°C$$

$$\text{Boiling point of sulphur} = 415.58℃$$

1.8. Types Of Thermometer

There are many types of thermometer

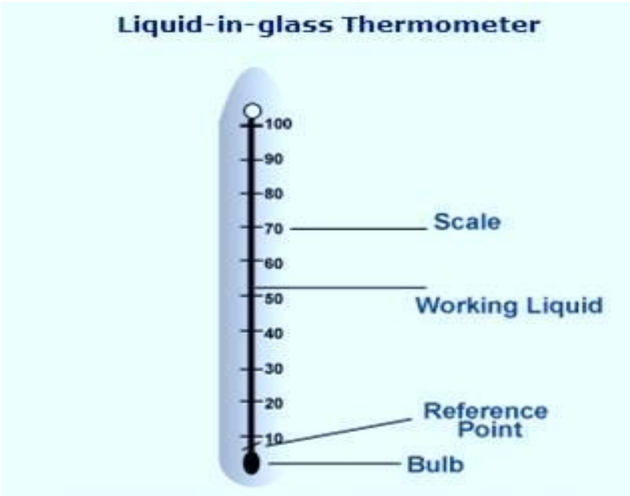
- Liquid in glass thermometer.
- Gas constant temperature thermometer.
- Thermocouple thermometer.
- Pyrometer thermometer.
- Platinum resistance thermometer.

1.8.1.Liquid In Glass Thermometer

They are most used liquid in glass thermometer used in mercury in glass thermometer.

Thermometric property under this thermometer is length (l)

$$\theta = \left(\frac{L_\theta - L_{0℃}}{L_{100℃} - L_{0℃}}\right)100℃$$



Advantage Of This Thermometer

- Can be used over wide range of temperature since its boiling point $375℃$ $and\ freezing\ point - 390℃$
- Its expansion is linear
- Can be easily seen through the glass
- Does not wets the glass
- It have low specific heat capacity.

Disadvantage of This Thermometer.

- It can be used to measure temperature above the boiling point and below freezing point

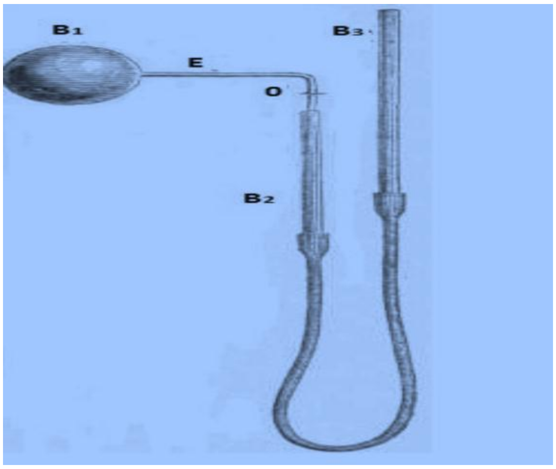
NB: Why Water Is Not Used In Making Liquid In Glass Thermometer?

i. Water have high specific heat capacity ($4200\ j/kgk$)
ii. It wets the glass
iii. Its expansion is not linear
iv. It is colorless hence it cannot be seen easily.

1.8.2. Gas Constant Thermometer

There are two categories

- Constant volume gas thermometer.
- Constant pressure gas thermometer.



Constant Volume Gas Thermometer

Is the gas thermometer whose measurement of temperature depends on variation of pressure.

$$\text{i.e}\quad \theta = \left(\frac{P_\theta - P_{0°C}}{P_{100°C} - P_{0°C}}\right)100°C$$

1.8.3. Pressure Constant Gas Thermometer

Is the type of gas thermometer whose measurement of temperature depends on variation of volume

$$\theta = \left(\frac{V_\theta - V_{0°C}}{V_{100°C} - V_{0°C}}\right)100°C$$

ADVANTAGE OF PRESSURE CONSTANT THERMOMETER

- They are more sensitive than liquid thermometer.
- The expansion is linear
- They can be used to measure very low temperature as well as very high temperature eg $200°C - 500°C$

Disadvantage Of Gas Constant Thermometer

- Indirect reading
- It is very complicated
- It consume time
- Cannot be used to measure liquids in large quantity.

1.8.4. Platinum Resistance Thermometer

Is the thermometer whose measurements of temperature depends on the variation of resistance

The resistance of platinum increases with increase in temperature, the variation of resistance of platinum wire and temperature and it is given by the following relation.

$$R_\theta = R_o(1 + a\theta + b\theta^2)$$

$$\text{where a and b are constants}$$

$$\theta \text{ is the temperature i.e}$$

$$\theta = \left(\frac{R_\theta - R_o}{R_{100℃} - R_o}\right) 100℃$$

Example 08:

The resistance Rt of platinum varies with temperature according to the equation $Rt = R_o(1 + 800bt - bt^2)$ where "b" is constant. Calculate temperature on a platinum scale corresponding to $400℃$ *on a gas scale*.

$$\text{solution}$$

$$\text{since t} = 400℃$$

$$\text{then from } Rt = R_o(1 + 800bt - bt^2)$$

$$\text{substituting the value of t to the eqn above}$$

$$R_\theta = R_o(1 + 800 \times 400b - 400^2b$$

$$R_\theta = R_o(1 + 320000b - 160000b)$$

$$R_\theta = R_o(1 + 160000b) ------ (i)$$

$$R_{100} = R_o(1 + 800 \times 100b - 100^2b)$$

$$R_{100} = R_o(1 + 80000b - 10000b)$$

$$R_{100} = R_o(1 + 70000b) --- (ii)$$

$$\text{but } \theta = \left(\frac{R_\theta - R_o}{R_{100} - R_o}\right) 100℃$$

$$\theta = \left(\frac{R_o - 160000R_ob - R_o}{R_o - 70000R_ob - R_o}\right) 100$$

$$\theta = \left(\frac{160000R_ob}{70000R_ob}\right)100$$

$$\theta = 228.57℃$$

Example 09:

The resistance R_t of platinum wire at temperature t℃ measured on the gas scale is given by $R_t = R_o(1 + at + bt^2)$ where $a = 3.8 \times 10^{-3}$and $b = -5 \times 10^{-3}$. What temperature will be platinum thermometer indicate when temperature on gas scale is 200℃

solution

$$\theta = 200℃$$

$$R_\theta = R_o\left(1 + 3.8 \times 10^{-3} \times 200 \pm 5 \times 10^{-3} \times 200^2\right)$$

$$R_\theta = -198.24R_o ---------(i)$$

$$R_{100} = R_o(1 + 3.8 \times 10^{-3} \times 100 - 5 \times 10^{-3} \times 100^2$$

$$R_{100} = -48.62R_o -------(ii)$$

$$\text{from} \quad \theta = \left(\frac{R_\theta - R_o}{R_{100} - R_o}\right)100$$

$$\theta = \left(\frac{-198.24 - R_o}{-48.62R_o - R_o}\right)100$$

$$\theta = 4.015 \times 100$$

$$\theta = 401.5℃$$

Example 10: A liquid in glass thermometer uses liquid of which the volume varies with temperature according to the equation $V_\theta = V_o(1+\propto\theta + b\theta^2)$ where $\propto = b \times 10^3$. What temperature will be indicated on liquid in glass scale when that gas thermometer if 80℃

solution

$$\text{given } \theta = 80℃$$

$$\propto = b \times 10^3$$

$$\text{from} \quad V_\theta = V_o(1+\propto\theta + b\theta^2)$$

$$V_\theta = V_o(1 + b \times 10^3 \times 80 + b \times 80^2)$$

$$V_\theta = V_o(1 + b \times 10^3 + b \times 6400)$$

$$V_\theta = V_o(1 + 86400b) ------(i)$$

$$V_{100} = V_o(1 + b \times 10^3 \times 100 + b \times 100^2)$$

$$V_\theta = V_o(1 + 110000b) --------(ii)$$

$$\text{from}\quad \theta = \left(\frac{V_\theta - V_o}{V_{100} - V_o}\right)100°C$$

$$\theta = \left(\frac{V_o + 86400bV_o - V_o}{V_o + 110000bV_o - V_o}\right)100$$

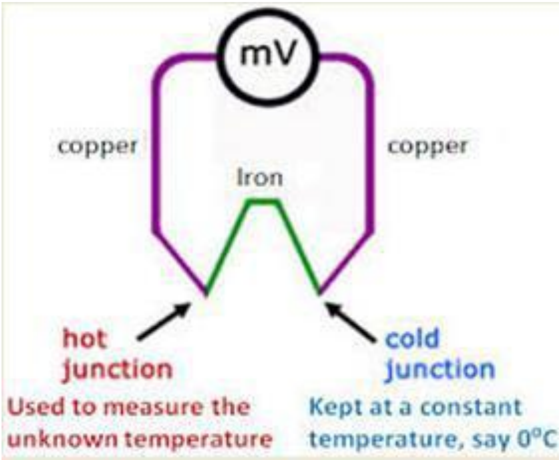
$$\theta = \left(\frac{86400}{110000}\right)100$$

$$\theta = 78.55°C$$

1.8.5. Thermocouple Thermometer

Is the type of thermometer whose measurements of temperature depends on the variation of electromagnetic force.

Thermometer is made up with two dissimilar wires metals for example iron (Fe) and copper (Cu).



The temperature of hot junction at which e.m.f is maximum is called **neutral temperature.**

The temperature of hot junction at which e.m.f is zero is called **Temperature of inversion.**

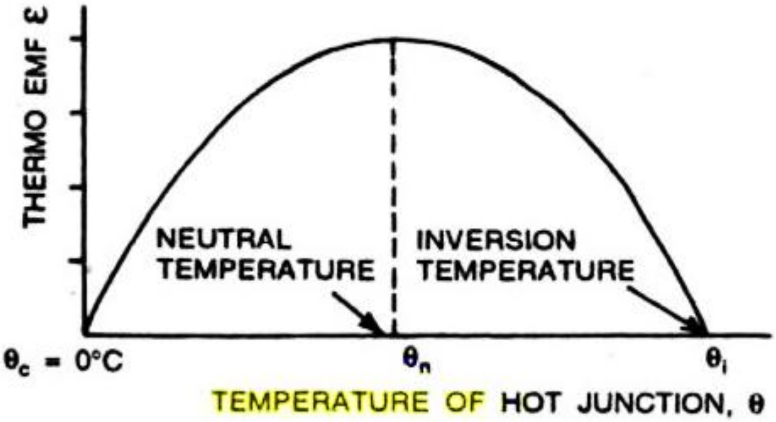
$$\text{then}\quad \theta = \left(\frac{E_\theta - E_o}{\mathbf{E_{100} - E_o}}\right)\mathbf{100}°C$$

The variation of e. m. f **in thermocouple** with temperature is given by the relation

$$E.m.f = A\theta + B\theta^2$$

where A and B are constant.

The graph of e. m. f against temperature is curve.



θ_N = Neutral temperature

θ_I = Temperature of inversion

from the graph

$$\theta_N - 0°C = \theta_I - \theta_N$$

$$\theta_N = \theta_I - \theta_N$$

$$\theta_I = 2\theta_N$$

inversion temperature is twice times the

neutral temperature

also $E = A\theta + B\theta^2$

differentiating the eqn above

$$dE = Ad\theta + 2\theta Bd\theta$$

$$\frac{dE}{d\theta} = A + 2\theta B$$

but when E is maximum $\theta = \theta_N$

since $\frac{dE}{d\theta} = 0$

$$0 = A + 2\theta B$$

$$0 = A + 2B\theta_N$$

$$2B\theta_N = -A$$

$$\theta_N = -\frac{A}{2B}$$

Again $E = A\theta + B\theta^2$

$$\text{where } E = 0 \text{ then } \quad \theta = \theta_I$$

$$0 = A\theta_I + B\theta_I^2$$

$$-A\theta = B\theta_I^2$$

$$-A = B\theta_I$$

$$\theta_I = -\frac{A}{B}$$

Example 11:

In a thermocouple thermometer the temperature of hot junction is $10°C$ while the neutral temperature is $270°C$. Find the temperature of inversion.

solution

$$\text{given } \theta_N = 270°C$$

$$\theta_I = ?, \qquad \text{tempearture of hot junction} = 10°C$$

$$\text{from } \theta_N - 0°C = \theta_I - \theta_N$$

$$\theta_I = (270 - 10) + 270$$

$$\theta_I = 530°C$$

hence the temperature of inversion is $530°C$

Example 12:

The $e.m.f$ of a certain thermocouple thermometer varies with temperature θ of a hot junction with a cold junction at $0°C$ in relation below

$E = 40\theta - \frac{\theta^2}{20}$, Determine

i. Neutral temperature
ii. Inversion temperature

solution

from the given equation above

$$E = 40\theta - \frac{\theta^2}{20}$$

$$dE = 40d\theta - \frac{2\theta d\theta}{20}$$

$$dE = 40d\theta - \frac{\theta d\theta}{10}$$

$$\frac{dE}{d\theta} = 40 - \frac{\theta}{10}$$

$$\frac{dE}{d\theta} = 0$$

$$0 = 40 - \frac{\theta}{10}$$

$$40 = \frac{\theta}{10}$$

$$\theta = 400°C$$

$$\text{when} \frac{dE}{d\theta} = 0 \text{ then } \theta = \theta_N$$

hence neutral temperature is 400°C

(ii)Inversion temperature obtained when

$$E = 0$$

Hence from the equation

$$E = 40\theta - \frac{\theta^2}{20}$$

$$0 = 40\theta - \frac{\theta^2}{20}$$

$$40\theta = \frac{\theta^2}{20}$$

$$40 = \frac{\theta}{20}$$

$$\theta = 800°C$$

or

$$\theta_N - 0°C = \theta_I - \theta_N$$

$$\theta_I = (400 - 0) + 400$$

$$\theta_I = 800°C$$

hence the inversion temperature = 800°C.

Example 13:

The e.m.f of Cu-Fe thermocouple varies with temperature of hot junction (cold junction 0°C) as given below

$E(mv) = 14\theta - 0.02\theta^2$. Calculate

(i) Inversion temperature
(ii) Neutral temperature

solution

(i)inversion temperature

from the given equation

$$E = 14\theta - 0.02\theta^2$$

$$\text{At } \theta_I, \quad E = 0$$

$$0 = 14\theta - 0.02\theta^2$$

$$14\theta = 0.02\theta^2$$

$$14 = 0.02\theta$$

$$\theta = \frac{14}{0.02}$$

$$\theta = 700°C$$

$$\text{inversion temperature} = 700°C$$

$$\text{(ii)} \quad \textbf{Neutral temperature}$$

$$E = 14\theta - 0.02\theta^2$$

$$dE = 14d\theta - 0.04d\theta$$

$$\frac{dE}{d\theta} = 14 - 0.04\theta$$

$$\text{At } \theta_N, \frac{dE}{d\theta} = 0$$

$$0 = 14 - 0.04\theta$$

$$0.04\theta = 14$$

$$\theta = \frac{14}{0.04}$$

$$\theta = 350°C$$

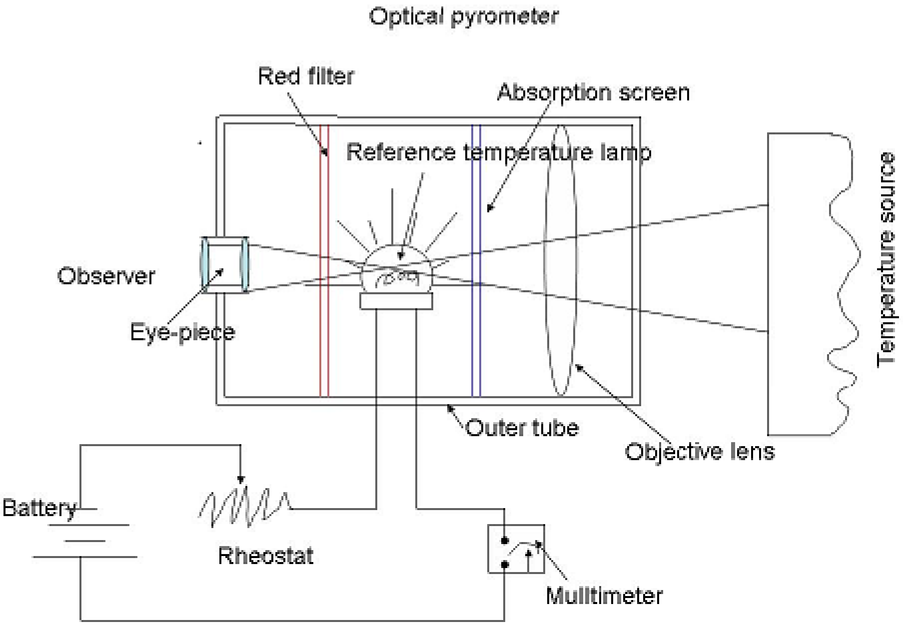
1.8.6. Pyrometer

Is the type of thermometer which is used to measure temperature of radiation from the hot objects, Example Furnace
The radiation of temperature depends on intensity (I) of a radiation.

$$\theta = \left(\frac{I_\theta - I_o}{I_{100} - I_o}\right) 100°C$$

Types Of Pyrometer

- Total radiation pyrometer Is the type of pyrometer which is used to measure the temperature of bodies which emits invisible radiation. Example Radiation from furnace
- Optical pyrometer Is the type of pyrometer which is used to measure temperature of bodies which emits visible radiation. Example Radiation from the sun.



1.9. Worked Examples Set 01

1. A constant volume gas thermometer and a thermocouple thermometer are used together to measure a boiling point of a certain liquid X. the reading of the two thermometers at the ice point, steam point and the boiling point of X were given in the table below

	Gas thermometer	Thermocouple thermometer
Ice point	101kPa	0mV
Steam point	138kPa	5.4mV
Boiling point X	124kPa	3.4mV

Calculate

(i) The boiling point of X
(ii) Its boiling point on the thermocouple centigrade scale.

Solution

$$\theta = \left(\frac{P_\theta - P_0}{P_{100} - P_o}\right) \times 100℃$$

$$= \left(\frac{124 - 101}{138 - 101}\right) \times 100℃$$

$$\theta = 62.16℃$$

(ii)the boiling point on the thermocouple

centigrade scale is

$$\theta = \left(\frac{E_\theta - E_o}{E_{100} - E_0}\right) \times 100℃$$

$$\theta = \left(\frac{3.4 - 0.0}{5.4 - 0.0}\right) 100℃$$

$$\theta = 62.96℃$$

2. The table below gives data for two thermometers at three different temperatures (the ice point, the steam point and room temperatures)

Type of thermometer	property	Ice point	Steam point	Room temperature
Gas	Pressure in mmHg	760	1040	795
thermistor	Current in mA	12.0	54.0	15.0

(i) Calculate the temperature of the room according to each thermometer
(ii) State why thermometers disagree in their value for room temperature
(iii) Explain why a gas thermometer is seldom used for temperature measurement in the laboratory.

solution

(a)the room temperature on the gas scale

$$\theta_{gas} = \left(\frac{P_\theta - P_o}{P_{100} - P_o}\right) 100℃$$

$$\theta_{gas} = \left(\frac{795 - 760}{1040 - 760}\right) 100℃$$

$$= 12.5℃$$

the room temperature on the resistance scale

$$\theta_{resistance} = \left(\frac{R_\theta - R_o}{R_{100} - R_o}\right) 100℃$$

$$= \left(\frac{15 - 12}{54 - 12}\right) 100℃$$

$$\theta_{resistance} = 7.1℃$$

(c)Because it is bulky, does not give direct reading and it is very slow in operation.

3. The relation connecting the magnitude of X and the absolute temperature T is given by

$$X = \frac{a}{T - 273}$$

When T is greater than 223K and a is constant. Derive an expression for the celcius temperature t based on this scale and establish the relation between t and T. What is the value of t corresponding to T = 423K?

solution

$$\text{from the relation } \theta = \left(\frac{X_\theta - X_o}{X_{100} - X_o}\right)100°C$$

X_θ is the value of property X at

temperature θ to be determined

$$X = \frac{a}{T - 223}$$

at ice point 0°C, T = 223K

$$X_o = \frac{a}{273 - 223} = \frac{a}{50}$$

at steam point 100°C, T = 373K

$$X_{100} = \frac{a}{373 - 273} = \frac{a}{150}$$

at temperature t°C, T = T

$$X_t = \frac{a}{T - 223}$$

the temperature on centigrade scale is

$$t = \left(\frac{X_\theta - X_o}{X_{100} - X_o}\right)100°C$$

$$t = \left(\frac{\frac{a}{T-223} - \frac{a}{50}}{\frac{a}{150} - \frac{a}{50}}\right)100°C$$

$$t = \frac{3}{2}\left(\frac{T - 273}{T - 223}\right)100°C$$

$$t = \frac{3}{2}\left(1 - \frac{50}{T - 223}\right)100°C$$

$$t = \frac{3}{2}\left(1 - \frac{50}{423 - 223}\right)100°C$$

$$t = 112.5°C$$

4. What type of thermometer would you use to measure each of the following. In each case explain the reason for your choice.

(i) The boiling point of water on the mountain
(ii) The temperature just after the ignition in a cylinder of an internal combustion engine.
(iii) The temperature of the filament of an electric lamp
(iv) The normal melting point of zinc.

Answers:

(i) Platinum resistance thermometer: it is very accuracy than all other thermometer except gas thermometer. It is also responds quickly than the gas thermometer.
(ii) Thermocouple thermometer: it respond quickly to varying temperature and it is also suitable to measure temperature of a point
(iii) Optical pyrometer: this respond to visible radiation emitted by the filament it is only thermometer for measuring high temperature
(iv) Platinum resistance thermometer: it is accuracy and it respond quickly compared to gas thermometer.

5. The resistance of the given wire at various temperatures on constant volume gas scale are as follows

t°C	0	10	20	30	40	50	60	70	80	90	100
R (Ohm)	5.00	5.08	5.16	5.23	5.31	5.4	5.5	5.61	5.73	5.86	6.00

Find

(i) The temperature on the resistance scale corresponding to 75°C on the gas scale.
(ii) The temperature on the scale corresponding to 35°C on the resistance scale

solution

$$\text{(i)at } 75°\text{C } R = \frac{5.73+5.61}{2} = 5.67$$

$$R_o = 5.0\Omega, \ R_{100} = 6.00\Omega$$

$$\theta = \left(\frac{R_\theta - R_o}{R_{100} - R_o}\right)100°\text{C}$$

$$\theta = \left(\frac{5.67 - 5.0}{6.00 - 5.0}\right)100°\text{C}$$

$$\theta = 67°\text{C}$$

(ii) the resistance corresponds to 35°C is

$$35 = \left(\frac{R - 5}{6 - 5}\right)100°\text{C}$$

$$0.35 = \frac{R-5}{1}$$

$$R = 5 + 0.35$$

$$R = 5.32\Omega$$

6. The apparent expansion of a liquid in glass is given by $V_t = V_o(1 + \alpha t + \beta t^2)$,where $\frac{\beta}{\alpha} = -8 \times 10^{-5}°C^{-1}$, and t is the centigrade temperature measured by constant volume air thermometer. Calculate the reading of the liquid thermometer if an air thermometer in the same enclosure reads 50°C.

$$\text{volume at } 0°C, \quad = \quad V_o$$

$$\text{volume at the steam point } V_{100} \quad = \quad V_o(1 + 100\alpha + 10000\beta)$$

$$\text{volume at } 50°C\ V_{50} \quad = \quad V_o(1 + 50\alpha + 2500\beta)$$

$$\text{then the liquid in thermometer is}$$

$$\theta \ = \ \left(\frac{V_\theta \ - \ V_o}{V_{100} \ - \ V_o}\right) \times 100°C$$

$$\theta \ = \ \left(\frac{V_o(1 + 50\alpha + \ 2500\beta) - V_o}{V_o(1 + 100\alpha \ + 10000\beta) - V_o}\right) \times 100°C$$

$$\theta \ = \ \left(\frac{\alpha + \ \ 50\beta}{\alpha \ + \ \ 100\beta}\right) \times 50°C$$

$$\beta = \ -8 \times 10^{-5}$$

$$\theta \ = \ \left(\frac{\alpha \ - (50 \times 8 \ \times 10^{-5}\alpha)}{\alpha - (100 \times 8 \times 10^{-5}\alpha)}\right) \times 50$$

$$\theta \ = \ \frac{0.996\alpha}{0.992\alpha} \times 50°C$$

$$\theta \ = \ \ 50.20°C$$

1.10. Competetitive Examination File Unit Set 01:

Problem 01

A thermometer uses mercury as liquid in glass experiments show that the length of mercury at 0°C and 100°C are 5cm and 7cm respectively. At a certain temperature the length of the mercury is found to be 6.5cm, find this certain temperature.
(Answer: $\boldsymbol{\theta = 75}$°C)

Problem 02

The pressure recorded by a constant volume gas thermometer at a Kelvin temperature T is 4.80 x 10^4Nm^{-2}. Calculate T if the pressure at triple point 273.16K is 4.20 x 10^4Nm^{-2}. **(Answer. T = 312K)**

Problem 03

The resistance of a platinum wire at a temperature θ°C, measured on gas scale is given by $R_\theta = R_o(1 + a\theta + b\theta^2)$ Where $a = 3.8 \times 10^{-3}$ and $b = -5.6 \times 10^{-7}$. What temperature will the platinum thermometer indicate when the temperature on a gas scale is 200°C

Problem 04

The pressure of air in a constant volume gas thermometer is 80cm and 109.3cm at $0°C$ and $100°C$ respectively. When the bulb is placed in hot water, the pressure is 100cm. calculate the temperature of hot water

Problem 05

The resistance of a platinum resistance thermometer is 100Ω at room temperature of 25°C. In an experiment for measurement of temperature, the resistance of the thermometer is found to be 115.68Ω. Find the value of temperature given that the temperature coefficient of resistance of platinum is 0.004/°C.

Problem 06

A constant mass of a gas maintained at constant pressure has a volume of $200\,cm^3$ at the temperature of melting ice, $273.2cm^3$ at the temperature of water boiling under standard pressure and $525.1cm^3$ at the normal boiling point of sulphur. A platinum wire has resistances of 2.00Ω, 2.778Ω, and 5.280Ω at these temperatures. Calculate the values of boiling- point of sulphur given by the two sets of observations and comment on the results.

Problem 07

In the thermocouple, the temperature of the cold junction is $10°C$ while the neutral temperature is $270^{°C}$. What is the value of temperature of inversion?.

Problem 08

In a certain thermocouple the thermo e. m. f E is given by $E = \alpha\theta + \frac{1}{2}\beta\theta^2$ where θ is the temperature of the hot junction and the cold junction being at 0°C. If $\alpha = 10\mu V/°C$ and $\beta = -\frac{1}{20}\,\mu V/°C$, find

(i) The neutral temperature

(ii) The temperature of inversion

Problem 09

(a) What does one require in order to establish a scale of temperature?

(b) A Copper – constant thermocouple with its cold junction at $0°C$ had an EMF of 4.28mV with its other junction at 100°C. The EMF becomes 9.29mV when the temperature of the hot junction was 200°C. If the EMF E is related to the temperature different θ by the equation $E = A\theta + B\theta^2$, Calculate

(i) The values of A and B

(ii) The range of temperature of which E may be assumed proportional to θ without incurring an error of more that 1%?

Problem 10

The resistance R_t of a platinum varies with temperature t according to the equation $R_t = R_0 (1 + 8000bt - bt^2)$ where "b" is a constant. Calculate the temperature on platinum scale corresponding to 400°C on the gas scale.

Problem 11

(a) Define the thermodynamic temperature scale

(b) The resistance of a platinum resistance thermometer is 1.20Ω when measuring a Kelvin temperature T of a body and 1.00Ω at the triple point of water. Find T and its centigrade equivalent.

Problem 12

(a) What do you understand by the terms

(i) Thermodynamic temperature scale

(ii) Triple point of water

(b) The resistance of a platinum wire at temperature T°C measured on a gas scale is given by $R(T) = R_0 (1 + aT + bT^2)$. What temperature will the platinum thermometer indicate when the temperature on the gas scale is 200°C?

(Take $a = 3.8 \times 10^{-3}$ and $b = -5.6 \times 10^{-7}$)

Problem 13

(a) Define

(i) Thermodynamic temperature scale

(ii) How thermodynamic temperature donated and what is its SI unit?

(iii) Explain why a gas thermometer is seldom used for temperature measurement in the laboratory?

(b) Study the table below and answer the questions which follow:

Type of thermometer	property	Values of property		
		Ice point	Steam point	Room temp
Gas	Pressure in mmHg	760.0	1240.0	892.0
Thermistor	Current in mA	12.0	70.0	28.0

(i) Calculate the temperature of the room for each thermometer

(ii) Explain why thermometers disagree in their values of room temperature.

(iii)What are the advantages of gas thermometer over liquid in-glass thermometers?

Problem 14

(a) (i) Describe how mercury in glass thermometer could be made sensitive.

(ii) A sensitive thermometer can be used to investigate the difference in temperature between the top and bottom of the waterfall. Calculate the temperature difference of the water fall 50m high.

(b) (i) Platinum resistance thermometer and constant volume gas thermometer are based on different thermometric properties but they are calibrated using the same fixed points. To what extent are the thermometers likely to agree when used to measure temperature near the ice point and near the steam point.
(ii)The resistance of the element of a platinum resistance thermometer is 2.0Ω at ice point and 2.73Ω at steam point. What temperature on the platinum resistance scale would correspond to resistance value of 8.34Ω and when measured on the gas scale the same temperature will correspond to a value of 1020°C? Explain the discrepancy.

Problem 15

(a) (i) What is meant by a thermometric property of a substance?
(ii) What qualities make a particular property suitable for use in practical thermometers

(b) Explain
(i) Why at least two (2) fixed points are required to define a temperature scale?
(ii) Mention the type of thermometer which is most suitable for calibration of thermometers.

2 Heat Transfer

Heat transfer is a flow of heat energy from one point to another due to temperature difference between the points.

OR

This is the transfer of heat energy from one body or system to another as a result of difference in temperature. In general heat energy transfers from the region of higher temperature to the region of lower temperature

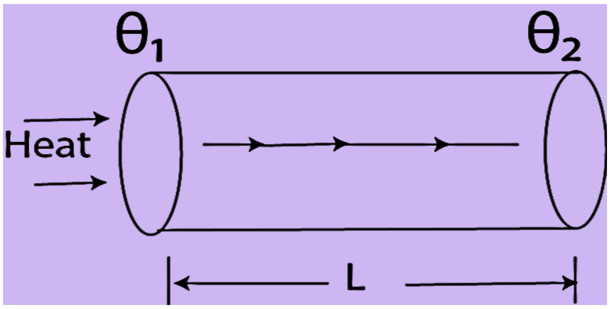
Ways Of Heat Transfer

- Thermal radiation
- Thermal convection.
- Thermal conduction

2.1. Thermal Conduction

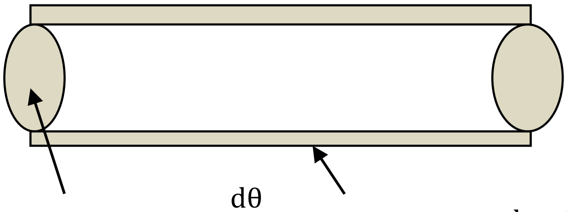
Thermal conduction is a flow of heat energy from one point to another point due to vibration of particles about their mean position.
Or Is the process in which heat flow from hotter region to the colder region of an objects without any next movement of substance particle itself.

Consider the figure below



As the energy applied in one side it makes the molecules collision of the conductor particle as the particle vibrate about their mean position it collide with neighbor, one some energy applied to them.

- **Conductor** Is the substance which allow heat energy to pass through it example iron and aluminum.
- **Insulator** Is the substance which does not allow heat energy to pass through it. Example Plastic, Glass, wood and rubber.
- **Steady state condition** Is the phenomena for which rate of heat flow of each cross section area is constant but not the same. hence $\frac{d\theta}{dt} = \text{constant}$
- **Lagging** Is the process of covering conductor with an insulator in order to prevent heat loss to the surrounding.



$\frac{d\theta}{dt}$ lagging material

- **Unlagged Conductor**
  Unlagged conductor is the conductor which do not covered by insulating materials. For unlagged conductor the rate of heat flow is not constant.
- **Rate of heat flow**
  Rate of heat flow is the flow of heat energy per unit time. The SI –unit is watt or J/s
- **Temperature difference**
  Temperature difference is the difference interval between two points $\Delta\theta = d\theta = \theta_2 - \theta_1$ if $\theta_2 > \theta_1$
- **Temperature gradient**
  Temperature gradient is the temperature difference per unit length of a conductor $g = \frac{\theta_2 - \theta_1}{l}$
- **Cross sectional area**
  Cross sectional area is the section of area on a conductor where heat is conducted
  NB: Due to experimental investigation the following observation observed
  a) The rate of heat flow $\frac{d\theta}{dt}$ is directly proportional to the temperature gradient $\frac{d\theta}{dt} \propto \frac{\theta_2 - \theta_1}{l}$ – – – – – (i)
  b) The rate of heat flow is directly proportional to the cross sectional area
  $\frac{d\theta}{dt} \propto A$ – – – – – – – – (ii)
  Combining the two equations
  $\frac{d\theta}{dt} \propto A\left(\frac{\theta_2 - \theta_1}{l}\right)$ iff $\theta_2 > \theta_1$

$$\frac{d\theta}{dt} = kA\left(\frac{\theta_2 - \theta_1}{l}\right)$$

k is a proportionality constant called Thermoconductivity

NB: Good conductor of heat have large value of thermo conductivity (k)
Then from the equation

$$\frac{d\theta}{dt} = kA\left(\frac{\theta_2 - \theta_1}{l}\right)$$

k can be obtained

$$\left(\frac{d\theta}{dt}\right)l = kA(\theta_2 - \theta_1)$$

$$k = \frac{\left(\frac{d\theta}{dt}\right)l}{A(\theta_2 - \theta_1)}$$

hence k is the measure how easily a substance

conduct heat

Or **k** is the rate of heat flow per cross section area per temperature gradient. The unit of K is w/mK and its dimensional variables with dimension $[MLT^{-3}\theta^{-1}]$

Consider a table below

substance	K in w/mk
Silver	420
Copper	380-400
Aluminum	200
Steel	40
Glass	0.84
Brick	0.84

This is the process in which heat flows from the hot end to the cold end of the solid body without there being any net movement of the particles of the solid.

2.1.1. Mechanism Of Thermal Conduction

Mechanism 1

The molecules of a solid vibrate about their fixed positions with an energy that increases with temperature. When a part of the solid is heated, the molecules there start vibrating more violently.

Since neighboring molecules are bound to each other, a molecule vibrating with larger energy will transfer some of its energy to its neighbors which in turn will transfer energy to the next neighbors and so on.

Mechanism 2

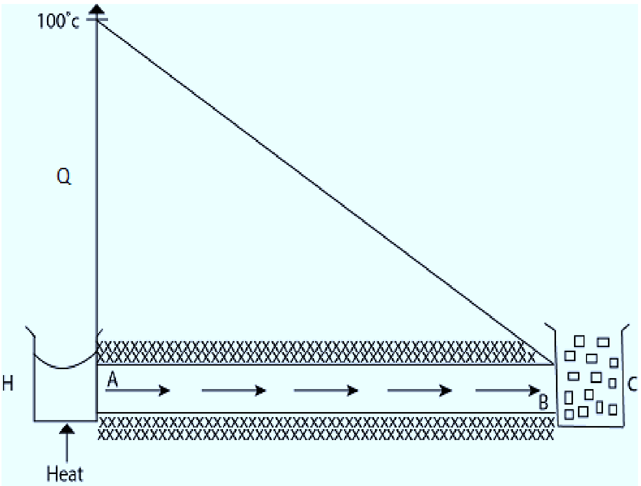
In case of metals heat energy can also be transported by the free electrons. Since the electrons are very small, they can travel rapidly around throughout the specimen transferring energy by collision to other electrons and other molecules. Hence, the electrons are more effective in transferring energy from the hotter part to the colder part of the material than the mechanism explained above (mechanism1) This explains why thermal conduction in metals is much more than that in insulator. In metals heat energy is mainly carried by the free electrons although some energy is carried by intermolecular vibration.

2.1.2. Temperature Distribution Along Conductor

(i) Lagged conductor

If the metal bar is well-lagged with a bad conductor of heat such as asbestos and wool the temperature now falls uniformly from the hot end to the cold end of the bar.

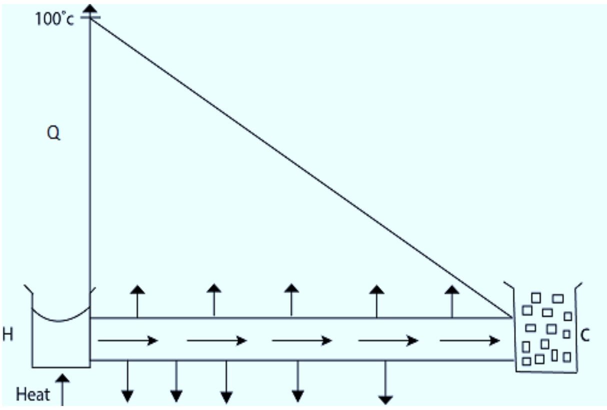
A graph of temperature against length of the bar is shown below:



$$\frac{d\theta}{dt} = \text{constant}$$

there is no heat loss from a conductor the radial heat lines are uniform

(ii) **Unlagged conductor**

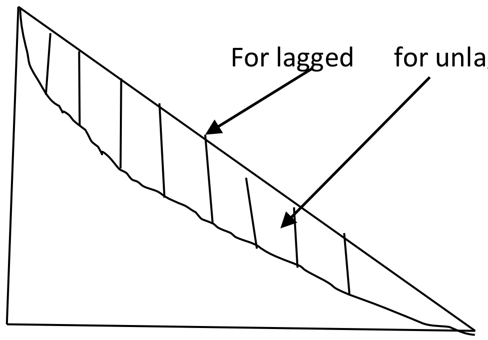


$$\frac{d\theta}{dt} \text{ is not constant}$$

there is heat given out (lost) from the conductor

The radial heat lines are not uniform they are randomly

Since the metal bar is well-lagged no heat is lost to the surrounding and a graph of fall of temperature against length of the bar is a straight line (see figure above). Sketch showing the temperature distribution along a conductor with length for both lagged and unlagged bar in one area.



The shaded part represent the heat lost by the bar

NB: for the formation of

$$\frac{d\theta}{dt} = kA\left(\frac{\theta_2 - \theta_1}{l}\right) \text{ is valid iff there is no}$$

heat lost (means the conductor is lagged)

2.1.3. Composite Bars (Rods)

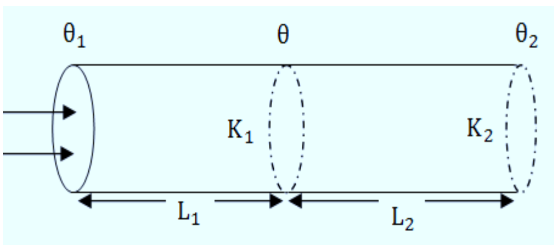
If two or more bars are connected in any way and then allowed to conduct heat, The rate of heat flow can be analyzed as follows

2.1.3.1 Series Connection

A composite bars is that bar consisting of two or more metal bars of different materials joined end to end.

Consider a composite bar made of different materials of coefficient of thermal conductivities K_1 and K_2 respectively

Consider the figure below



if $\theta_1 > \theta > \theta_2$ in series connection

the rate of heat flow is constant in

each bar

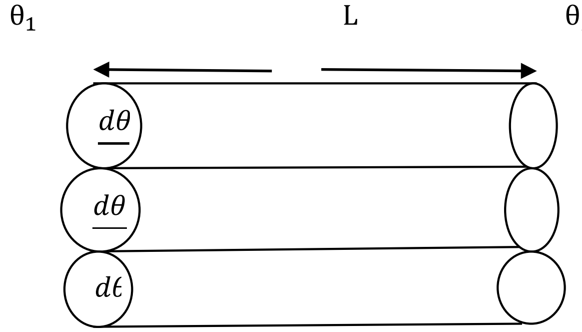
$$\text{hence } \left(\frac{d\theta}{dt}\right)_1 = \left(\frac{d\theta}{dt}\right)_2 = \left(\frac{d\theta}{dt}\right)_3$$

$$\text{but } \frac{d\theta}{dt} = kA\left(\frac{\theta_1 - \theta_2}{l}\right)$$

$$\frac{k_1A(\theta_1 - \theta)}{L_1} = \frac{k_2A(\theta - \theta_2)}{L_2}$$

2.1.3.2 Parallel Connection

In parallel connection the total rate of heat flow is the sum of individual rate of heat flow of each conductor as the figure below show



$$\left(\frac{d\theta}{dt}\right)_T = \left(\frac{d\theta}{dt}\right)_1 + \left(\frac{d\theta}{dt}\right)_2 + \left(\frac{d\theta}{dt}\right)_3$$

$$\text{since } \frac{d\theta}{dt} = \frac{kA(\theta_1 - \theta_2)}{L}$$

$$\left(\frac{d\theta}{dt}\right)_T = \frac{k_1A(\theta_1 - \theta_2)}{L} + \frac{k_2A(\theta_1 - \theta_2)}{L} + \frac{k_3A(\theta_1 - \theta_2)}{L}$$

$$\left(\frac{d\theta}{dt}\right)_T = \frac{A(\theta_1 - \theta_2)}{L}(k_1 + k_2 + k_3)$$

2.1.4 Thermoresistance

Thermo resistance is the opposition offered by a conductor to the flow of heat through it.

Or is the temperature difference per unit rate of heat flow.

$$R_\theta = \frac{\theta_1 - \theta_2}{\frac{d\theta}{dt}} \text{ iff } \theta_1 > \theta_2$$

R_θ is related with conductivity k by the following formula

$$\text{from } \frac{d\theta}{dt} = \frac{kA(\theta_1 - \theta_2)}{L} - - - - - \text{(i)}$$

$$\text{also } R_\theta = \frac{\theta_1 - \theta_2}{\frac{d\theta}{dt}} - - - - \text{(ii)}$$

substituting eqn (i)into eqn)ii)

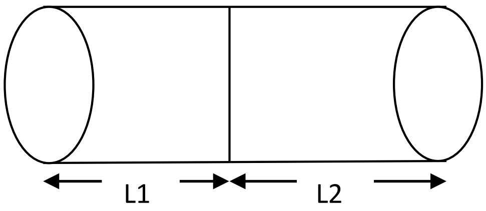
$$R_\theta = \frac{\theta_1 - \theta_2}{\left(kA\left(\frac{\theta_1 - \theta_2}{L}\right)\right)}$$

$$R_\theta = \frac{1}{kA\left(\frac{1}{L}\right)}$$

$$R_\theta = \frac{L}{kA}$$

Thermo resistance (R_θ)is inversely proportional to the thermo conductivity if L and A remain constant then the unit R_θ is wk

Thermo resistance in series



$$\frac{d\theta}{dt} = \frac{k_1A(\theta_1 - \theta_2)}{L_1}$$

$$\theta_1 - \theta_2 = \frac{L_1}{k_1A}\left(\frac{d\theta}{dt}\right) - - - - - (i)$$

for conductor 2

$$\frac{d\theta}{dt} = \frac{k_2A(\theta_2 - \theta_3)}{L_2}$$

$$\theta_2 - \theta_3 = \frac{L_2}{k_2A}\left(\frac{d\theta}{dt}\right) - - - - - (ii)$$

add the two equations

$$+\begin{cases}\theta_1 - \theta_2 = \frac{L_1}{k_1A}\left(\frac{d\theta}{dt}\right)\\ \theta_2 - \theta_3 = \frac{L_2}{k_2A}\left(\frac{d\theta}{dt}\right)\end{cases}$$

$$\theta_1 - \theta_3 = \frac{d\theta}{dt}\left(\frac{L_1}{k_1A} + \frac{L_2}{k_2A}\right)$$

$$\frac{\theta_1 - \theta_3}{\frac{d\theta}{dt}} = \left(\frac{L_1}{k_1A} + \frac{L_2}{k_2A}\right)$$

$$\text{but} \quad \frac{L_1}{k_1A} + \frac{L_2}{k_2A} = R_\theta$$

$$R_\theta = (\theta_1 - \theta_3)/\frac{d\theta}{dt}$$

$$R_{\theta_1} = \frac{L_1}{k_1A}$$

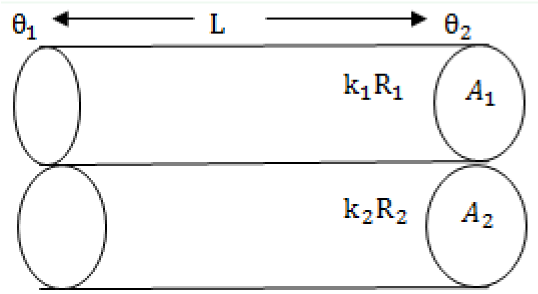
$$R_{\theta_2} = \frac{L_2}{k_2A}$$

$$\text{Since} \quad R_{\theta_T} = R_{\theta_1} + R_{\theta_2}$$

$$R = R_1 + R_2$$

for parallel bars

In parallel bars total rate of heat flow is the sum of bars rate of heat flow.



$$\left(\frac{d\theta}{dt}\right)_1 = \frac{k_1A_2(\theta_1 - \theta_2)}{L} ----(i)$$

$$\left(\frac{d\theta}{dt}\right)_2 = \frac{k_2A_2(\theta_1 - \theta_2)}{L} ----(ii)$$

$$\left(\frac{d\theta}{dt}\right)_T = \frac{k_1A_1(\theta_1 - \theta_2)}{L} + \frac{k_2A_2(\theta_1 - \theta_2)}{L}$$

$$\frac{1}{R_T} = \left[\frac{k_1A}{L} + \frac{k_2A}{L}\right]$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

2.1.5 Competitive Examination File Unit Set 02:

Problem 01

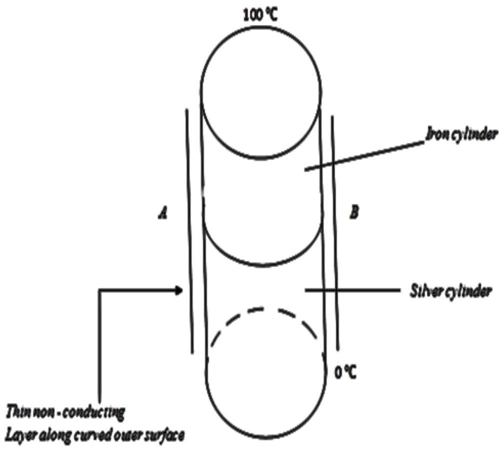
Calculate the quantity of heat conducted through $2m^2$ of brick-wall 12cm thick in 1 hour if the temperature on one side is 8℃ and the other side is 28℃. Given that thermal conductivity of brick = $0.13Wm^{-1}K^{-1}$

Problem 02

Estimate the rate at which ice melts in a wooden box 2cm thick and inside measurements 60cm × 60cm × 60cm, assume that external temperature is 27℃ and coefficient of thermal conductivity of wood = 0.1674 $Wm^{-1}K^{-1}$. Specific latent heat of fusion of ice = 336×10^3 Jkg^{-1}

Problem 03

Two cylinders of equal physical dimensions are placed one on top of the other as illustrated below.



The lower surface of the silver cylinder is kept at 0℃ and the upper surface of the iron cylinder is kept at 100℃. Given that the thermal conductivity of silver is eleven times that of iron, calculate the temperature of the surface AB

Problem 04

A composite bar is made of a bar of copper 10cm long, a bar of iron 8cm long and a bar of aluminium 12cm long, all having the same cross – sectional area. If the extreme ends of the bars are maintained at 100℃ and 10℃ respectively, find the temperature at the two junctions. Given that thermal conductivity of copper, iron and aluminium are 400, 40 and 20 $Wm^{-1}K^{-1}$ respectively.

Problem 05

A window of height 1.0m and 1.5m contains a double glazed unit consisting of two single glass panes, each of thickness 4.0mm separated by an air gap of 70mm. Calculate the heat energy per second conducted through the window when the temperature difference across the unit is 10K.

Problem 06
An electric heater is used in a room of total wall area $137m^2$ to maintain a temperature of 20°C inside it when the outside temperature is 10°C. The walls have three layers of different materials. The innermost layer is of wood of thickness 2.5cm, the middle layer is of cement of thickness 1.0cm and the outermost layer is of brick of thickness 25cm. Find the power of electric heater. Assume that there is no heat loss through the floor and ceiling. Thermal conductivity of wood, cement and brick are 1.25 $Wn^{-1}K^{-1}$,1.5 $Wm^{-1}K^{-1}$ and 1.0 $Wn^{-1}K^{-1}$ respectively.

Problem 07
(a) What does it mean by thermal conductivity of substance.
(b) Find the heat lost per square meter through a cavity wall when the temperature difference between the inside and outside is 15°C, given that each of the two brick layers is 100mm thick and the cavity is also 100mm across. Brick = 1.0 $Wm^{-1}K^{-1}$
Air = 0.025 $Wm^{-1}K^{-1}$

Problem 08
(a) Assuming you are managing a metal box company what requirements for thermal conductivity, specific heat capacity and coefficient of expansion would you want a material to be used as a cooking utensil to satisfy.
(b) A hot water boiler consists of iron wall of thickness 2.0cm and effective inner area of $2.5m^2$. The boiler is heated by a furnace and generates high pressure steam of temperature 170°C at the rate of $1.2kgmin^{-1}$. The latent heat of steam at 170°C is $2.09 \times 10^6\ Jkg^{-1}$. Assuming the outer face of the boiler to be at a temperature of 178°C, what is the coefficient of thermal conductivity of iron?

Problem 09
A copper kettle has a circular base of radius 10cm and thickness 3.0mm. The upper surface of the base is covered by a uniform layer of scale 1.0mm thick. The kettle contains water which is brought to the boil over an electric heater. In the steady state condition, 5.0g of steam is produced each minute. Determine the temperature of the lower surface of the base assuming the condition of heat along the surface of the cattle can be neglected. Thermal conductivities: -

Copper = $3.8 \times 10^2\ Wm^{-1}k^{-1}$
Scale = 1.34 $Wm^{-1}k^{-1}$
Specific latent heat of steam = $226 \times 10^3\ JKg^{-1}$

Problem 10
(a) Define thermal conductivity of a material
(b) Heat is supplied at the rate of 80W to one end of a well – lagged copper bar of uniform cross – sectional area $10cm^2$ having a total length of 20cm. The heat is removed by water cooling at the other end of the bar. Temperature recorded by two thermometers T_1 and T_2 at distance 5cm and 15cm from the hot end are 48°C and 28°C respectively.

(i) Calculate the thermal conductivity of copper

(ii) Estimate the rate of flow (in g/min) of cooling water sufficient for the water temperature to rise by 5

(iii) What is the temperature of the cold end of the bar.

Problem.11

(a) (i)The thermal conductivity β of a substance may be defined by the end equation $\frac{dq}{dt} = -\beta A \frac{d\theta}{dx}$

(ii) Identify briefly each term in this equation and explain the minus sign.

(iii) Describe briefly one method of measuring thermal conductivity of a bad conductor in the form of disc.

(b) One end of a well lagged copper rod is placed in a steam chest and a 0.6kg mass of copper is attached to the other end of the rod with an area of 2cm^2. When steam at $100℃$ is passed into the chest and a steady-state is reached the temperature of the mass of copper rises by $4℃$ per minutes; if the temperature of the surrounding is $15℃$. Calculate the length of the rod. Given that: Specific heat capacity of copper = $400\text{Jkg}^{-1}\text{K}^{-1}$. Thermal conductivity of copper = $360\text{Wm}^{-1}\text{K}^{-1}$

2.1.6 Growth Of Ice In Pond

Freezing of ponds from a surface downwards depends on two facts

(i) The water is densest at about 4℃.
(ii) Heat is abstracted from the top only.

If the air above a pond is at, say $-10℃$, convection will be maintained normally until the whole of the water in the pond is at 4℃. Next, below this temperature the coolest liquid stays at the top and a thin layer of ice forms. The eventual thickness of the coating of the ice is determined by conduction and the length of time for which this operates

ASSUMPTIONS

a) Heat is transferred by conduction through the layer of ice
b) The temperature above the surface of ice is constant at $-\theta$, the temperature of air above ice .
c) The temperature below the surface of the ice is constant at $0℃$
d) There is no absorption of heat from the bulky of the surface.

Let θ be the temperature of ice contact with air and θ_1 be the temperature of ice in contact with water.

Then let x be the thickness of ice at time t and dx be rise in the thickness after small interval of time dt

Then the rate of heat of fusion of ice will be given

$$\frac{d\theta}{dt} = L_f\left(\frac{dm}{dt}\right) ------(i)$$

$$\text{where } L_f = \text{latent heat of fusion}$$

$$\frac{dm}{dt} = \text{rate of mass increase}$$

$$\text{but mass (m)} = \rho v$$

$$\text{Volume (v)} = Adx$$

$$m = \rho Adx ----(ii)$$

$$\text{substituting into equation (i)}$$

$$\frac{d\theta}{dt} = \frac{L_f \rho Adx}{dt}$$

$$\frac{d\theta}{dt} = L_f \rho A\left(\frac{dx}{dt}\right) ----- (iii)$$

$$\text{The rate of heat conduction}$$

$$\frac{d\theta}{dt} = \frac{kA(\theta_1 - \theta_2)}{dx}$$

$$\text{since the system is in series}$$

$$\text{heat of conduction} = \text{heat of fusion}$$

$$\frac{kA(\theta_1 - \theta_2)}{dx} = L_f \rho A\left(\frac{dx}{dt}\right)$$

$$\frac{dx}{dt} = \frac{kA(\theta_1 - \theta_2)}{L_f \rho dx}$$

2.1.7 Application Of Thermal Conductivity In Daily Life

- Cooking utensils are made up of metal with handles of high conductivity source metal have k
- Thick walls are used in construction of houses.
- To prevent ice melting if is placed in gunny bags.
- In cold countries windows have two pores of glass with layer in between.
- In thermos flask stopper is made up of woody or plastic materials to prevent heat loss by conduction
- When water is poured into the beaker of thick walls glass cracks. The inner surface expand on heating but glass is bad conductor of heat so heat does not reach outside.

Example 14:

Calculate the quantity of heat conducted through $2m^2$ of brick wall 12cm thickness an 1hour if the temperature of one side is 8°C and on other side is 28°C given that ($k = 0.13wm^{-1}k^{-1}$)

solution

$$A = 2m^2,\ \theta_1 = 8°C$$

$$\theta_2 = 28°C,\ L = 12cm \text{ or } 0.12m$$

$$k = 0.13wm^{-1}k^{-1}$$

$$\text{from} \frac{d\theta}{dt} = \frac{kA(\theta_1 - \theta_2)}{L}$$

$$\frac{H}{t} = \frac{kA(\theta_1 - \theta_2)}{L}$$

$$H = \frac{kA(\theta_1 - \theta_2)}{L} \times t$$

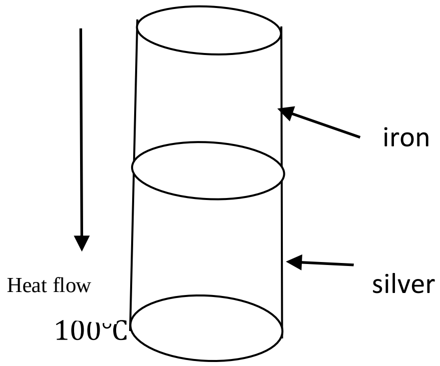
$$H = 0.13 \times \frac{2(28-8)}{0.12} \times 3600$$

$$H = 156000J$$

$$\text{Heat requred} = 156KJ$$

Example 15:

Two cylinder of equal physical condition are placed on top of each other as shown below



Given that conductivity of silver is eleven times that of iron . Find the temperature of surface AB and state the assumption made from your solution.

solution

let conductivity of silver be x

and that of iron be y

$$x = 11y$$

$$\theta_1 \ 100°C$$

$$\theta_2 = 0°C$$

$$\text{length of iron} = \text{length of silver since the bars are in series}$$

$$\text{hence in series connection: rate of heat flow in iron} = \text{rate of heat flow in silver}$$

$$\text{from} \frac{d\theta}{dt} = \frac{kA(\theta_1 - \theta_2)}{L}$$

$$\text{for iron}$$

$$\frac{yA(100 - \theta)}{L} = \frac{xA(\theta - 0°C)}{L}$$

$$\text{since} \quad x = 11y$$

$$\frac{yA(100 - \theta)}{L} = \frac{11yA(\theta - 0°C)}{L}$$

$$100 - \theta = 11\theta$$

$$100 = 11\theta + \theta$$

$$12\theta = 100$$

$$\theta = \frac{100}{12}$$

$$\theta = 8.3°C$$

$$\text{assumption the cylinder was lagged}$$

Example 16:

A temperature gradient in a 0.5m long rod is $80\ °C/m$. The temperature of hot end is 30°C. Find the temperature of the cold end

$$\text{solution}$$

$$L = 0.5cm, \qquad \text{Temp gradient} = 80\ °C/m$$

$$\theta_1 = 30°C, \quad \theta_2 = ?$$

$$\text{from temp gradiend} = \frac{\theta_1 - \theta_2}{L}$$

$$80 = \frac{30 - \theta_2}{0.5}$$

$$80(0.5) = 30 - \theta_2$$

$$40 - 30 = -\theta_2$$

$$\theta_2 = -10°C$$

Example 17:

The opposite faces of cubical block of iron of cross section area $4cm^2$ and of thickness of 0.5m are kept in contact with steam and melting ice. Calculate the quantity of ice melted in 10minutes. Given of thermal conductivity of iron is $83wm^{-1}C^{-1}$ and latent heat of fusion of ice is $336 \times 10^3 Jkg^{-1}$

solution

Rate of conduction

$$\frac{d\theta}{dt} = \frac{kA(\theta_1 - \theta_2)}{L} ------(i)$$

rate of fusion

$$\frac{d\theta}{dt} = \frac{mL_f}{t} -------(ii)$$

rate of conduction = rate of fusion

$$\frac{kA(\theta_1 - \theta_2)}{L} = \frac{mL_f}{t}$$

required mass of ice

$$m = \frac{kAt(\theta_1 - \theta_1)}{L_f L}$$

$$m = \frac{83 \times 4 \times 10^{-4} \times 600(100 - 0)}{336 \times 10^3 \times 0.5}$$

mass of ice melted = 0.01187kg

Example 18:

Two solid copper spheres of diameter 10cm and 5cm are at temperatures which are respectively 10°C and 5°Caove that of surrounding. Assuming Newton's law and condition to apply, compare the rate of fall of temperatures of two spheres. Indicate assumption made in your calculation.

solution

the rate of loss of heat by spheres is given by

$$mc\frac{d\theta}{dt} = -kA(\theta - \theta_s)$$

but $m = \rho v$

$$m = \frac{4\pi r^3 \rho}{3} \text{ and area } A = 4\pi r^2$$

$$\frac{d\theta}{dt} = -\frac{3k(\theta - \theta_s)}{r\rho c}$$

for the first sphere

$$\left(\frac{d\theta}{dt}\right)_t = -3k\frac{\theta_1 - \theta_s}{r_1\rho c} - - - - - - - (i)$$

for the second sphere

$$\left(\frac{d\theta}{dt}\right)_2 = \frac{-3k\,(\theta_2 - \theta_s)}{r_2\rho c} - - - - - - - (ii)$$

dividing eqni by ii

$$\frac{(\frac{d\theta}{dt})_1}{\left(\frac{d\theta}{dt}\right)_2} = \left(\frac{r_2}{r_1}\right)\left(\frac{\theta_1 - \theta_s}{\theta_2 - \theta_s}\right)$$

$$= \left(\frac{5}{10}\right)\left(\frac{10}{5}\right)$$

$$= 1$$

$$\frac{\left(\frac{d\theta}{dt}\right)_1}{\left(\frac{d\theta}{dt}\right)2} = 1$$

Example 19:

A brass boiler has a base area of 0.15m^2 and the thickness of 1cm. It boils water at the rate of 6kg/m when placed on the gas stone. What is the temperature of a flame contact with boiler Given conductivity of brass $109\text{wm}^{-1}\text{k}^{-1}$ latent heat of vaporization of water $2.25 \times 10^6\text{J/kg}$

solution

$$A = 0.15\text{m}^2, \quad l = 1\text{m}$$

$$\text{k of brass} = 109\text{wm}^{-1}\text{k}^{-1}$$

$$\text{latent of vaporization} = 2.25 \times 10^6\ \text{J/kg}$$

$$\theta_2 = 100°\text{C}, \quad \theta_1 = ?$$

$$\text{from}\frac{d\theta}{dt} = \frac{kA(\theta_1 - \theta_2)}{L}\ \text{for conduction}$$

for vaporizatio

$$\frac{\theta}{t} = \frac{mL_V}{t}$$

$$\frac{kA(\theta_1 - \theta_2)}{L} = \frac{L_v dm}{dt}$$

$$109 \times \frac{0.1(\theta_1 - 100)}{0.01} = 2.25 \times 10^6 \times 6$$

$$\theta_1 - 100 = 137.6$$

$$\theta_1 = 137.6 + 100$$

$$\text{temperature will be} = 237.6℃$$

Example 20:

The closed metal vessel contain water at $30℃$. The vessel has surface area of $0.5m^2$ and uniform thickness of 40mm if the outside temperature is $15℃$. Calculate the heat lost per minute by conduction give thermal conductivity of metal is $400wm^{-1}k^{-1}$

solution

$$L = 0.04m, \qquad k = 400wm^{-1}k^{-1}$$

$$\theta_1 = 30℃, \qquad \theta_2 = 15℃$$

$$\frac{d\theta}{dt} = \frac{kA(\theta_1 - \theta_2)}{L}$$

$$\frac{d\theta}{dt} = \frac{400 \times 0.5(30 - 15)}{0.04}$$

$$\frac{d\theta}{dt} = 200\left(\frac{15}{0.04}\right)$$

$$\frac{d\theta}{dt} = 75000 \text{ J/min}$$

Example 21. The temperature of the two outer surface of a composite slab shown below, consisting of two materials having coefficients of thermal conductivity K and 2K and thickness x and 4x respectively are T_1 and T_2 $(T_2 > T_1)$. Find the expression for the rate of transfer of heat through the slab in the steady state.

Solution

Let T be the temperature of the interface of the two slabs. Let H_1 and H_2 be the rate of flow of heat through the two slabs

Case 1: for slab of thermal conductivity K and thickness x

$$\frac{dH}{dt} = \frac{K\,A\,(T_2 - T)}{x} \ldots\ldots\ldots\ldots\ldots..(i)$$

Case 2: For slab of thermal conductivity 2K and thickness 4x

$$\frac{dH}{dt} = \frac{2K\,A\,(T - T_1)}{4x} \dots\dots\dots\dots\dots (ii)$$

Then, in steady state condition, the rate of heat flow id the same for both slabs

$$\frac{K\,A\,(T_2 - T)}{x} = \frac{2K\,A\,(T - T_1)}{4x}$$

Solving for the temperature T

$$[KA(T_2 - T)] \times 4x = x \times [2KA(T - T_1)]$$

$$4xKAT_2 - 4xKAT = 2xKAT - 2xKAT_1$$

$$\text{collecting the like terms, results into}$$

$$4xKAT_2 + 2xKAT_1 = 2xKAT + 4xKAT$$

$$T(6xKA) = 2xKA(2T_2 + T_1)$$

$$T = \frac{2xKA\,(2T_2 + T_1)}{6xKA}$$

$$T = \frac{T_1 + 2T_2}{3}$$

$$\text{substituting for T in the equation (i), we get}$$

$$H_1 = \frac{K\,A}{x}\left[T_2 - \left(\frac{T_1 + 2T_2}{3}\right)\right]$$

$$H_1 = \frac{K\,A\,(T_2 - T_1)}{3x}$$

Example 22; A layer of ice 20 cm thick has formed on a pond. The temperature of air is $-10°C$. Find how long will it take for another 0.1 cm layer of water to freeze? Given that thermal conductivity of ice is $2.1Wm^{-1}K^{-1}$. Latent heat of ice is $3.36 \times 10^5 Jkg^{-1}$ and density of ice is $1{,}000\ kgm^{-3}$. **Ans. t =3208second**

Example 23; An electric heater is used in a room of total wall area $137m^2$ to maintain temperature of $20°C$ inside it. When the outside temperature is $-10°C$. The walls has three layers of different materials. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 25 cm. find the power of electric heater. Assume that there is no heat loss through the floor and ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 W/mK.

Solution.

NB: *(i)* *When conduction of heat takes place across a composite slab made of different materials, temperature at the interface keeps on changing.*

(ii) *When steady state is reached, the rate of conduction of heat across any interface of the composite slab is the same.*

Suppose that in the steady state, the temperature at the interface of wooden and cement layers become θ_2 and that at the interface of cement and brick layers become θ_3.

The rate of heat flow cross the wooden layer when $k_1 = 0.125$ and $d_1 = 2.5\text{cm}$

$$\frac{dQ_1}{dt} = \frac{K_1 A(\theta_1 - \theta_2)}{d_1}$$

$$\frac{dQ_1}{dt} = \frac{0.125A\,(20 - \theta_2)}{2.5 \times 10^{-2}}$$

$$\frac{dQ_1}{dt} = 5A(20 - \theta_2) \qquad \ldots\ldots(i)$$

Similarly, the rate of heat flow across the cement layer, when $K_2 = 1.5$ and $d_2 = 1.0$ cm

$$\frac{dQ_2}{dt} = \frac{K_2 A(\theta_2 - \theta_3)}{d_2}$$

$$\frac{dQ_2}{dt} = \frac{1.5 \times A(\theta_2 - \theta_3)}{1 \times 10^{-2}}$$

$$\frac{dQ_2}{dt} = 150A(\theta_2 - \theta_3) \qquad \ldots\ldots(ii)$$

Also, the rate of heat flow across the brick layer is, when $k_3 = 1.0$ and $d_3 = 25.0$ cm

$$\frac{dQ_3}{dt} = \frac{K_3 A(\theta_3 - \theta_4)}{d_3}$$

$$\frac{dQ_3}{dt} = \frac{1.0 \times A\big(\theta_3 - (-10)\big)}{25 \times 10^{-2}}$$

$$\frac{dQ_3}{dt} = 4\,A(\theta_3 + 10) \qquad \ldots\ldots(iii)$$

In a steady state condition; the rate of heat flow is the same.

$$\frac{dQ_1}{dt} = \frac{dQ_2}{dt} = \frac{dQ_3}{dt}$$

$$5A(20 - \theta_2) = 150A(\theta_2 - \theta_3)$$

$$(20 - \theta_2) = 30\,(\theta_2 - \theta_3)$$

$$30\theta_2 + \theta_2 = 20 + \theta_3$$

$$31\theta_2 = 20 + \theta_3$$

$$\theta_2 = \frac{20 + 30\theta_3}{31}$$

Equating equation (i) and equation (ii)

$$5A(20 - \theta_2) = 4A(\theta_3 + 10)$$

$$5(20 - \theta_2) = 4(\theta_3 + 10)$$

$$5\left(20 - \frac{20 + 30\theta_3}{31}\right) = 4(\theta_3 + 10)$$

$$100 - \frac{100 + 150\theta_3}{31} = 4\theta_3 + 40$$

$$3100 - 100 - 150\theta_3 = 124\theta_3 + 1240$$

$$3100 - 1240 = 124\theta_3 + 150\theta_3$$

$$1860 = 274\theta_3$$

$$\theta_3 = \frac{1860}{274}$$

$$also\ \theta_2 = \frac{20 + 30\theta_3}{31} = \frac{20 + 30 \times 6.8}{31} = 7.2℃$$

$$\theta_2 = 7.2℃$$

$$Power,\quad P = \frac{dQ_1}{dt}$$

$$but\ \frac{dQ_1}{dt} = 5A(20 - \theta_2)$$

$$P = 5 \times 137(20 - 7.2)$$

$$P = 8768 \approx 9000\text{W}$$

$$\text{Power} = 9\text{KW}$$

2.2. Thermal Convection

Thermal convection is the process of heat transfer from one point to another due to actual movement of heated body molecules. Thermal convection it mostly occurred in fluid liquids and gases.

Rate of heat loss is the quantity of heat lost per unit time $\left(\frac{d\theta}{dt}\right)$

The SI- unit is watt or (J/s)

Excess temperature is the difference between body temperature and surrounding temperature.

$$\text{Excess temp} = \theta - \theta_s \text{ iff } \theta > \theta_s$$

$$\text{where } \theta \text{ is body temperature}$$

$$\theta_s \text{ surrounding temperature.}$$

Thermal convections classified into two types.

i. Natural or free convection
ii. Forced or artificial convection

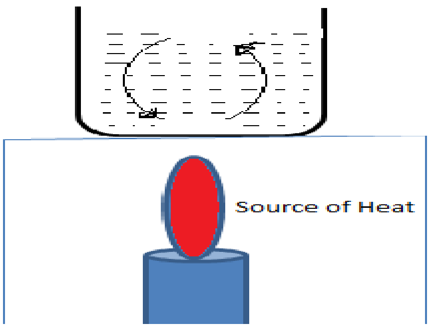
2.2.1. Natural Convection

Is the type of convection in which heated body molecules flow due to difference in density or pressure example boiling point of liquid. It remove heat naturally. OR

This is a type of convection in which a heated fluid flows from the hot region to the cold region due to differences in density.

Example

When a fluid is heated from below, the lower part of the fluid become hot and therefore expands.
Its density decreases due to the increase in volume of fluid molecules. Its position is displaced by cold fluid from the top . This in turn gets heated and rises to the top and this process continues as shown the figure below

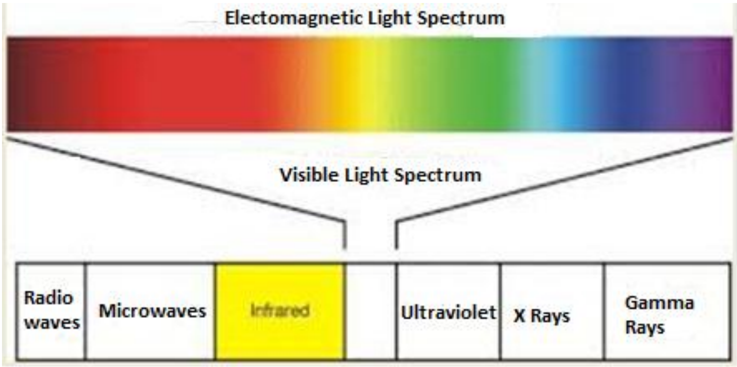


2.2.2. Forced Convection

Is the type of convection in which heated body molecules are forced to move by pumps or blowers. It influenced by external factor such as wind blowers in cooling forced.

Forced convection is used in electrical motors for efficiency cooling.

This is the transfer of heat energy from one point to another without the requirement of any material medium. It is like a throw of radiant energy. Thermal radiation consist of electromagnetic waves with a range of wavelengths covering the infra-red and visible regions of the electromagnetic spectrum



All bodies continuously emit and absorb thermal radiation in the form of electromagnetic waves. A body at higher temperature than the surrounding units it emits more radiation than it absorbs.

Thus, there is a continuous exchange of radiation between the body and the surrounding with the result that there will be a rise or fall in temperature of the body.

2.2.3 Newton's Law Of Cooling

This law applies when a body is cooling under conditions of natural convection and the temperature difference between the body and surroundings is small (<30℃)

The law of cooling state that

"Under the condition of natural convection, the rate of heat loss is directly proportional to excess temperature over surrounding"

$$\frac{d\theta}{dt} \propto (\theta - \theta_s)$$

$$\frac{d\theta}{dt} = -k(\theta - \theta_s)$$

the negative sign show that heat lost to the surrounding.

Proof of Newton's law of cooling from Stefan's law

Consider a body maintained at temperature **T.** let T_o be the temperature of the surroundings. According to Stefan's law

$$\text{Heat lost per second per area}, E = \varepsilon\sigma(T^4 - T_o^4)$$

$$\text{where } \sigma = \text{Stefan's constant.}$$

$$\varepsilon = \text{Emmisivity of surface of the body.}$$

$$\text{let } \Delta T \text{ be the temperature difference}$$

$$\text{therefore } T = T_o + \Delta T$$

$$E = \varepsilon\sigma[(T_o + \Delta T)^4 - T_o^4)]$$

$$\text{dividing by } T_o$$

$$E = \varepsilon\sigma\, T_o^4 \left[\left(\frac{T_o}{T_o} + \frac{\Delta T}{T_o}\right)^4 - \frac{T_o^4}{T_o^4}\right]$$

$$E = \varepsilon\sigma\, T_o^4 \left[\left(1 + \frac{\Delta T}{T_o}\right)^4 - 1\right]$$

$$\text{Using binomial theorem to expan, neglect the high power}$$

$$E = \varepsilon\sigma T_o^4 \left[\left(1 + \frac{4\Delta T}{T_o} + \text{highest term}\right) - 1\right]$$

$$E = \varepsilon\sigma T_o^4 \left(\frac{4\Delta T}{T_o}\right)$$

$$E = 4\varepsilon\sigma T_o^3 \Delta T$$

$$\text{where } \Delta T = T - T_o$$

$$E = 4\varepsilon\sigma T_o^3\,(T - T_o)$$

$$\text{hence the term } (4\varepsilon\sigma T_o^3) \text{is constant}$$

$$E = k(T - T_o)$$

$$E \propto (T - T_o) \text{ hence } E = \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} \propto (T - T_o)$$

Therefore, when a temperature difference between a body and a surrounding is small, the rate of cooling of a body is directly proportional to the temperature difference between the body and the surroundings.

2.2.3.1 Five- Fourth Power Law Of Cooling

This was a modification of Newton's law of cooling

$$\frac{d\theta}{dt} = -k(\theta - \theta_s)^{\frac{5}{4}}$$

This was done by **PETIT** and **DULONG**. Hence Newton's law of cooling modified to

"The rate of heat loss is directly proportional to the excess temperature power five over four"

$$\frac{d\theta}{dt} \propto (\theta - \theta_s)^{\frac{5}{4}}$$

2.2.3.2 Limitation Of Newton's Law Of Cooling

- The law it carry under excess temperature between 50k and 300k
- The law is not used for forced convection.

2.2.3.3.Factors Affecting Rate Of Cooling

a) Excess temperature

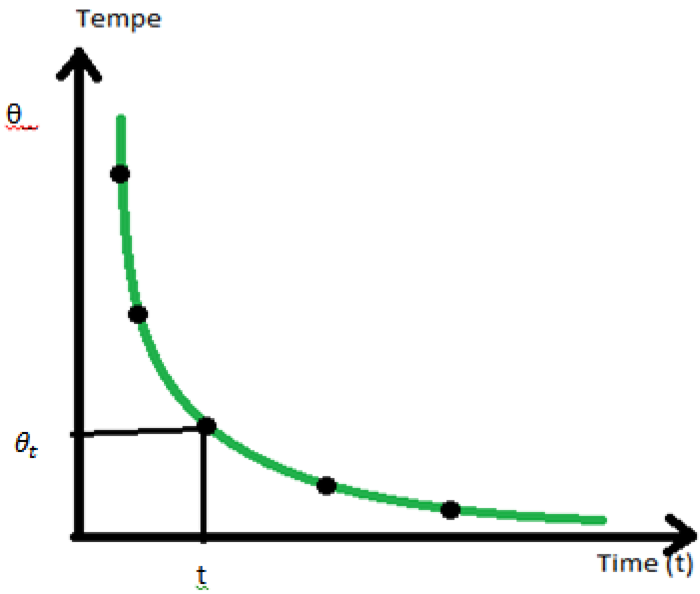
i.e $\frac{d\theta}{dt} \propto (\theta - \theta_s)$

b) Surface area $\frac{d\theta}{dt} \propto A$

c) Volume of a liquid $\frac{d\theta}{dt} \propto \frac{1}{V}$

2.2.4 Equation Of Cooling.

Consider the body initially having the temperature θ_o, then if the body cools after a given time $'t'$ the temperature of the body changes and become θ_t. See the cooling curve below



from Newton's law of cooling

$$\frac{d\theta}{dt} \propto (\theta - \theta_s)$$

$$\text{But} \quad \frac{\theta}{t} = \frac{mC\theta}{t}$$

$$\frac{d\theta}{dt} = \frac{mCd\theta}{dt}$$

$$\text{if}\frac{d\theta}{dt} = -k(\theta - \theta_s)$$

$$\frac{d\theta}{\theta - \theta_s} = -kdt$$

Integrating both sides of the equation

$$\int_{\theta}^{\theta t} \frac{d\theta}{\theta - \theta_s} = \int_{0}^{t} -kdt$$

$$[\text{In}(\theta - \theta_s)]_{\theta}^{\theta t} = -k\int_{0}^{t} dt$$

$$\text{In}(\theta t - \theta_s) - \text{In}(\theta_o - \theta_s) = -kt$$

$$\text{In}\left(\frac{\theta t - \theta_s}{\theta_o - \theta_t}\right) = -kt$$

$$\log_e\left(\frac{\theta_t - \theta_s}{\theta_o - \theta_s}\right) = -kt$$

$$\frac{\theta_t - \theta_s}{\theta_o - \theta_s} = e^{-kt}$$

$$\theta_t - \theta_s = (\theta_o - \theta_s)e^{-kt}$$

$$\boxed{\theta_t = \theta_s + (\theta_o - \theta_s)e^{-kt}}$$

this is the equation of cooling

2.2.5 Worked Examples Set 02:

Example 21:

A body cools in 7minutes for 60°C to 40°C. What will be its temperature after next 7minutes? The room temperature is 10°C. Assume Newton's law of cooling holds good throughout the process

$$\text{Initial temperature } (\theta_o) = 60°C$$

$$\text{Temperature at time } t(\theta)_t = 40°C$$

$$\text{temperature of surrounding } (\theta_s) = 10°C$$

$$\text{time for cooling } t = 7\text{minutes}$$

from the equation of cooling

$$\theta_t = \theta_s + (\theta_o - \theta_s)e^{-kt}$$

$$40 = 10 + (60 - 10)e^{-7k}$$

$$30 = 50e^{-7k}$$

$$\frac{30}{50} = e^{-7k}$$

Apply In both sides

$$\text{In}\left(\frac{3}{5}\right) = -7k\text{Ine}$$

$$k = \frac{\text{In}\left(\frac{3}{5}\right)}{-7}$$

$$k = 0.073$$

since required to calculate temperature after

7minutes next means (θ_t)

$$\theta_t = 10 + 50e^{-0.073\times14}$$

$$\theta_t = 10 + 17.97$$

$$\theta_t = 27.97℃$$

Example 22.

The temperature of the body falls from 30℃ to 20℃ in 5minutes. The air temperature is 13℃. Find the temperature after further 5minutes

solution

$$\theta_o = 30℃, \quad \theta_t = 20℃, \quad \theta_s = 13℃$$

from Newton′slaw of cooling

$$\theta_t = \theta_s + (\theta_o - \theta_s)e^{-kt}$$

$$e^{-kt} = \frac{\theta_t - \theta_s}{\theta_o - \theta_s}$$

for the first 5minutes

$$-5k = \text{In}\left(\frac{\theta_t - \theta_s}{\theta_o - \theta_s}\right)$$

$$-5k = \text{In}\left(\frac{20 - 13}{30 - 13}\right)$$

$$-5k = \text{In}\left(\frac{7}{17}\right)$$

$$k = 0.1775$$

but required to find temperature after

5minutes (θ_t)

$$\theta_t = 13 + (20 - 13)e^{-(5\times 0.1775)}$$

$$\theta_t = 13 + 7 \times 0.4118$$

$$\theta_t = 15.88°C$$

Example 23.

A liquid in calorimeter cools from 80°C to 70°C in 5minutes. How long it will take the liquid to cool from 60°C to 55°C if the surrounding temperature is 30°C?. Assume Newton's law of cooling hold true

consider when

$$\theta_o = 80°C, \theta_t = 70°C, \qquad \theta_s = 30°C$$

$$\text{the } \theta_t = \theta_s + (\theta_o - \theta_s)e^{-kt}$$

$$\left(\frac{\theta_t - \theta_s}{\theta_o - \theta_s}\right) = e^{-kt}$$

$$-5k = \text{In}\left(\frac{70 - 30}{80 - 30}\right)$$

$$-5k = \text{In}\left(\frac{4}{5}\right)$$

$$k = 0.0446$$

$$\text{required t when } \theta_o = 60°C, \qquad \theta_t = 55°C, \qquad \theta_s = 30°C$$

$$-0.0446t = \text{In}\left(\frac{55 - 30}{60 - 30}\right)$$

$$-0.0446t = \text{In}\left(\frac{25}{30}\right)$$

$$-0.0446t = -0.1823$$

$$t = \frac{0.1823}{0.0446}$$

$$t = 4.085\text{minutes.}$$

Example 24:

The body is initially at 75°C cools to 65°C in 5minutes. The body has cooled to 57°C after next 5minutes. Determine the surrounding temperature of a body and temperature of a body after 12minutes.

$$\text{solution}$$

$$\text{from Newton's law of cooling}$$

$$-kt = \text{In}\left(\frac{\theta_t - \theta_s}{\theta_o - \theta_s}\right)$$

$$\text{case 1: when } \theta_o = 75°C, \quad \theta_t = 60°C, \theta_s = ?$$

$$\text{for first 5minutes}$$

$$-5k = \text{In}\left(\frac{65 - \theta_s}{75 - \theta_s}\right) ----- (i)$$

$$\text{also case 2: when}$$

$$\theta_o = 65°C, \quad \theta_t = 57°C \quad \theta_s = ?$$

$$\text{for other 5minutes}$$

$$-5k = \text{In}\left(\frac{57 - \theta_s}{65 - \theta_s}\right) ------(ii)$$

$$\text{equating the two eqns}$$

$$\text{In}\left(\frac{65 - \theta_s}{75 - \theta_s}\right) = \text{In}\left(\frac{57 - \theta_s}{65 - \theta_s}\right)$$

$$\frac{65 - \theta_s}{75 - \theta_s} = \frac{57 - \theta_s}{65 - \theta_s}$$

$$(65 - \theta_s)^2 = (75 - \theta_s)(57 - \theta_s)$$

$$4225 - 130\theta_s + \theta_s^2 = 4275 - 132\theta_s + \theta_s^2$$

$$4225 - 130\theta_s = 4275 - 132\theta_s$$

$$50 = 2\theta_s$$

$$\theta_s = 25°C$$

$$\text{calculating the value of k from eqn (i)}$$

$$-5k = \text{In}\left(\frac{65 - 25}{75 - 25}\right)$$

$$-5k = \text{In}\left(\frac{4}{5}\right)$$

$$\text{now at } t = 12$$

$$-12k = In\left(\frac{\theta_t - 25}{75 - 25}\right)$$

$$-12\left(-\frac{1}{5}In\left(\frac{4}{5}\right)\right) = In\left(\frac{\theta_t - 25}{50}\right)$$

$$-0.5355 = In\left(\frac{\theta_t - 25}{50}\right)$$

$$e^{-0.5355} = \frac{\theta_t - 25}{50}$$

$$0.5854 = \frac{\theta_t - 25}{50}$$

$$29.27 = \theta_t - 25$$

$$\theta_t = 29.27 + 25$$

$$\theta_t = 54.27℃$$

Example 25: A body at 80℃ cools to 64℃ in 5 minutes and to 52℃ in the next 5 minutes. What will be its temperature after 5 minutes? Calculate temperature of the surroundings.

solution

from Newton′slaw of cooling

for first 5 minutes

$$In\left(\frac{\theta_t - \theta_s}{\theta_o - \theta_s}\right) = -kt$$

$$In\left(\frac{64 - \theta_s}{80 - \theta_s}\right) = -5k - - - - - (i)$$

consider second 5 minute

$$In\left(\frac{52 - \theta_s}{64 - \theta_s}\right) = -5k - - - - - -(ii)$$

equating the two equations

$$In\left(\frac{52 - \theta_s}{64 - \theta_s}\right) = In\left(\frac{64 - \theta_s}{80 - \theta_s}\right)$$

$$\left(\frac{52 - \theta_s}{64 - \theta_s}\right) = \left(\frac{64 - \theta_s}{80 - \theta_s}\right)$$

$$(64 - \theta_s)^2 = (52 - \theta_s)(80 - \theta_s)$$

$$4096 - 128\theta_s + \theta_s^2 = 4160 - 132\theta_s + \theta_s^2$$

$$4096 - 128\theta_s = 4160 - 132\theta_s$$

solving for θ

$$4\theta_s = 64$$

$$\theta_s = \frac{64}{4} = 16℃$$

surrounding temperature is 16℃

also require temp after next 5 min

$$\theta_t = \theta_s + (\theta_o - \theta_s)e^{-kt}$$

if $\theta_s = 16℃$ then solve for k

$$\text{In}\left(\frac{52-16}{64-16}\right) = -5k$$

$$-5k = \text{In}\left(\frac{36}{48}\right)$$

$$k = -\frac{1}{5}\text{In}\left(\frac{3}{4}\right)$$

$$k = 0.05754$$

$$\theta_t = 16 + (52-16)e^{-0.05754\times5}$$

$$\theta_t = 16 + 27$$

$$\theta_t = 43℃$$

hence temp after next 5 min is 43℃

2.2.6 Application Of Convection

- ✓ **Heating of rooms :**A room heater warms air closer to it which then rises and cool air (denser) moves down to take their place. This cool air is in turns heated abd rise upward. In this way convectional currents are set up in the room which transfers heat to different parts of the room.
- ✓ **Cooling of transformer :**A transformer is always kept in a tank containing oil so as to remove the heat generated on it due to the flow of current. The heat is removed by convection.
- ✓ **Water is poor** conductor of heat but water in a beaker can be warmed quickly due to convection currents.
- ✓ **Refrigerator use convectional currents** of air to cool food and beverage. Cooling unit cools the air around it and cause it to sink down thus displacing the bottom warm air upward, which is then cooled and establish convectional currents.

- ✓ **Land and sea breeze ,** are examples of convectional current in nature

2.2.7 Competitive Examination File Unit Set 03:

Problem 01

A body cools from 40°C to 30°C in 5 minutes.The temperature of the room being 15°C, what will be the temperature of the body after another 5 minutes?

Problem 02

In a room at 15°C a body cools from 35°C to 30°C in 4 minutes. Find the further time elapse before the temperature of the body is 20°C

Problem 03

Wind blows over a hot liquid placed in a beaker in the laboratory whose average room temperature is 27°C. The liquid rate of cooling is 15°C/ min when it is at a temperature of 87°C. Calculate the liquid rate of cooling when it is at a temperature of 57°C.

Problem 04

A body initially at 80°C cools to 64°C in 5 minutes and 52°C in 10 minutes. What will be the temperature after 15 minutes and what is the temperature of the surrounding?

Problem 05

(a) State Newton's law of cooling and give one limitation of the law.

(b) A body initially at 70°C cool to a temperature of 55°C in 5minutes. What will be its temperature after 10minutes given that the surrounding temperature is 31°C. (Assume Newton's law of cooling holds true)

2.3. Thermal Radiation

Thermal radiations the way of heat transfer by invisible electromagnetic radiation.

Heat produced by this process is known as **radiant energy**. An object above 0k emits thermal radiation continuously.

At low temperature body emits infrared radiation and at high temperature bodies emit ultraviolet radiation (UV) together with infrared Properties of radiant energy due to thermal radiation. Hence due to thermal radiation a body can

- Can travel via vacuum
- Travel in a straight line
- Obey law of reflection
- Can be refracted
- Travel with a speed of light

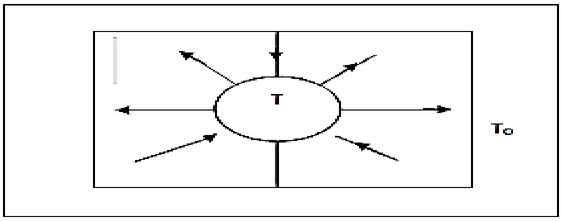
- Exhibit phenomena of diffraction in interference.

When thermal radiation is incident on a body some of the radiation can be reflected, Transmitted and some can be absorbed and cause heating.

2.3.1. Prevost's Theory Of Heat Exchange

When the temperature of a body is constant the body is losing heat by radiation and gaining it by absorption at equal rates

Consider a black body at a temperature T to be placed in an enclosure having temperature T_0



Energy radiated /sec = $A\delta T^4$

Energy absorbed /sec = $A\sigma\ T_0^4$

If the body is not at the same temperature as its surrounding temperature, the net flow of energy between the surrounding and the body is not constant because of unequal emission and absorption.

If the temperature of the body is greater than that of the surrounding, then the net energy will flow from the body to the surrounding.

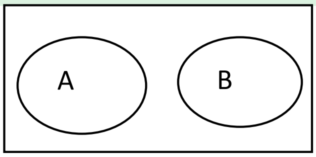
Net energy emitted /sec = $A^{\sigma}T^4 - A^{\sigma}T_0^4$

Therefore

Net energy emitted /sec $=A^{\sigma}\ (T^4 - T_0^4)$

The theory state that **'When the temperature of the body is constant, at this condition will absorb and emit radiation at equal rate"**

Consider the body A and B which are at different temperature and suppose the temperature of A and temperature of B are above 0k. And let T_A be a temperature of body A and T_B be a



Temperature of body B considering the diagram below.

- If $T_A > T_B$ the body the body B will absorb more radiation than it emits.
- If $T_A = T_B$ then the two bodies will absorb and emit radiation at equal rate and they are said to be in thermo equilibrium.
- The body which absorbs radiation is called **Adiathermanous or absorber.**
- The body which emits radiation is called **Diathermanous or emitters**

2.3.2. The Black Body

BLACK BODY Is the body which absorb radiation from the surroundings

A perfectly black body is the one which absorbs completely all the radiation falling on it and reflects none

Since a perfectly black body is a perfect absorber, it will also be a perfect radiator.

When a perfectly black body is heated to a high temperature, it emits thermal radiation of all possible wavelengths.
Practical examples of perfectly black body,

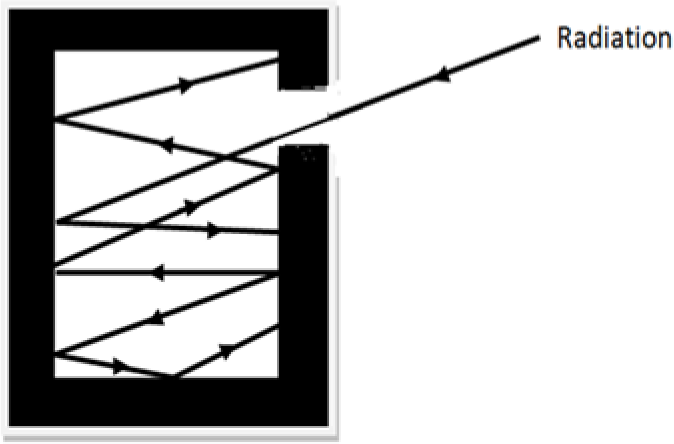
(1) The sun

(2) A surface coated with lamp- black

This surface can absorbs 96% to 98% of the incident radiation and may be considered as a perfect black body for all practical purposes

2.3.3 How To Realize A Black Body

A good black body can be realized simply by punching a small hole in the lid of a closed empty tin.



The hole looks almost black, although the shining tin is a good reflector.

Reason

The hole looks almost black ,although the shining tin is a good reflector because the radiation that enters through it is reflected from the inside walls several times and is partially absorbed at each reflection and loses energy until no radiation is reflected back. Hence the hole absorbs all radiation falling on it.

2.3.4 Black Body Radiation (Bbr)

Is that thermal radiation emitted by a black body at a given temperature. Any object at a temperature greater than absolute zero emits thermal radiation of all wavelengths within a certain range.

The amount of thermal energy radiated for different wavelength intervals is different and depends on temperature and nature of the surface.

perfect black body Is the body which absorb all radiation falling on it and reflect none.

2.2.5 Relative Intensity

Is the power of radiation per unit area per unit wavelength of the radiation and it is denoted by letter ($E\lambda$)

$$E\lambda = \frac{P}{A\lambda}$$

This is the rate at which radiant energy is transferred per unit area.

$$I = \frac{\text{Energy emitted /Time}}{\text{Area}}$$

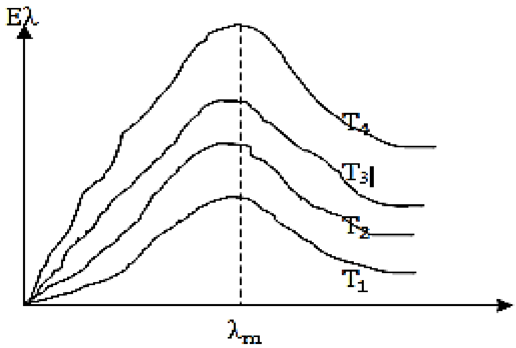
$$I = \frac{\text{power emitted}}{\text{Area}}$$

$$I = \frac{P}{A}$$

The SI unit used is Watt/metre^2 (Wm^{-2})

2.2.6 Spectra Curve

These are graph of curve which show the relationship between relative intensity and wavelength at a particular temperature.



$$T_4 > T_3 > T_2 > T_1$$

The curve provide the following features,

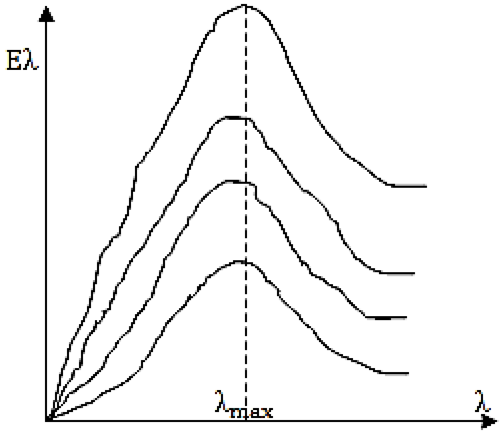
- ❖ The total energy emitted by black body increase rapidly with increase in temperature.
- ❖ The area enclosed by a particular curve represents total radiant energy per unit area emitted by black body.
- ❖ As a temperature increase a peak of curve shift towards lower wavelength
- ❖ At a given temperature with increase in wavelength the (Eλ) first increase and reaches at top then start decreasing

maximum wavelength Is the wavelength corresponding to maximum relative intensity.

2.3.7. Laws Of Black Body Radiation

2.3.7.1. Wien's Displacement Law

The law state that **" The absolute temperature is inversely proportional to the wavelength corresponding to maximum wavelength"**



$$T \propto \frac{1}{\lambda_{max}}$$

2.3.8 Energy Distribution In The Spectrum Of A Black Body

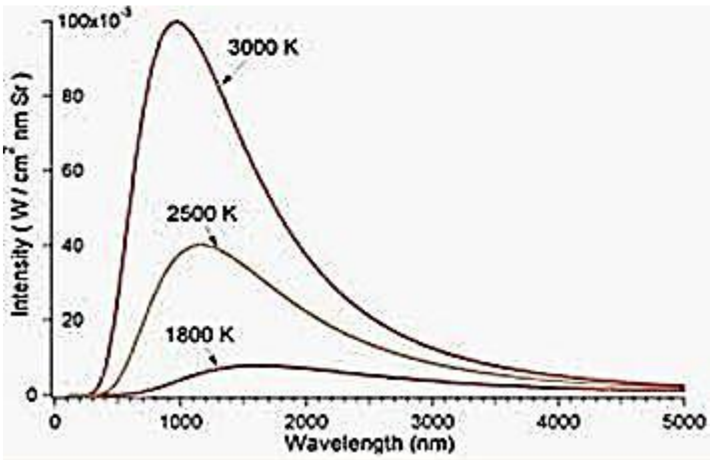
The energy radiated by a black body at constant temperature contains a continuous range of wave lights

The energy carried by the radiation is not distributed evenly across the wavelength range

However the distribution changes if the source temperature alters

The proportion of energy carried by shorter wave lengths increases as the source temperature increases

The figure below shows how the energy is distributed over the wave length range for several values of source temperature.



Deductions

The total energy emitted by a black body increases rapidly with the increase in temperature for any wavelength.

For a given temperature, the radiant energy emitted by a black body is the maximum for a particular wave length as the temperature of the body increases the peak of the curve shifts towards shorter wave length

This is in accordance to Wien's displacement law

i.e. $\lambda_{max} \propto \frac{1}{T}$

This explains why a heated metal i.e. iron changes colours from red through yellow to white. When a metal i.e. iron is heated it first emits invisible radiation of longer wave length in infrared region. With increasing temperature the wave length of the emitted radiation becomes shorter and the metal appears red

With further increasing temperature the wave length becomes shorter and shorter and the metal emits all the colors of the visible spectrum and finally it appears white.

The area enclosed by a particular curve represent the radiant energy [of all wave length] per second per unit area emitted by the black body at that temperature

When the area enclosed by a particular curve is measured, it is found to be directly proportional to the fourth power of the corresponding absolute temperature

$$E \propto T^4$$

This is in accordance to Stefan's law

2.3.9 Emissive Power(E_λ)

Means the emissive power of a body at a particular temperature is the total energy of all wave lengths radiated per second per unit area of the body.

Is the ratio of energy radiated by the body to the energy radiated by perfect black body.

The emissivity (ε) of a surface is the ratio of the power radiated by a surface of a given body to that radiated by a black body at the same temperature

$$\varepsilon = \frac{\text{energy radiated by black body}}{\text{Energy radiated by perfect black body}}$$

$$\text{Emmisivity } (\varepsilon) \text{does not exceed one } (1)$$

$$0 \leq \varepsilon < 1$$

2.3.10 Absorptive Power (a_λ)

The absorptive power of a body at a given temperature and for a particular wavelength is the ratio of thermal energy absorbed by it in a given time to the total thermal energy incident on it for the same time , both in the unit wave length around

$$a_\lambda = \frac{\text{thermal energy absorbed}}{\text{Total incident energy}}$$

2.3.11 Kirchooff's Law Of Black Body Radiation

The law state that

"The ratio of the emissive power to the absorptive power of radiation of a given wavelength is the same for all bodies at the same temperature and is equal to the emissive power of a perfectly black body at the temperature".

$$\frac{e_\lambda}{a_\lambda} = E_\lambda = constant$$

Where

e =Emissive power of a body corresponding to wave length

$a_\lambda =$ Absorptive power of a body corresponding to wave length

$E_\lambda =$ Emissive power of a perfectly black body at the same temperature corresponding to wave length

From Kirchhoff's law

$$\frac{e_\lambda}{a_\lambda} = E_\lambda$$

$$e_\lambda = E_\lambda a_\lambda$$

since E_λ is constant

$$e_\lambda \propto a_\lambda$$

Therefore

If a body emits strongly the radiation of a particular wavelength, then it must also absorb the same wave length strongly.

i.e. good emitters of heat also good absorbers and vice- versa

$$T = \frac{b}{\lambda_{max}}$$

$$b = T\lambda_{max}$$

Example 25:

An indirect heating filament is radiating maximum energy of wavelength 2.16×10^{-7}m. Find the absolute temperature of the body if $b = 2.88 \times 10^{-3}$km.

$$\text{solution}$$

$$\lambda = 2.16 \times 10^{-7}\text{m}$$

$$b = 2.88 \times 10^{-3}\text{km}$$

$$\text{reqquired temperature}$$

$$\text{from } b = T\lambda_{max}$$

$$T = \frac{b}{\lambda_{max}}$$

$$T = \frac{2.88 \times 10^{-3}}{2.16 \times 10^{-7}}$$

$$T = 13333.33\text{k}$$

Example 26:

Radiation from the moon gives maximum at $\lambda = 4700\text{A}^{o}$ and $\lambda = 14 \times 10^{-6}$m.What conclusion can you draw.

$$\text{solution}$$

$$b = 2.9 \times 10^{-3}\text{mk}$$

$$\lambda_1 = 4700\text{A}^{o}$$

$$\lambda_2 = 14 \times 10^{-6}\text{m}$$

$$\text{from } T = \frac{b}{\lambda_{max}}$$

$$T = \frac{2.9 \times 10^{-3}}{4.7 \times 10^{-7}}$$

$$T = 6170\text{k}$$

$$\text{again when } \lambda = 14 \times 10^{-6}\text{m}$$

$$T = \frac{2.9 \times 10^{-3}}{14 \times 10^{-6}}$$

$$T = 207.14\text{k}$$

2.3.12 Stefan's Law Of Black Body

Stefan's law state that "The total rate at which the body radiate energy per unit area is directly proportional to the fourth power of absolute temperature"

$$\frac{P}{A} \propto T^4$$

$$P \propto AT^4$$

$$P = kAT^4$$

k is proportionality constant which is called

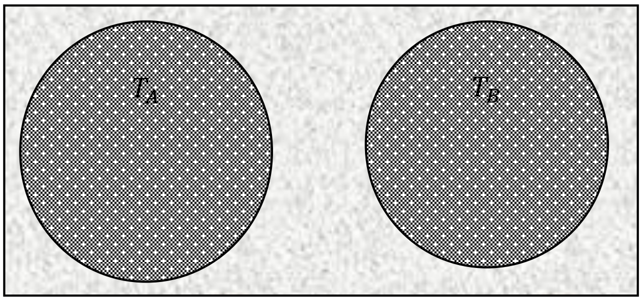
stefen′sconstant (δ)

$$P = \delta\varepsilon AT^4$$

for perfect black body $\varepsilon = 1$

$$P = \delta AT^4$$

Consider the two bodies with temperature (T) above 0k in an enclosure as shown below.



If $T_A > T_B$the physical composition and the size is the same since bodies radiate energy at different rate, a there is net power loss.

$$\text{net power loss} = P_A - P_B$$

$$P_{net} = \delta\varepsilon AT_A^4 - \delta\varepsilon AT_B^4$$

$$P_{net} = \delta\varepsilon A\left(T_A^4 - T_B^4\right)$$

2.3.13 Worked Examples Set 03A:

Example 27:

A spherical black body of radius 12cm radiate 450watt at 500k. If the radius were halved and temperature doubled. What will be the power radiated

solution

$$\text{radius (r)} = 12\text{cm} = 0.12\text{m}$$

$$P_1 = 450\text{w}$$

$$T_1 = 500\text{k}$$

$$T_2 = 1000\text{k}$$

$$P_2 = ?$$

$$\text{Area } A = 4\pi r^2$$

$$P_1 = 4\delta\varepsilon\pi r_1^2 T_1^4 ------ (i)$$

$$P_2 = 4\delta\varepsilon\pi r_2^2 T_2^4 ------ (ii)$$

$$\frac{P_1}{P_2} = \frac{4\delta\varepsilon\pi r_1^2 T_1^4}{4\delta\varepsilon\pi r_2^2 T_2^4}$$

$$\frac{P_1}{P_2} = \frac{r_1^2 T_1^4}{r_2^2 T_2^4}$$

$$P_2 = \left[\left(\frac{r_1}{r_2}\right)^2 \left(\frac{T_1}{T_2}\right)^4\right] P_1$$

$$P_2 = 450\left(\frac{6}{12}\right)^2\left(\frac{1000}{500}\right)^4$$

$$P_2 = 450 \times \frac{1}{4} \times 16$$

$$P_2 = 1800\text{w}$$

Example 28:

A metal sphere of temperature 127°C is placed in an enclosure of 27°C. The sphere has a surface area of of 0.1m^2 and is attached to radiate heat of 100w. If it behaves as black body determine the value of stefan's constant

solution

$$\text{from } P = \delta\varepsilon A T^4$$

$$\delta = \frac{P}{\varepsilon A(T_1^4 - T_2^4)}$$

$$\delta = \frac{100}{0.1(400^4 - 300^4}$$

$$\delta = 5.7 \times 10^{-8}\text{wm}^{-1}\text{k}^{-4}$$

Example 29: A tungsten filament of 60w electric lamp has length of 0.5m and diameter of 6×10^5m. The radiation energy from the lamp is 80% of its power. Calculate the average power per unit area radiated from the filament given that ($\varepsilon = 1$).

solution

$$L = 0.5\text{m}, \quad r = 3 \times 10^5\text{m},$$

$$P = 80\%, \qquad A = 2\pi rl$$

$$\frac{P}{A} = \frac{48}{9.4 \times 10^{-5}}$$

$$\frac{P}{A} = 5.1 \times 10^5 wm^{-2}$$

$$\text{intensity} = \frac{P}{A}$$

Example 30:

A tungsten filament of an electric lamp has length of 0.5m and diameter of 6×10^{-3}m .The power rating is 60w. Assume that the radiation from the filament is equivalent to 80% of that of the perfect black body radiator at the same temperature. Estimate the steady temperature of the filament.

solution

$$\varepsilon = \frac{80}{100} = 0.8, \quad l = 0.5m$$

$$r = 3 \times 10^{-3}m, \quad p = 60w$$

$$\delta = 5.7 \times 10^{-8} wm^{-2}k^{-4}$$

Required temperature

$$P = \delta\varepsilon A T^4$$

$$T^4 = \frac{P}{\delta\varepsilon A}$$

$$T^4 = \frac{P}{2\pi r l \varepsilon}$$

$$T = \sqrt[4]{\frac{P}{2\pi r l \delta\varepsilon}}$$

$$T = \sqrt[4]{\frac{60}{2\pi \times 0.5 \times 3 \times 10^{-3} \times 5.7 \times 10^{-8} \times 0.8}}$$

$$T = 611.27k$$

Example 31:

An iron ball having surface area of $400m^2$ at a temperature of 727°C is placed in an enclosure 227°C. If the surface emission of iron is 0.4. Find the heat radiated by the ball per second. Given that stefan's constant is $5.7 \times 10^{-8} wm^{-2}k^{-4}$

$$\varepsilon = 0.4, \quad \delta = 5.7 \times 10^{-8} wm^{-2}k^{-4}$$

$$T_1 = 727 + 273 = 1000K$$

$$T_2 = 227 + 273 = 500K$$

$$A = 400 \times 10^{-4} m^2$$

$$P = \delta\varepsilon A(T_1^4 - T_2^4)$$

$$P = 0.4 \times 5.7 \times 10^{-8} \times 4 \times 10^{-2}(1000^4 - 500^4)$$

$$\text{Power radiated} = 850\text{J}$$

2.3.14 Worked Examples Set 03B;

Example 1: A small hole is made in hollow sphere whose wall are at 723°C. find the total energy radiated per second per square cm.

solution

$$\text{given } \sigma = 5.7 \times 10^{-5}\text{crgcm}^{-2}\text{s}^{-1}\text{K}^{-4}$$

$$\text{temperature T} - (723 + 273) = 996\text{K}$$

$$\text{total energy radiated per second per cm}^2$$

$$= \sigma T^4$$

such that

$$= 5.7 \times 10^{-5} \times 996^4\text{erg}$$

$$= 5.61 \times 10^7\text{erg} = 561\text{J}$$

Example 2:

Calculate the temperature in kelvin at which perfect black body radiates at the rate of 5.67W/cm^2 given $\sigma = 5.67 \times 10^{-5}\text{ergs}^{-1}\text{cm}^{-2}$

solution

from the relation

$$E = \sigma T^4$$

$$T^4 = \frac{E}{\sigma} = \left(\frac{E}{\sigma}\right)^{\frac{1}{4}}$$

$$T = \left(\frac{5.67 \times 10^7}{5.67 \times 10^{-5}}\right)^{\frac{1}{4}}$$

$$T = 1000\text{K}$$

Example 3:

Due to change in the mains voltage, the temperature of an electric bulb rises from 3000K to 4000K. What is the percentage rise in electric power consumed?. solution

electric power consumed in the first case

$$P_1 = \sigma T_1^4 - - - - - - -(i)$$

electric power consumed in second case

$$P_2 = \sigma T_2^4 - - - - - - -(ii)$$

$$\frac{P_1}{P_2} = \frac{T_1^4}{T_2^4}$$

$$= \frac{(4 \times 10^{12})^4}{(3 \times 10^{12})^4} = \frac{256}{81}$$

$$\frac{P_1}{P_2} = \frac{256}{81}$$

Example 4:

Consider a sun as perfect black body sphere of radius 6.8×10^8m. calculate the energy radiated by the sun in one minute. Surface temperature of the sun is 6200K. Stefan's constant 5.67×10^{-8}

solution

$$\text{radiation from the sun} = 6.8 \times 10^8\text{m}$$

$$\text{surface area of the sun} = 4\pi R^2$$

$$= 4\pi \times (6.8 \times 10^8)^2$$

$$= A = 5.8 \times 10^{18}$$

$$\text{time t} = 1\text{minute} = 60\text{sec}$$

$$\text{total energy radiated} = \sigma T^4 \times A \times t$$

$$E = 5.67 \times 10^{-8} \times 6200^4 \times 5.8 \times 10^{18} \times 60$$

$$E = 2.92 \times 10^{28}\text{J}$$

Example 5:

An electric tungsten filament having an area 0.3m^2 is raised to a temperature of 2727℃ when a current passes through it. Calculate the electric power consumed in watt if the emissivity of the filament is 0.35. stefan's constant $\sigma = 5.7 \times 10^{-8}\text{ergs}^{-1}\text{K}^{-4}$. given that $1\text{W} = 10^7\text{ergs}^{-1}$. solution

$$\text{Area A} = 0.3\text{cm}^2 = 0.3 \times 10^{-4}\text{m}^2$$

$$\text{Temperature } T = (2727 + 273) = 3000\text{K}$$

$$\text{then Power consumed} = Ae\sigma T^4$$

$$P = 0.3 \times 10^{-4} \times 0.35 \times 5.7 \times 10^{-8} \times 3000^4$$

$$P = 48.22 \text{ W}$$

Example 6:

An indirectly heated filament radiating maximum energy of wavelength of 2.16×10^{-5}cm. Find the net amount of heat energy lost per second unit area. The temperature of a surrounding air is 13°C.

$$\text{solution}$$

$$\text{apply Wien's displacement law}$$

$$b = \lambda_{max} T$$

$$T = \frac{b}{\lambda_{max}} = \frac{0.288}{2.16 \times 10^{-5}}$$

$$T = 13333.3K$$

$$\text{temperature of surroundings,}$$

$$T_o = (13 + 273) = 286K$$

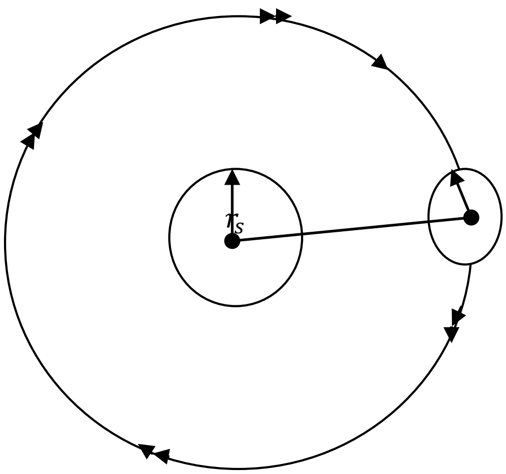
$$E = \sigma(T^4 - T_o^4)$$

$$= 5.7 \times 10^{-5}[13333.3^4 - 286^4)$$

$$18.24 \times 10^8 Js^{-1}$$

2.3.15 Application Of Stefan's Law Of Black Body

(i) Estimation of temperature of the earth and other planet. Consider the figure below



power consumed from the earth

$$P_e = \frac{A_e}{A_{imag}} \times P_T$$

$P_T = \delta\varepsilon A_s T_s^4$

This is total power radiated by a system

$$P_e = \frac{A_e}{4\pi R^2} \times \delta\varepsilon A_s T_s^4$$

where

A_e = Area of Earth

r_e = Radius of earth

T_s = Temperature of sun

T_e = Earth's temperature

A_{imag} = Area of imaginary sphere

P_T = Total power radiated from the sun

Assume the earth and the sun are perfect black,
The power absorbed by the earth = area of the earth over area of imaginary sphere times total power radiated from the sun.

$$P = \delta\varepsilon A T^4$$

$$P_e = \left(\frac{A_e}{A_{imag}}\right) \times P_T$$

$$\text{But} = A_e = \pi r_e^2$$

$$A_s = 4\pi r_s^2$$

$$P_e = \left(\frac{A_e}{A_s}\right) \times P_T$$

$$P_e = \left(\frac{\pi r_e^2}{4\pi R^2}\right) \times 4\pi\delta\varepsilon r_s^2 T_s^4$$

$$4\varepsilon\delta\pi r_e^2 T_e^4 = \left(\frac{\pi r_e^2}{4\pi R^2}\right) \times 4\pi\delta\varepsilon r_s^2 T_s^4$$

apply mathematical simplification
the equation become

$$4T_e^4 = \frac{T_s^4 \times r_s^2}{R^2}$$

$$T_e^4 = \frac{\frac{T_s^4 \times r_s^2}{R^2}}{4}$$

$$T_e = \sqrt[4]{\frac{T_s^4 r_s^4}{4R^2}} \quad T_e = \frac{T_s\sqrt{\frac{r_s}{R}}}{\sqrt{2}}$$

$$T_s = 6000K$$

$$r_s = 7 \times 10^8 m$$

$$R = 1.5 \times 10^{11}\text{m}$$

$$T_e = \frac{6000\sqrt{\frac{7\times10^8}{1.5\times10^{11}}}}{\sqrt{2}}$$

$$T_e = 289.82\text{k}$$
$$T_e \approx 290\text{K}$$
$$\text{Hence the temperature of the earth} = 290\text{K}$$

2.3.16 Solar Constant

Is the maximum energy radiated by the sun per unit area reached at the certain place on the planet from the sun.

The SI unit is watt/m^2

$$\text{solar constant } (s) = \frac{P}{A}$$

Example 32:

The solar constant which the energy is arriving per second at the earth from the sun is about 1400wm². Estimate the surface temperature of the sun given that the radius of the sun is 1.5×10^{11}m and stefan's constant $5.7 \times 10^{-8}\text{wm}^{-2}\text{K}^{-4}$

$$\text{solution}$$
$$\text{solar constant } S = 1400\text{wm}^{-2}$$
$$r_s = 7 \times 10^8\text{m}$$
$$R = 1.5 \times 10^{11}\text{m}$$
$$\delta = 5.7 \times 10^{-8}\text{wm}^{-2}\text{K}^{-4}$$
$$\text{required temperature}$$
$$S = \frac{P_e}{A_{imag}}$$
$$S = \frac{\delta\varepsilon A_s T_s^4}{4\pi R^2}$$
$$\text{assume the sun is perfect black body}$$
$$\varepsilon = 1$$
$$S = \frac{\delta A_s T_s^4}{4\pi R^2}$$
$$S = \frac{\delta r_s^2 T_s^4}{R^2}$$
$$T_s = \sqrt[4]{\frac{SR^2}{\delta r_s^2}}$$
$$T_s = \sqrt[4]{\frac{1400 \times (1.5 \times 10^{11})^2}{(5.7 \times 10^{-8})(7 \times 10^8)^2}}$$
$$\text{temperature of the sun} = 5800\text{K}$$

Example 33:

A solar furnace have a concave mirror of collecting area $0.6m^2$. A solid x of mass 0.5kg and specific heat capacity of $700\,J/kgK$and temperature of $18°C$ is placed at the focus of the mirror. What temperature is reached by x in 0.5minutes? If the average solar constant is $1400\,w/m$.

solution

$$A_x = 0.6m^2, \; m_x = 0.5kg$$

$$C_x = 700\,J/kgK, \; \theta_i = 18°C$$

$$\theta_f = ?, \; S = 1400\,w/m$$

$$\text{Time} = 0.5\text{minutes} = 30\text{sec}$$

$$\text{Heat gained by x} = \text{Energy due to radiation}$$

$$Pt = m_x C_x(\theta_f - \theta_i)$$

$$\text{but Power } P = SA$$

$$SAt = m_x C_x(\theta_f - \theta_i)$$

$$0.5 \times 700(\theta - 18) = 1400 \times 0.6 \times 30$$

$$350(\theta - 18) = 25200$$

$$350\theta - 6300 = 25200$$

$$350\theta = 31500$$

$$\theta = \frac{31500}{350}$$

$$\theta = 90°C$$

$$\text{temp} = 90 + 273$$

$$\text{temperature required} = 363K$$

2.3.17 Worked Examples 03C:

1 .A solid sphere has a temperature of 773K. The sphere is melted and recast into a cube that has the same emissivity and emit the same radiant power as the sphere. Calculate the temperature of the cube.

solution

$$\text{Let T be a temperature of cube}$$

$$\text{volume of sphere} - \text{volume of cube}$$

$$\frac{4}{3}\pi r^3 = L^3$$

$$\text{simple mathematics}$$

$$\frac{L}{r} = \sqrt[3]{\frac{4\pi}{3}}$$

consider a stefan′slaw of black body

$$\varepsilon A\delta T^4 = \varepsilon A\delta T_c^4$$

$$(4\pi r^2)\varepsilon\sigma \times 773^4 = 6L^2\varepsilon\sigma T^4$$

$$(4\pi r^2)773^4 = 6L^2T^4$$

$$T = 773 \times \sqrt[4]{\frac{4\pi}{6} \times \left(\frac{r}{L}\right)^2}$$

$$\text{since } \frac{r}{L} = \sqrt[3]{\frac{4\pi}{3}}$$

$$T = 773 \times \sqrt[4]{\frac{4\pi}{6} \times \left(\sqrt[3]{\frac{4\pi}{3}}\right)^2}$$

$$T = 1181K$$

2. A solid cylinder is radiating a energy. It has length that is ten times its radius. It is cut into smaller cylinders, each of the same length. Each small cylinder has the temperature as the original cylinder. The total radiant power emitted by the pieces is twice that emitted by the original cylinder. How many smaller cylinders are there?

solution

Given $L = 10r$ and let n be number of small cylinder

produced the applying stefan′s law

let total radiant power emitted by "n" number of pieces of cylinder be P_c also let P_o be radiant power emitted by original cylinder such that

$$2P_o = nP_c$$

$$2\varepsilon\sigma T^4(2\pi r^2 + 2\pi rL) = n\varepsilon\sigma T^4\left(2\pi r^2 + \frac{2\pi rL}{n}\right)$$

simplifying the equation results

$$2\pi r^2(n-2) = 2\pi rL$$

$$r(n-2) = L$$

$$\frac{L}{r} = n - 2$$

$$\text{but } \frac{L}{r} = 10$$

$$10 = n - 2$$

$$n = 12$$

3 (a) What is meant by Stefan's constant.

(b) A sphere of radius 2cm with a black surface is cooled and then suspended in a large evacuated enclosure the black walls of which are maintained at 25°C. If the rate of change of thermal energy of the sphere is 1.85Js^{-1} when its temperature is −73°C. calculate the value of Stefan's constant.

solution

By stefan's law

$$P = \sigma\varepsilon A(T_1^4 - T_2^4)$$

$$\sigma = \frac{P}{\varepsilon A(T_1^4 - T_2^4}$$

$$\sigma = \frac{P}{4\pi r^2\varepsilon(T_1^4 - T_2^4)}$$

$$T_1^4 - T_2^4 = (273 + 27)^4 - (-73 + 273)^4$$

$$= 6.5 \times 10^9 K$$

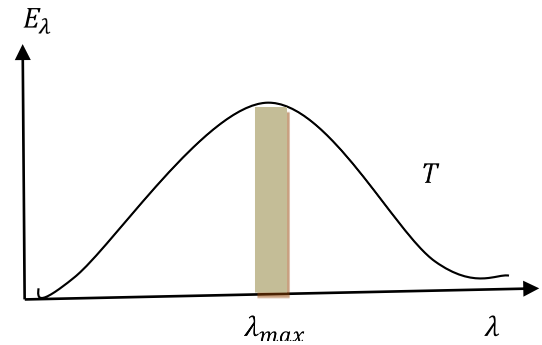
$$\sigma = \frac{1.85}{4\pi \times (0.02)^2 \times 6.5 \times 10^9}, \qquad \varepsilon = 1$$

$$\sigma = 5.66 \times 10^{-8} Wm^{-2}K^{-4}$$

4 (a) State Stefan's law and draw diagram to show how the energy is distributed against wavelength in the spectrum of a black body for two different temperature. Show which temperature is higher

(b) Draw a sketch showing how the energy E_λ in a narrow band of wavelengths of mean value λ, emitted by a black body radiator at a constant temperature, varies with λ. In your diagram showing

(i). The wavelength λ_{max} with maximum energy
(ii).The area which is measure of total energy emitted by a black body.



- The maximum wavelength is as indicated in the diagram.
- The are under the whole curve.

5.(a) State Prevost's theory of heat exchange. Refer to notes

(b) A solid copper sphere, of diameter 10mm, is cooled to a temperature of 150K and is then placed in an enclosure maintained at 290K. Assuming that all interchange of heat is by radiation, calculate the initial rate of rise of temperature of the sphere. The sphere may be treated as black body. Given that density of copper is $8930kgm^{-3}$, specific heat capacity of copper is $370Jkg^{-1}K^{-1}$ and stefan's constant is $5.7 \times 10^{-8}Wm^{-2}K^{-4}$

$$\text{By stefan's law of blackbody}$$

$$\text{from} \quad Q = mc\Delta T$$

$$\frac{dQ}{dt} = mc\left(\frac{dT}{dt}\right) ------(i)$$

$$P = \varepsilon\sigma A(T_o^4 - T_1^4) -----(ii)$$

$$P = \frac{dQ}{dt}$$

$$mc\left(\frac{dT}{dt}\right) = 4\pi r^2\varepsilon\sigma(T_o^4 - T_1^4)$$

$$\frac{dT}{dt} = \frac{4\pi r^2\varepsilon\sigma(T_o^4 - T_1^4)}{mc}$$

$$m = \rho v = \frac{4}{3}\pi r^3\rho$$

$$\frac{dT}{dt} = \frac{4\pi\varepsilon\sigma r^2(T_o^4 - T_1^4)}{\frac{4}{3}\pi r^3\rho c}$$

$$\frac{dT}{dt} = \frac{3\sigma\varepsilon}{r\rho c}(T_o^4 - T_1^4)$$

$$= \frac{3 \times 1 \times 5.7 \times 10^{-8}}{10^{-3} \times 8930 \times 370}(290^4 - 150^4)$$

$$\frac{dT}{dt} = 0.068\text{Ks}^{-1}$$

6. The silica cylinder of radiant wall heater is 0.6m long and has a radius of 5mm. If it is rated at 1.5kW estimate its temperature when operating. State two assumption made during temperature estimation.

$$\text{solution}$$

$$\text{consider stefan's law of black body}$$

$$P = \varepsilon\sigma A T^4$$

$$T = \sqrt[4]{\frac{P}{\varepsilon\sigma A}}$$

$$\text{Area for cylinder} = 2\pi r L$$

$$T = \sqrt[4]{\frac{P}{2\pi r L \sigma\varepsilon}}$$

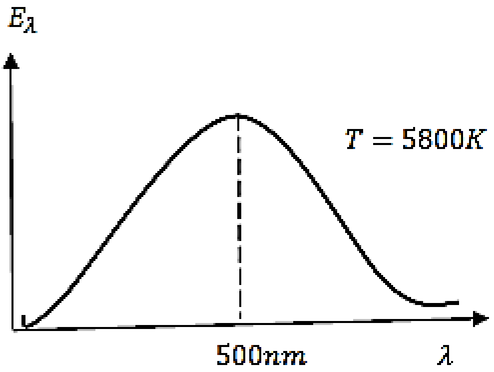
$$= \sqrt[4]{\frac{1500}{1 \times (2\pi \times 0.005 \times 0.6) \times 5.7 \times 10^{-8}}}$$

$$T = 1072\text{K}$$

Assumptions made from estimation

- The body behaves as blackbody.
- The heat is radiated only on the curved surface of cylinder but not on its sides.

7. The diagram below show how E_λ, the energy radiated per unit area per second per unit wavelength interval, varies with wavelength λ for radiation from the sun's surface.



Calculate the wavelength at λ_{max} at which the corresponding curve peak for.

- Radiation in the sun's core where the temperature is approximately 15×10^5K
- Radiation in interstellar space which corresponds to a temperature of approximately 2.7K. Name the part of the electromagnetic spectrum to which the calculated wavelength belongs in each case.

solution

Applying Wien'sdisplacement law

$$(i)\lambda_{max}T = 15 \times 10^6\lambda$$

$$500 \times 5800 = 15 \times 10^6\lambda$$

$$\lambda = \frac{500 \times 5800}{15 \times 10^6}$$

$$= 0.193\text{nm}$$

$$\lambda = 1.93 \times 10^{-10}\text{m}$$

$$\text{X} - \text{ radiation}$$

$$(ii)\ 500 \times 5800 = 2.7\lambda$$

$$\lambda = \frac{500 \times 5800}{2.7}$$

$$\lambda = 1.07 \times 10^{-3}\text{m}$$

$$\text{Infra} - \text{red region}$$

8. Calculate the apparent temperature of the sun from the following given information; sun's radius 4.4×10^5miles. Distance from earth is 9.2×10^7miles. Solar constant= 1.4kWm^{-2} and stefan's constant $= 5.7 \times 10^{-8}\text{Wm}^{-2}\text{K}^{-4}$

solution

$$\text{Solar constant} = \frac{\text{Power from the sun}}{\text{Area of earth}}$$

$$S = \frac{P}{A}$$

$$P = AS$$

$$\varepsilon A\sigma T^4 = 4\pi r^2 S$$

$$T^4 = \frac{4\pi r^2 S}{4\pi R^2 \sigma} \quad \text{when } \varepsilon = 1$$

$$T^4 = \sqrt[4]{\left(\frac{r}{R}\right)^2 \left(\frac{S}{\sigma}\right)}$$

$$T = \left(\frac{r}{R}\right)^{\frac{1}{2}} \left(\frac{S}{R}\right)^{\frac{1}{4}}$$

$$T = \left(\frac{9.2 \times 10^7}{4.4 \times 10^5}\right)^{\frac{1}{2}} \left(\frac{1400}{5.7 \times 10^{-8}}\right)^{\frac{1}{4}}$$

$$T = 5724\text{K}$$

9. A body which has surface area 5.00cm^2 and temperature of $727°\text{C}$ radiate 300J energy in one minute. What is emissivity? Given that stefan's constant is $5.67 \times 10^{-8}\text{Wm}^{-2}\text{K}^{-4}$

solution

$$P = A\varepsilon\sigma(T^4)$$

$$\text{since POwer} = \frac{\text{Energy radiated}}{\text{time (t)}}$$

$$\frac{E}{t} = \varepsilon\sigma A(T^4)$$

$$E = \varepsilon\sigma At(T^4)$$

$$\varepsilon = \frac{E}{\sigma A T^4}$$

$$\varepsilon = \frac{300}{5.67 \times 10^{-8} \times 5 \times 10^{-4} \times (727 + 273)^4}$$

$$\varepsilon = 0.18$$

10.(a) A tungsten wire of length 100cm and perimeter of 0.2cm behaves as black body at 2000K. Calculate the rate at which the wire is radiating energy at 2000K. Given that stefan's constant $\sigma = 5.67 \times 10^{-8}\mathrm{Wm^{-2}K^{-4}}$.

solution

The rate at which the tungsten wire radiates energy is given by $P = \varepsilon\sigma T^4 A$

$$\varepsilon = 1; \quad A = \text{perimeter} \times \text{length}$$

$$A = 0.2 \times 100 = 20\mathrm{cm^2}$$

$$A = 20 \times 10^{-4}\mathrm{m^2}$$

$$P = 1 \times 5.67 \times 10^{-8} \times 2000^4 \times 20 \times 10^{-4}$$

$P = 1814.4\ \mathrm{W}$

(b) An aluminum foil of relative remittance 0.2 is placed between two concentric spheres at temperature 300K and 200K respectively. Calculate the temperature of the foil after the steady state is reached. Also calculate the rate of energy transfer between one of the sphere and the foil. Given Stefan's constant $5.67 \times 10^{-8}\mathrm{Wm^{-2}K^{-4}}$

$$T_1 = 300\mathrm{K}, \qquad T_2 = 200\mathrm{K}$$

At the steady state let T be the absolute temperature of the foil. Under steady state, Heat gained per second by foil from first sphere = heat lost per second by foil to the second sphere. $\varepsilon\sigma A(T_1^4 - T^4) = \varepsilon\sigma A(T^4 - T_2^4)$

simple mathematics

$$T_1^4 - T_2^4 = 2T^4$$

$$T^4 = \frac{T_1^4 - T_2^4}{2}$$

$$T = \sqrt[4]{\frac{300^4 - 200^4}{2}}$$

$$T = 263.9\mathrm{K}$$

The rate at which energy is transferred from the first sphere to the foil.

$$= \varepsilon A\sigma(T_1^4 - T^4)$$

$$= 0.2 \times 5.67 \times 10^{-8} \times (300^2 - 263.9^4)$$

$$= 36.85\mathrm{Wm^{-2}}$$

11.(a) Calculate the energy radiated in one minute by a blackbody of surface area 200cm^2 maintained at $127°\text{C}$. Given stefan's constant $= 5.67 \times 10^{-8}\text{Wm}^{-2}\text{K}^{-4}$.

solution

the energy radiated per sec by a body is given by

$$P = \varepsilon\sigma A(T^4 - T_o^4)$$

$$\varepsilon = 1 \text{ for perfect black body}$$

$$P = 1 \times 5.67 \times 10^{-8} \times 200 \times 10^{-4} \times 400^4$$

$$P = 29.184 \text{ J/s}$$

energy radiated in one minute

$$= 29.184 \times 60$$

$$\text{Energy} = 1751\text{J}$$

(b) At what temperature will the filament of a 100W lamp operate if it is supposed to be a perfectly black body of area 1cm^2? Given that $\sigma = 5.67 \times 10^{-8}\text{Wm}^{-2}\text{K}^{-4}$

solution

consider Stefan's law of radiation

$$P = \varepsilon\sigma A T^4$$

$$T^4 = \frac{P}{\varepsilon\sigma A}$$

$$= \frac{100}{1 \times 5.67 \times 10^{-8} \times 10^{-4}}$$

$$T = 2049\text{K}$$

2.3.17. Conceptual Questions

12. (a) why the pupil of the eye black?

- Any radiation that enters the pupil of the eye is reflected from inside the eye many times and is partly absorbed at each reflection until none remain

(b) Why does a good absorber of radiant energy appear black?

- **Answer;** a good absorbing surface remove all visible colors when it converts all the radiant energy incident upon it to heat. It cannot absorb and reflect the same wavelengths at the same time.

(c) Why is the energy of thermal radiation is less than that of visible light?

- **Answer;** the energy of electromagnetic wave is given by $E = hf$ since the frequency of thermal radiation is less than that of visible light, the energy associated with thermal radiation is less than associated with visible light.

(d) The tile floor feels colder than wooden floor, even though both floor materials are at the same temperature. Explain.

- **Answer;** It is because tile is a better heat conduct than wood. The heat transferred from foot to the wood is not conducted away rapidly. So the wood quickly heats up on its surface to the temperature of your foot.

(e) Why are steam pipes wrapped with insulating material?

- **Answer;** to minimize the loss of heat due to radiation.

13. (a) A red glass heated in a furnace and then taken out appears green. Explain.

- **Answer;** At low temperature, red glass appears red because it absorbs green light strongly. Therefore at high temperature, it emits green colour strongly and therefore appear green.

(b) A body is at 0°C. Is it radiating heat?

- **Answer;** yes, a body radiate heat even at 0°C

2.3.18 Competitive Examination File Unit Set 04:

1. Find the rate of heat flow through a square iron plate each sides equal to 4cm and thickness of 5mm. its opposite faces are kept at 90°C and 40°C respectively. K for iron is 0.15cgs
   [**ans. 240cal/s]**
2. Calculate the thermal reeistance of an alminium rod of length 0.20m and diameter of 0.04m the thermal conductivity is 0.50cal/cmdeg. The temperature difference along the length of the rod is 50°C . Also calculate the rate of heat transfer along the length of the rod. **[ans. 3.18sdeg/cal; 15.7cal/sec]**
3. Calculate the difference in temperature between two sides of an iron plate 2cm thick when heat is conducted at the rate of 6×10^5cal/min per square metre. The coefficient of thermal conductivity of metal is 0.2cgs units. **[ans. 10°C]**
4. Calculate the rate of heat loss through a glass window of area 1000cm^2 and thickness of 4mm when a temperature inside is 37°C and outside is -5°C. Coefficient of thermal conductivity of glass is 0.0022cal/scmK. **[[ans. 231cal/s]**
5. Air in the room is at 25°C and outside temperature is 0°C. The window of the room has an area of 2m^2 and thickness of 2mm. Calculate the rate of loss of heat by conduction through window. Thermal conductivity of glass is 1.0W/mdegree. **[ans. 25kW]**

6. Calculate the radiant emittance of black body at temperature of 400K. Given that $\sigma = 5.672 \times 10^{-8}\text{Js}^{-1}\text{m}^{-2}\text{K}^{-4}$.
[ans. $1.452 \times 10^{7}\text{Js}^{-1}\text{m}^{-2}$]
7. Black body at temperature 400K radiates at the rate of $1.452 \times 10^{10}\text{erg/sm}^2$. Calculate the value of stefan's constant
[ans. $5.67 \times 10^{-5}\text{erg/scm}^{-2}\text{K}^{-4}$]
8. The energy per second emitted by a black body at $1227°\text{C}$ is E. If the temperature of the black body is increased to $2727°\text{C}$, calculate the energy emitted per second in terms of E in second case. **[ans. 16E]**
9. At what temperature will the filament of 100W lamp operate, if it is supposed to be perfectly black body of area 1cm^2? Given $\sigma = 5.67 \times 10^{-8}\text{Js}^{-1}\text{m}^2\text{K}^4$. **[ans. 2049K]**
10. A copper ball 2cm in radius is heated in a furnance to $400°\text{C}$. if its emissivity is 0.3, at what rate does it radiate energy? Given $\sigma = 5.67 \times 10^{-8}\text{Js}^{-1}\text{m}^2\text{K}^4$.**[ans. 17.54J/s]**
11. To what temperature must a black body be raised in order to double total radiation if original temperature is $727°\text{C}$.
[ans. 916.2°C)
12. Calculate the temperature at which a perfect black body radiates energy at the rate of 1W/cm^2. Given $\sigma = 5.67 \times 10^{-8}\text{Js}^{-1}\text{m}^2\text{K}^4$. **[ans. 648K]**
13. Two body A and B are kept in evacuated vessel maintained at a temperature of $27°\text{C}$. The temperature of A is $527°\text{C}$ and that of B is $127°\text{C}$. Compare the rates at which heat is lost from A and B. **[ans. 22.9]**
14. Radiation from the moon gives two maxima at wavelengths of $4700\text{A}°$ and at $14 \times 10^{-4}\text{cm}$. What conclusion can you draw from this? Given that $\text{b} = 0.2898\text{cmK}$.
15. A body cools from $50°\text{C}$ to $40°\text{C}$ in 6minutes, when its surrounding temperature is $30°\text{C}$. What will be its temperature 12 minutes after the start of the experiment. **[ans. 35°C]**
16. The total energy arriving from the sun and falling on the upper atmosphere of the earth is 1400W/m^2. The distance of the earth from the sun is $1.5 \times 10^{11}\text{m}$. Find the power out put of the sun. **[ans. 10.68cal]**
17. For T = 1000K, $\lambda_{\text{m}}=$ (i)$5 \times 10^{-4}\text{cm}$ (ii) $4 \times 10^{-4}\text{cm}$ for a blak body spectrum. Calculate the corresponding values of wavelength for temperature T = 2000K **[ans. 2.5×10^{-4}; $2\times 10^{-4}\text{cm}$]**
18. Calculate the temperature corresponding to maximum intensity of $4800\text{A}°$. **[ans 6037.5K]**

19. The surface temperature of the body is 1500K. Find the wavelength at which it radiates maximum. Given $b = 0.2898$cmK **[ans.19320A°]**

20. An electric bulb with tungsten filament having an area of 0.25cm^2 is raised to a temperature of 3000K when a current passes through it. Calculate the electric energy being consumed in watt, if the emissivity of the filament is 0.35. stefan's constant $\sigma = 5.67 \times 10^{-5}\text{crgs}^{-1}\text{cm}^{-2}\text{K}^{-4}$. If due to fall in mains voltage, the filament temperature falls to 2500K. What will be the wattage of the bulb. **[ans. 40.19W; 19.38W]**

21. A thin brass rectangular sheet of sides 15cm and 12cm is heated in furnance to 600℃ and taken out. How much electric power is needed to maintain the sheet at this temperature, given that its emissivity is 0.250? **[ans. 296W]**

22. A body cools from 60℃ to 40℃ in 7 minutes when placed in a surrounding which is maintained at a temperature of 10℃. What is the temperature of the body after the expiry of next 7 minutes? **[ans. 28℃]**

23. A room is maintained at 20℃ by a heater of resistance 20Ω connected to 200V mains. The temperature is uniform throughout the room and the heat is transmitted through a glass window of area 1m^2 and thickness 0.2cm. Calculate the temperature outside. Given that thermal conductivity of glass is 0.2cal/ms℃ and mechanical equivalent heat is 4.2/cal. **[ans. 15.24**℃]

24. The temperature of the body increased from 27℃ to 127℃. By what fraction would the radiation emitted by it increase? **[ans. 256/81]**

25. An aluminium foil of relative emittance 0.2 is placed between two concentric sphere at temperatures 300K and 200K respectively. Calculate the temperature of the foil after the steady state is reached. Also calculate the rate of enrgy transfer between one of the spheres and the foil. Given Stefan's constant $\sigma = 5.67 \times 10^{-8}\text{Wm}^{-2}\text{K}^{-4}$. **(ans. 263.9K, 36.85W/m²)**

26. A body which has surface area of 5cm^2 and temperature 727℃ radiates 300J of energy in one minute. What is its emissivity. Given Stefan's-Boltzman constant is $5.67 \times 10^{-8}\text{W/m}^2\text{K}^4$ (ans. 0.18)

CONCEPTUAL TARGETS

- *The tiles floor feels colder than the wooden floor, even though both floor materials are at the same temperature. Why?*

  Answer;

  Because tile is a better heat conductor than wood. the heat transferred from your foot to the wood is not conducted away rapidly.

- *Thermal conductivity of air is less than that of felt but felt is a better heat insulator in comparison to air. Why?*

  Answer;

  Free air transmit heat by convection currents. In case of felt, there are fine pores which trap air and do not allow its movement.

2.3.19 Competitive Examination File Unit Set 05:

Problem 1

Assuming the total surface area of human body is $1.25m^2$ and the surface temperature is 30°C.

Find the total rate of radiation of energy from the human body. Given that Stefan's constant, σ = 5.67 x 10^{-8} $Wm^{-2}K^{-4}$

Problem 2

A black ball of radius 1m is maintained at a temperature of 30°C. How much heat is radiated by the ball in 4second. Given that Stefan's constant = 5.67 x 10^{-8} $Wm^{-2}K^{-4}$

Problem 3

Two spheres made of the same material have radii 2.0cm and 3.0cm and their temperature are 627°C and 527°C respectively. If they are black bodies, compare:

(a) The rate at which they are losing heat

(b) The rate at which their temperature are falling.

Problems 4

A tungsten filament of total surface area of $0.45km^2$ is maintained at a steady temperature of 2227 °c. Calculate the electrical energy dissipated per second if all this energy is radiated to the surrounding. Given that emissivity of tungsten at 2227^0C = 0.3 and Stefan's constant = 5.67 x 10-^{8} $Wm^{-2}K^{-4}$ **[Ans. 29.9W]**

Problem 5

The tungsten filament of an electric lamp has a length of 0.5m and a diameter of 6 x 10-5m. The power of rating of the lamp is 60W. Assuming the radiation from the filament is equivalent to 80% that of a perfect black body radiator at the same temperature, estimate the steady temperature of the filament given that. Stefan's constant = 5.67 $\times$ 10^{-8} $Wm^{-2}K^{-4}$ $\mathbf{T = 935.5K}$

Problem 6

The total external surface area of the dog's body is 0.8 m^2 and the body temperature is 37 °C at what rate is it loosing heat by radiation when it is in a room whose temperature is 17 °C? Assume that dog's body behaves as a black body and given that Stefan's constant is 5.67 x 10-8 Wm^{-2} K^{-4} **[Ans. P = 98.086W]**

Problem 6

The cathode of a certain diode valve consists of a cylinder 2 x 10^{-2}m long and 0.1 x 10^{-2} m in diameter. It is surrounded by a co-axial anode of diameter larger than that of the cathode. The anode remains at a rate temperature of 127 °C when the power of 4 watts is dissipated in heating the cathode. Estimating the temperature of cathode. [ans. **1035.29]**. List the assumption you have made in arriving at your estimate

Problem 7

A metal sphere with a black surface and radius 30 mm is cooled to 200k and placed inside an enclosure at a temperature of 300k. Calculate the initial rate of temperature rise of the sphere, assuming the sphere is a black body given that Density of metal=8000 kgm-3. Specific heat capacity metal = 400$Jkg^{-1}K^{-1}$ Stefan's constant =$5.67\times 10^{-8}$$Wm^{-2}K^{-4}$ **[answer: dθ/dt = 0.012 c / sec]**

Problem 9

The energy arriving per unit area on the earth's surface per second from the sun is 1.34 x 10^3 Wm^{-2} the average distance from the earth to the sun is 215 times as great as the sun's radius. Given that both the earth and the sun are black bodies Stefan's constant = 5.67 x 10-8 $Wm^{-2}K^{-4}$. **[Answer: T = 575k]**

Problem 10

The amount of radiant heat received by the earth from the sun is $1.38 \times 10^3 Wm^{-2}$. Suppose all these radiations on the earth are re emitted by the earth. Calculate the temperature of the earth. Given Stefan's constant =5.67 x 10-8 Wm^{-2} K^{-1}

[ans. T = 395K]

Problem 11

The surface temperature of the sun is 6000K. If we consider it as a perfect black body, calculate the energy radiated by the sun per second. Given that the radius of the sun = 6.92 × 108m and δ = 5.67 × 10-8$Wm^{-2}K^{-4}$ **[ans.P = 4.42 × 1026W]**

Problem 12

The temperature of a furnace is 2324°C and the intensity is maximum in its radiation spectrum nearly at 12000A^0. If the intensity in the spectrum of a star is maximum is nearly at 4800A^0 then calculate the surface temperature of the star. **[T_2=6492.5K]**

Problem 13

The wavelength corresponding to maximum energy for the moon is 14 x 10^{-6}m estimate the temperature of the moon if b = 2.884 × 10-3 mK **[T = 206K]**

2.3.20. Calorimetry

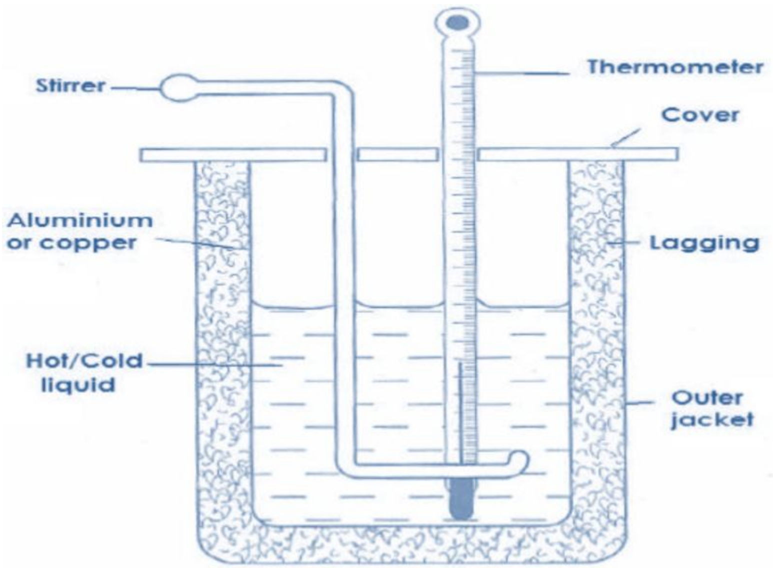
Calorimetry is a science which deals with heat and measurement.
Calorimeter is a device which is used to measure quantity of heat.

2.3.21 Law Of Calorimetry

The law states that "If two bodies with different temperature are mixed together then heat lost by a hotter one is equal to heat gained by cold ones"

$$\text{Heat lost} = \text{Heat gained}$$

In this case we move interested in determination of heat capacities
Thermometer



$$\text{By principle of calorimeter}$$

$$\text{Heat lost by metal} = \text{heat gained by calorimeter and water}$$

$$\text{let heat lost by metal be } H_m$$

$$\text{Heat gained be water be } H_w$$

$$\text{Heat gained by calorimeter be } H_c$$

$$\text{Then } H_m = m_m c_m(\theta_m - \theta_f)$$

$$H_w = m_w c_w(\theta_f - \theta_i)$$

$$H_c = m_c c_c(\theta_f - \theta_i)$$

$$H_m = H_c + H_w$$

$$m_m c_m(\theta_m - \theta_f) = m_c c_c(\theta_f - \theta_i) + m_w c_w(\theta_f - \theta_i)$$

$$c_m = \frac{m_c c_c(\theta_f - \theta_i) + m_w c_w(\theta_f - \theta_i)}{m_m(\theta_m - \theta_f)}$$

Example 34:

When a block of metal of mass0.11kg and specific heat capacity of 400J/kgK heated to 100°C and quickly transferred to a calorimeter containing 0.2kg of liquid at 10°Cthe resulting temperature after stirring is 18°C. On repeating the experiment with 0.4kg of liquid in the same container at the same initial temperature the resulting temperature is 14.5°C. Calculate

i. Specific heat capacity of liquid.
ii. Thermal heat capacity of calorimeter.

solution

Experiment 1

$$m_b = 0.11\text{kg}, \quad m_w = 0.2\text{kg}$$

$$\theta_b = 100°\text{C}, \quad \theta_f = 18°\text{C}$$

$$c_b = 400\,\text{J/kgK}, \quad \text{Require } C_w$$

$$\text{Heat lost by block} = \text{Heat gained by water}$$

$$H_b = H_w$$

$$m_b c_b(\theta_b - \theta_f) = m_w c_w(\theta_f - \theta_i)$$

$$C_w = \frac{m_b c_b(\theta_b - \theta_f)}{m_w(\theta_f - \theta_i)}$$

$$C_w = \frac{0.11 \times 400(100 - 18)}{0.2 \times (18 - 14.5)}$$

$$C_w = \frac{44 \times 82}{0.7}$$

$$C_w = 5154.29$$

$$\text{Specific heat capacity of watr } (C_w) = 5154.29\,\text{J/kgK}$$

Experiment 02

$$m_w = 0.4\text{kg}, \quad \theta_i = -10°\text{C}$$

$$\theta_f = 15.5°\text{C}, \text{Require } C_c$$

$$H_b = H_c$$

$$m_b c_b(\theta_b - \theta_f) = m_c c_c(\theta_f - \theta_i)$$

$$C_c = \frac{m_b c_b(\theta_b - \theta_f)}{m_c(\theta_f - \theta_i)}$$

$$C_c = \frac{0.11 \times 400(100 - 14.5)}{0.4(14.5 — 10})$$

$$C_c = 383.88$$

$$\text{Specific heat capacit of calorimeter } (C_c) = 383.88 \text{ J/kgK}$$

$$\text{(ii)Thermal heat capacity of calorimeter}$$

$$H_c = m_c c_c(\theta_f - \theta_i)$$

$$H_c = 0.09 \times 383.88(14.5 — 10)$$

$$H_c = 846.46\text{J}$$

$$\text{heat capacity of calorimeter} = 846.46\text{J}$$

- **Specific heat capacity** Is the quantity of heat required to rise a temperature of unit mass of substance by 1K or 1℃
- **Heat capacity** is the quantity of heat require to rise a temperature of a body with a whole mass by 1K
- **Latent heat** is the heat required to change the state of a substance without change in temperature.
- **Specific latent heat** is heat energy required to change a unit mass of 1kg from one state to another at constant temperature. The SI-unit is J/kg.
- **Specific latent heat of fusion** is the heat required to change unit mass of a solid to liquid at constant temperature.

$$\mathbf{H = mL_f}$$

- **Specific latent heat of vaporization** is the quantity of heat required to change unit mass 1kg of a substance from liquid to vapour at constant temperature.

$$H = mL_v$$

Example 35:

Calculate the heat required to melt ice of mass 10kg at 0℃ given specific latent heat of ice is 3.4×10^4 J/kg.

$$\text{solution}$$

$$\text{from} \quad \text{Heat H} = mL_f$$

$$H = 10 \times 3.4 \times 10^4$$

$$\text{Heat} = 3.4 \times 10^5 \text{ J}$$

Example 36:

Calculate the heat required to turn 500g of ice at 0°C to water at 100°C. Given that specific latent heat 3.4×10^5 J/kgKand specific heat capacity of water 4200 J/kgK

Example 37:

It takes for an electric kettle to heat a certain quantity of water from 0°C to boiling point 100°Cin 15minutes. It require 80minutes all the water at 100°C into steam. Calculate the latent heat of steam.

solution

let the mass of water be m

A quantity of heat liberated in 15minutes

$$= m_c(\theta_f - \theta_i)$$

$$= m \times (100 - 0)$$

$$= 100\text{mcal} ----- (i)$$

A quantity of heat liberated in 80minute

$$= 100m \times \frac{80}{15}$$

$$= \frac{8000}{15} m ----- (ii)$$

let L_v be latent heat of steam

$$mL_v = \frac{8000m}{15}$$

$$L_v = \frac{8000}{15}$$

$$L_v = 533.33 \text{ cal/g}$$

Example 38:

0.75g of petroleum burnt in a bomb calorific factor which contained 200g of water and had a water equivalent 750g. The rise of temperature was 3.08°C. Calculate the calorific value of petroleum.

solution

heat gained by water

$$= m_w c_w(\theta_f - \theta_i)$$

$$= 200 \times 1(3.08 - 0)$$

$$= 2000 \times 3.08$$

$$\text{heat gained} = 6160\text{cal}$$

heat gained by calorimeter

$$H = m\Delta\theta$$

$$\text{Heat} = 750 \times 3.08$$

$$\text{Heat gained by calorimeter} = 2310\text{cal}$$

$$\text{Total heat gained} = 6160 + 2310$$

$$= 8470\text{cal}$$

This heat is gained due to combustion of 0.75g

of petroleum

$$\text{Calorific value} = \frac{\text{total heat}}{\text{mass of petroleum}}$$

$$= \frac{8470}{0.75}$$

$$= 11293 \text{ cal/g}$$

$$\text{Calorific value of petroleum} = 11293 \text{ cal/g}$$

3 Thermodynamics

Thermodynamics is the study of relationship between heat and other forms of energy. In Other hand **Thermodynamics** can be defined as the science of energy conversion involving heat and other forms of energy, most notably mechanical work.

It studies and interrelates the macroscopic variables (temperature, volume and pressure

Examples

- Rise the temperature of water in kettle
- Burn some fuel in the combustion chamber of an aero engine to propel an aircraft.
- Cool our room on a hot humid day.
- Heat up our room on a cold winter night.
- Have our beer cool.
- Some coal/gas in a power plant to generate electricity.
- Petrol in a car engine

It explain that at any isolated system in the universe is associated internal energy.
THERMAL ENERGY (U)

Internal energy U is the total average kinetic energy of all gases molecule in the system.

$$\text{mathematically}$$

$$U = K.E \times N$$

$$\text{where}$$

$$K.E = \text{kinetic energy of the molecules}$$

$$N = \text{Number of molecules}$$

$$\text{Hence } K.E = \frac{3}{2}KT$$

$$K = \frac{R}{L_A}$$

$$U = \frac{3}{2}NkT$$

$$\text{where } N = nL_A$$

$$U = \frac{3}{2} nL_A kT$$

$$\text{Since } R = kL_A$$

$$U = \frac{3}{2} nRT$$

$$du = \frac{3}{2} nRdT$$

$$R = \text{Unversal gas constant} \approx 8.314$$

$$k = \text{Boltzman constant} \approx 1.38 \times 10^{-28}$$

$$n = \text{Number of moles}$$

Example 39: Find the internal energy of a gas of volume 20cm^3 at STP

Solution

$$\text{given volume } v = 20\text{cm}^3$$

$$\text{absolute temp} = 273\text{k}$$

$$\text{volume at stp} = 22.4\text{dm}^3$$

$$R = 8.314$$

then required to find the value of U

$$U = \frac{3}{2} nRT$$

$$\text{but } n = \frac{V}{V \text{ at stp}}$$

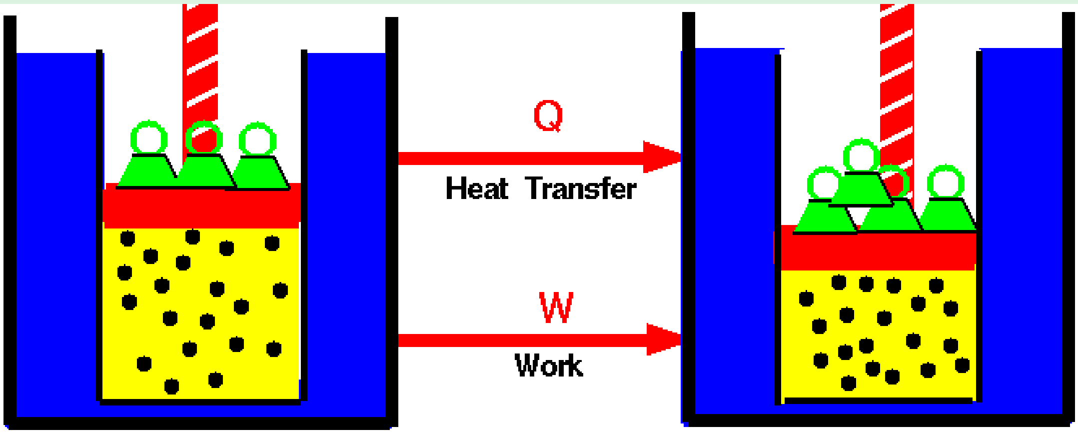
$$n = \frac{20000}{22.4} = 892.86\text{moles}$$

$$u = \frac{3}{2} \times 892.86 \times 8.314 \times 273$$

$$\text{Energ } u = 3.03 \times 10^6 \text{J}$$

3.1. First Law Of Thermodynamics

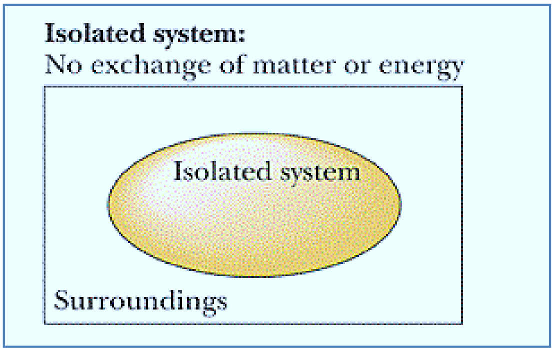
When the heat is supplied an isolated thermodynamic system the internal energy is produced in the gas which will cause the rise of pressure of the piston and the load will be raised up. Considering the given system below.



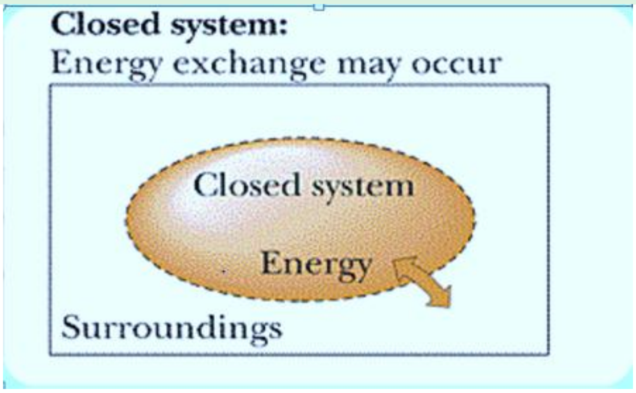
Any thermodynamics system in an equilibrium state possesses a state variable called an **Internal energy.** Between any two equilibrium states, the changes in internal energy is equal to the difference of the heat transfer into the system and work done by the system.

Isolated system is thee system designed for a certain work, hence from here we can state the first law of thermodynamics

- Isolated systems Are the one in which no interaction between system and the surroundings.
- Are completely isolated from their environment.
- They do not exchange heat, work or matter with their environment.
- An example is a completely insulated rigid container, such as a completely insulated gas cylinder.

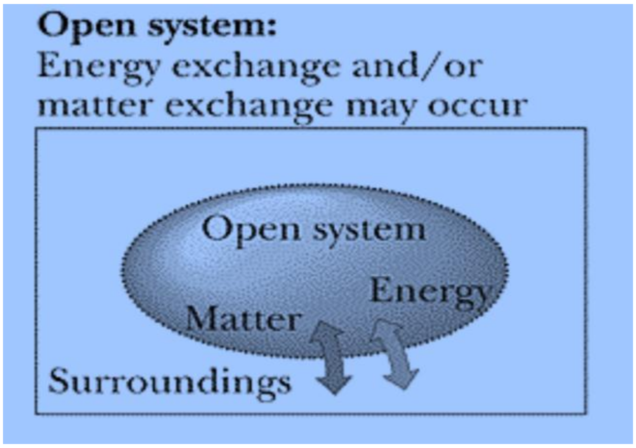


Closed systems are able to exchange energy (heat and work) but not matter with their environment. A greenhouse is an example of a closed system exchanging heat but not work with its environment.



1. **Open systems** may exchange any form of energy as well as matter with their environment (from the system to surroundings or vice versa).

2. A boundary allowing matter exchange is called permeable. The ocean would be an example of an open system and most of the engineering devices are open system**.**

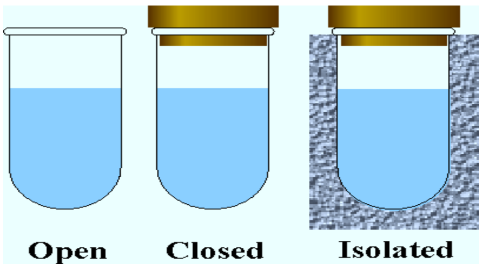


Summary of Open, closed and isolated system

Open systems can exchange both matter and energy with the environment.

Closed systems exchange energy but not matter with the environment.

Isolated systems can exchange neither energy nor matter with the environment.



Internal energy

1. **Internal energy** is defined as the energy associated with the random, disordered motion of molecules.

2. It is separated in scale from the macroscopic ordered energy associated with moving objects; it refers to the invisible microscopic energy on the atomic and molecular scale.

3. The internal energy is the total energy contained in a thermodynamic system.

- It is the energy necessary to create the system, but excludes the energy associated with a move as a whole, or due to external force fields.
- Internal energy has two major components, kinetic energy and potential energy.

First law of thermodynamics

The first law of thermodynamics is the application of the conservation of energy principle to heat and thermodynamic processes:

Heat added to the thermodynamic system goes to change the internal energy and to do the work by the system.

The law state that *"When heat is supplied to the isolated thermodynamic system it will be equal to the sum of internal energy and external work done of the system".*

$$\text{mathematically}$$

$$Q = U + w$$

$$\text{where}$$

$$Q = \text{Heat supplied to the system}$$

$$w = \text{workdone}$$

$$U = \text{internal energy}$$

$$\text{since } dQ = du + dw$$

$$\text{but } dw = pdv$$

$$du = \frac{3}{2}nRT$$

The internal energy of a system can be changed by heating the system or by doing work on it.

- If the system is isolated, its internal energy cannot change.
- An isolated system which does not interact with the surroundings Q=0 and W=0. Therefore, E remains constant for such a system.

- If we apply the first law of thermodynamics to the human body (Human Metabolism):
- we know that the body can do work. If the internal energy is not to drop, there must be energy coming in. It isn't in the form of heat; the body loses heat rather than absorbing it. It is the chemical potential energy stored in foods.

3.2. Molar Heat Capacity

Is the amount of heat required to raise the temperature of one mole of a gas through one Kelvin $C = \frac{\text{Heat supplied}}{\text{no of moles}\times\Delta T}$

$$C = \frac{Q}{ndT}$$

$$\text{The SI} - \text{unit of molar heat capacity } {}^{J}/_{molK}$$

$$\text{then heat supplied} = nCdt$$

There are two molar principle heat capacities namely

- Molar heat capacity at constant volume (C_v)
- Molar heat capacity at constant pressure (C_p)

3.2.1. Molar Heat Capacity At Constant Volume (C_V)

Is the heat required to raise the temperature of the mole of a gas through 1k at a constant volume

Molar heat capacity at constant pressure (C_p) is the amount of heat required to raise a temperature of of one mole of a gas through 1k at constant pressure

3.3. Meyer's Equation

Meyer's equation is given by the relation

$$C_p - C_v = R$$

The absolute difference between molar heat capacity at constant pressure and at constant volume is equal to the numerical value Universal gas constant (R)

$$\text{Proof}$$

$$\text{consider the expansion of gas at constant}$$

$$\text{pressure and volume in isolated system}$$

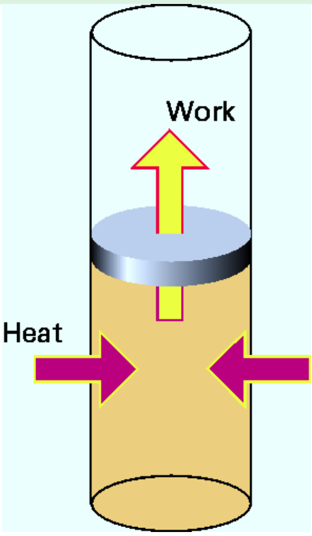
$$\textbf{case 1}$$

$$\text{At constant volume for nnumberof of moles or gas in cointainer which}$$

$$\text{does not expand}$$

$$\text{hence in this case } dv = 0$$

$$\text{consider the figure below}$$



$$dw = pdv$$

$$\text{but } dv = o$$

$$Q = nCdT$$

$$dQ = nC_v dT$$

From the first law of thermodynamic

$$Q = U + w$$

$$dQ = du + dw$$

$$nC_v dT = du + pdv$$

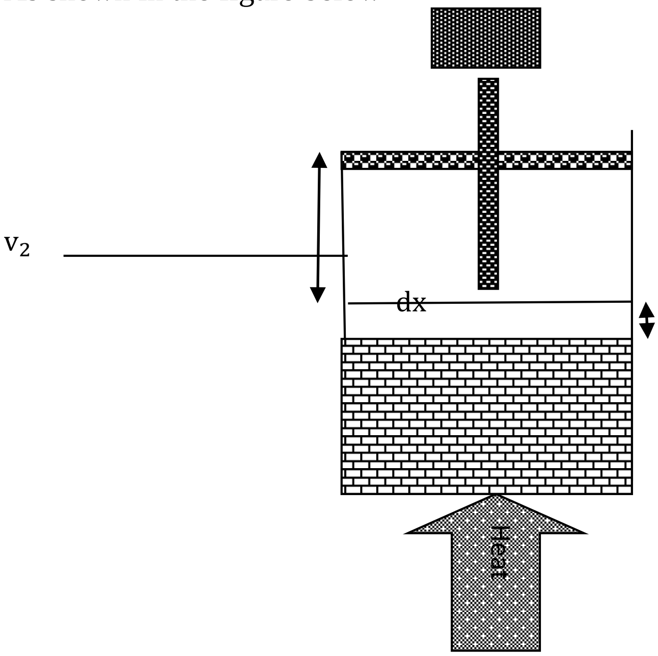
$$nC_v dT = du + 0$$

$$nC_v dT = du - - - - - (i)$$

case 2

consider a gas expand at constant pressure

As shown in the figure below



The volume will change from v_1 to v_2 (there is change in volume)

$$dw = pdv$$

$$dQ = nC_pdT$$

from 1st law of thermodynamics

$$dQ = du + pdv - - - - - (ii)$$

substitung into eqn (i)into (ii)

$$nC_vdT = du$$

$$nC_pdT = nC_vdT + pdv$$

But from ideal gas

$$pv = nRT$$

differentiating equation with respect to T

$$\frac{pdv}{dT} = \frac{nRdT}{dT}$$

$$pdv = nRdT$$

plug eqn iii

$$nC_pdT = nC_vdT + nRdT$$

$$nRdT = nC_pdT - nC_vdT$$

$$nRdT = ndT(C_p - C_v)$$

concelling ndT both side we get

$$R = C_p - C_v$$

where

C_p = molar heat capacity at constant pressure

C_v = molar heat capacity at constant volume

R = Universal gas constant

3.3.1. Ratio Of Molar Heat Capacity At (γ)

Consider this ratio for gases classified as

- Monotonic gases example Helium, Argon, Krypton
- Diatonic gases example Oxygen, water, dintrogen
- Polyatomic gases example Ammonia, Carbon dioxide, water, methane gas

Hence this ratio sometimes is called atomicity

Atomicity is the ratio of molar heat capacity at constant pressure over molar heat capacity at constant volume

mathematically

$$\gamma = \frac{C_p}{C_v}$$

for monotonic gases

$$\gamma = 1.67$$

For diatomic gas

$$\gamma = 1.4$$

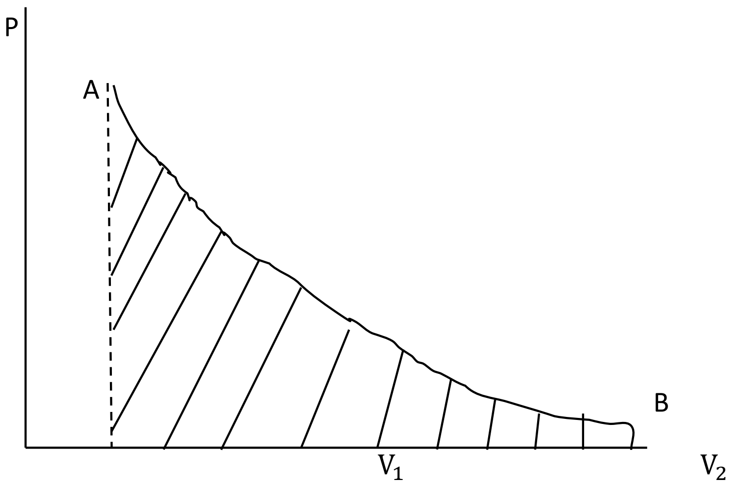
For triatomic

$$\gamma = 1.3$$

3.4. P – V CURVES

These are curves which show the relationship between pressure and volume in isolated system

Example



Under P-V curve the area represented in the curve is equal to the total work done(dw)

3.5. Thermodynamics Processes:

There are processes on ideal gases which occur continuously namely:

a) Isothermal process

b) Isobaric process

c) Isochoric/isovolumetric process

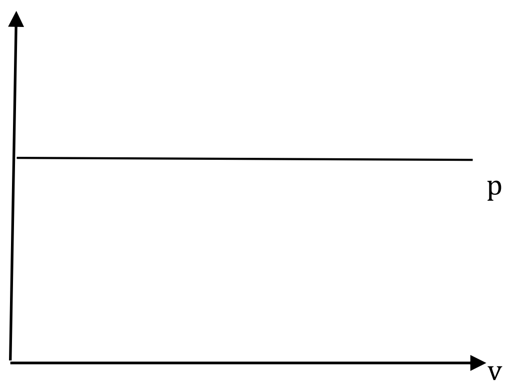
d) Adiabatic process

3.5.1. Isobaric Process

Is a thermodynamic process which occur at constant pressure

Example

In $p - v$ curve it is represent y straight line and it measure the line along the $x - $ axis



the workdone in this proces

from

$$dw = pdv$$

intergrating from v_1 to v_2

$$\int_0^w dw = \int_{v_1}^{v_2} pdv$$

$$w.d = p\int_{v_i}^{v_f} dv$$

$$w.d = p[v]_{v_1}^{v_2}$$

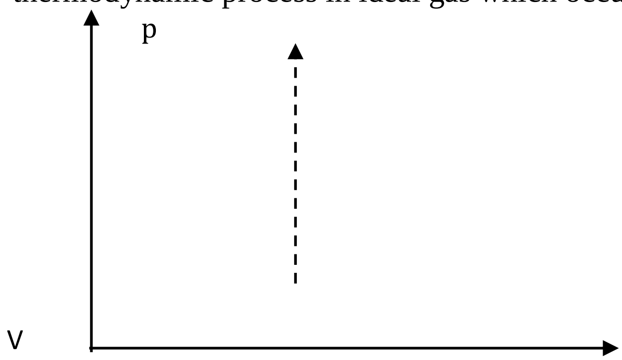
$$w.d = p(v_2 - v_1)$$

hence the workdone in this process

is $\boxed{w.d = v_2 + v_1}$

3.5.2. Isochoric Process

Is any thermodynamic process in ideal gas which occur at constant volume



this process obey pressure law

since workdone in this process is zero

because $dv = 0$

$$w.d = pdv$$

$$w.d = p \times 0$$

$$w.d = 0J$$

3.5.3. Isothermal Process

Is a thermodynamic process which occur at constant temperature Or Is a thermodynamic process which obey Boyle's law

$$pv = k$$

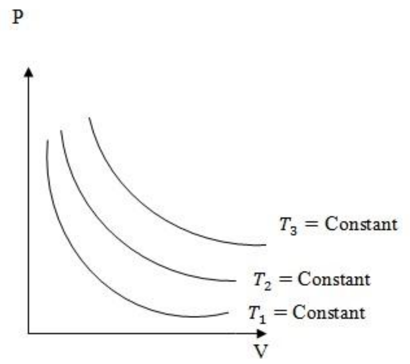
In such a process heat is, if necessary, supplied or removed from the system at just the right rate to maintain constant Temperature.

Conditions for isothermal change

(1) The gas must be held in a thin-walled, highly conducting vessel, surrounded by a constant temperature bath.
(2) The expansion or contraction must take place slowly. So that the heat can pass in or out to maintain the temperature of the gas at every instant during expansion or contraction.

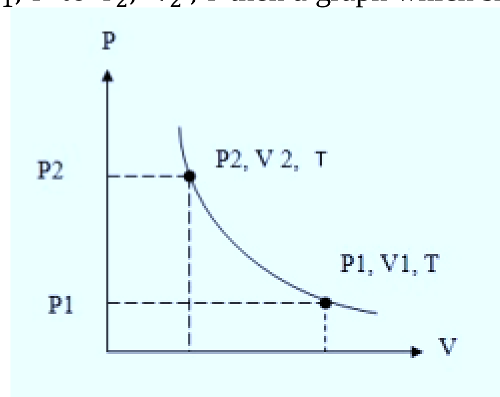
Isothermal change represented graphically

When the temperature is constant the pressure of a gas varies with volume and a graph which shows this variation is a curve known as isothermal curve.



Where $T_3 > T_2 > T_1$

This graph is also called PV - curve or PV – Indicator diagram
When a gas expands, or is compressed, at constant temperature, its pressure and volume vary along the appropriate isothermal, and the gas is said to undergo an isothermal compression..expansion
Isothermal reversible change When the gas is compressed isothermally from P_1, V_1, T to P_2, V_2 , T then a graph which show this variation is:



If the gas is allowed to expand isothermally so that the state of the gas is brought back from (P_2, V_2, T) through exactly the same intermediate stage then the gas is said to undergo isothermal reversible change:
Definition

An isothermal reversible change is that change which goes to and from through exactly the same intermediate stages at constant temperature.

Isothermal reversible change equation

Since the temperature is constant, and is isothermal change obeys Boyle's law.

PV = constant.

$\therefore$ $P_1V_1 = P_2V_2$ Isothermal reversible change equation

the workdone in isothermal process

workdone in this process

$$p_1v_1 = p_2v_2 = p_3v_3 = k$$

let

$$pv = nRT$$

$$\text{if } n = 1$$

$$pv = RT - - - - - (i)$$

$$\text{but w.d} = pdv - - - -(ii)$$

from equation (i)

$$p = \frac{RT}{v}$$

substitute into eqn

$$\text{w.d} = pdv$$

$$dw = \left(\frac{RT}{v}\right)dv$$

integration the equation

$$\int_0^w dw = RT\int_{v_1}^{v_2}\frac{dv}{v}$$

$$\text{w.d} = RT[\ \text{Inv}\ ]_{v_1}^{v_2}$$

$$\text{w.d} = RT(\text{Inv}_2 - \text{Inv}_1)2$$

$$\text{w.d} = RT\ \text{In}\left(\frac{v_2}{v_1}\right)$$

if will be given

$$p_1\&p_2$$

$$p_1v_1 = p_2v_2$$

$$\frac{p_1}{p_2} = \frac{v_2}{v_1}$$

mathematically

$$\text{w.d} = RT\ \text{In}\left(\frac{p_1}{p_2}\right)$$

Example 40 :

Calculate amount of heat necessary to raise a temperature of 2moles of the gas from 20°C to 50°C at

a) Constant volume
b) Constant pressure

solution

$$R = 8.314$$

$$C_p = 2.49R$$

$$C_v = 1.5R$$

(i)at constant volume

$$Q = nc_v dT$$

$$n = 2\text{moles}$$

$$C_v = 1.5R$$

$$R = 8.314$$

$$Q = 2 \times 1.5 \times (50 - 20)$$

$$Q = 3 \times 30 \times 8.314$$

$$Q = 90 \times 8.314$$

$$\text{Heat supplied} = 748\text{J}$$

(ii)At constant pressure

$$Q = nC_p dT$$

$$Q = 2 \times 2.49 \times 8.314 \times 30$$

$$Q = 1242\text{J}$$

$$\text{heat supplied} = 1242\text{J}$$

Example 41:

One gram of water ($1cm^3$) become $1671cm^3$ steam at pressure oof 1atm (1.013×10^5)pa . The Latent heat of vaporization at this pressure is $2256Jg^{-1}$. Calculate

a) External work
b) Internal energy

$$\text{solution}$$

$$v_1 = 1cm_3 = 1 \times 10^{-6}m^3$$

$$v_2 = 1671 \times 10^{-6}m^3$$

$$p = 1.013 \times 10^5 pa$$

$$L_v = 2256Jg^{-1}$$

$$\text{mass} = 1g$$

$$\text{workdone} = pdv$$

$$w.d = 1.013 \times 10^5 \times (1671 - 1) \times 10^{-6}$$

$$w.d = 169J$$

$$\text{hence the external workdone} = 169J$$

$$\text{(ii)internal energy}$$

$$\text{from first law of thermodynamics}$$

$$dQ = mL_v$$

$$Q = 1g \times \frac{2256J}{g^{-1}}$$

$$Q = 2256J$$

$$\text{from first law of thermodynamics}$$

$$dQ = du + dw$$

$$Q = u + w$$

$$u = Q - w$$

$$u = 2256 - 169$$

$$u = 2087$$

$$\text{The internal energy} = 2087J$$

Example 42:

Determine the amount of work done when a system containg Determine the amount of work done when a system containg $1cm^3$of a liquid water at 0℃ free under constant pressure of 1.0atm and forms $1.1cm^3$ of ice and name the process formed .

The process is called isobaric process

solution

$$v_i = 1cm^3 = 1.0 \times^{10} - 6m^3$$

$$v_f = 1.1\ cm^3 = 1.1 \times 10^{-6} m^3$$

$$p = 1.013 \times \frac{10^5 N}{m^2}$$

from workdone

$$w.d = pdv$$

$$w.d = p(v_f - v_i)$$

$$w.d = 1.013 \times 10^5 (1.1 \times 10^{-6} - 1.0 \times 10^{-6})$$

$$\text{workdone} = 0.1\ J$$

3.5.4. Adiabatic Process

Adiabatic is a thermodynamic process which occur when there is no heat enter or leave the system (i.e heat is constant). An adiabatic change is the change which takes place without exchange of heat between inner and outer of the system. It is the one which takes place at constant heat.

In general, an adiabatic change involves a fall or rise in temperature of the system.

Condition for adiabatic change

No heat is allowed to enter or leave the gas.

therefore.

- The gas must be held in a thick – walled, badly, conducting vessel.
- The change in volume must take place rapidly to give as little time as possible for heat to escape.

Examples of adiabatic process/change

- The rapid escape of air from a burst Tyre.
- The rapid expansions and contractions of air through which a sound wave is passing.

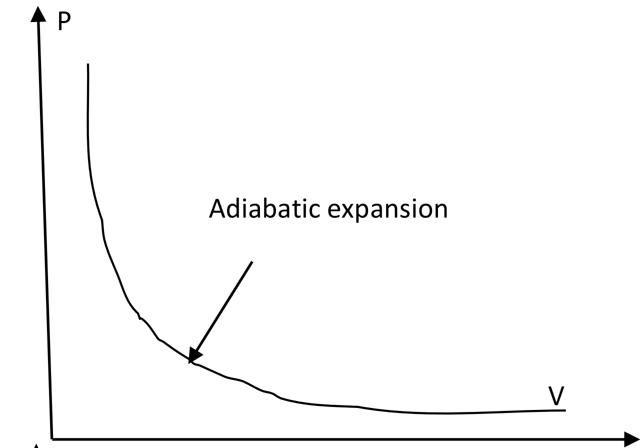
Adiabatic change represented graphically

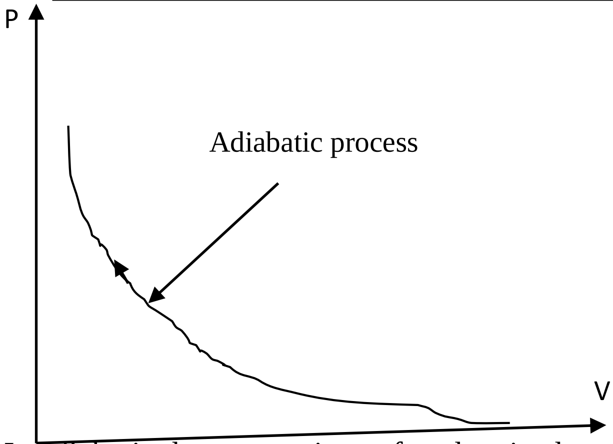
A curve which relates the pressure and volume when the heat content of the gas is kept constant is called an adiabatic. Adiabatic curves and isothermal curves are similar except that adiabatic are steeper than isothermals. If the gas is compressed adiabatically from volume V_o to volume V_1 its temperature rises to T_2 so that its new position is P_1V_1 on the new isothermal. Similarly, if the gas is left to expand adiabatically from volume V_o to volume V_2 its temperature is lowered to T_1 so that its new position is ( ) on P_2V_2 the new isothermal

Adiabatic reversible change

An adiabatic reversible change is the change which goes to and fro through exactly the same intermediate stages without exchange of heat between inner and out of the system.

In this process all vaiable pressure, volume, temperature are changing





InAdiabatic the process is too fast than isothemal process

Hence the equation of adiabatic process

$$pv^{\gamma} = k$$

3.6. Differences Between Adiatic Process And Isothermal Process

ISOTHERMAL PROCESS	ADIABATIC PROCESS
Temperature remain constant	Heat is constant
The process occurs slowly	The process occur suddenly (faster)
Obey the equation of boyle's law	Obey the equation of $PV^{\gamma} = k$
At any point on the isothermal process the slope is less than that of adiabatic process	At any point the slope is larger than isothermal at the same poin.

Derivation

$$pv^{\gamma} = k$$

from first law of thermodynamic

$$Q = U + w$$

$$dQ = dU + dw$$

$$\text{But } dw = pdv$$

$$dQ = dU + pdv$$

but for adiabatic process heat is constant = 0

$$0 = dU + pdv$$

$$dU = -pdv$$

$$dU = nc_v dT$$

$$0 = nc_v dT + pdv$$

from ideal gas equation

$$pv = nRT$$

differentiate w.r.t. T

$$\frac{pdv}{dT} + \frac{vdp}{dT} = nR$$

$$pdv + vdp = nRdT$$

$$dT = \frac{pdv + vdp}{nR} ----(i)$$

substituting to the equation

$$0 = nc_v dT + pdv$$

$$0 = nc_v\left(\frac{pdv + vdp}{nR}\right) + pdv$$

$$0 = c_v\left(\frac{pdv + vdp}{R}\right) + pdv$$

$$-pdv = c_v\left(\frac{pdv + vdp}{R}\right)$$

hence from Meyer's equation

$$R = c_p - c_v$$

$$-pdv = \frac{c_v pdv + c_v vdp}{c_p - c_v}$$

$$-pdv\left(c_p - c_v\right) = p\, c_v pdv + c_v vdp$$

$$-\frac{c_p pdv}{pv} = \frac{c_v vdp}{pv}$$

$$-\frac{\frac{c_p dv}{c_v}}{v} = \frac{\frac{c_v dp}{c_v}}{p}$$

$$\frac{dp}{p} = -\frac{\left(\frac{c_p}{c_v}\right)dv}{v}$$

$$\text{but } \frac{c_p}{c_v} = \gamma$$

$$-\frac{\gamma dv}{v} = \frac{dp}{p}$$

$$\frac{dp}{p} + \frac{\gamma dv}{v} = 0$$

$$\int \frac{dp}{p} + \int \frac{\gamma dv}{v} = \int 0$$

$$\text{Inp} + \gamma\, \text{Inv} = c$$

$$\text{Inp} + \text{Inv}^{\gamma} = c$$

$$\text{In}\,(pv^{\gamma}) = c$$

$$\log_e pv^{\gamma} = c$$

$$pv^{\gamma} = e^{c}$$

$$pv^{\gamma} = K$$

Hence proved

since for ideal gas

$$pv = \text{Rt for 1mole}$$

$$p = \frac{RT}{v}$$

$$K = \frac{RTv^{\gamma}}{V}$$

also

$$\frac{pv}{p} = \frac{RT}{P}$$

$$v = \frac{RT}{P}$$

$$\frac{K}{R} = \frac{RTV^{\gamma}}{RV}$$

$$K = TV^{\gamma-1}$$

And the expression for the workdone

$$dw = pdv$$

$$\text{But} \quad pV^{\gamma} = k$$

$$p = \frac{k}{V^{\gamma}}$$

$$dw = \left(\frac{k}{v^{\gamma}}\right)dv$$

integrating both sides

$$\int_0^w dw = \int_{v_1}^{v_2} \frac{k}{V^{\gamma}} dv$$

$$\text{workdone} = k\int_{v_1}^{v_2} \frac{dv}{v^{\gamma}}$$

$$w.d = k\int_{v_1}^{v_2} V^{-\gamma+1} dv$$

$$w.d = k\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{v_1}^{v_2}$$

$$w.d = \frac{k}{-\gamma+1}\left(V_2^{-\gamma+1} - V_1^{-\gamma+1}\right)$$

but

$$PV^{\gamma} = k$$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} = k$$

$$w.d = \frac{1}{1-\gamma}\left(kV_2^{1-\gamma} - kV_1^{1-\gamma}\right)$$

$$w.d = \frac{1}{1-\gamma}\left(P_2V_2^{\gamma}V_2^{1-\gamma} - P_1V_1^{\gamma}V_1^{1-\gamma}\right)$$

$$w.d = \frac{1}{1-\gamma}(P_2V_2 - P_1V_1)$$

but

$$pv = Rt$$

$$P_1V_1 = RT_1$$

$$P_2V_2 = RT_2$$

$$w.d = \frac{1}{1-\gamma}(RT_2 - RT_1)$$

$$w.d = \frac{R}{1-\gamma}(T_2 - T_1)$$

3.7. Application Of First Law Of Thermodynamic

- It is used to show the relationship between molar heat capacity at constant pressure and molar heat capacity at constant volume (C_p and C_v)
- **Boiling process**. Suppose a liquid of mass, m vaporizes at a constant pressure P. Its volume in liquid state is V_1 and its volume in the vapor state is V_v. the work done in the expansion and change in internal energy of the system. Since expansion takes place at constant pressure, the work done by the system is given by

$$w = \int_{v_1}^{v_2} pdv$$

$$w = P\int_{v_1}^{v_2} dv = p(v_2 - v_1)$$

- **Melting process**. When a solid changes into liquid state, its internal energy increases. This can be calculated from the first law of thermodynamics.

$$\text{Let } m = \text{mass of the solid}$$

$$L = \text{specific latent heat of fusion}$$

$$\text{heat absobed during melting process}$$

$$Q = mL$$

- **Isobaric process**. A process that occurs at constant pressure is called isobaric process. Suppose a gas expands at constant pressure

$$Q = \Delta U + w$$

In this case, the heat added increases the internal energy of the gas as well as the gas does external work.

- **Isochoric process**. A process that occurs at constant volume is called isochoric process, work done is zero because there is no change of volume.

We conclude that, if the heat is added to a system at constant volume, all the heat goes into increasing the internal energy of the system.

3.8. Limitation Of First Law Of Thermodynamics

The first law of thermodynamics based on the conservation of energy. any process which would violet it cannot occur. The first law of thermodynamics failed to the following

- The first law of thermodynamics does not indicate the direction of transfer of heat.
- The first law does not tell anything about the conditions under which heat can be converted into work.
- The first law does not tell us why the whole heat energy cannot be converted into mechanical work continuously.

3.9. Worked Examples Set 04a:

Example 43:

The ideal gas at 17°C has a pressure of 760mmHG and is compressed.

i. Isothermally
ii. Adiabatically

Until its volume halved in each case reversibly. Calculate the final presure and temperature of the gas assuming that $c_p = 2100$ and $c_v = 1500$

solution

$$\text{Given } T_1 = 17°C$$

$$P_1 = 760\text{mmHG}$$

$$v_i = v$$

$$v_2 = \frac{v}{2}$$

$$\textbf{(i)Isothermally}$$

$$P_1V_1 = P_2V_2$$

$$P_2 = \frac{P_1V_1}{V_2}$$

$$P_2 = \frac{760 \times V}{V/2}$$

$$P_2 = 760 \times 2$$

$$P_2 = 1520\text{mmHg}$$

For isothermally the temperature is constant

hence $T_2 = 17°C = 290k$

(ii)Adiabatically

final temperature

from $PV^{\gamma} = k$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} = k$$

But atomicity $(\gamma) = \frac{C_p}{C_v}$

$$(\gamma) = \frac{2100}{1500}$$

$$\gamma = 1.4$$

$$P_2 = \frac{P_1V_1^{\gamma}}{V_2^{\gamma}}$$

$$P_2 = P_1[\frac{V_1}{V_2}]^{\gamma}$$

$$P_2 = 760(\frac{V}{V/2})^{1.4}$$

$$P_2 = 760\ (2)^{1.4}$$

$$P_2 = 2005\text{mmHG}$$

The final temperature for adiabatic process

from $TV^{\gamma-1} = k$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} = k$$

$$T_2 = \frac{T_1V_1^{\gamma-1}}{V_2^{\gamma-1}}$$

$$T_2 = T_1(\frac{V_1}{V_2})^{\gamma-1}$$

$$T_2 = 290(\frac{V}{V/2})^{\gamma-1}$$

$$T_2 = 290(2)^{1.4-1}$$

$$T_2 = 290 \times 2^{0.4}$$

$$T_2 = 382.65k$$

Hence the final temperature $T_2 = 382.65k$

Example 44:

A gas is suddenly compressed to 1/3 to its volume. Compute the rise in temperature if original temperature is 27°C. Use atomicity $(\gamma) = 1.5$

solution

$$v_i = v$$

$$v_f = \frac{1}{3}v$$

$$T_1 = 27°C = 300k$$

required to calculate T_2

from $TV^{\gamma-1} = k$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} = k$$

$$T_2 = T_1(\frac{v_i}{v_2})^{\gamma-1}$$

$$T_2 = 300(\frac{v}{v/3})^{\gamma-1}$$

$$T_2 = 300\ (3)^{1.5-1}$$

$$T_2 = 519.62k$$

hence the rise in temperature

$$dt = 519.62 - 300$$

$$dt = 219.62k$$

Example 45:

A quantity of perfect gas at 15°C is compressed adiabatiacally to one fourth of its volume. Calculate the final temperature use $\gamma = 1.4$

$$T_1 = 15°C = 288k$$

$$v_1 = v$$

$$v_2 = \frac{1}{4}v$$

$$\gamma = 1.4$$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T_2 = T_1(\frac{v_1}{v_2})^{\gamma-1}$$

$$T_2 = 288(\frac{v}{\frac{v}{4}})^{\gamma-1}$$

$$T_2 = 288(4)^{1.4-1}$$

$$T_2 = 501.4k$$

The final temperatute of the system is 501.4k

Example 46:

Ten moles of water gas at N.T.P are compressed adiabatically so that its temperature become 400°C . Calculate

i. The work done by the gas
ii. The increase in internal energy of the gas given that ($R = 8.314$ and $\gamma = 1.4$)

solution

$$T_1 = 273k$$

$$T_2 = 400 + 273 = 673k$$

$$\gamma = 1.4$$

$$R = 8.314$$

$$\text{from } w.d = \frac{1}{1-\gamma}(nRT_2 - nRT_1)$$

$$w.d = \frac{nR}{1-\gamma}(T_2 - T_1)$$

$$w.d = \frac{10 \times 8.314}{1-1.4}(673 - 273)$$

$$w.d = \frac{81.34}{-0.4}(400)$$

$$w.d = -8.314 \times 10^4 J$$

the negative sign means the work was done on

the gas

(ii)the increase in internal energy

fron fisrt law of thermodynamics

$$dQ = du + dw$$

but for adiabatic process $dQ = 0$

$$du + dw = 0$$

$$du = -dw$$

$$du = -(-8.314 \times 10^4)$$

$$du = 8.314 \times 10^4 J$$

hence the internal energy $= 8.314 \times 10^4 J$

3.10. Worked Examples Set 04b

Example 1:

1kg of water at 373K is converted into steam at same temperature. Volume of $1cm^3$ of water become $1671cm^3$ of boiling. What is change in the internal energy of the system if the latent heat of vapourization of water is $5.4 \times 10^5 cal/kg$? **[MPPTET 2002]**

solution

volume of 1kg of water

$$= 1000cm^3 = 10^{-3}m^3$$

volume of 1kg of steam $10^3 \times 1671cm^3 = 1.671m^3$

change in volume $dV = (1.671 - 10^{-3})m^3$

$$dV = 1.670m^3$$

pressure P $0.76 \times 13600 \times 9.8Nm^{-2}$

$\therefore$ density of mercury $= 13600kgm^{-3}$

since $dw = PdV$

$$dw = 0.76 \times 13600 \times 9.8 \times 1.67$$

$$dw = 1.691 \times 10^5 = \frac{1.691 \times 10^5}{4.2} cal$$

$$dw = 4.026 \times 10^4 cal$$

$$\text{but} \quad dU = dQ - dw$$

$$dU = 5.4 \times 10^5 - 0.4026 \times 10^5 \text{cal}$$

$$dU = 4.9974 \times 10^5 \text{cal}$$

Example 2:

1mole of an ideal gas undergoes a cyclic change ABCD. Calculate the net work done in the process. Given that 1atm = 10^6dynes/cm^2

solution

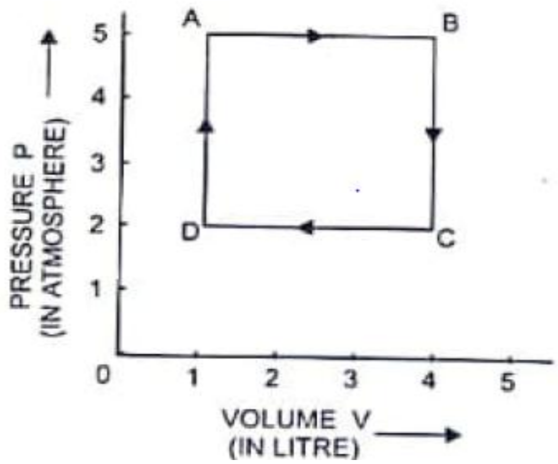
Since the loop ABCD is traced in the clockwise direction therefore the work done is positive.

$$\text{work done} = +\text{area of ABCD}$$

$$W = DC \times DA$$

$$= (4 - 1)\text{litre} \times (5 - 2)\text{atm}$$

$$= 3\text{litre} \times 3\text{atm}$$



$$w = 3 \times 10^3 \times 3 \times 10^6$$

$$\text{workdone} = 9 \times 10^9 \text{ erg.}$$

Example 3:

Consider the figure shown below, when a system is taken from state B along the path ACB, 80kcal of heat flows into the system and 30kcal of work done.

(i) How much heat flows into the system along path ADB, if the work done is 10kcal.
(ii) When the system is returned from B to A along the curved path, the work done is 20kcal. Does the system absorb or liberate heat and how much?
(iii) If $U_A = 0$ and $U_D = 40$kcal, find the heat absorbed in the processes AD and DB.

solution

Heat flowing into the system when it is taken from A to state B along the path ACB.

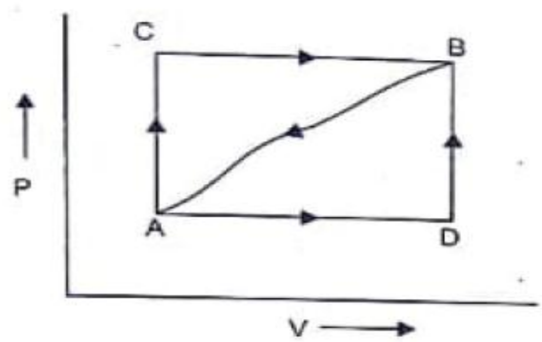
$dQ_{ACB} \;=\; +80\text{kcal}$ positive sign indicate the heat flows into the system

Corresponding work done

$dQ_{ACB} \;=\; +30\text{kcal}$, positive sign taken because work is done by the system

Let the change in internal energy along the path ACB be dU_{ACB}

$$\text{now} \quad dQ_{ACB} \;=\; dU_{ACB} \;+\; dw_{ACB}$$



$$dU_{ACB} \;=\; dQ_{ACB} - dW_{ACB}$$

$$dU_{ACB} \;=\; (80 \;-\; 30)\text{kcal}$$

$$dU_{ACB} \;=\; 50\text{kcal}$$

Example 4:

A certain gas at atmospheric pressure is compressed adiabatically so that its volume becomes half of its original volume. Calculate the resulting pressure in dynescm^{-2}. Given that $\gamma = 1.4$

solution

$$\text{let initial volume } V_1 \;= V$$

$$\text{final volume be } V_2 \;=\; V$$

$$\text{initial pressure } P_1 \;= 76\text{cmHg}$$

$$= 76 \times 13.6 \times 981\text{dyne cm}^{-2}$$

$$\text{final pressure } P_2 \;=\; ?$$

consider the equation for adiabatic expansion

$$P_1 V_1^{\gamma} \;=\; P_2 V_2^{\gamma}$$

$$\text{such that } P_2 \;=\; P_1 (\frac{V_1}{V_2})^{\gamma}$$

$$P_2 = P_1\left(\frac{V}{\frac{V}{2}}\right)^{1.4}$$

$$P_2 = 76 \times 13.6 \times 981 \times 2^{1.4}\ \text{dyne cm}^{-2}$$

$$P_2 = 2.7 \times 10^6\ \text{dyne cm}^{-2}$$

Example 5.

A motor tyre pumped to a pressure of 3 atm suddenly bursts. Calculate the fall in the temperature due to adiabatic expansion. The temperature of air before expansion is 27°C. Given that $\gamma = 1.4$

solution

initial pressure $P_1 = 3\text{atm}$

initial temperature $T_1 = 27 + 273 = 300\text{K}$

final preesure $P_2 = 1\ \text{atm}$

Final temperature let be T_2

fall in temperature $= T_1 - T_2$

form the relation

$$T_1^{\gamma} P_1^{1-\gamma} = T_2^{\gamma} P_2^{1-\gamma}$$

$$\left(\frac{T_2}{T_1}\right)^{\gamma} = \left(\frac{P_1}{P_2}\right)^{1-\gamma}$$

$$\left(\frac{T_2}{300}\right)^{1.4} = \left(\frac{3}{1}\right)^{1-1.4}$$

$$\left(\frac{T_2}{300}\right)^{1.4} = \left(\frac{1}{3}\right)^{0.4}$$

simplify and solve for T_2

$$T_2 = 219.2\text{K}$$

fall in temperature

$$T_1 - T_2 = 300 - 219.2$$

$$= 80.8\text{K}$$

Example 6:

A gram molecule of a gas at 127°C expands isothermally until its volume is doubled. Find the amount of work done and heat absorbed.

solution

$$\text{temperature } T = 127 + 273 = 400\text{K}$$

molar gas constant $R = 8.3 \times 10^7$ erg/mol °C

$$\text{let initial volume } V_1 = V$$

$$\text{let final volume } V_2 = 2V$$

$$\text{the work done } W_{iso} = RT\ In\left(\frac{V_2}{V_1}\right)$$

$$W_{iso} = 8.3 \times 10^7 \times 400 \times In\left(\frac{2V}{V}\right)$$

$$w_{iso} = 2.301 \times 10^{10}\text{erg}$$

$$\text{heat absorbed } Q = \frac{w_{iso}}{J}$$

$$Q = \frac{2.301 \times 10^{10}}{4.2 \times 10^7}$$

$$Q = 547.9\text{cal}$$

Example 7:

Why the brake drum of an automobile gets heated up when the automobile moves down a hill at constant speed.

- Since the speed is constant therefore there is no change kinetic energy. The loss in gravitational potential energy is partially the gain in the heat energy of the brakes drum.

Example 8

Why water at the base of waterfall is slightly warmer than at the top?

- The potential energy at the top gets converted into kinetic energy at the bottom. The kinetic energy gets partially converted into heat energy when the water hits the ground. Consequently, there is slightly increase in the temperature of water.

END OF TOPIC CONCEPTUAL QUESTIONS

- Which is the only way of heat transfer through a solid?
  Ans.
  Conduction.
- Why are steam pipes wrapped with insulating material? OR Tea pots are generally covered with tea – cosy made of felt or wool. Why?
  Ans.

So as to minimize the loss of loss heat due to radiation.

- The roof of buildings are often painted white during summer. Why?
  Ans.
  When painted white, the roofs will absorb very small amount of heat and the house will become hot in summer
- Why do two layers of cloth of equal thickness provide warmer covering than a single layer of cloth of double the thickness? OR Why two think blankets are warmer than a single blanket of double the thickness?
  Ans.
  When we use two thin blankets, air gets trapped between them. Since air is poor conductor of heat, two thin blankets prove warmer than a single blanket of double thickness.
- Why felt is used for thermal insulation in preference to air? OR If air is bad contactor of heat, why do we not feel warm without clothes?
  Ans.
  No doubt, air is bad conductor of heat. But if we are without clothes, air carriers away heat from our body due to convention and hence we feel cold.
- Is it possible than when one end of a rod is heated; then after some time, the temperature of the entire rod becomes the some? Under what conditions would this happen?
  Ans.
  No, the temperature of the entire rod cannot become the same. However, if the rod is covered with non-contacting material, the temperature of the entire rod will become same, when the steady state is reached.
- In which method of heat transfer, gravity does not play any party?
  Ans.
  Conduction and radiation.
- In which process of heat transfer, mass of the object heated does not play any party?
  Ans.
  Conduction and radiation.
- Why is it hotter at the same distance over the top of a fire than in front of it?
  Ans.
  At a point in front of the fire, heat is reserved due to process of radiation only, while at a point above the fire, heat reaches both due to radiation and conducting.
- Ventilator are provided in room just below the roof. Why?
  Ans.
  The air respired out is hotter and hence rarer. As such as, it rises up in the room. So that it can its way out of the room, the ventilators are provided just below the roof.

- A body is at 0°C. Is it radiating heat?
  Ans.
  Yes, even at 0°C, a body radiates heat.
- At what temperature does a body stop radiating?
  Ans.
  At 0 K.
- If kelvin temperature of an ideal black body is doubled, what will be the effect on the energy radiated by it?
  Ans.
  The energy radiated by a body is directly proportional to fourth power of its absolute temperature. As the temperature is doubled, the energy radiated will become (2) i.e.16 times.
- A blanket, which keeps us warm in the winter, is also able to protect ice from melting. Explain. OR A flannel keeps the ice cold but keeps people warm.
  Ans.
  A blanket only prevents the flow of heat due to the air trapped in the poles. In winter, it keeps us warm as heat cannot flow from our body to the surroundings. On the other hand, ice is saved from melting, as heat cannot flow from the surrounding to the ice.
- Birds often swell their feathers in winter. Why?
  Ans.
  By doing so, birds are able to form an envelope of air around their bodies. Since air is bad conductor of heat, it prevent heat from flowing their bodies to the surroundings. As a result, the birds do not feel cold.
- A marble floor appear colder than cemented floor in winter, though both are at the same temperature. Explain. Why. OR The iron chairs appear to be colder than the wooden chair in winter. Explain, why.
  Ans.
  When we touch a marble floor in winter, heat flows from our body to the marble floor quickly (as the thermal conductivity of marble floor is very high) and has a result, it appear colder. On the other hand, as the cemented floor has comparatively low thermal conductivity, heat does not flow that quickly to the cemented floor from our body on touching it.
- Piece of copper and glass are heated to the same temperature. Why does a piece of copper feel hotter on touching?
  Ans.
  When we touch the hot piece of cooper, heat readily flows from the piece of copper of our fingers, whereas it does not happen so in case of the hot piece of glass. It is because, copper is a better conductor of the heat than glass.
- Stainless steel cooking pans are preferred with extra copper bottom. Why?

Ans.

The coefficient of thermal conductivity of copper is quite large as compared to that of steel. When a cooking pan is fitted with extra cooper bottom, it will arrow more heat to flow into the pan, when paced over flame. It will result in faster cooking of the food.

- Water can be boiled in a paper cup. Explain.

  Ans.

  When a paper cap containing water is placed over burner, heat from the flame is taken it up by the water, before it can burn the paper cup. It is because, in comparison to paper, conductivity of water is quite high.

- A piece of paper wrapped tightly on a wooden rod is found to get charred, when held over a flame compared to similar piece of paper when wrapped around a brass rod. Why?

  Ans.

  When a piece of paper is wrapped tightly on a wooden rod and is held over flame, the heat reserved over the flame does not wholly pass to the wooden rod. It is because, wood is bad conductor of heat. As sufficient amount of heat is left with the paper, it gets charred.

  On the other hand, when a piece of paper is wrapped on a brass rod and is held over the flame, the heat from the flame immediately flows to the brass rod. It is because, brass is a good conductor of heat. Due to this fact, the paper does not get charred in this case.

- If a drop of water falls on a very hot iron, it does not evaporate for a long time. Explain, Why.

  Ans.

  When a drop of water falls on a very hot iron, it gets insulated from the hot iron due to the formation of the thin layer of water vapors, which is bad conductor in nature. It takes quite long to evaporate as heat is conducted from hot iron to the drop through the layer of water vapors very slowly. On the other hand, if a drop of water falls on an iron which is not very hot, then it comes in direct contact with the iron and evaporates immediately.

- A wire gauze is generally used, when a glass vessel is heated over a flame. Why?

  Ans.

  When the wire is used over the flame of a burner, it conducts away a lot of heat from the flame and the gas reaching above the gauze fails to acquire its ignition temperature. As such, no flame appears above the wire gauze. The glass vessel receives heat through the wire gauze uniformly instead of receiving directly from the flame. It saves the glass vessel from breaking due to the intense heating of a particular small portion of its bottom.

- A liquid is generally heated from below. Why?

Ans.

When a liquid is heated, it becomes rarer due to decrease in density and its rises up. The liquid from the upper part of the vessel comes down to take its place and thus convection currents are set up. If the liquid is heated at the top, no such convection currents will be set up and only the liquid in the upper part of the vessel will become hot. However, the temperature in the lower part of the vessel will rise slightly due to a small amount of heat conducted by the hot liquid in the upper part of the vessel.

- Why do lump black and platinum black serve as perfect black body only for absorption of heat radiation?

  Ans.

  A body coated with lump back (or platinum black) absorbs the hurt radiation of all the wavelength falling on it. But when a body is heated, it does not radiate heat radiation of all the wavelengths. Hence, a body coated with lamp black or platinum black serves as a perfect black body only for absorption of heat radiation.

- Is it necessary that all black colored objects should be considered black bodies?

  Ans.

  It is not necessary that all the black colored objects should be considered as a black bodies. For example, if we take a black surface but highly polished, it will not behave as a perfect black body. On the other hand, the sun, which is a shining hot sphere, behave as a perfect black body.

- If all objects radiate electromagnets radiation, why do not the objects around us in everyday life glow colder and colder?

  Ans.

  An object radiating heat also receives heat from the surrounding. A stage comes, when let of loss of heat from the objects becomes just equal to the rate at which it receives heat from the surroundings. At that stage, the temperature of the object becomes constant.

- Tea in thermos flask remains hot for a long time. Why?

  Ans.

  Thermos flask is a double walled glass bottle and the space between the two walls is evacuated. When hot tea is poured into the flask, heat cannot be conducted away due to vacuum between the two walls. Further, the walls of the flask are highly polished. Due to this, the outer surface of the inner will can radiate only a very little amount of heat. On the other hand, the highly polished inner surface of the outer walls reflects back the heat radiated by the outer surface of the inner wall. Due to this, tea remains hot in the flask for a long time

- Animals curl into a ball, when they feel cold.
  Ans.
  The total energy radiated by the body depends on its surface area. When the animals feel cold, they curl their bodies into a ball in order to decrease in surface area of their bodies. Due to decrease in surface area, the loss of heat due radiation also becomes small.
- The earth constantly reserves heat radiation from the sun and gets warmed up. Why does the earth not get as hot as the sun?
  Ans.
  Due to the large distance of the earth from the sun, it receives only a small part of the heat radiation radiated by the sun. Further, due to loss of heat from the surface of the earth due to the radiation and convention, the earth cannot maintain the temperature, it attains due to the heat received from the sun.
- Why the clear nights are colder than clouds nights? OR On a winter night you feel warmer, when clouds over the sky than when the sky is clear. Why?
  Ans.
  The surface of the earth absorbs sun rays during day time and gets heated. At night, the earth radiates heat but the clouds reflect the heat radiation back to the earth. Therefore, on a cloudy night, the heat radiation are reflected back to the surface of earth. However, when the sky is clear, the heat radiation escape the surface of the earth. For this reason, the cloudy nights are warmer than the nights, when the sky is clear.
- The cooling unit of a refrigerator is fitted near the top. Explain, why.
  Ans.
  As the air gets cooled in a refrigerator, it becomes denser and goes down. Therefore, when cooling unit is fixed near the top, the whole of the refrigerator will get cooled.
- Heat is generated continuously in an electric heater but its temperature remains constant after some time. Why?
  Ans.
  When heat is generated in an electric heater, its temperature continuously rises. The temperature of the electric heater rise in the beginning due to the reason that the rate of production of heat is greater than the rate at which heat is lost to the surroundings. At a particular temperature of the heater, the rate of production of the heat in the heater becomes equal to the rate at which heat is lost to the surroundings. Then, the temperature of the heater becomes constant.
- On a hot day, the surface water of a pond is warmer than the water below; but on a day when it is nearly freezing, the surface water is colder. Why?

Ans.

The density of water is maximum at 4℃ On a hot day, the temperature of whole of the water in the pond is more than 4℃. The warmer water, being rarer, reaches the top and hence on a hot day, water o the pond is warmer at the top. On the other hand, on an extremely day, when temperature is below 4℃, the nearly freezing water (having temperature just above 0℃, but below 4℃) being rarer, appears at the top but water having temperature close to 4℃, being heavier, goes down. Therefore, on a day, when it is nearly freezing, water of the pond is colder at the top than at the bottom.

- A body with large reflectivity is a poor emitter.

  Ans.

  A body with large reflectivity is a poor absorber of heat. Since poor absorbers of heat is poor emitters also, a body with large reflectivity is a poor emitter.

- A brass tumbler feel much colder than a wooden try on a chilly day.

  Ans.

  When we touch a brass tumbler on a chilly day heat flows from our body to the tumbler quickly (as thermal conductivity of brass is very high) and as a result, it appears colder. On the other hand, as the wood has very low thermal conductivity, heat does not flows to the wooden try form our body on touching it.

- An optical pyrometer (for measuring high temperature) calibrated for an ideal black body radiation gives too low a value for a temperature of a red hot iron piece on the open, but gives a correct value for the temperature, when the same piece is in the furnace.

  Ans.

  When a hot iron at temperature Tis in the furnace, the heat radiated per second per unit area E= $\sigma\ T^4$ when the hot body is placed in open, say at temperature T_0, then the heat radiated per second per unit area, $E' = \sigma\,(T^4 - T_0^4$ As $E' < E$,the optical pyrometer gives too low a value for the temperature of the hot iron piece in the open.

- The earth without its atmosphere would be inhospitably cold.

  Ans.

  The atmosphere does not allow the loss of heat from the surface of earth in the form of radiation. In other words, the heat radiation received by the earth from the sun during the day are kept trapped by the atmosphere. Therefore, the earth without its atmosphere would be inhospitably cold.

- Heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water.

  Ans.

Steam at 100°C possesses more heat than water at 100°C. The heat possessed by one gram of steam at 100°C is greater by an amount 540 calories than that possessed by 1 g of water at 100°C. For this reason, heating system based on circulation of steam are more efficient than those based on circulation of hot water.

- Is the bulb of a thermal of a thermometer made of diathermic or adiabatic wall?
  Ans.
  The bulb of a thermometer should attain the temperature of the body (whose temperature is to be measured) quickly, when brought in contact with it. If the bulb has an adiabatic wall, no transfer of heat energy will take place from the body to the bulb. On the other hand, the diathermic wall will quickly allow the transfer of the heat energy from the body the bulb. For this reason, the bulb of a thermometer is made of a diathermic wall.
- A student records the initial length l , change in temperature Δ T and change in length Δl of a rod as follows:

S.NO.	L (m)	ΔT (°C)	Δℓ (m)
1.	2	10	$4x10^{-4}$
2.	1	10	$4x10^{-4}$
3.	2	20	$2x10^{-4}$
4.	3	10	$6x10^{-4}$

  If the first observation is correct, what can you say about the observation 2,3, and 4.
  Ans.
  Now, change in length is directly proportional to the rise in temperature of the rod. In view of this, second and third observation are wrong, while the fourth observation is correct.
- Why does a metal bar appear hotter than a wooden bar at the same temperature? Equivalently it also appear cooler than wooden bar, if they are both colder than room temperature.
  Ans.
  When we touch a metal bar, heat flows from the metal bar to our body quickly (as thermal conductivity of metal is very high) and a result, it appear hotter. On the other, as the wooden bar has comparatively low thermal conductivity, heat does not flow that quickly from the wooden bar to our body on touching it and appears less hot.

3.13. END OF TOPIC GENERAL SOLVED QUESTIONS

1. If a copper rod is 5 cm longer than a steel rod and their difference in length is to be maintained at constant temperature, find their actual lengths. Given linear expansivity of steel = $1.7 \times 10^{-5}K^{-1}$ and that of copper is $1.1 \times 10^{-5}K^{-1}$

Solution

Let ℓ_s and ℓ_c be the lengths of steel and copper respectively and let $\Delta\ell_s$ and $\Delta\ell_c$ be their difference in lengths for steel and copper respectively.

$\ell_c = \ell_s + 5$

$\ell_c - \ell_s = 5$ ……………………..(i)

The difference in lengths

$\Delta\ell_s - \Delta\ell_c = 0$

$\Delta\ell_c = \Delta\ell_s$ …………………… (ii)

From the definition

Linear expansivity of copper rod is given by

$$\propto_c = \frac{\Delta\ell_c}{\ell_c \Delta T}$$

$\Delta\ell_c = \propto_c \ell_c \Delta T$ ………………………(iii)

Linear expansivity of steel rod is given by

$$\propto_s = \frac{\Delta\ell_s}{\ell_s \Delta T}$$

$\Delta\ell_s = \propto_s \ell_s \Delta T$ ……………………...(iv)

Dividing the two equation

Then at constant temperature

$\Delta\ell_c = \Delta\ell_s$

$\propto_s \ell_s \Delta T = \propto_c \ell_c \Delta T$

$\frac{\propto_s}{\propto_c} = \frac{\ell_c}{\ell_s}$ given that $\propto_s = 1.7 \times 10^{-5}K^{-1}$, $\propto_c = 1.1 \times 10^{-5}K^{-1}$

$$\frac{1.7 \times 10^{-5}}{1.1 \times 10^{-5}} = \frac{\ell_c}{\ell_s}$$

$$\frac{1.7}{1.1} = \frac{\ell_c}{\ell_s}$$

$$\frac{17}{11} = \frac{\ell_c}{\ell_s}$$

$$\ell_c = \left(\frac{17}{11}\right)\ell_s$$

then through back equation (i)

$\ell_c - \ell_s = 5$

$$\left(\frac{17}{11}\right)\ell_s - \ell_s = 5$$

$$\ell_s\left(\frac{17}{11} - 1\right) = 5$$

$$\ell_s\left(\frac{6}{11}\right) = 5$$

$$\ell_s = \frac{5 \times 11}{6}$$

$$\ell_s = \frac{55}{6} = 9.17$$

$$\therefore\ \ell_s = 9.17\ \text{cm}$$

then substitute in equation (i)

$$\ell_c - \ell_s = 5$$

$$\ell_c - 9.17 = 5$$

$$\ell_c = 5 + 9.17$$

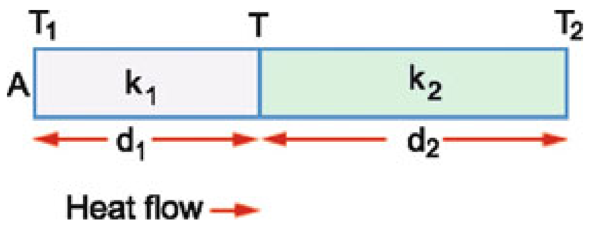
$$\ell_c = 14.17\ \text{cm}$$

$$\therefore\ \ell_c = 14.17\ \text{cm}$$

2. Two slabs of cross-section area A and of thickness d_1 and d_2 and thermal conductivities k_1 and k_2 are arranged in contact face to face. The out face of the first slab is maintained at T_1°C, that of the second one at T_2°C and the interface at T°C. Calculate

a) Rate of flow of heat through the composite slabs
b) The interface temperature
c) The equivalent conductivity.

solution



consider the figure below

(i) (a)for the first slab, heat flow is given by

$$\frac{dQ_1}{dt} = \frac{k_1 A(T_1 - T)}{d_1} - - - - 1$$

for the second slab, heat flow is given by

$$\frac{dQ_2}{dt} = \frac{k_2 A(T - T_2)}{d_2} - - - - - 2$$

now heat flow must be the same in both slabs

$$\frac{dQ_1}{dt} = \frac{dQ_2}{dt} = \frac{dQ_3}{dt}$$

$$T_1 - T = \frac{d_1}{k_1 A} \cdot \frac{dQ}{dt} \ldots\ldots\ldots\ldots 1$$

$$T - T_2 = \frac{d_2}{k_2 A} \cdot \frac{dQ}{dt} ----2$$

add the two eqns results

$$T_1 - T_2 = \frac{d_1}{k_1 A} \cdot \frac{dQ}{dt} + \frac{d_2}{k_2 A} \cdot \frac{dQ}{dt}$$

$$T_1 - T_2 = \frac{1}{A}\left(\frac{d_1}{k_1} + \frac{d_2}{k_2}\right)\frac{dQ}{dt}$$

simple mathematics

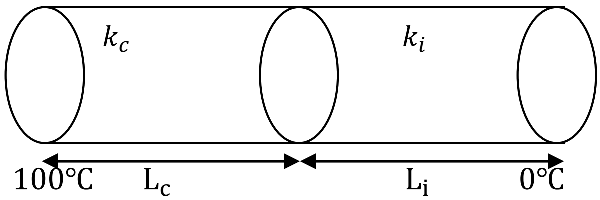
the rate of heat flow

$$\frac{dQ}{dt} = \frac{A(T_1 - T_2)}{\frac{d_1}{k_1} + \frac{d_2}{k_2}}$$

2. A bar of copper and a bar of iron equal length are welded together end to end and are lagged. Determine the temperature of interface when the free end of the copper bar is at 100°C and the free end of the iron is at 0°C and the condition are steady. Given that thermal conductivity of copper is 92 and that of iron is 16 $^{cal}/_{msc}$

solution

consider the bar below



suppose

k_c − thermal conductivity of copper

k_i − themal conductivity of iron

L_c − length of copper bar

L_i − length of iron bar

but given that $L_c = L_i$

then the bars are connected is series

such that rate of heat flow is the same

for both bars

then $$\frac{k_c A(100 - \theta)}{L_c} = \frac{k_i A(\theta - 0)}{L_i}$$

θ − temperature of interface

$$k_c(100 - \theta) = k_i(\theta - 0)$$

$$92(100 - \theta) = 16\theta$$

$$9200 - 92\theta = 16\theta$$

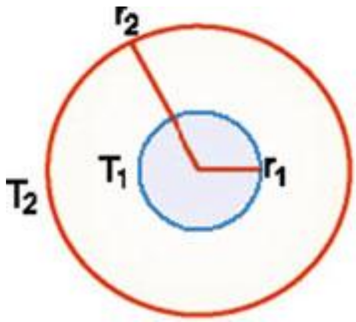
$$108\theta = 9200$$

$$\theta = \frac{9200}{108}$$
$$\theta = 85.19°C$$

3. Find radial flow of heat in a material of thermal conductivity placed between two concentric spheres of radii r_1 and r_2 $(r_1 < r_2)$ which are maintained at temperature T_1 and T_2 $(T_1 > T_2)$.

solution

Consider the concentric sphere below



$$\frac{dQ}{dt} = -kA\left(\frac{dT}{dr}\right)$$
$$dT = -\frac{1}{k}\left(\frac{dQ}{dt}\right)\left(\frac{dr}{4\pi r^2}\right)$$

when a steady stady state is reached

$\frac{dQ}{dt}$ will be independent of r and is constant hence

$$dT = \frac{-1}{4\pi k}\left(\frac{dQ}{dt}\right)\frac{dr}{r^2}$$

integrating both sides

$$\int_{T_1}^{T_2} dT = \int_{r_1}^{r_2} \frac{-1}{4\pi k}\left(\frac{dQ}{dT}\right)\frac{dr}{r^2}$$

integrating on the left side

$$T_2 - T_1 = \frac{-1}{4\pi k}\left(\frac{dQ}{dT}\right)\int_{r_1}^{r_2}\frac{dr}{r^2}$$
$$T_2 - T_1 = \frac{-1}{4\pi k}\left(\frac{dQ}{dT}\right)\left[-\frac{1}{r}\right]_{r_1}^{r_2}$$
$$T_2 - T_1 = \frac{-1}{4\pi k}\left(\frac{dQ}{dT}\right)\left(\frac{-1}{r_2} + \frac{1}{r_1}\right)$$

$$T_2 - T_1 = \frac{-1}{4\pi k}\left(\frac{dQ}{dT}\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

simple mathematics

$$T_2 - T_1 = \frac{-1}{4\pi k}\left(\frac{dQ}{dT}\right)\left(\frac{r_1 - r_2}{r_1 r_2}\right)$$

$$\frac{dQ}{dT} = -\frac{4\pi k(r_1 r_2)(T_2 - T_1)}{r_1 - r_2}$$

$$\frac{dQ}{dT} = 4\pi k\left(\frac{r_1 r_2}{r_2 - r_1}\right)(T_1 - T_2)$$

4. Find the radial rate of heat flow in a material of thermal conductivity k placed between co-axial cylinder of length L and radii r_1 and r_2 respectively ($r_1 < r_2$) maintained at temperature T_1 and T_2 respectively($T_1 > T_2$).

solution

$$\frac{dQ}{dt} = -kA\left(\frac{dT}{dr}\right)$$

neglecting the area of the faces, the area of the cylinder is given by

$$A = 2\pi rL$$

$$\text{also } \frac{dQ}{dt} = \text{constant}$$

$$\frac{dQ}{dt} = -2\pi rLk\left(\frac{dT}{dr}\right)$$

apply mathematical operations

$$\frac{dr}{r} = \left(\frac{2\pi Lk}{\frac{dQ}{dt}}\right)dT$$

apply integral both sides

$$\int_{T_1}^{T_2}\left(\frac{2\pi Lk}{\left(\frac{dQ}{dt}\right)}\right)dT = \int_{r_1}^{r_2}\frac{dr}{r}$$

$$\left(\frac{2\pi Lk}{\left(\frac{dQ}{dt}\right)}\right)\int_{T_1}^{T_2} dT = \int_{r_1}^{r_2}\frac{dr}{r}$$

$$\left(\frac{2\pi Lk}{\left(\frac{dQ}{dt}\right)}\right)(T_2 - T_1) = \text{In}(r_2 - r_1)$$

$$\left(\frac{2\pi Lk}{\left(\frac{dQ}{dt}\right)}\right)(T_2 - T_1) = \text{In}\left(\frac{r_2}{r_1}\right)$$

$$\text{simplify and solve for}\frac{dQ}{dt}$$

$$\boxed{\frac{dQ}{dt} = \frac{2\pi Lk(T_2 - T_1)}{\text{In}\left(\frac{r_2}{r_1}\right)}}$$

5. A small pond has layer of ice on the surface that is 1cm thick. If the air temperature is $-10°C$, Find the rate (in m/h) at which ice is added to the bottom of the layer. The density of ice is 917kg/m^3, thermal conductivity of ice is 0.59W/mK and the latent heat of fusion is 333KJ/kg. Assume that the underlying water is at 0°C.

solution

$$\frac{dQ}{dt} = \frac{kA}{d}(T_1 - T_2)$$

$$\frac{dQ}{dt} = \frac{0.59A(0 - -10)}{0.01}$$

$$\frac{dQ}{dt} = 590A\ ^{J}/_{s} - - - - - - (i)$$

$$\text{let x}\left(\frac{m}{s}\right)\text{ ice be added at the bottom}$$

$$\text{of the layer. mass of ice formed}$$

$$\text{per second}$$

$$m = \rho \times A - - - - - - (ii)$$

$$\text{the required energy per second}$$

$$m = \rho \times AL - - - -(iii)$$

$$\text{equating eqn i and iii}$$

$$x = \frac{590A}{\rho AL}$$

$$x = \frac{590}{917 \times 333 \times 10^3}$$

$$x = 1.932 \times 10^{-6}$$

$$x = 0.00695\ ^{m}/_{h}$$

6. At low temperature say below 50K, the thermal conductivity of a metal is proportional to the absolute temperature, that $k = aT$ where a is constant with a numerical value of that depends on the particular metal. Show that the heat flow through a rod of length l and cross- sectional area A and whose ends are at temperatures T_1 and T_2 are given by

$$Q = \frac{aA}{2L}(T_1^2 - T_2^2)$$

solution

$$\text{tHermal conductivity } k = aT$$

$$\text{also tHe Heat flow in a conductor}$$

$$dQ = kA\frac{dT}{L}$$

but $k = aT$

$$dQ = AaT\frac{dT}{L}$$

$LdQ = AaTdT$

integrating both sides

$$\int_{T_2}^{T_1} AaTdT = L\int_0^Q dQ$$

$$\frac{Aa}{2}[T^2]_{T_2}^{T_1} = QL$$

$$\frac{Aa}{2}(T_1^2 - T_2^2) = QL$$

$$Q = \frac{Aa}{2L}(T_1^2 - T_2^2)$$

proved

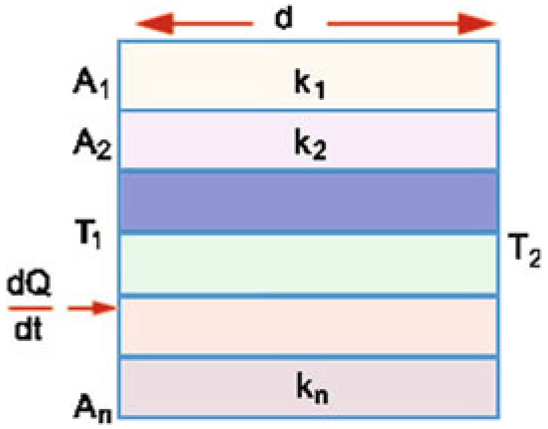
7. n slabs of the same thickness, the cross sectional area $A_1, A_2 \dots\dots A_n$ and thermal conductivities $k_1, k_2 \dots\dots k_n$ are placed in contact in parallel and maintained at temperature T_1 and T_2. Calculate

1. The rate of heat flow through the composite slabs
2. The equivalent conductivities.

solution

the rate of heat flow through the opposite slab

consider the figure below



(i)the rate of heat flow through a composite bar

$$\frac{dQ}{dt} = \frac{kA}{L}(T_1 - T_2)$$

but $k = k_1, k_2 \ldots\ldots k_n$

$A = A_1, A_2 \ldots\ldots A_n$

hence slab have constant change in temperature and thickness L

$$\frac{dQ}{dt} = \frac{T_1 - T_2}{L}\sum_{i=1}^{n} k_i A_i$$

(ii)the equivalent conductivities

taking the above eqn

$$\frac{dQ}{dt} = \left(\frac{T_1 - T_2}{L}\right)\sum_{i=1}^{n} k_i A_i$$

$$k_{eq} = \frac{\sum k_i A_i}{\sum A_i}$$

8. Two perfectly lagged metal bars A and B are arranged in series and parallel. When the bars are in series the hot end of A is maintained at 90°C and the cold end B is maintained at 30°C. When the bars are in parallel the hot end of each is maintained at 90°C and the cold end of each is maintained at 80°C. Calculate the ratio of the total rate flow of heat in the parallel arrangement to that in the series arrangement. The bars have the same length and the same cross sectional area. Thermal conductivity of A is 400W/mK and that of B is 200W/mK.

the rate of conduction of heat through

the bars when in series is

$$\left(\frac{dQ}{dt}\right)_{series} = \frac{\text{temperature difference}}{\text{total thermal resistance}}$$

$$\left(\frac{dQ}{dt}\right) \text{in series} = \frac{\theta_1 - \theta_2}{\frac{l}{k_1 A} + \frac{l}{k_2 A}}$$

$$\left(\frac{dQ}{dt}\right) \text{series} = \left(\frac{A}{l}\right)\left(\frac{\theta_1 - \theta_2}{\frac{1}{k_1} + \frac{1}{k_2}}\right)$$

$$\left(\frac{dQ}{dt}\right) \text{series} = \left(\frac{A}{l}\right)\left(\frac{90 - 30}{\frac{1}{400} + \frac{1}{200}}\right)$$

$$\frac{dQ}{dt} = 8000\left(\frac{A}{l}\right)$$

(ii)the total rate of flow of heat through the bars in parallel is

$$\left(\frac{dQ}{dt}\right) \text{parallel} = \left(\frac{dQ}{dt}\right)_A + \left(\frac{dQ}{dt}\right)_B$$

$$\frac{k_A A(90-80)}{l} + \frac{k_B A(90-80)}{l}$$

$$= \left(\frac{A}{l}\right)(400 \times 10 + 200 \times 10)$$

$$\left(\frac{dQ}{dT}\right) \text{ in parallel } = 6000\left(\frac{A}{l}\right)$$

the ratio of heat flow

$$\frac{\left(\frac{dQ}{dt}\right)_{\text{parallel}}}{\left(\frac{dQ}{dt}\right)_{\text{series}}} = \frac{6000\left(\frac{A}{l}\right)}{8000\left(\frac{A}{l}\right)}$$

$$= 0.75$$

9. Assuming the thermal conductivity of copper is two times that of aluminum and four times that of brass. Three metal rods, made of copper, aluminium and brass respectively are each 15cm long and 3cm in diameter. The rods are placed end to end, with the aluminium between the other two. The free end of the copper and brass are maintained at laced end to end, with the aluminium between the other two. The free end of the copper and brass are maintained at $100°C$ and $0°C$ respectively. Find the equilibrium temperatures of the copper- aluminium junction and the aluminium – brass junction.

solution

Let θ_1 and θ_2 be the temperature at the copper - aluminium junction and aluminium brass junction respectively. And let k be thermal conductivity of copper, then that of alunium is ½ k and that of brass is ¼ k

At equilibrium the rate of heat flow is the same

$$\frac{dQ}{dt} = k_{cu}A\left(\frac{100-\theta_1}{15}\right) = k_{Al}A\left(\frac{\theta_1 - \theta_2}{15}\right) = k_B A\left(\frac{\theta_2 - 0}{15}\right)$$

$$k(100-\theta_1) = \frac{1}{2}k(\theta_1 - \theta_2) = \frac{1}{4}(k\theta_2)$$

$$400 - 4\theta_1 = 2\theta_1 - 2\theta_2 = \theta_2$$

$$4\theta_1 + \theta_2 = 400 ----(i)$$

$$2\theta_1 - 3\theta_2 = 0 ----(ii)$$

adding the two eqns

and solve for θ

$$\theta_1 = 85.7°C \quad \text{and} \quad \theta_2 = 57.1°C$$

10. A copper rod has a length of 1.5m and a cross section area of $4.0 \times 10^{-4}\text{m}^2$. One end of the rod is in contact with boiling water and the other with mixture of ice and water. What is the mass of ice per second that melts? Assume that no heat is lost through the side surface of the rod. Latent heat of fusion of ice is no heat is lost through the side surface of the rod. Latent heat of fusion of ice is 3.35×10^5J/kg and thermal conductivity of copper = 390W/mK.

solution

rate of heat flow through the copper

bar = heat gained by ice per second

$$\frac{dQ}{dt} = \frac{kA(\theta_1 - \theta)}{x} = \left(\frac{m}{l}\right)L$$

$$\frac{m}{t} = \frac{390 \times 4 \times 10^{-4} \times 100}{1.5 \times 3.35 \times 10^5}$$

$$\frac{m}{t} = 1.86\,\text{g/min}$$

11. Three building materials, plaster board ($k = 0.3\, {}^{W}/_{mK^4}$, brick ($k = 0.6\text{Wm}^{-1}\text{K}^{-4}$ and wood ($k = 0.1\text{Wm}^{-1}$, are sandwitched together, such that the brick is in the middle of the plasterboard and wood. The plasterboard being at the inside surface. The temperatures at the inside and outside surfaces are25°C and 0°C, respectively. Each material has the same thickness and cross sectional area. Find the temperature at

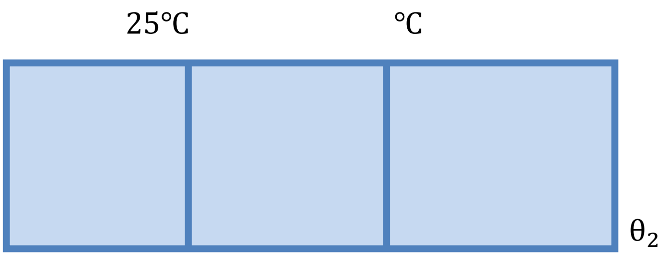
(i). Plasterboard – brick interface.
(ii). Brick- wood interface

solution

Let θ_1 and θ_2 be the temperature at

the plasterboard – brick interface

and brick – wood interface respectively



At steady, the rate of flow of heat is the same

$$\frac{dQ}{dt} = k_p A\left(\frac{25-\theta_1}{x}\right) = k_{\mathbf{B}} A\left(\frac{\theta_1-\theta_2}{x}\right) = k_w A\left(\frac{\theta_2-0}{x}\right)$$

$$0.3(25-\theta_1) = 0.6(\theta_1-\theta_2)$$

simple mathematics

$$\theta_1 = 19.4°C \text{ and } \theta_2 = 16.7°C$$

NB: thermal conductivity depend on

(i). Temperature of the material.
(ii). Nature of the material itself.

NECTA 2007

12 . (a) (i) What is meant by thermometric property of a substance?

Answer; thermometric property of the substance is the property of that substance which varies with temperature and can be in the construction of thermometer.

(ii) What qualities make a particular property suitable for use in practical thermometers?

Answer.

- The sensitivity of the property to temperature change.
- The linearity of the change. The change of temperature should give a proportional change in the property.

(b)(i) Explain why at least two (2) fixed points are required to define a temperature scale.

Answer; At least two fixed points are required to define a temperature scale because the two fixed points can be used to create the fundamental interval of that scale.

(ii) Mention the type of thermometer which is most suitable for calibration of thermometers.

Answer; A gas thermometer is the most suitable thermometer for the calibration of thermometers.

(c) When a metal cylinder of mass 2×10^{-2}kg and specific heat capacity $500\text{Kkg}^{-1}\text{K}^{-1}$ is heated at constant power, the initial rate of rise of temperature is 3Kmin^{-1}. After a time the heater is switched off and the initial rate of fall of temperature is 0.3Kmin^{-1}. What is the rate at which the cylinder gains heat energy immediately before the heater is switched off?

solution

$$\text{from} \quad H = mc\theta$$

$$\frac{dH}{dt} = \frac{d(mC\theta)}{dt} = mc\frac{d\theta}{dt}$$

$$\text{given that} \frac{d\theta}{dt} = 3 - 0.3 = 2.7\ \text{K/min}$$

$$\text{then} \quad \frac{dH}{dt} = 2 \times 10^{-2} \times 500 \times 2.7$$

$$= 27\ \text{J/min} = \frac{27}{60}\text{Js}^{-1}$$

since the cylinder gain heat at the

rate of 0.45Js^{-1}

13. (a) (i) What is black body radiation of a given body?

Answer; Black body radiation is the radiation emitted by a perfectly non – reflecting body as a consequence of its temperature property (alone)

- Explain why heat may just mean infrared.
  Answer; Heat is may just mean infrared wave because infrared radiation is the major constituent of the radiation emitted by a body with temperature above 0K.
- State Prevost's theory of heat exchange.
  (ans; refer to your notes)

(b) (i) Explain why in cold climates, windows of modern building are double grazed, i.e there are two pieces of glass with a small air space between them.

Answer; because the air in the middle acts as insulator for it i.e air ia bad conductor of heat hence it prevent the transfer of heat from the inside to outside hence keeping the inside warm.

(ii). what is Wien's displacement law? (refer to your notes)

(c) The sun's surface temperature is about 6000K. The sun's radiation is with maximum at wavelength of 0.5×10^{-6}m. A certain light bulb filament emits radiation with maximum wavelength of 2×10^{-6}m. if both the surface of the sun and of the filament have the same emissive characteristics, What is the temperature of the filament?

$$\text{solution}$$

$$\text{Let } T_1 \text{ be sun's temp} = 6000\text{K}.$$

$$T_2 \text{ be the filament temp}$$

$$\lambda_1 \text{the wavelength of sun's radiation } = 0.5 \times 10^{-6}\text{m } \lambda_2 \text{ be wavelength of the}$$

$$\text{filament radiation} = 2 \times 10^{-6}\text{m}$$

$$\text{consider wien's law displacement}$$

$$\lambda_{\text{max}} \propto \frac{1}{T} \; ; \quad \lambda_{\text{max}} = \frac{k}{T}$$

$$\lambda_1 T_1 = \lambda_2 T_2$$

$$T_2 = \frac{0.5 \times 10^{-6} \times 6000}{2 \times 10^{-6}}$$

$$\text{temperature of filament} = 1500\text{K}$$

(d) (i) State Newton's law of cooling and give one limitation of the law.

The law states that "the rate of loss of heat from a body is proportional to the excess temperature of the body over the temperature of its surrounding"

Limitation of the law;

The surroundings should be in forced convection, but provided the temperature excess is small, the law is fairly well obeyed even in the case of free or natural convection.

(ii). A body initially at 75°C cools to a temperature of 55°C in 5 minutes. What will be its temperature after 10 minutes given that the surrounding temperature is 31°C? (assume Newton's law of cooling holds true)

$$\text{solution}$$

$$\text{fron Newton's law } \frac{d\theta}{dt} = -k(\theta - \theta_s)$$

$$\text{negative value show temperature decreasing}$$

$$\frac{d\theta}{\theta - \theta_s} = -kdt$$

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta - \theta_s} = \int_0^t -kdt$$

$$[\text{In}(\theta - \theta_s)]_{\theta_1}^{\theta_2} = -kt$$

$$\text{simple mathematics}$$

$$\text{In}\left(\frac{\theta_2 - \theta_s}{\theta_1 - \theta_s}\right) = -kt$$

$$\left(\frac{\theta_2 - \theta_s}{\theta_1 - \theta_s}\right) = e^{-kt}$$

$$\text{simplifying results}$$

$$\theta_2 = \theta_s + (\theta_1 - \theta_s)e^{-kt}$$

$$55 = 31 + (70 - 31)e^{-5k}$$

$$\text{solving for k}$$

$$k = 0.097$$

$$\text{let } \theta_t \text{ be temperature of the body after 10 min}$$

$$\theta_t = \theta_s + (\theta_2 - \theta_s)e^{-0.097(10)}$$

$$\theta_t = 31 + (55 - 31)e^{-0.097\times10}$$

$$= \quad 40.09℃$$

the temperature will be 40.09℃

NECTA 2008.

14 . (a) (i) What is meant by reference temperature as applied in thermocouple?

Answer; The reference temperature of thermocouple is the temperature of its cold junction.

(ii). The emf (in microvolt) in a lead iron thermocouple, one junction of which is at 0℃ is given by V =1784t – 2.4t^2, when t is the temperature of the hot junction in ℃. Calculate the neutral temperature.

- ✓ The neutral temperature is that temperature at which the emf generated by thermocouple is maximum.

$$\text{then} \quad \text{if } V = 1784t - 2.4t^2$$

$$\frac{dV}{dt} = 1784 - 4.8t$$

$$\text{at maximum } \frac{dV}{dt} = 0 = \text{neutral temp}$$

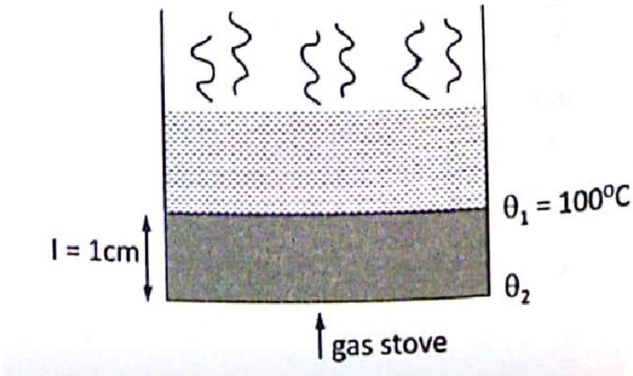
$$\theta_N = \frac{1784}{4.8} = 371.67℃$$

(iii). When a particular temperature is measured on scales based on different properties, it has different numerical value on each scale except at a certain points. Explain why and state at what points the value agree.

Answer; the numerical values are different because the different thermo metrical properties vary differently with temperature. The numerical value disagree at the fixed points.

(b) A brass boiler has a base area of 0.15m^2 and thickness of 1cm It boils water at the arte of 6kg/min when placed on the gas stove. What is the temperature of the part of the flame in contact with the boiler?

Consider the diagram of the brass boiler below



As water evaporates, it consume latent heat of vaporization Q. Latent heat of vaporization is given by as $Q = mL$

Where L = specific latent heat of vaporization

The rate of consumption of latent heat

$$\frac{Q}{t} = \frac{mL}{t} = \left(\frac{m}{t}\right)L$$

$$\frac{Q}{t} = \left(\frac{6}{60}\right) \times 2.25 \times 10^6$$

$$\frac{Q}{t} = 2.25 \times 10^5 \text{ J/s}$$

this heat comes from the gas stove,

since the rate of flow of heat through the brass

is also $\frac{Q}{t}$

$$\frac{Q}{t} = \frac{kA(\theta_2 - \theta_1)}{l}$$

simple mathematics

$$\theta_2 = \frac{\left(\frac{Q}{t}\right)l}{kA} + \theta_1$$

$$= (2.25 \times 10^5 0 \times \left(\frac{1 \times 10^{-2}}{109 \times 0.15}\right) + 100$$

$$= 237.6℃$$

Therefore the temperature of the part of flame in contact with boiler is $237.6℃$

9. (a) (i) Differentiate between forced and natural convection and state the laws governing these processes.

Forced convection	Natural convection
The body cools in an environment where air surrounding the body is made to move example by fanning and blowing	the air around the cooling body is stay stationary. i.e no external fanning or blowing air
law governing is a Newton law of cooling rate of cooling $\propto (\theta - \theta_s)$	Law governing is a Doulong and Petit law Rate of cooling $\propto (\theta - \theta_s)^{\frac{5}{4}}$

(ii) A piece of copper of mass 50g is heated to $100°C$ and then transferred to a well insulated copper calorimeter of mass 25g containing 100g of water at $10°C$. Neglecting heat loss calculate the final steady temperature of water after it has been well stirred,

solution

After stirring each body will have a temperature θ. The piece of copper will have lost heat while the calorimeter and water will gain heat.

Conserve energy

Heat lost by copper = heat gained by calorimeter + water

H(copper) = H(calorimeter)+H(water)

Since $H = mc\Delta\theta$

$$H = MC\Delta\theta\ (\text{copper}) = MC\Delta\theta\ (\text{calorimeter}) + MC\Delta\theta\ (\text{water})$$

$$M_{cu}C_{cu}\ (100 - \theta) = M_{ca}C_{ca}(\theta - 10) + M_wC_w(\theta - 10)$$

$$0.05 \times 4 \times 10^2(100 - \theta) = 0.025 \times 4 \times 10^2(\theta - 10) + 0.1 \times 4.2 \times 10^3(\theta - 10)$$

$$20(100 - \theta) = 10(\theta - 10) + 420(\theta - 10)$$

$$2000 - 200 = 10\theta - 100\theta 420\theta - 4200$$

$$6300 = 450\theta$$

$$\theta = \frac{6300}{450} = 14°C$$

Therefore the final heat steady temperature is $\theta = 14°C$

(b) A blackened sphere of radius 2cm is contained within a hollow evacuated enclosure the wall of which are maintained at $27°C$. Assuming that the sphere radiate like a black body, calculate the rate at which the sphere loses heat when its temperature is $227°C$.

from Stefan's law of black body

$$P_1 = \sigma A T_1^4$$

$$\text{temp of surrounding } T_1 = 27 + 273$$

$$T_1 = 300K$$

$$\text{temp of sphere } T_2 = 273 + 227$$

$$T_2 = 500k$$

$$\Delta P = \sigma A(T_2^4 - T_1^4)$$

$$\Delta P = 4\pi r^2 \sigma(T_2^4 - T_1^4)$$

$$= 4\pi \times 0.02^2 \times 5.7 \times 10^{-8} \times (500^4 - 300^4)$$

$$= 15.59\, J/s$$

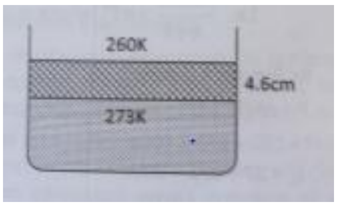
15. (i) State two important thermal characteristics of an ideal cooking pot.**(necta 2009)**

(ii) Ice is forming on the surface of pond. When is at 4.6cm thick the temperature of the ice in contact with the air is 260K while the surface in contact with the water is at temperature 273K. Calculate the rate of loss of heat per unit area from the water. Find the rate at which the thickness of the ice is forming.

solution

(i) – High thermal conductivity to ensure fast transfer of heat from the source to the object.
- Low specific heat capacity to ensure that the amount of heat energy required to raise the temperature of a unit mass of the pot in minimum so that little heat is used to raise the temperature of the pot.
- small coefficient of expansion to ensure that there is no considerable expansion of the pot so that the handles of the pot are not disturbed.
- high melting point to ensure that the pot will stand high temperature.

(ii) consider the illustration below.



Rate of conduction of heat across ice = rate of loss of latent heat from water to form ice

$$k_{ice}A\frac{dT}{l} = \frac{ml_f}{t}$$

$$\text{since } m = Ah\rho$$

where h is new thickness of ice formed

$$k_{ice}A\frac{dT}{l} = A\rho_{ice}l_f\left(\frac{h}{t}\right)$$

$$\frac{h}{t} = k_{ice}\left(\frac{13}{4.6 \times 10^{-7} \times \rho_{ice}l_f}\right)$$

The value for the thermal conductivity of ice, (= 1.6W/mK) density of ice (= 920kg/m^3) and specific latent heat of fusion (= 334000J/kg) should be provided to complete the answer.

$$\frac{h}{t} = \frac{13 \times 1.6}{0.046 \times 920 \times 334000}$$

$\frac{h}{t} = 1.47 \times 10^{-6}$ m/s

The rate of increase of thickness of ice = 1.47×10^{-6}m/s.

16. (a) State Stefan's law.

(b) A metal sphere of temperature 127°C is placed in an enclosure of temperature 27°C. The sphere has a surface area of 0.1m^2 and is found to radiate heat at a rate of 100W If it behaves as a black body. Calculate the value of Stefan's constant.

17 (a) Differentiate between **Heat capacity** and **specific heat capacity.**

(b) Explain briefly how you can measure the temperature (T) of furnace ($\approx$ 300°C) using the normal laboratory apparatus (Without using a pyrometer)

18. (a) The resistance and a gas thermometers may show different values in measuring the temperature of the surrounding. Explain the reason behind.

(b) The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law $R = R_o[1 + (5 \times 10^{-4})(T - T_o)]$. The resistance is 101.6**Ω** at the triple point of pure water and 165.5**Ω** at the normal melting of load (600.5K). Determine the temperature when the resistance is 123.4**Ω**. (**ans. 111.2°C; necta 2009)**

NECTA 2010.

19. (a) (i) Define thermal convection.

Thermal convection is the process by which heat is transmitted through fluids (gases and liquids) from one point to another due to the actual bodily movement of heated particles of the fluid.

(b) In a special type thermometer a fixed mass of a gas has a volume of 100cm^3 at a pressure of 81.6cmHg at the ice point and volume of 124cm^3 and pressure of 90cmHg at steam point. Determine the temperature if it volume is 120cm^3 and pressure of 85cmHg.

solution

$$\theta = \left(\frac{P_\theta V_\theta - P_0 V_o}{P_{100}V_{100} - P_o V_o}\right) \times 100°C$$

$$\theta = \left(\frac{120 \times 85 - 100 \times 81.6}{124 \times 90 - 100 \times 81.6}\right) \times 100$$

$$= \frac{100 \times 2040}{300} = 68°C$$

$$\theta = 68°C$$

The temperature at the given pressure and volume is 68°C°C

(a) What value does the scale of this thermometer give for absolute zero?

solution

$$\text{at absolute temperature} \quad P_o = 0; \; V_o = 0$$

$$\theta = 100\left(\frac{P_\theta V_\theta - P_o V_o}{P_{100}V_{100} - P_o V_o}\right)$$

$$\theta = 100\left(\frac{0 - 100 \times 81.6}{124 \times 90 - 100 \times 81.6}\right)$$

$$\theta = \frac{-100 \times 8160}{3000} = -272°C$$

Absolute zero temperature is − 272°C on this scale

20. (a) Stefan's law of thermal radiation.

(ans; refer to your notes)

(b) A solid copper sphere cools at the rate of 2.8°C/min when its temperature is 127°C. At what rate will a solid copper sphere of twice the radius cool when its temperature is 227°C? In both cases the surrounding are kept at 27°C and conditions are such that Stefan's law may be applied.

solution

By Prevost's theory

$$P = \sigma A(T^4 - T_o^4) = mc\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{\sigma A(T^4 - T_o^4)}{mc}$$

$$\frac{d\theta}{dt} = \frac{4\pi r^2\sigma(T^4 - T_o^4)}{\frac{4\pi r^3}{3}\rho c}$$

$$\frac{d\theta}{dt} = \frac{3\sigma(T^4 - T_o^4}{\rho cr}$$

$$\frac{d\theta_1}{dt} = \frac{k(T^4 - T_o^4)}{r_1}$$

$$\text{so that } T_1 = 127 + 273$$

$$= 400K$$

$$T_o = 27 + 273$$

$$= 300K$$

$$T_2 = 227 + 273 = 500K$$

$$2.8 = \frac{k(400^4 - 300^4)}{r_1}; \quad r_2 = 2r_1$$

$$\frac{d\theta_2}{dt} = \frac{k(T_1^4 - T_o^4}{r_2}$$

$$\frac{d\theta_2}{dt} = \frac{k(500^4 - 300^4)}{2r_1}$$

$$\frac{d\theta_2}{dt} = \frac{2.8(400^4 - 300^4)}{2(400^4 - 300^4)}$$

the new rate of cooling $= 4.352°C/min$

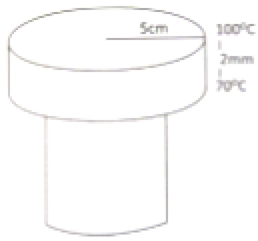
21. (a) State Newton's law of cooling.

(b) Explain the observation that a piece of wire when steadily heater up appears reddish in color before turning bluish.

22. (a) A glass disc of radius 5cm and uniform thickness of 2mm had one of its sides maintained at 100°C while a copper block in good thermal contact with this side was found to be 70°C. The copper block weighs 0.75kg. The cooling of copper was studied over a range of temperature and the rate of cooling at 70°C was found to be 16.5K/min. Determine thermal conductivity of glass.

solution

consider the figure below



$$\frac{dH}{dt} = \frac{KA(\theta_2 - \theta_1)}{l}$$

$$\frac{dH}{dt} = mc\frac{d\theta}{dt}$$

$$\frac{dH}{dt} = \frac{K\pi r^2 \times (100 - 70)}{0.002} = 117.8K$$

$$117.8 = 0.75 \times c \times \left(\frac{16.5}{60}\right)$$

$K = 0.7W/mk$

thermal conductivity $K = 0.7\ W/mk$

(b) A cylindrical element of 1KW electrical fire is 30cm long and 1.0cm in diameter. If the temperature of surroundings is 20°C, estimate the working temperature of the element.

solution

from Stefan's law of black body radiation

$$P = \sigma A(T^4 - T_o^4)$$

$$P = 1000W;\ \sigma = 5.67 \times 10^{-8}$$

$$\text{and} \quad A = 2\pi rh + \pi r^2$$

$$A = \pi(2 \times 0.005 \times 0.3 + 0.005^2)$$

$$A = 9.5033 \times 10^{-3} m^2$$

$$1000 = 5.67 \times 10^{-8} \times 9.5033 \times 10^{-3}(T^4 - 293^4)$$

solving for the value of T

$$T = 894°C$$

the working temperature $T = 894°C$

NECTA 2011

23. (a) Briefly explain what it means by thermal conduction and define the coefficient of thermal conductivity.

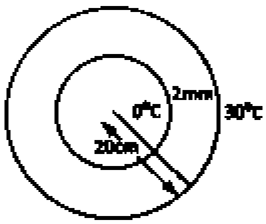
Thermal conduction is the process by which heat flows from the hot region to the cold region of a body without there being net movement of the particle of the body. Thermal conduction is caused by free movement of electrons or friction between adjacent atoms or molecules

Coefficient of thermal conductivity is the rate of flow of heat per unit area per unit temperature gradient when the heat flow is at right angles to the faces of thin parallel sided slab of the material

(b) Ice cubes of mass 5g at 0°C are placed inside a spherical container having an outside diameter of 40cm, 2mm thick and of thermal conductivity $5.0 \times 10^{-4}Wm^{-1}K^{-4}$. How long will it take for all the ice cubes o melt if the room temperature is 30°C?

solution

Consider the illustration below



the rate of heat flow across a spherical shell

$$\text{Rate} = \frac{4\pi k(\theta_2 - \theta_1)}{\frac{1}{r_1} - \frac{1}{r_2}}$$

$$\text{Rate} = \frac{4\pi \times 5 \times 10^{-4}(30 - 0)}{\frac{1}{0.198} - \frac{1}{0.2}}$$

$$\text{Rate} = 73\text{ W}$$

$$R = \frac{ml_f}{t}$$

$$3.73 = \frac{5 \times 10^{-4} \times 3.33 \times 10^5}{t}$$

$$t = 44.116\text{s}$$

$$t = 7.43\text{ minutes}$$

It will take 7 minutes 26 second for the ice cube to melt

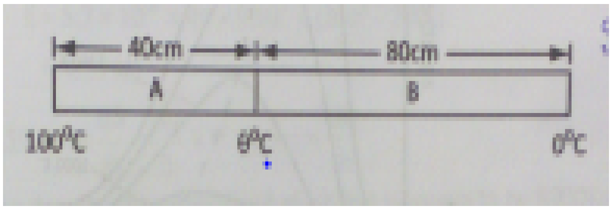
24.Two metal rods A and B of lengths 40cm and 80cm respectively the same cross section area of 10cm^2 are joined end to end. If the composition is perfectly lagged and the free of A is fixed at 100℃ while that of B is pressed to a point at 0℃. Calculate

(i.) The junction temperature of the two metal rods;

(ii.) The quantity of heat that flows per minute in steady state (thermal conductivities of rods A and B are 360 and $80\text{Wm}^{-1}\text{K}^{-4}$)

solution

(ii)consider the illustration below



let the junction temperature be t

for series connection

rate for conductor A = rate for B

$$\frac{k_A A(100-t)}{l_A} = \frac{k_B A(t-0)}{l_B}$$

$$\frac{360(100-t)}{0.4} = \frac{80\,(t-0)}{0.8}$$

$$900(100-t) = 100t$$

$$t = 90℃$$

the junction temp is 90℃

$$\text{(ii) Rate} = \frac{360 \times 10 \times 10^{-4}(100-90)}{0.4}$$

$$\text{Rate} = 9\text{watt}$$

$$\text{heat flow} = \text{rate} \times \text{time}$$

$$\text{heat} = 9 \times 60$$

$$= 540\text{J of heat flows in a minute.}$$

NECTA 2004 P2.

25. (a) A gas expands adiabatically and its temperature falls while the same gas compressed adiabatically its temperature rises. Explain.

Answer.

- ✓ When a gas expands, it does work. example in driving a piston. The molecules of the gas bombarded the piston, and if the piston moves they gives up some of their kinetic energy to it. When a molecules bounces off the moving piston, it does so with a velocity less than that with which it truck. The change in velocity is small because the piston moves much more slowly than the molecules but there are many molecules striking the piston at any instant and their total loss of of kinetic energy is equal to the work done in driving the piston forward. The work done by a gas expanding, therefore is at expense of its internal energy. The temperature of the gas will consequently fall during expansion.
- ✓ **Simultaneously:** if a gas is compressed, its temperature rises. The molecules now rebound from the forward – moving piston with a velocity greater than their incidence velocity. The total increase in kinetic energy of all molecules is equal to the work done in moving the piston.

(b) A mole of oxygen is insulated in an infinitely flexible container. The atmospheric pressure outside the container is $5 \times 10^5 \mathrm{Nm}^{-2}$ when $580\mathrm{J}$ of heat is supplied to the oxygen the temperature increases to 300K and the volume of the container increases by $3.32 \times 10^{-2}\mathrm{m}^3$. Calculate the value of principle molar heat capacities and the specific universal gas constant. Given molar mass of oxygen = 32×10^{-3}kg.

solution

consider first law of thermodynamics

$$\mathrm{dQ} = \mathrm{dU} + \mathrm{dw}$$

$$\mathrm{dQ} = \text{total heat supplied}$$

$$\mathrm{dU} = \text{increase in internal energy}$$

$$\mathrm{dw} = \mathrm{pdv} = \text{external workdone}$$

$$\mathrm{dQ} = \mathrm{nC_p dT}$$

$$\mathrm{C_p} = \frac{\mathrm{dQ}}{\mathrm{ndT}}$$

$$\mathrm{C_p} = \frac{580}{1 \times (300 - 280)\mathrm{K}}$$

$$\mathrm{C_P} = 29\,\mathrm{J/Kmol}$$

molar hat capacity at constant pressure is $29\,\mathrm{J/Kmol}$

$$\text{from } dQ = dU + dw$$

$$dU = nC_v dT$$

$$C_v = \frac{dQ - pdv}{ndT}$$

$$C_v = \frac{580 - (5 \times 10^5)(3.32 \times 10^{-4})}{1 \times (300 - 280)}$$

molar heat capacity at constant volume is 20.7 J/Kmol

from Meyer'sequation

$$C_p - C_v = R$$

$$R = (29 - 20.7)\ \text{J/Kmol}$$

$$R = 8.3\ \text{J/Kmol}$$

required to find R in J/kgK gas constant r

$$r = \frac{R}{M_r} = \frac{8.3}{32 \times 10^{-3}}$$

$$= 259.375\ \text{J/kgK}$$

specific universal gas constat = 259.375 J/kgK

NECTA 2009 P2

26. When water is boiled under a pressure of 2atm the boiling point is 120°C. At this pressure 1kg of water has a volume of $10^{-3}m^3$ and 2kg of steam have a volume of $1.648m^3$. Compute the

(i). Work done when 1kg of steam is formed at this temperature.
(ii). Increase in the internal energy.

solution

(i) workdone when 1kg of steam is formed = pdv

$$w = p(V_2 - V_1)$$

$$w = 2 \times 1.01 \times 10^5 (0.824 - 10^{-3})$$

$$w = 166246\text{J} = 166.246\text{kJ}$$

$$\text{workdone} = 166.246\text{kJ}$$

(ii)from the first law of themodynamic

$$dQ = dU + dw$$

$$dQ = mL_v$$

$$dQ = 1 \times 2.3 \times 10^6$$

$$dQ = 2.3 \times 10^6 J$$

$$dU = dQ - dw$$

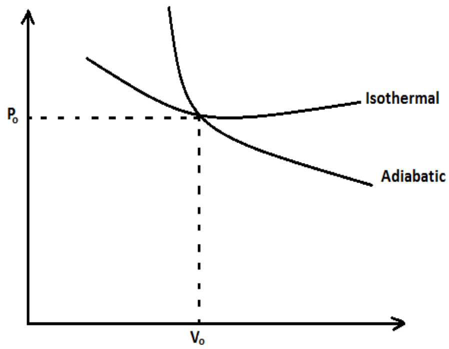
$$dU = 2.3 \times 10^6 - 1.66 \times 10^5$$

$$dU = 2.134 \times 10^6 J$$

$$\text{increase in internal energy} = 2.134 \times 10^6 J$$

NECTA 2008 P2

27.



In a $P-v$ diagram above, an adiabatic and an isothermal curve for an ideal gas intersect. Show that the absolute value of the slope of the adiabatic $\left|\left(\frac{dP}{dV}\right)_{ad}\right|$ is ɣ times that of the isothermal. Hence the adiabatic curve is steeper because the specific heat ratio ɣ is greater than one.

solution

for isothermal process

$$PV = K$$

differentiating w.r.t. V

$$\frac{d}{dV}(PV) = \frac{d}{dV}(K)$$

$$P + V\frac{dP}{dV} = 0$$

$$V\frac{dP}{dV} = -P$$

$$\left|\left(\frac{dP}{dV}\right)\right|_{iso} = -\frac{P}{V} - - - - - - - (*)$$

for adiabatic process

$$PV^{\gamma} = K$$

differentiating w.r.t. V

$$\frac{d}{dV}(PV^{\gamma}) = \frac{d}{dV}(K)$$

$$\frac{Pd}{dV}(V^{\gamma}) + \frac{V^{\gamma}dP}{dV} = 0$$

$$\gamma PV^{\gamma-1} + \frac{V^{\gamma}dP}{dV} = 0$$

$$\frac{V^{\gamma}dP}{dV} = -\gamma P\frac{V^{\gamma}}{V}$$

$$\frac{dP}{dV} = -\gamma\left(\frac{P}{V}\right)$$

$$\left|\left(\frac{dP}{dV}\right)\right|_{adi} = -\gamma\left(\frac{P}{V}\right) - - - (**)$$

dividing the two equations

$$\frac{\left|\left(\frac{dP}{dV}\right)\right|_{adi}}{\left|\left(\frac{dP}{dV}\right)\right|_{iso}} = \frac{\gamma\left(\frac{P}{V}\right)}{\left(\frac{P}{V}\right)}$$

$$\frac{\left|\left(\frac{dP}{dV}\right)\right|_{adi}}{\left|\left(\frac{dP}{dV}\right)\right|_{iso}} = \gamma$$

$$\therefore \left|\left(\frac{dP}{dV}\right)_{adi}\right| = \gamma\left|\left(\frac{dP}{dV}\right)_{iso}\right|$$

hence shown

NECTA 2004

28. (a) (i) What is the difference between Kelvin temperature scale and the Celcius temperature scale?

(ii) A copper – constantan thermocouple with its junction at ice point had an emf of 4.28mV with its junction at 100°C. The emf became 9.29mV when the temperature difference was 200°C. Find the values of A and B in the equation $E = A\theta + B\theta^2$ where E is the emf and θ the temperature difference.

(b) (i) What is meant by temperature gradient?

(ii) The end of the straight uniform metal rod are maintained at temperature of 100°C and 20°C, the room temperature being below 20°C. Draw sketch graphs of the variation of temperatures of the rod along its length when the surface of the rod is lagged, coated with soot and polished. Give a qualitative explanation of the form of graphs.

solution

(a) (i) **Kelvin temperature scale** uses the triple point of water as its only one fixed point where as the celcius scale uses the temperature of the melting ice and temperature of pure boiling water (at 1atm pressure) as its two fixed points.

$T = 273 + \theta°C$ Where T is the temperature in Kelvin and θ°C temperature in celcius.

(ii) solution

$$\text{emf is } 4.28\text{mV when } T = 100°C$$

$$\text{emf is } 9.29\text{mV when } T = 200°C$$

given that

$$E = A\theta + B\theta^2$$

$$\text{case 1; when } \theta = 100°C \text{ and emf} = 4.28\text{mV}$$

$$4.28 \times 10^{-3} = 100A + B(100)^2$$

$$100A + 10000B = 4.28 \times 10^{-3} --- (*)$$

$$\text{also case 2; when } T = 200°C \text{ and emf } 9.29\text{mV}$$

$$200A + 40000B = 9.29 \times 10^{-3} --- (**)$$

$$\text{solving equation } (*) \text{and } (**)$$

$$100A + 10000B = 4.28 \times 10^{-3}$$

$$200A + 40000B = 9.29 \times 10^{-3}$$

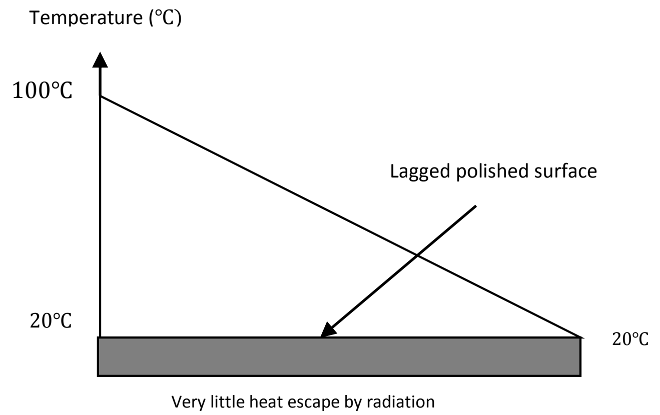
then solving for A and B

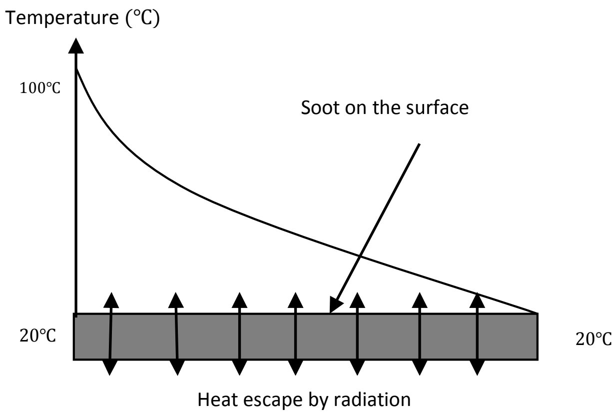
$$A = 3.915 \times 10^{-5} V^o C^{-1} \text{ and } B = 3.65 \times 10^{-5} V^o C^{-1}$$

(b) (i) The temperature gradient is the temperature difference between two points divide by the length of the conductor between these points.

i.e temp gradient $= \frac{d\theta}{L}$

(ii) The graphs of a lagged rod and that of a polished rod are similar, since polishing prevent heat to escape by radiation for the lagged rod or polished rod.





From the graph.

- **A lagged rod** will have a straight line indicating that no heat lost to the surrounding and so heat is conducted uniformly.
    - **A rod coated with a soot is** good radiator of heat, so heat is lost as it is conducted from the hot end to the cold end hence the graph is curve.
    - **A polished rod;** this is similar to the lagged one. A polished surface has a very low tendency of losing heat. Hence graph is straight line indicating that the flow of heat is uniform.

NECTA 2005

29. (a) (i) What is black body?

(ii) State Wien's la for black body radiation.

(iii) If the radiated power per nanometer wavelength from the sun to peaks at 490nm, estimate the temperature of the sun's surface assuming the sun to radiate as a black body and that Wien's constant is 2.93×10^{-3}mK.

(b) What is Prevost's theory of heat exchange?

(c) A cube of side 0.01m has a surface which gives 50% of the emission of black body at the same temperature. If the temperature of the cube is 700°C,

(i) Calculate the power radiated by the cube.

(ii) If the same power above is given by a black body sphere at 300°C, what would its diameter be? Suppose that Stefan's constant is $5.7 \times 10^{-8}\text{W/m}^2\text{K}^4$.

solution

(a) (i) **A black body** is an ideal body which absorbs all radiation that falls on it and reflect none; its radiation therefore is characteristic of its temperature alone.

(ii) **Wiens' law** state that " The peak wavelength of a particular radiation emitted by a black body is inversely proportional to the black body's absolute temperature".

Stefan's law: state that " the total energy E radiated per second per unit surface area at temperature T is proportional to the fourth power of the body's absolute temperature.

(iii) **solution**

$$\text{consider Wien's law}$$

$$\boldsymbol{\lambda_m T = C}$$

$$T = \frac{C}{\lambda_m} = \frac{\mathbf{2.93} \times 10^{-3}}{490 \times 10^{-9}}$$

$$T \approx 5980\text{K}$$

(b) **Prevost's Theory** of heat exchange states that " A body radiate heat at a rate which depends on the nature of its surface and its temperature and absorb heat at a rate which depends on the nature of its surface and temperature of its surrounding"

(c) (i) solution

$$\text{side of a cube} = 0.01\text{m}, \text{emissivity} = 50\% = 0.5$$

$$\text{temp of cube} = 700 + 273 = 973\text{K}$$

$$\text{Total surface area } A = 6 \times (\text{side} \times \text{side})$$

$$= 6 \times (0.01 \times 0.01)$$

$$A = 6 \times 10^{-4}\text{m}^2$$

$$\text{from Stefan's law}$$

$$P = A\sigma\varepsilon T^4$$

$$P = 0.5 \times 6 \times 10^{-4} \times 5.7 \times 10^{-8} \times 973^4 15.33\text{watts}$$

$$\text{power radiated by cube} = 15.33\text{watts}$$

(ii) solution

$$\text{power} = 15.33\text{w}; \ T = 300 + 273 = 573\text{K}$$

$$\text{let R be radius of the sphere}$$

$$\text{such that} \quad A = 4\pi R^2$$
$$\text{if} \quad P = A\varepsilon\sigma T^4 \quad \text{then}$$
$$P = 4\pi R^2 \varepsilon\sigma T^4$$
$$R = \left(\frac{P}{4\pi\sigma}\right)^{\frac{1}{2}} \quad \text{where } \varepsilon = 1$$
$$R = \left(\frac{15.33}{4\pi \times 5.7 \times 10^{-8} \times 573^4}\right)^{\frac{1}{2}}$$
$$R = 0.0141\text{m but diameter} = 2R$$
$$d = 2 \times 0.0141 = 0.0282$$
$$\text{therefore diameter } d = 0.0282\text{m}$$

NECTA 2005

30. (a) (i) What is the difference between ice point and triple point of water?

(ii) Several cooking utensils for sale are rated at "HIGH" or "LOW" in terms of their thermal conductivity, specific heat capacity, coefficient of expansion, and melting point. Explain briefly the thermal ratings you would observe with respect to each property in purchasing a cooking utentsil.

(b) A calorimeter of thermal capacity 30J/K contains 100cm^3 of glycerin and it cools from 80°C to 70°C in 3.5min room temperature being 20°C. When the glycerin is replaced by 100cm^3 of water, the water cools from 43°C to 33°C in 6.5min. Determine the specific heat capacity of glycerin. density of glycerin is $1.2 \times 10^3\text{kg/m}^3$

solution

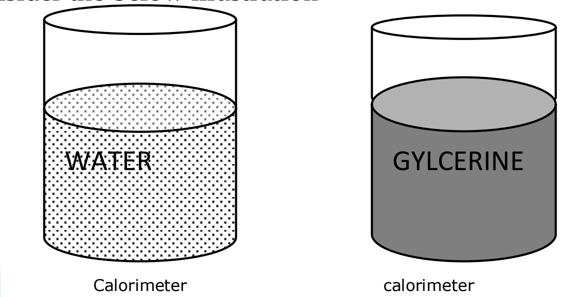
Differences between ice point of water and triple point of water

Ice point of water	Triple point of water
Is the temperature of the pure melting ice	Is the temperature at which three states of water exist together in equilibrium
It is at 273.15K	It is at 273.16K

(ii) Thermal rating to be observed for

- Thermal conductivity; **HIGH** – To ensure fast transfer of energy from the source to the object.
- Specific heat capacity: **LOW** – to ensure that the amount of heat energy required to raise the temperature of a unit mass of a vessel is minimum so that little heat is used to rise the temperature of the vessel.
- Coefficient of expansion: **LOW** – to ensure that there is no considerable expansion of the vessel as not to disturb handling of the vessel.
- Melting point: **HIGH –** this is to ensure that vessel will stand high temperature.

(c) Consider the below illustration



let surrounding temp be $\theta_s = 20°C$

heat lost by glycerine $H = m_g c_g \Delta\theta$

$$\frac{H}{t} = m_g c_g \Delta\theta$$

but also $\frac{dH}{dt} = m_g c_g \frac{d\theta}{dt} ----(*)$

consider Newton'slaw of cooling

Heat lost by Glycerine

$$\frac{dH}{dt} \propto A(\theta - \theta_s)$$

such that $\frac{dH}{dt} = kA(\theta - \theta_s) --- (**)$

k depends on the nature of surface

A surface area of calorimeter

substitute equation $*$ into equation $**$

$$m_g c_g \frac{d\theta}{dt} = kA(\theta - \theta_s)$$

separating variablea and integrate

$$\frac{d\theta}{\theta - \theta_s} = \frac{kAdt}{m_g c_g}$$

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta - \theta_s} = \frac{kA}{m_g c_g}\int_0^{t_1} dt$$

$$[\,\text{In}(\theta - \theta_s)]_{\theta_1}^{\theta_2} = \frac{kAt_1}{m_g c_g}$$

$$\text{In}\left[\frac{\theta_2 - \theta_s}{\theta_1 - \theta_s}\right] = \frac{kAt_1}{m_g c_g} ----(***)$$

also for water

$$\text{In}\left[\frac{\theta_4 - \theta_s}{\theta_3 - \theta_s}\right] = \frac{kAt_2}{m_w c_w} ----(****)$$

dividing equation (****)by (***)

$$\frac{\text{In}\left[\frac{\theta_4 - \theta_s}{\theta_3 - \theta_s}\right]}{\text{In}\left[\frac{\theta_2 - \theta_s}{\theta_1 - \theta_s}\right]} = \left(\frac{t_2}{m_w c_w}\right)\left(\frac{m_g c_g}{t_1}\right)$$

$$\frac{\text{In}\left(\frac{33 - 20}{43 - 20}\right)}{\text{In}\left(\frac{70 - 20}{80 - 20}\right)} = \left(\frac{16.5}{4.2 \times 10^3 \times \rho_w V_w}\right)\left(\frac{\rho_g V_g c_g}{3.5}\right)$$

$$\frac{\text{In}\left(\frac{13}{23}\right)}{\text{In}\left(\frac{50}{60}\right)} = \left(\frac{16.5}{10^3 \times 100 \times 4.2 \times 10^3}\right)\left(\frac{1.2 \times 10^3 \times 100 \times c_g}{3.5}\right)$$

$$3.12933 = 1.34693 \times 10^{-3} c_g$$

solving for c_g

$$c_g = \frac{3.12933}{1.34693 \times 10^{-3}} = 2323.29\text{Jkg}^{-1}\text{k}^{-1}$$

31. (a) (i) How does heat transfer by convection differ from that by conduction?

(ii) State Newton's law of cooling and Stefan's law. For each law state one significant limitation.

(iii) State and illustrate how an increase of temperature affects the radiation spectrum of a black body.

(b) Given that the solar constant has value of 1350W/m^2.

(i) Estimate the total direct solar energy which enters the Tanzania atmosphere from 06.55a.m to 05.05p.m a sunny day. Neglect changes in solar beam between the Earth's atmosphere and the sun – earth midpoint.

(ii) What is the total rate at which the sun emits out energy.

solutions

(a)(i) Differences between heat transfer by convection and by conduction

Heat transfer by convection	Heat transfer by conduction
In this way heat energy is transferred through the convection fluid currents that are set within the fluid and moving from hotter to cooler parts	In this way heat energy is transferred by either vibratory motion of particles constituting a solid or through free electrons

(iii) **Newton's law of cooling:** " The rate of loss of heat energy to the surrounding is directly proportional to the excess temperature"

Limitation: It is applied only under forced convection.

Stefan's law: "The amount of heat energy radiated by a black body per second per unit surface area is directly proportional to the fourth power of its absolute temperature".

Limitation: it is applied for black body radiation only.

(b) (i) solution

$$\text{time} = \ 10\text{hrs} + 10\text{minute}$$

$$= (10 \times 60)\text{minutes} = (10 \times 60) + 10$$

$$\text{t} = 610\text{minutes} = 36600\text{seconds}$$

$$\text{power receirced} = \ \text{solar constant} \times \text{Area of Tanzania}$$

$$\text{P}_{\text{rec}} \ = \ 1350 \ \times 94500 \times 10^6$$

$$= \ 1.27575 \times 10^{15}\text{watts}$$

$$\text{Power} \ = \frac{\text{energy}}{\text{time}}$$

$$\text{Energy received on Tz} = \text{Power} \times \text{time}$$

$$\text{E} \ = \ 1.27575 \times 10^{15} \times 36600$$

$$\text{energy} \ = \ 4.67 \times 10^{19}\text{J}$$

$$\text{energy from sun that enter Tanzania} = 4.67 \times 10^{19} \text{ J}$$

(ii) Total rate at which the sun emits out energy. The sun emits total power P, this power is distributed in a sphere of radius R by the time it reaches the earth. Since given power unit area (solar constant) = 1350W/m^2 on the earth.

$$\frac{\text{P}}{\text{area of sphere}} \ = \ 1350$$

$$\frac{\text{P}}{4\pi\text{R}^2} \ = \ 1350$$

$$\text{P} \ = \ 1350 \times 4\pi\text{R}^2$$

$$\text{P} \ = \ 1350 \times 4\pi \times (1.5 \times 10^{11})^2$$

$$P = 3.8151 \times 10^{26}$$

the rate of energy emitted by the sun is

$$3.8151 \times 10^{26}$$

NECTA 2006

31. (a) (i) Why is heat needed to change liquid water into vapor? What amount of energy is needed?

(ii) The molar heat capacity of hydrogen at constant volume is 20.2J/mol°C. What is the molar heat capacity at constant pressure?

(b) (i) In industrial refrigerator, ammonia is vaporized in the cooling unit to produce a low temperature. Why should the evaporation of ammonia reduce the temperature in the refrigerator?

(ii) How much energy is needed to convert 150g of water at $20°\text{C}$ into a steam at $100°\text{C}$? Given Latent heat of vaporization $2.2 \times 10^3\text{J/g}$.

solution

(a)(i) Heat is needed to change water into vapor in order to:

- ✓ Breaks intermolecular bonds of water molecules and become free molecules.
- ✓ Work against the atmosphere as the molecules evaporate. (the amount of energy needed is the Latent heat of vaporization)

(ii) From mayor's equation

$$c_p - c_v = R$$

$$\text{such that} \quad c_p = c_v + R$$

$$\text{where } R = 8.3\text{Jmol}^{-1}\text{C}^{-1}$$

$$c_p = 20.2 + 8.3$$

$$= 28.5\text{Jmol}^{-1}\,°\text{C}^{-1}$$

The molar heat capacity of hydrogen at constant pressure is 28.5J/mol°C.

(b)(i) The evaporation of ammonia reduces the temperature of the refrigerator since, as the ammonia vapourizes, it absorbs its latent heat of vapourization from the surrounding hence cools the refrigerator.

(ii) solution

let H be energy needed to convert 150g of

water at 20°C to steam at 100°C

$$H = ML + MC\Delta\theta$$

$$C = 4.2Jg^{-1};\ L = 2.2 \times 10^3 J\,g^{-1}$$

$$\Delta\theta = 100 - 20 = 80°C$$

$$H = ML + MC\Delta\theta$$

$$= M(L + C\Delta\theta)$$

$$= 150(2.2 \times 10^3 + 4.2 \times 80)$$

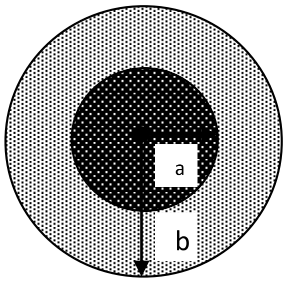
$$= 3.804 \times 10^5 \text{Joules}$$

Therefore 380.4KJ is needed to convert 150g of water at 20°C to steam 100°C

32. A vessel in shape of spherical shell has an inner radius "a" and outer radius "b". The wall has a thermal conductivity k. If the inside is maintained at a temperature T_1 and the outside is at a temperature T_2. Show that the rate of flow of heat between the surfaces is given by the relation;

$$\frac{dQ}{dt} = \left(\frac{4\pi kab}{b-a}\right)(T_1 - T_2)$$

Suppose the vessel appeared as shown below,



the rate of heat flow in the vessel

$$\frac{dQ}{dt} = \frac{kAdT}{dr}$$

$$\frac{dQ}{dt} = \frac{4\pi r^2 k}{dr} dT$$

$$dr = \frac{4\pi r^2 k}{\frac{dQ}{dt}} dT$$

$$\frac{dr}{r^2} = \frac{4\pi k}{\frac{dQ}{dt}} dT$$

$$\int_a^b \frac{dr}{r^2} = \int_{T_2}^{T_1} \frac{4\pi k}{\frac{dQ}{dt}} DT$$

$$\left[-\frac{1}{r}\right]_a^b = \frac{4\pi k}{\frac{dQ}{dt}}\int_{T_2}^{T_1} dT$$

$$-\frac{1}{b} + \frac{1}{a} = \frac{4\pi k}{\frac{dQ}{dt}}\left[\ T\ \right]_{T_2}^{T_1}$$

$$\frac{b-a}{ab} = \frac{4\pi k}{\frac{dQ}{dt}}(T_1 - T_2)$$

$$\frac{dQ}{dt} = \frac{4\pi kab}{b - a}(T_1 - T_2)$$

NECTA 2016;

33. Briefly explain why:

(i) A body with large reflectivity is poor emitter.

(ii) It would be too cold to live on the earth without its atmosphere.

Answer:

(i) A body of large reflectivity is poor emitter because the higher the reflecting ability the more difficult it is for heat to penetrate across the surface, whether into the body (absorption) or out of the body (emission)

(ii) The earth without its atmosphere would be too cold to live because.

- ❖ The atmosphere a poor conductor and so it acts as an insulator of heat from the earth's surface thereby the sun is not directly above an area.
- ❖ The atmosphere itself absorbs heat and preserves it through its heat capacity. Thus, without the atmosphere heat would escape quickly to space leaving the earth surface very cold during the night.

34. (a) Identify two factors on which the coefficient of thermal conductivity of material depends.

(b) A brass boiler area $1.5 \times 10^{-1} m^2$ and thickness of 1.0cm boils a water at the rate of 0.6kg/min when placed on the gas stove. Estimate the temperature of the part of the flame in contact with the boiler.

Answers:

(i) Factors affecting coefficient of thermal conductivity of material;

- Nature of the material
- Free electrons available in the material
- Efficiency of lagging**.**

(ii) Temperature of the flame;

in this case the rate of heat through the base is the same as the rate of boiling of water

$$\left(\frac{dQ}{dt}\right)_{base} = \left(\frac{dQ}{dt}\right)_{boiling}$$

$$\text{then}\quad \frac{kA(\theta_f - \theta_c)}{L} = L_V\frac{dm}{dt}$$

where

θ_f – flame temp; θ_c – boiling water tem

A – area of base of boiler

L – thickness of base;

$\frac{dm}{dt}$ – rate of boiling of water

$$\frac{dm}{dt} = 6\,\text{kg/min} = \frac{6}{60}\text{kg/s} = 0.1\,\text{kg/s}$$

$$\theta_f = \frac{L\frac{dm}{dt}L_V}{kA} + \theta_c$$

$$\theta_f = \frac{0.1 \times 2.25 \times 10^6 \times 0.01}{109 \times 0.15} + 100$$

$$\theta_f = 238°C$$

35. (a) Briefly describe the working principle of an thermocouple.

(b) In a certain thermocouple thermometer the emf is given by $E = a\theta + \frac{1}{2}b\theta^2$ where θ is the temperature of hot junction. If $a = 10mV°C^{-2}$, $b = \frac{1}{20}mV°C^{-1}$ and the cold junction is at 0°C, calculate the neutral temperature.

Answers:

(b) Required neutral temperature

$$E = a\theta + \frac{1}{2}b\theta^2$$

differentiating the equation

$$\frac{dE}{d\theta} = a + \frac{2b}{2}\theta$$

$$\frac{dE}{d\theta} = a + b\theta$$

for neutral temperature $\frac{dE}{d\theta} = 0$

then $\theta_N = -\frac{a}{b}$

$$= -10 \times -20$$

$$\theta_N = 200°C$$

36. (a) what is meant by thermal radiation?

(b) Briefly explain why forced convection is necessary for excess temperature less than 20K.

Answer:

(a) Thermal radiation is the transfer of heat in form of electromagnetic waves.

(b) Forced convection is necessary for excess temperature less than 20K because at such small temperature difference the various layers of air/fluid above a hot surface do not acquire enough kinetic energy for convection currents to all substantial air movement. Thus it is necessary for air to be set in motion using external force.

37. (a) why is the energy of thermal radiation less than that of visible light?

(b) A body with a surface area of 5.0cm^2 and temperature of $727°\text{C}$ radiate 300Joules of energy in one minute. Calculate the emissivity of the body.

Answers:

(a) Energy of thermal radiation is less than that of visible light because the wavelength of thermal radiation (which is in the infra – red region of the spectrum) is longer than that of visible spectrum. Longer wavelengths have lower energy.

(b) Required emissivity of the body.

$$\frac{dQ}{dt} = \sigma AeT^4$$

$$e = \frac{\frac{dQ}{dt}}{\sigma AT^4}$$

$$\text{but } \frac{dQ}{dt} = \text{power} = \frac{300}{60} = 5\text{Watts}$$

$$e = \frac{5}{5.67 \times 10^{-8} \times 5 \times 10^{-4} \times (727 - 273)^4}$$

$$e = 0.18$$

38. (a) State Newton's law of cooling.

(b) A body cools from $70°\text{C}$ to $40°\text{C}$ in 5 minutes. If the temperature of the surrounding is $10°\text{C}$, calculate the time it takes to cool from $50°\text{C}$ to $20°\text{C}$.

Answers:

Newton's law of cooling "the rate of cooling of a body is directly proportional to excess temperature above its surrounding"

(b) time to cool from $50°\text{C}$ to $20°\text{C}$

$$k = -\frac{1}{t}\text{In}\left(\frac{\theta_2 - \theta_s}{\theta_1 - \theta_s}\right)$$

$$k = -\frac{1}{5}\text{In}\left(\frac{40 - 10}{70 - 10}\right)$$

$$k = 0.1386\text{min}^{-1}$$

$$t = -\frac{1}{k}\text{In}\left(\frac{\theta_2 - \theta_s}{\theta_1 - \theta_s}\right)$$

$$t = -\frac{1}{0.1386}\text{In}\left(\frac{20 - 10}{50 - 10}\right)$$

$$t = 10\text{min}$$

NECTA 2015.

39. (a) What is meant by a thermometric property?

(b) Mention three qualities that makes a particular property suitable for use in practical thermometer.

Answers:

(a) Thermometric property is a physical property that varies linearly with temperature variation.

(b) Qualities of a suitable thermometric property

- Linearity: it must vary linearly with temperature variation.
- Sensitivity: it must change with temperature change.
- Measurability: its variation with temperature must be measurable.
- Visibility: the thermometer property.

40. Study the value in table below which represent the observations of a particular room temperature obtained by using two types of thermometers and then answer the questions that follows.

Temp in °C	Resistance measured by resistance thermometer Ω	Pressure recorded by constant volume gas thermometer NM^{-2}
Steam, 100°C	75	1.1×10^7
Ice point, 0°C	63	8.00×10^6
	64.992	8.51×10^6

(i) Calculate the value of unknown room temperature on the scales of resistance thermometer and constant volume gas thermometer.

(ii) Why do the answers above differ slightly.

(i) Unknown room temperature

$$\theta = \left(\frac{X_\theta - X_o}{X_{100} - X_o}\right) \times 100°C$$

for resistance thermometer scale

$$\theta = \left(\frac{64.992 - 63}{75 - 63}\right) \times 100°C$$

$$\theta = 16.6°C$$

for the gas thermometer

$$\theta = \left(\frac{8.51 \times 10^6 - 8 \times 10^6}{1.1 \times 10^7 - 8 \times 10^6}\right) \times 100°C$$

$$\theta = 17°C$$

(iii) The answer differ slightly because the function of the variation of resistance with temperature and that of volume with temperature are not identical.

41. (a) Define coefficient of thermal conductivity.

(b) Write down two characteristics of perfect lagged bar.

42. A thin copper wall of a hot water tank having a total surface area of $5.0m^2$ contains $0.8m^3$ of a water at 350K and is lagged with a 50mm thick layer of a material of thermal conductivity 4.0×10^{-2}W/mK. If the thickness of copper wall is neglected and the temperature of the outside surface is 290K.

(i) Calculate the electrical power supplied to an immersion heater.

(ii) If the heater were switched off, how long would it take for the temperature of hot water to fall by 1K?

43. The element of an electric fire with an output of 1000W is cylinder of 250mm long and 15mm in diameter. If it behaves as black body, estimate its temperature.

NECTA 2014.

44. (a) Give two ways in which the internal energy of the system can be changed.

(i) Ways of changing internal energy of a system

- By changing its temperature
- By changing its pressure (for gaseous system)

(b) List down two simple applications of the first law of thermodynamics in our daily life.

(ii) Application of first law of thermodynamics.

- The combustion of fuel of fuel in a diesel engine.
- Warning a hot air balloon.

45. (a) define thermal convection.

Thermal convection is the transfer of heat in fluids by bulk movement of molecules.

(b) Prove that at a very small temperature difference $\Delta T = T_b - T_s$ Newton's law of cooling obey's the Stefan's whereby T_b is the temperature of the body and T_s is the temperature of the surrounding.

solution

Proof that Newton's law of cooling obey's Stefan's law.

$$-mc\frac{d\theta}{dt} = \sigma A(T_b^4 - T_s^4) ----(i)$$

where T_b − temp of the body; T_s − surrounding

$$\frac{d\theta}{dt} = \text{rate of cooling}$$

T_b and T_s that have small difference,

$$\text{say} \quad T_b = T_s + \Delta T ----- (ii)$$

substituting (ii)into (i)

$$-mc\frac{d\theta}{dt} = \sigma A[(T_s + \Delta T)^4 - T_s^4]$$

$$-mc\frac{d\theta}{dt} = \sigma A[T_s^4 + 4T_s^3\Delta T + 6T_s^2\Delta T^2 + 4T_s\Delta T^3 + \Delta T^4 - T_s^4)$$

$$-mc\frac{d\theta}{dt} = \sigma A[4T_s^3\Delta T + 6T_s^2\Delta T^2 + 4T_s\Delta T^3 + \Delta T^4$$

ΔT is small, that higher power can be neglected

$$-mc\frac{d\theta}{dt} = 4\sigma AT_s^3\Delta T$$

$$-\frac{d\theta}{dt} = \frac{4\sigma AT_s^3}{mc}\Delta T$$

since $\frac{4\sigma AT_s^3}{mc}$ is constant

thus $-\frac{d\theta}{dt} = k\Delta T$

$$-\frac{d\theta}{dt} \propto \Delta T$$

hence shown!!!!!

NECTA 2013

46. Name the temperature of thermocouple at which the thermo,

(i) emf changes its sign.

(ii) Electric power becomes zero.

Answers:

(i) The temperature at which emf changes its sign is called neutral temperature.

(ii) The temperature at which electric power becomes zero is called inversion temperature.

47. A person sitting on a bench on a calm hot summer day is aware of a cool breeze blowing from the sea. Briefly explain why there is a natural convection.

Answers;

There is a natural convection of cool air from the sea during the day because during the day, land is warmer than the ocean, as a result hot air on the land raises up in atmosphere and is replaced by cool air moving from the sea. This cool air what brings cool breeze known is sea breeze.

48. A nichrome – constantan thermocouple gives about $70\mu V$ for each $1°C$ difference in temperature of between the junctions. If 100 such thermocouples are made into thermopile, what voltage is produced when the junction are $20°C$ and $240°C$?.

solution

voltage produced

given sensitivity, $70\mu V$ per $°C$

from the information given

$$V = \Delta\theta \times \text{sensitivity of junction}$$

$$V = (240 - 20) \times 70 \times 10^{-6} \times 100$$

$$V = 15.4V$$

49. A black body of temperature θ is placed in blackened enclosure maintained at a temperature of 10°C. When its temperature rises to 30°C the net rate of loss of energy from the body was found to be 10W. find the power generated by the body at 50°C if the energy exchange takes place solely the process of forced convection.

solution

$$\text{rate of change of heat; } \frac{dQ}{dt} = P = mc\,(T_s - T_e)$$

but mc is constant

$$P \propto T_s - T_e$$

$$\frac{P_1}{P_2} = \frac{T_{s1} - T_e}{T_{s2} - T_e}$$

$$P_2 = P_1\left(\frac{T_{s2} - T_e}{T_{s1} - T_e}\right)$$

$$P_2 = 10\left(\frac{50 - 10}{30 - 10}\right)$$

$$P_2 = 20W$$

50. (a) Write down the laws governing the black body radiation.

(b) A cup of tea kept in a room with a temperature of 22℃ cools from 66℃ to 63℃ in one minute. How long will the same cup of tea take to cool from the temperature of 43°C to 40°C under the same condition?

Answers:

Laws of radiations:

Stefan's law: The rate of emission of heat per unit area is directly proportional to the fourth power of thermodynamic temperature.

Wien's displacement law: The wavelength of radiation emitted at maximum intensity is inversely proportional to temperature.

Prevost theory of heat exchange: A body emits heat at a rate which depends on the nature of its surface and its temperature and absorbs heat at a rate which depends on the nature of its surface and the temperature of its surrounding.

51. A lagged copper rod is uniformly heated by a passage of an electric current. Show by considering a small section dx that the temperature θ varies with a distance x along a rod in a way that, K = $\left(\frac{d^2T}{dx^2}\right) = -H$, where K is a thermal conductivity and H is the rate of heat generation per unit volume.

solution

$$\text{consider } P = -kA\frac{d\theta}{dx}$$

differentiate w.r.t. x

$$\frac{dP}{dx} = -\frac{d}{dx}\left(kA\frac{d\theta}{dx}\right)$$

$$\frac{dP}{dx} = -kA\frac{d}{dx}\left(\frac{d\theta}{dx}\right)$$

$$\frac{dP}{dx} = -kA\frac{d^2\theta}{dx^2}$$

$$\frac{dP}{Adx} = -k\frac{d^2\theta}{dx^2}$$

$$\text{since} \quad Adx = dv$$

$$\frac{dP}{dv} = -k\frac{d^2\theta}{dx^2}$$

$$\frac{dP}{dv} = H$$

rate of heat transfer per unit volume

$$H = -k\frac{d^2\theta}{dx^2}$$

$$k\frac{d^2\theta}{dx^2} = -H$$

NECTA 2012

52. What is difference between Kelvin temperature scale and celcius temperature scale.

Kelvin temp scale	Celcius temp scale
Has its zero at $-273.15°C$	Has its zero at ice point
There is one fixed point, the triple point of water (273.16°C)	There are two fixed points (ice and steam point)

53. Mention three basic advantages of gas thermometer.

Advantages of gas thermometer

- It is very accuracy.
- It is very sensitive.
- It can measure a wide range of temperature $(-270°C)$ to $1500°C$

54. (a) What is meant by a “a perfect thermal source” as used in thermal radiation.

(b) Define thermal conduction.

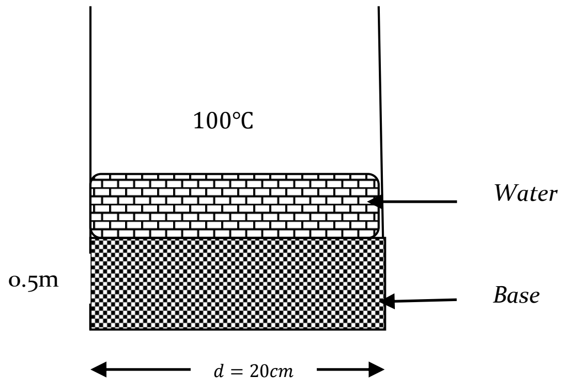
Answers:

A perfect thermal source Is the body that radiates all the heat generate in it and absorb none that falls on it.

Thermal conduction is the process by which heat flows from the hot end to the cold end of a solid body without there being net movement of the particles of the body.

55. An aluminium saucepan in contact with a hot plate has a base of diameter 20.0cm and thickness of 0.5cm if the saucepan contains water boiling away at the rate of 0.15g/s, estimate the temperature at the lower surface of the saucepan vessel.

NOTE:
"water boiling way" implies that the surface in contact with water is at 100°C. This further implies that the rate of conduction of heat by the aluminum sauce pan.



$$Q = \frac{kA(\theta_2 - \theta_1)}{l} = L_V \frac{dm}{dt}$$

$$\frac{\pi r^2 k(\theta_2 - \theta_1)}{l} = L_V \frac{dm}{dt}$$

$$\frac{210 \times \pi \times 0.1^2\ (\theta_2 - 100)}{0.5 \times 10^{-2}} = 0.15 \times 10^{-3} \times 2.25 \times 10^6$$

$$\theta_2 - 100 = \frac{1.695}{6.597}$$

$$\theta_2 = 0.2569 + 100$$

$$\theta_2 = 100.257°C$$

56. Define the term perfect black body. Give one limitation of Newton's law of cooling.

Answers:

A perfect blackbody is the body which absorbs all the radiation falling on it and reflects none.

Limitation of Newton's law of cooling

- It can only be applied when the excess temperature over the surroundings is small (less than 30°C)

57. Briefly explain the following

(i) Why does a good absorber of radiant energy appear black?

(ii) Why do two sheets of similar glass insulated much more effectively when separated by a thin layer of air than when they are in contact.

Answers:

(i) A good absorbs of radiant energy appear black since it absorbs all radiation falling including visible light. Objects appear of the colour they reflect.

(ii) Two sheet of similar glass insulate much more effectively when separated by a thin layer of air than in contact because air is a better insulator than glass, thus;
Air being a better insulator it has very low conductivity which reduces the rate of conduction of heat.

58. A roof which measures $20\text{m} \times 50\text{m}$ is blackened. If the temperature of the sun's surface is 6000K, calculate the solar energy incident on the roof per minute, assuming that half of it is lost when passing through the earth's atmosphere.

solution

Rate of radiation by sun is equal to power

$$P = \sigma A_s T_s^4$$

$$P = 4\pi R_s^2 \sigma T_s^4$$

but constant

$$S = \frac{\text{power from the sun}}{A_o}$$

since $A_o =$ is area of sphere containing the orbit

$$S = \frac{\sigma A_s T_s^4}{A_o}$$

$$S = \frac{4\pi R_s^2 \sigma T_s^4}{4\pi R^2}$$

$$S = \sigma T_s^4 \left(\frac{R_s^2}{R}\right)$$

$$S = \sigma T_s^4 \left(\frac{R_s}{R}\right)^2$$

$$S = 5.67 \times 10^{-8} \times 6000^4 \times (\frac{7.04 \times 10^8}{1.5 \times 10^{11}})^2$$

$$S = 1618.64 \text{ J/s}$$

since half of heat is lost

$$\text{the rate of heat gain} = \frac{1}{2} A_s S$$

$$P = 0.5 \times 1618.64 \times 20 \times 50$$

$$P = 809{,}321 \text{ J/s}$$

Heat Power $\times$ time

$$\text{Heat} = 809321 \times 60$$
$$\text{Heat} = 48.6 \times 10^6 \text{J}$$
$$48.6 \times 10^6 \text{ is incident on the roof per minute}$$

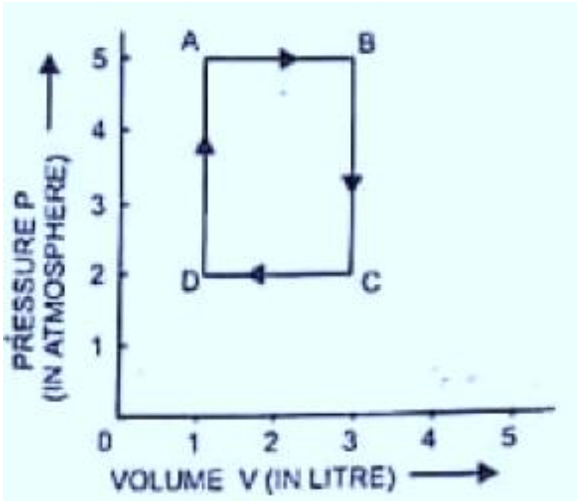
3.11. Competitive Examination File Unit Set 06:

1. One mole of an ideal gas undergoes cyclic change ACBD. Using the figure below to calculate

    a. Work done along AB, BC CD and DA.

    b. Net work done in the process

    c. Net change in internal energy of the gas. (1atm = $1.01 \times 10^5 \text{N/m}^2$)



[ans, (a) 1010J;0J; -404J; 0J (b)606J; (c)zero]

2. **A** gas occupying 1litre at 80 cm pressure is expanded adiabatically to 1190cm^3. If the pressure falls to 60cm in the process, deduce the value of γ. **[ans. 1.66]**
3. **A** gas suddenly compressed to half of its original volume. Calculate the rise in temperature, the original temperature being 27°C given that $\gamma = 1.5$ **[ans. $\mathbf{124.2}$°C]**
4. Calculate the fall in the temperature of helium initially at 15°C when it is suddenly expanded to 8 times its volume. The ration of specific heats id 5/3. **[ans. $\mathbf{216}$°C]**
5. Ideal gas at 75 cmHg pressure is compressed isothermally until its volume reduced to 3/4th of its original volume. It is then allowed to expand adiabatically to a volume of 20% greater than its original volume. If the initial temperature of a gas is 17°C, calculate the final pressure and temperature such that $\gamma = 1.4$. **[ans. 51.79cmHg; $\mathbf{-32.7}$°C]**
6. A tyre is pumped to a pressure of 6atmosphere suddenly bursts. Room temperature is 15°C. Calculate temperature of the escaping air. Given $\gamma = 1.4$ for air. **[ans. $\mathbf{172.6}$K]**

7. Calculate the final volume of one mole of a gas after isothermal expansion at $27°C$, if its original volume is $150cm^3$. Given that the amount of work done by the gas during expansion is 2.303×10^9erg and 8.31×10^7erg/molK. **[ans. $164.5cm^3$]**

8. the volume of an ideal gas in vessel is two litre at normal pressure. The gas is compressed under adiabatic conditions to such an extent as to reduce its volume to 1litre. What is the final pressure of the gas? Given that $\gamma = 1.4$ **[ans. $2.7 \times 10^5 Nm^{-2}$]**

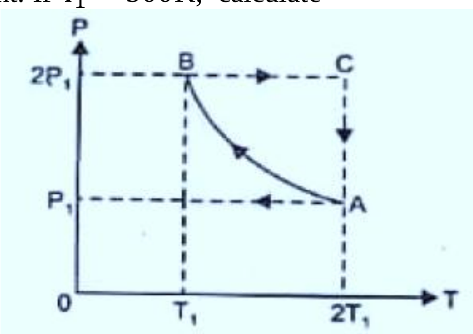
9. 2 moles of Helium gas (γ =5/3) of 20 litre volume at $27°C$ subjected to constant pressure is expanded to double the initial volume. Then it is adiabatically taken to initial temperature $27°C$. What will be the final pressure, final volume and work done in isobaric and adiabatic process. **[ans. 112.8litre, $45 \times 10^3 Nm^{-2}$; 7470 J]**

10. $2m^3$ of volume of a gas at a pressure of $4 \times 10^5 Nm^{-2}$ is compressed adiabatically so that the volume becomes $0.5m^3$. Find the new pressure. Compare this with the pressure, if the compression was isothermal. Calculate the work done in each process. Given that$\gamma = 1.4$. **[ans. $2.79 \times 10^6 Nm^{-2}$; 1.74; -1.49×10^6J; -1.11×10^6J]**

11. At 27°C, two moles of an ideal monatomic gas occupy a volume V. the gas expands adiabatically to volume 2V. Calculate
    (i) Final temperature of the gas.
    (ii) The change in its internal energy.

    The work done by the gas during the process.**[ans. 189K; -2767.2J; 2767.2J]**

12. Two moles of an ideal monatomic gas is taken through a cycle ABCA as shown below in P – T diagram. During the process AB, pressure and temperature of the gas vary such that PT = constant. If $T_1 = 300K$, calculate



    (i) The work done on the gas in the process AB
    (ii) The heat absorbed or released by the gas in each of the process. Give answers in terms of the gas constant R. **[ans. -1200R; 1500R (B to C); 1200Rlog2 (C to A); -2100R (A t o B).**

13. 20000J of heat energy is supplied to a metallic object of mass 1kg at atmospheric pressure at 20°C. Find

(i) The final temperature of the metal.

(ii) The work done by the metal.

(iii) Change in internal energy of metal. Specific heat of metal $= 400\text{J/kg°C}$. Density of metal is 9000kg/m^3, coefficient of expansion $= 9 \times 10^{-5}/°\text{C}$ and atmospheric pressure $= 10^5\text{N/m}^2$. **[ans.** $\mathbf{70}$°C; $\mathbf{0.05J}$; $\mathbf{19999.95J}$

3.12. Competitive Examination File Unit Set 07:

Problem 01

5 moles of hydrogen initially at STP are compressed adiabatically so that the temperature becomes 400°C. Find:

(i) The work done on the gas

(ii) The increase in internal energy of the gas

Given that $\gamma = 1.4$ for diatomic gas.

Problem 02

At 27°C two moles of an ideal monatomic gas occupy a volume V. the gas expands adiabatically to a volume 2V. Calculate:

(i) The final temperature of the gas

(ii) The change in its internal energy

(iii)The work done by the gas during this process

Given that $\gamma = 1.67$

Problem 03

A metallic cylinder contains 10 litres of air at 3 atmospheres of pressure and temperature of 300K.

(a) If the pressure is suddenly doubled, what are the new values of volume and temperature.

(b) If the pressure is slowly doubled, what are the new values of volume and temperature.

Problem 04

(a) Define the principle molar heat capacities of a gas.

(b) Why the energy needed to raise the temperature of a fixed mass of a gas by a specific amount is greater if the pressure is kept constant than when the volume is kept constant.

(c) Find the two principal heat capacities for oxygen (diatomic molecule) whose ratio of $\frac{c_p}{c_v}$ is 1.4 at STP.

Problem 05

A quantity of oxygen is compressed isothermally until its pressure is doubled. It is then allowed to expand adiabatically until its volume is restored. Find the final pressure in terms of the initial pressure. Given that $\gamma = 1.4$

Problem 06

(a) With the help of sketch diagram distinguish between an "isothermal change" and an "adiabatic change". Illustrate your answer with an example of a gas changing from state A to state B.

(b) Argon gas (specific heat capacity ratio 1.67) is contained in a 250 cm^3 versel at a pressure of 750mmHg and a temperature of 0°C The gas is expanded isothermally to a final volume of 400cm^3

(i) Calculate the final pressure of the gas

(ii) By how much will the pressure will be lowered if the change is made adiabatically instead?

Problem 07

(i) Define mean "free path" for a molecule of a gas

(ii) How is the means free path of the molecule of a gas affected by temperature.

(b) The heat capacity C_v at constant volume for 8 moles of oxygen gas is 166.2KJ^{-1} . Find the heat capacity at constant pressure for 8 moles of oxygen.

Problem 08

(a) What is the difference between an "isothermal process" and "adiabatic process"?

(b) How, much work is required to compress 5 moles of air at 20 °C and 1 atmosphere pressure at $\frac{1}{10}$ of the original volume by:

(i) An isothermal process

(ii) An adiabatic process

(c) What are the final pressure for case (b) (i) and (b) (ii) above?

(d) In a diesel engine, the cylinder compresses air from approximately standard temperature and pressure to about one sixteenth of the original volume and a pressure of about 5 atmospheres. What is the temperature of the compressed air?

γ = 1.403

R = 8.31 $mol^{-1}K^{-1}$

C_V = 20.68 $Jmol^{-1}K^{-1}$

Problem 09

100g of a gas are enclosed in a cylinder which is fitted with a movable frictionless piston.

When a quantity of heat is supplied to the gas it expands at constant pressure doing 8400J of work and heating up by 20°C. Calculate:

(i) The change in internal energy of the gas

(ii) The specific heat capacity of the gas at constant volume C_v

Given that C_P = 1.26 x $10^3 JKg^{-1}K^{-1}$

Problem 10

Given that the molar heat capacities of hydrogen at constant value and constant pressure are respectively $20.5 Jmol^{-1}K^{-1}$ and $28.8 Jmol^{-1}K^{-1}$. Calculate

(i) The molar gas constant

(ii) The heat needed to raise the temperature of 8g of hydrogen from 10°C to 15°C at constant pressure.

(iii) The increase in internal energy of the gas

(iv) The external work done

Problem 11

The density of a gas is $1.775 kgm^{-3}$ at 27°C and $1\times 10^5 Nm^{-2}$ pressure and its specific heat capacity at constant pressure is $846 Jg^{-1}K^{-1}$. Find the ratio of its specific heat capacity at constant pressure to that at constant volume.

Problem 12

A gas of volume $500 cm^3$ and pressure $1.0 \times 10^5 Nm$ expands adiabatically to $600 cm^3$. Calculate

(i) The final pressure

(ii) The work done by the gas

(iii) The final temperature if the initial temperature of the gas before expansion was 23°C Given that $\gamma = 1.4$

Problem 13

One gram of water becomes 1671 of steam at a pressure of 1 atmosphere $(= 1.013 \, x \, 10^5 Pa)$. The latent heat of vaporization at this pressure is $2256 Jg^{-1}$. Calculate the external work done and the increase in internal energy.

Problem 14

$1.0 m^3$ of water is converted into $1671 cm^3$ of steam at atmospheric pressure and 100 °C temperature. The latent heat of vaporization of water is $2.3 \times 10^6 Jkg^{-1}$. If 2.0Kg of water is converted into steam at atmospheric pressure and 100 $^{\circ}C$ temperature, then how much will be the increase in its internal energy?

Given that

Density of water = $1.0 \times 10^5 kgm^{-3}$

Atmospheric pressure = $1.01 \times 10^5 Nm^{-2}$

Problem 15

(a) What is an isothermal change?

(b) A cylinder fitted with a frictionless piston holds a volume of $1000 cm^3$ of air at a pressure of 1.10×10^5 Pa and temperature of 300 K. The air is then heated to 375K at constant pressure. Determine the new volume of the gas. The gas is then compressed isothermally to a volume of $1000cm^3$. Calculate the new pressure.

Problem 16

(a) (i) What is the difference between an isothermal and an adiabatic process?

(ii) Show that an adiabatic change follows an adiabatic equation.

$$PV^{\gamma} = \text{constant}$$

(b) (i) Distinguish between the specific heat capacity and the molar heat capacity. Given the unit of each.

(ii) Calculate the two principal molar heat capacities of oxygen and explain why the specific heat capacity of the gas at constant volume is less than that at constant pressure.

Problem 17

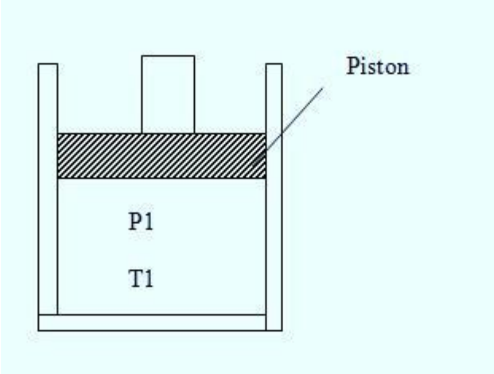
(i) What is reversible change?

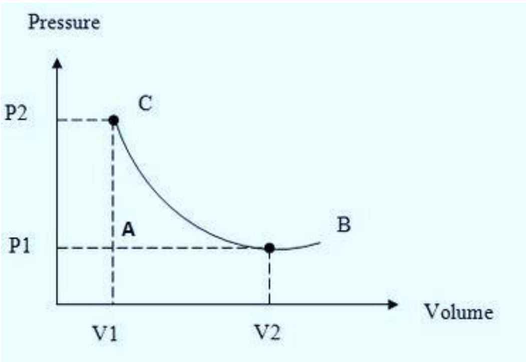
(ii) State the condition for a reversible change to occur.

(iii)A litre of air at 10^5 Pa pressure expands adiabatically and reversibly to twice its volume. Calculate the work done by the gas.

Problem 18

A cylinder in the figure below holds a volume V_1 = $1000cm^3$ of air at an initial pressure of $P_1 = 1.1 \times 10^5 Pa$ and temperature T_1 = 300K. Assume the air behaves like an ideal gas.





(i) AB – the air heated to 373 K at constant pressure. Calculate the new volume.

(ii) BC – the air is compressed isothermally to volume V_1. Calculate the new pressure P_2

(iii) Calculate the root mean square speed of nitrogen molecules at a temperature of 27 ℃

Problem 19

a. State the 1st law of thermodynamics and write its equation.

b. A liter of air initially at 25℃ and 760mmHg is heated at constant pressure until the volume is doubled. Determine:

(i) The final temperature

(ii) The external work done by the air in expanding it.

(iii)The quantity of heat supplied

Problem 20

0.15 mol of an ideal mono atomic gas is enclosed in a cylinder at a pressure of 250 KPa and a temperature of 320K. The gas is allowed to expand adiabatically and reversibly until its pressures is 100KPa

(a) Sketch a P – V curve for the process.

(b) Calculate the final temperature and the amount of work done by the gas.

Problem 21

(i) Define the bulk modulus of a gas

(ii) Find the ratio of the adiabatic bulk modulus of a gas to that of its isothermal bulk modulus in terms of the specific heat capacities of the gas.

Problem 22

(a) A gas expands adiabatically and its temperature falls while the same gas when compressed adiabatically its temperature rises. Explain giving reasons why this happens.

(b) A mole of oxygen at 280K is insulated in an infinitely flexible container is $5\times 10^{5}\text{Nm}^{-2}$. When 580J of heat is supplied to the oxygen the temperature increases to 300K and the volume of the container increases by $3.32\times 10^{-4}\text{m}^{3}$. Calculate the values of the principal molar heat capacities and the specific universal gas constant.

Given that molar mass of oxygen = 32×10^{-3}kg

Problem 23

(a) (i) Why is heat needed to change liquid water into vapour?

What amount of energy is needed

(ii) The molar heat capacity of hydrogen at constant volume is $20.2\text{Jmol}^{-1}\text{K}^{-1}$. What is the molar heat capacity at constant pressure?

(b) In an industrial refrigerator ammonia is vaporized in the cooling unit to produce a low temperature. Why should the evaporation of ammonia reduce the temperature in the refrigerator?. How much energy is needed to convert 150g of water at 20℃ into steam at 100℃

Problem 24

An ideal gas is kept in thermal contact with a very large body of constant temperature T and undergoes an isothermal expansion in which its volume changes from V_1 to V_2. Derive an equation for the work done by the gas.

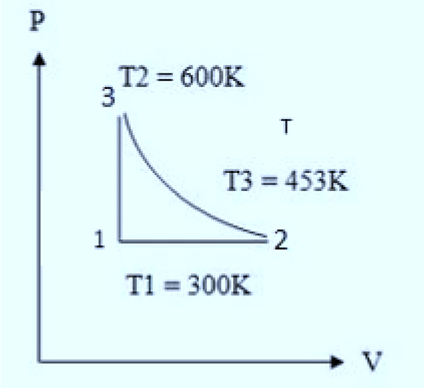
Problem 25

A heat engine carries I mole of an ideal gas around a cycle as shown in the figure below. Process 1 – 2 is at constant volume, process 2 – 3 is adiabatic and process 3 – 1 is at a constant pressure of 1 a.t.m. The value of γ for this gas is $\frac{5}{3}$.

Find:

i.)The pressure and volume at points 1, 2 and 3

ii) The net work done by the gas in the cycle.



Problem 26

The door of a working refrigerator is left open.

What effect will this have on the temperature of the room in which the refrigerator is kept?. Explain

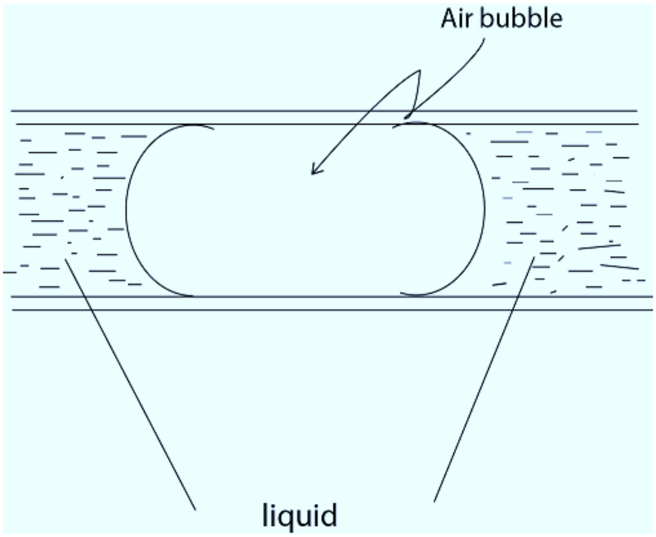
Problem 27

(a) What do you understand by the terms:

(i) Critical temperature?

(ii) Adiabatic change?

(b b) An air bubble is observed in a pipe of the braking system of a car. The pipe is filled with an incompressible liquid (see figure below). When the brakes are applied, the increased pressure in the pipe causes the bubble to become smaller.



Before the brakes are applied the pressure is $110 \times 10^3 Nm^{-2}$, the temperature is 290K and the length of the bubble is 15mm. When the brakes are applied quickly, the air bubble is compressed adiabatically and if the change in its length exceed 12mm the brakes fail. If the internal cross-sectional area of the pipe is $2 \times 10^{-5} m^2$

(i) Explain briefly why the compression of the bubble is considered to be adiabatic.

(ii) What is the maximum safe pressure in the system during rapid braking if the bubbles change in length does not exceed 12mm? Take γ_{air} = 1.4

(iii)Determine the temperature of the air in the bubble at the end of the adiabatic compression.

Problem 28

(a) Find the number of molecules and their mean kinetic energy for a cylinder of volume $4 \times 10^{-4}m^3$containing oxygen at a pressure of 2×10^5Pa Pa and a temperature of 300K

(b) When the gas is compressed adiabatically to a volume of $2\times 10^{-4}m^3$, the temperature rises to 434K. Determine the γ, the ratio of the principal heat capacities.

Given that:

Molar gas constant = $8.31 Jmol^{-1}K^{-1}$ and $N_A = 6 \times 10^{23} mol^{-1}$

Problem 29

(a) The first law of thermodynamics is a consequence of the law of conservation of energy. Explain briefly.

(b) What is the difference between isochoric process and isobaric process?

(c) Why is the energy needed to raise the temperature of a fixed mass of a gas by a specific amount is greater if the pressure is kept constant than if the volume is kept constant?

(d) A certain volume of a dry air at S.T.P is allowed to expand four times its original volumes under:

i. Isothermal conditions

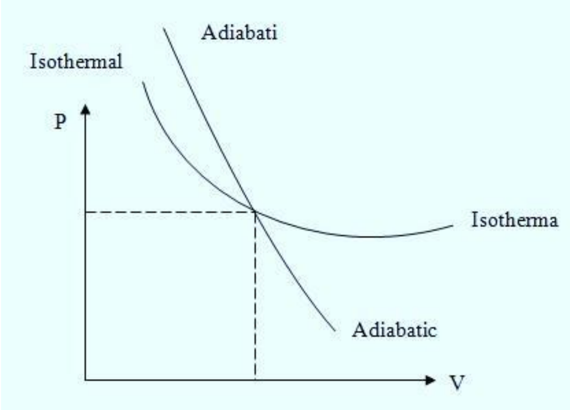
ii. Adiabatic conditions

Calculate the final pressure and temperature in each case

Given that γ = 1.4

Problem 30

In a P – V diagram shown below, an adiabatic and an isothermal curve for an ideal gas intersect. Show that the absolute value of the slope of the adiabatic $\left|[\frac{dp}{dv}]\right|$ is γ times that of the isothermal



Hence the adiabatic curve is steeper because the specific heat ratio γ is greater than 1

Problem 31

A Tyre has air pumped at a pressure of 4 atmospheres at room temperature of 27°C. If the Tyre bust suddenly, calculate the final temperature (take $\gamma = 1.4$)

Problem 32

Two moles of oxygen are initially at a temperature of 27°C and volume of 20 litres. The gas expanded first at constant pressure until the volume has doubled, and then adiabatically until the temperature returns to the original value.

(i) What is the total increase in internal energy?

(ii)What is the final volume?

Given that $\frac{C_P}{C_v} = \gamma = 2$

Problem 33

The Specific heat capacity of hydrogen at constant volume is $1.01 \times 10^4 \mathrm{Jkg^{-1}K^{-1}}$. If the density of hydrogen at S.T.P is $0.09\mathrm{kg^{-3}}$, calculate the specific heat capacity of hydrogen at constant pressure.

Problem 34

(a) Does a gas do work when is expands adiabatically? If so what is the course of energy needed to do this work.

(b) Derive a relation between the bulk modulus K and density ρ of a perfect gas under isothermal conditions and adiabatic conditions. A mass of air at 27°C and 750mmHg pressure occupies a volume of 8litres. If the air expands first isothermally until its volume increases by 50% and then adiabatically until its volume again increases by 50% each time reversibly. Calculate

i. The final pressure
ii. The final temperature

(c) An ideal gas expands adiabatically from initial temperatures T_1 to a final temperature T_2, prove that the work done by the gas is $C_v(T_1 - T_2)$

Problem 35

An ideal gas at 760mmHg is compressed isothermally until its volume is reduced to 75% of its original volume. The gas is then allowed to expand adiabatically to a volume 120% of its original volume. If the temperature of the gas is 20℃

(a) Construct the P – V indicator diagram.

(b) Calculate the final pressure and temperature

Given that:

C_P = 3600 J/kg K

C_V = 2400 J/kg K

Problem 36

A mono atomic gas initially at the temperature T = 25℃ and pressure of 2 atmospheres is expanded to a final pressure of 1.0 atmosphere.

(a) Isothermally and reversibly

(b) Isothermally against a constant pressure of 1.0 atmosphere. Calculate for each case:

i. The final temperature of the gas

ii. The increase of internal energy

3.14. General Competitive Examination File Unit Set 07:

1. The platnum thermometer has resistance of 1330Ω at $50℃$. What is the resistance at 500℃ if the temperature coefficient of resistance is $4.5 \times 10^{-3}℃^{-1}$
2. The resistance of platnum wire at temperature T℃ is measured on a gas scale given by equation $R_t = R_o(1 + aT + bT^2)$ what temperature will the platinum thermometer indicate when a temperature of a gas scale is $200℃$ given ($a = 3.8 \times 10^{-3}$ and $b = 5.6 \times 10^{-7}$
3. A copper constant thermocouple with its $0℃$ has an E.m.f of 4.28mV when its hot junction is $10℃$ the E.m.f become 9.29mV when temperature at hot junction $200℃$ if the emf is related to the temperature difference θ between the hot and cold junction by the equation $E = A\theta + b\theta^2$ Calculate;
   i. The value of A and B
   ii. Calculate the range of temperature which emf may be assumed proportional to θ without error of more than 1%
4. A cylinder contain 3 moles of oxygen at temperature of $27℃$. The cylinder is provided with frictionless piston which maintain a constant pressure of 1atm on the gas. The gas is heated unless its temperature rises to $127℃$.
   i. Calculate the energy supplied to the system
   ii. What is the change in internal enrgy of the gas
   iii. How muuch woork done in the system is required
5. (i) Define Isothermal and adiabatic changes and give the equation relating the pressure and volume of an ideal gas for each type of change.

(ii) Why has it been concluded that the pressure and volume change accompanying the passage of sound waves through a gas are adiabatic?

(b) Density of oxygen at s.t.p. is 1.43 kgm^{-3} and its molar mass is $32 \times 10^{-3} kgmol^{-1}$. Calculate the two principle molar heat capacities of oxygen if their ratio is 1.40?

(c) (i) State the first law of thermodynamics.

(ii) When applied to a fixed mass of a gas this law can be written in the form $du = dQ - pdv$. Explain the meaning of each of the three terms.

(iii) If the density of nitrogen at s.t.p. is 1.25kgm^{-3}. Calculate the root-mean square speed of nitrogen molecule at 227^0C.

6. (a)(i)The first law of thermodynamics is a consequence of the law at conservation of energy. Explain briefly.

(ii) What is the difference between isochoric process and isobaric process

(iii) Why is the energy needed to raise the temperature of a fixed mass of a gas by a specific amount is greater if the pressure is kept constant than if the volume is kept constant?

(b) A certain volume of dry air at STP is allowed to expand four times its original volume under

(i) Isothermal conditions
(ii) Adiabatic conditions.

Calculate the final pressure and temperature in each case (take $\gamma = 1.4$)

(c) (i)Calculate the change in the internal energy of 1g mass of water at atmospheric pressure when $1cm^3$ of water at its boiling point becomes $1671cm^3$ of water vapours

7(a) What do you understand by the terms

(i) Thermodynamic temperature scale?
(ii) Triple point of water?

(c) (i) What is the coefficient of thermal conductivity of a material?

(ii) The temperature difference between the inside and outside of a room is 25˚C. The room has a window of an area $2m^2$ and the thickness of the window material is 2mm. Calculate the heat flow through the window if the coefficient of the thermal conductivity of the window material is 0.5 SI units.

(d) (i)The temperature of a furnace is 2324˚C and the intensity is maximum in its radiation spectrum nearly at 12000Å. If the intensity in the spectrum of a star is maximum nearly 4800Å then calculate the surface temperature of the star.

(ii)A patient waiting to be seen by his physician is asked to remove all is clothes in an examination room that is at 16˚C. Calculate the rate of heat loss by radiation from the patient, given that his skin temperature is 34˚C and his surface area is 1.6m^2. Assume emissivity = 0.80.

(iii)Estimate the temperature of the surface of the sun from the following data: average radius of the earth's orbit $=1.5\times10^{8}km$ average radius of the sun $=7.0\times10^{5}Km,$ solar radiant power on earth at noon $=1400Wm^{-2}$. Assume the sun to be a perfectly black body?

8.(a) (i)What is fundamental interval? How would you use it to establish a scale of temperature?

(iii) Explain how the Kelvin absolute thermodynamic scale of temperature is defined (1mk)

(b) (i)State the three laws of black body radiation

(ii)The roof which measures 20m by 50m is blackened. Find the solar energy incident onto the roof per minute if the temperature of the sun's surface is about 6000K, given that half of the energy is absorbed while passing through the atmosphere, the roof being normal to the sun's rays (Radius of sun $R_s=7.5\times10^{8}m;$ distance of sun to earth $d=1.5\times10^{11}m.$

(c) (i) Define thermal conductivity of a material and state its units

(ii) What is the rate of flow of heat through a plaster ceiling of dimensions $5m\times3m\times15mm$ with $45mm$ thick layer of an insulting fiber glass if the inside and outside are at the surrounding air temperatures of 15˚C and 5˚C respectively?

9(a) (i) Briefly explain what it means by thermal conduction and define the coefficient of thermal

Conductivity.

(ii) Ice cubes of mass 5.0g at 0˚C are placed inside a spherical container having an outside Diameter of 40cm, 2mm thick and of thermal conductivity $5\times10^{-4}wm^{-1}k^{-1}$. How long will it take for all the ice cubes to melt if the room temperature is 30˚C?

(b) Two metal rods A and B of lengths 40 and 80cm respectively having the same cross sectional area of 10cm^2 are joined end to end. If the composition is perfectly lagged and the free end of A is fixed at 100˚C while that of B is pressed to a point 0˚C. Determine

(i) The junction temperature of the two metal rods. (2mks)

(ii) The quantity of heat that flows per minute in steady state. (Thermal conductivities of rod A and B are 360 and $8wm^{-1}k^{-1}$ respectively)

10.(a) Define the following terms as used in thermometry

(i) Lower fixed point

(ii) Upper fixed point
(iii) Fundamental interval
(iv) Temperature
(v) Temperature scale

(b) A constant mass of a gas maintained at a constant pressure, has a volume of 200m^3 at the
Temperature of heating ice, 273cm^3 at the temperature of water boiling point of sulphur. Calculate the value of the boiling point of sulphur. Also calculate the absolute zero temperature of the sulphur.

(c) Define
(i) Thermocouple thermometer
(ii) Reference temperature
(iii) Neutral temperature (θ_N)
(iv) Inversion temperature (θ_i)

(d) In a thermocouple thermometer, the e.m.f varies with temperature according to the following
relation.
$E_\theta = a\theta + b\theta^2$ where a and b are constants. Show that neutral temperature θ_N is given by $\theta_N = -\frac{a}{2b}$ if cold junction temperature is 0˚C.

(e) Given that $E_\theta = a\theta + b\theta^2$ where $a = 4\times10^{-3}mv/°C,\ b = -7.6\times10^{-6}mv/°C$
Calculate the following
(i) Inversion temperature
(ii) Neutral temperature
(iii) Cold junction temperature

11.(a) Define the following terms as used in heat transfer
(i) Lagged conductor
(ii) Unlagged conductor
(iii) Steady temperature
(iv) Temperature gradient(G)
(v) Thermal conductivity (K)
(vi) Rate of heat flow(R).

(b) A room has a $4m\times4m\times10cm$ roof $\left(k_1 = 1.26wm^{-1}k^{-1}\right)$. At some instant, the temperature outside is 46˚C and inside is 32˚C.
(i) Neglecting convection, calculate the amount of heat flowing per second into the room through the roof
(ii) If bricks $\left(k_2 = 0.65wm^{-1}k^{-1}\right)$ of thickness 7.5cm are laid down on the roof, calculate the new rate of heat flow under the same temperature conditions.

12.(a) Define the following laws
(i) Dulong's and petit law

(ii) Newton's law of cooling
(iii) Wiens displacement law
(iv) Stefan's law
(v) Prevost's theory of heat exchange

(b) (i) Under critical condition, the rate of heat lost to the environment is equal to the rate of heat flowing in the liquid. Show that $(\theta_t - \theta_s) = (\theta_o - \theta_s)\ell^{-kt}$. Where θ_t is the temperature of the body after time t, θ_s is the temperature of the surrounding (room temperature), θ_o is the initial temperature of the body, k is a constant and t is time taken to cool the body.

(ii) Proof the Newton's law of cooling from Stefan's law?

(iii) A body is placed inside a room where temperature is about 25˚C. If the body was placed with temperature of 75˚C and after 1 minute its temperature was found to be 71˚C. Find the time elapse for the temperature of the body to be 27˚C.

13.(a) State the following
(i) Thermodynamics
(ii) Zeroth law of thermodynamics
(iii) First law of thermodynamics
(iv) Open system
(v) Closed system
(vi) Isolated system

(b) Show that $C_p = C_v + R$ when C_p and C_v are molar heat capacities at constant pressure and constant volume of a gas respectively and R is the universal gas constant

(c) Define the following processes
- Isochoric
- Isobaric
- Isothermal
- Adiabatic

(d) Show that $PV^{\gamma} = K$ (constant) in adiabatic process.

NB $\gamma = \frac{C_P}{C_V}$

(d) Show that work done in adiabatic process when considering one mole of a gas is given by $w = \frac{R}{1-\gamma}(T_2 - T_1)$

14. (a) A mole of a gas at 127˚C expands isothermally until its volume is doubled. Find the amount of workdone and heat absorbed.

(c) A cylinder containing one gram molecule of the gas was compressed adiabatically until its temperature rose from 27˚C to 97˚C. Calculate the work done and heat produced in the gas $(\gamma = 1.5)$

(d) A quantity of oxygen is compressed isothermally until its pressure is doubled. Its then allowed to expand adiabatically until its original volume is restored. Find the final pressure in terms of the initial.

3.15. Competitive Examinaton File Unit Set 08:

Problem 01

A mono atomic gas initially at the temperature T = 25℃ and pressure of 2 atmospheres is expanded to a final pressure of 1.0 atmosphere.

a) Isothermally and reversibly

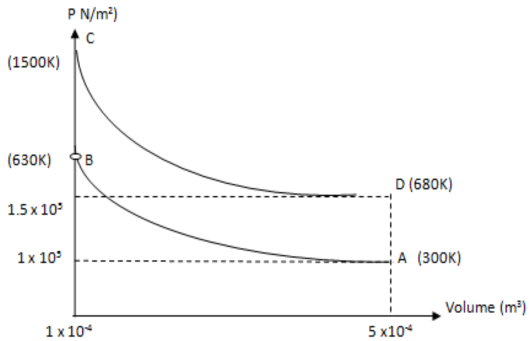
b) Isothermally against a constant pressure of 1.0 atmosphere. Calculate for each case:

(i) The final temperature of the gas

(ii) The increase of internal energy

Problem 02

The figure below shows some details concerning the behavior of a fixed mass of a gas assumed to be an ideal one in a petrol engine. The gas starts at A with a volume $5\times 10^{-4}\mathrm{m}^3$, temperature 300 K and a pressure of $1 \times 10^5\mathrm{Nm}^{-2}$. In the change from A to B it is compressed to volume of $1 \times 10^{-4}\mathrm{m}^3$, the pressure rises to $1.5 \times 10^5\mathrm{Nm}^{-2}$. And temperature 630K



a) Using the equation of state for an ideal gas, find the number of molecules in the fixed mass of a gas.

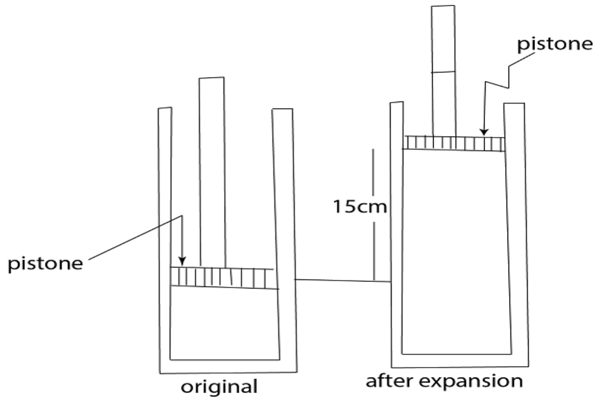
b) In the change from B to C the temperature of a gas rises from 630 K to 1500 K. The molar heat capacity at constant volume of the gas is $21\mathrm{JK}^{-1}\mathrm{mol}^{-1}$. Calculate the internal energy of the gas.

c) How much work is done by the gas in changing from B to C?

d) In the change from C to D, the gas expands to its original volume; the temperature at D is 680 K. Calculate the pressure at D.

Problem-03

The figure below shows a sample of gas enclosed in a cylinder by a frictionless piston of area $100cm^2$. The cylinder is now heated, so that 250J of energy is transferred to the gas, which then expands against atmospheric pressure of $1.00 \times 10^5 Nm^{-2}$. And pushes the piston 15.0 cm along the cylinder as shown



Calculate:

(a) The external work done by the gas
(b) The increase in internal energy of the gas.

Problem 04

When 1.50kg of water is converted to steam (at 100°C) at standard atmospheric pressure of $1.01 \times 10^5 Nm^{-2}$, 3.39MJ of heat are required. During the transformation from liquid to vapor state, the increase in volume of the water is $2.50m^3$. Calculate the work done against the external pressure during the process of vaporization. Explain what happens to the rest of the energy.

Problem05

A fixed mass of gas is cooled, so that its volume decreases from 4.0 liters to 2.5 liters at a constant pressure of $1.0 \times 10^5 Pa$.
Calculate the external work done by the gas.

Problem 06

The specific latent heat of vaporization of steam is 2.26 MJ Kg^{-1}. When $50cm^3$ of water is boiled at standard atmospheric pressure of $1.01 \times 10^5 Pa$, $83 \times 10^3 cm^3$ of steam are formed.

Calculate

(a) The mass of water boiled
(b) The heat input needed

(c) The external work done during vaporization

(d) The increase in internal energy

Given that density of water 1000 kgm^{-3}

Problem07

56.0×10^3kg of nitrogen is to be heated from 270 K to 310K. When this occurs in an insulated freely extensible container, 2.33 Kj of heat is required when contained in an insulated rigid container, 1.66KJ of heat is required. Calculate the principal molar heat capacities of nitrogen.

Problem 08

The specific heat capacity of a diatomic gas at constant volume is 0.410 $KJkg^{-1}K^{-1}$

Calculate

(a) The specific heat capacity of the gas at constant pressure.

(b) The specific gas constant for the gas.

Problem 09

The amount of heat required to raise the temperature of 3.00 mole of a polyatomic gas, at constant pressure, from 320 K to 370 K is 4.99 KJ.

Calculate

(a) C_p and C_v

(b) The value of γ

(c) The heat required to raise the temperature of 4.00 mole from 300 K to 400 K at constant volume

Problem 10

Argon has a molar heat mass of 40×10^3kg and a principal molar heat capacity, at constant volume, of $12.5 Jmol^{-1}K^{-1}$.

Calculate:

(a) The valve of γ

(b) The specific heat capacity at constant volume

(c) The amount of heat required to raise the temperature of 1.00kg of argon by 80 K at constant volume.

Problem 11

2.00 mole of nitrogen, at 300K are in an insulated, freely extensible container, and the pressure outside the container is $1.00\times 10^5 Nm^{-2}$. The principal molar heat capacity of nitrogen at constant pressure is 29.0 $Jmol^{-1}K^{-1}$.

Calculate:

(a) The heat required to raise its temperature to 340 K.

(b) The increase in volume of the gas during this process.
(c) The external work done
(d) The internal energy change
(e) The heat required to effect the temperature change at constant volume
Compare (d) and (e) and comment

Problem 12

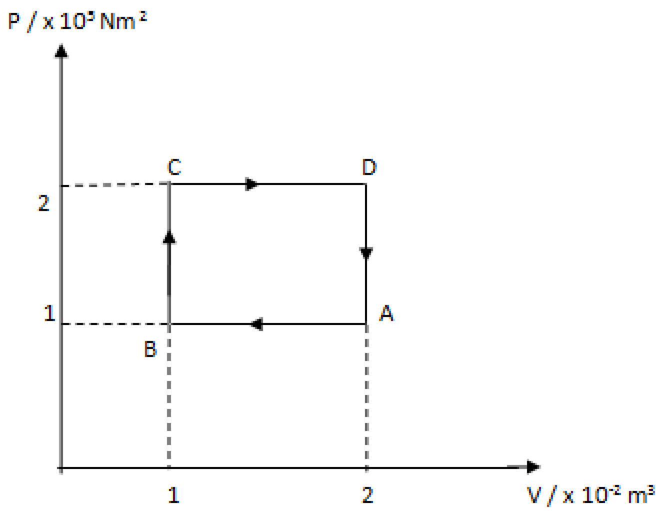
The piston of a bicycle pump is slowly moved in until the volume of air enclosed is one-fifth of the total volume of the pump and is at room temperature (290K). The outlet is then sealed and the piston suddenly drawn out to full extension. No air passes the piston. Find the temperature of the air in the pump immediately after withdrawing the piston assuming that air is a perfect gas with $\gamma = 1$.

Problem 13

A fixed mass of gas, initially at 7°C and a pressure of $1.00 \times 10^5\ \mathrm{Nm^{-2}}$, is compressed isothermally to one – third of its original volume. Calculate the final temperature and pressure, assuming $\gamma = 1.40$

Problem 14

A fixed mass of gas is taken through the closed cycle A→ B → C → D → A as shown in the figure below



(a) Calculate the work done in the cycle.
(b) How much heat transferred in the cycle?
(c) Is the heat absorbed or emitted by the gas?

Problem 15

One mole of water, occupying a volume of $1.8\times 10^{-5}cm^3$, is turned into steam in a boiler at a temperature of 373 K and a pressure of 1.0×10^5Pa. The volume of steam generated is $0.031cm^3$. The energy required is 41,000J.

Calculate the work done (in Joules) against the atmospheric pressure in the production of steam.

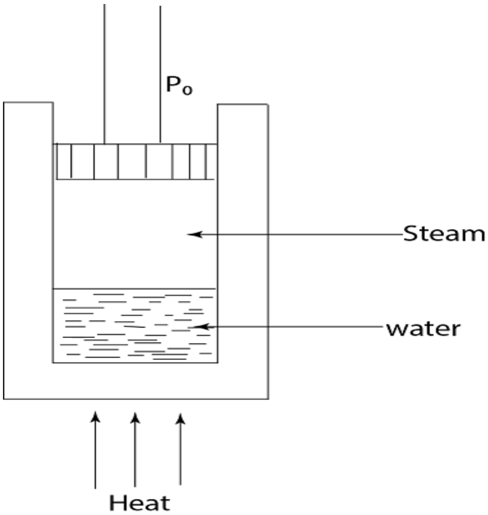
Problem 16

The figure below shows water changing into steam at constant pressure and held in a cylinder by a free – sliding piston. 1.00 kg of water at 100°C is changing into steam at atmospheric pressure. Calculate:

(a) The external work done

(b) The increase in internal energy

(c) What happens to the internal energy absorbed during the vaporization process?



Given that:

Density of water at 100°C = 960 kgm^{-3}

Density of steam at 100°C and at atmospheric pressure = 0.59

Atmospheric pressure, $P_o = 1.01\times 10^5$ Pa.

Specific latent heat of vaporization of water = $2.26\times 10^6 Jkg^{-1}$

Problem 17

At a temperature of 100 and a pressure of 1.01×10^5Pa, 1.00Kg of steam occupies $1.67m^3$, but the same mass of water occupies only $1.04\times 10^{-3}cm^{-3}$. The specific latent heat of vaporization o water at 100°C is $2.26\times 10^6 Jkg^{-1}$. For a system consisting of 1.00kg of water changing to steam at 100°C and 1.01×10^5Pa, find :

(a) The heat supplied to the system

(b) The work done by the system

(c) The increase in internal energy of the system.

Problem 18

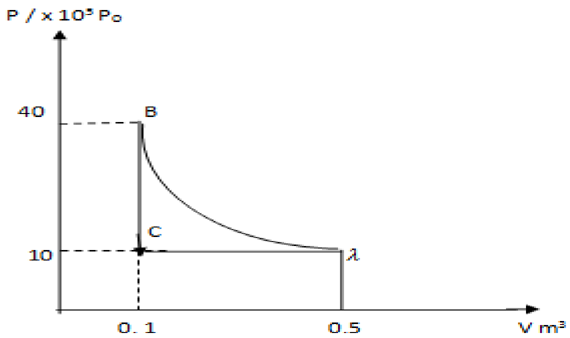
The ratio of the principal heat capacities of an ideal gas is γ, and the molar gas constant is R. Show that the molar heat capacity at constant pressure of the gas is $C_p = \frac{\gamma R}{\gamma - 1}$

Problem 19

The specific heat capacity at constant volume of a certain ideal gas is $6 \times 10^5 KJg^{-1}K^{-1}$ and is independent of temperature. Find the internal energy of 5.0×10^{-3}kg of the gas at 27°C.

Problem 20

Helium may be assumed to behave as an ideal gas. A sample of 20 moles of the gas are taken through the cycle of changes ABC as shown in the figure above.



(i) Use the data from the figure to show that the change from A to B must take place at constant temperature.

(ii) The temperature for this change is 300K. What is the temperature of the gas at C?

(iii) What energy process takes place between B and C?

(iv) The change in internal energy of the sample in the process from B to C is 56KJ. Calculate the molar heat capacity at constant volume for helium.

(v) Calculate the work done during the change from C to A. State and explain whether work is done on or by the gas during this part of the cycle. Justify your answer.

(vi) Determine the value of molar heat capacity at constant pressure for helium. Show Clearly how you arrive at your answer.

(vii) Use the figure to estimate the net work done during one complete cycle.

Problem 21

A fixed mass of an ideal gas has a volume V_o at an initial temperature of 300 K and an initial pressure of 1.2×10^5 Pa.

It is made to undergo the following cycle of process.

A. Isothermal expansion from its initial volume V_o to a volume $2V_o$
B. Expansion at constant pressure to a volume $4V_o$
C. Isothermal compression
D. Compression at constant pressure to its initial state.

(a) Sketch a cycle on a P – V diagram
(b) Determine:
(i) The pressure at the end of process A
(ii) The temperature at the end of process B
(iii) The volume at the end of process C

Problem 21

A fixed mass of an ideal gas at an initial temperature of 20°C and at a pressure of 1.00×10^5 Pa is compressed until its volume is one- quoter of its original volume. Calculate the final temperature and pressure of the gas, assuming:

(a) The compression is isothermal
(b) The compression is adiabatic

Given that $\frac{C_p}{C_v} = \gamma = 1.40$

Problem 22

In a diesel engine, fuel oil is injected into a cylinder in which air has been heated by adiabatic compression to above the ignition temperature of the oil. The ignition temperature of a certain fuel is 630°C, and the air enters the cylinder, which has an initial volume of $5.0 \times 10^{-4} m^3$ at a pressure of 1.0×10^5 Pa and a temperature of 28°C

(a) What minimum compression ratio (the ratio of the initial to the final volume of the cylinder) is required to heat the air to the fuel ignition temperature?
(b) How much work is done in compressing the air?

Given that for air $\gamma = 1.40$

Problem 23

(a) A cylinder fitted with a piston which can move without friction contains 0.05 mole of a mono atomic ideal gas at a temperature of 27°C and a pressure of 1.0×10^5 Pa.

Calculate:

(i) The volume of the gas.

(ii)The internal energy of the gas
(b)The temperature of the gas in (a) above is raised to 77°C, the pressure remaining constant.

Calculate:

(i) The change in internal energy

(ii)The external work done

(iii) The total heat energy supplied

Given that molar gas constant = $8.3\text{Jmol}^{-1}\text{K}^{-1}$.

Problem 23

(a) Give one practical example of each of the following:

(i) A process in which heat is supplied to a system without causing an increase in temperature.

(ii) A process in which no heat enters or leaves a system but the temperature changes.

(b) What happens to the energy added to an ideal gas when it is heated:

(i) At constant volume?

(ii) At constant pressure?

(c) Deduce an expression for the difference between the specific heat capacities of a gas at constant pressure and at constant volume.

(d) If the ratio of the principal specific heat capacities of a certain gas is 1.40 and its density at S.T.P is 0.09kgm^{-3}, calculate the values of the specific heat capacity at constant pressure and at constant volume. Standard atmospheric pressure $= 1.01 \times 10^5\text{Nm}^{-2}$

Problem 24

A steel pressure vessel of volume $2.2\times 10^{-2}\text{m}^3$ contains 4.0×10^5 Pa and temperature 300 K. An explosion suddenly releases 6.48×10^4J of energy, which raises the pressure instantaneously to 1.0×10^6 Pa. Assuming no loss of heat to the vessel, and ideal gas behaviour,

Calculate:

(a) The maximum temperature attained

(b) The two principal specific heat capacities of the gas.

What is the velocity of sound in this gas at a temperature of 300 K?

Problem 25

(a) Explain why an ideal gas can have infinity number of molar heat capacities and define the principal values.

(b)A thermally – insulated tube through which a gas may be passed at constant pressure contains an electric heater and thermometers for measuring the temperature of the gas as it enters and as it leaves the tube. $3.0\times 10^{-3}\text{m}^3$ of gas of density 1.8

kgm^{-3} flows into the tube in 90 seconds and, when electrical power is supplied to the heater at a rate of 0.16W, the temperature difference between the out let and inlet is 2.5 K. Calculate a value for the specific heat capacity of the gas at constant pressure.

Problem 26

(a) Explain clearly and concisely why, for a fixed mass of a perfect gas:

(i) The internal energy remains constant when the gas expands isothermally.

(ii) The heat capacity at constant pressure is greater than the heat capacity at constant volume.

(b) A vessel of volume 1.0 $\times 10^{-2} m^3$ contain an ideal gas at a temperature of 300 K and pressure 1.5 $\times 10^5$ Pa. Calculate the mass of gas, given that the density of the gas at temperature 285 K and pressure 1.0 $\times 10^5$ Pa is 1.2 kgm^{-3}

(c) 750 J of heat is suddenly releases in the gas, causing an instantaneous rise of pressure to 1.8 $\times 10^5$ Pa. Assuming ideal gas behavior, and no loss of heat to the containing vessel, Calculate the temperature rise, and hence the specific heat capacity at constant volume of the gas.

Problem 27

(a) What is an adiabatic change?

A vessel of volume 8.00 $\times 10^{-3} m^3$ contains an ideal gas at a pressure of 1.14 $\times 10^5$ Pa. A stopcock in the vessel is opened and the gas expands adiabatically, expelling some of its original mass, until its pressure is equal to that outside the vessel 1.01 $\times 10^5$ Pa. The stopcock is then closed and the vessel is allowed to stand until the temperature returns to its original value; in this equilibrium state, the pressure is 1.06 $\times 10^5$ Pa.

(i) Explain why there was a temperature change as a result of the adiabatic expansion.

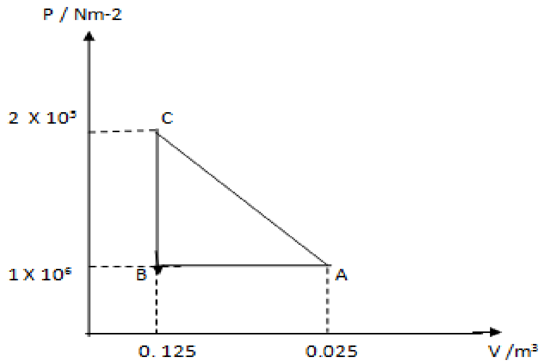
(ii) (ii)Find the volume which the mass of gas finally left in the vessel occupied under the original conditions.

(iii)Sketch a graph showing the way in which the pressure and volume of the mass of gas left in the vessel changed during the operations described above:

(iv)What is the value of γ, the ratio of the principal heat capacities of the gas.

(v) What can you deduce about the molecules of the gas? Give your reasons.

Problem 28



The diagram above represents an energy cycle whereby a mole of an ideal gas is firstly cooled at constant pressure $(A \rightarrow B)$ then heated a constant volume $(B \rightarrow C)$ and returned to its original state $(C \rightarrow A)$.

(a) Calculate the temperature of the gas at A, at B and at C

(b) Calculate the heat given out by the gas in the process A $\rightarrow$ B

(c) Calculate the heat absorbed in the process B $\rightarrow$ C

(d) Calculate the net amount of heat transferred in the cycle. Given that R = 8.3 $Jmol^{-1}k^{-1}$. and

$C_V = \frac{5}{2} R$

(e)

Problem 29

The specific latent heat of vaporization of particular liquid at 130 ^{o}C and a pressure of 2.60 $\times 10^5$ Pa is 1.84 $\times 10^6\ Jkg^{-1}$.

The specific volume of the liquid under these conditions is 2.00 $\times 10^{-3}\ m^{-3}kg^{-1}$. And that of the vapor is $5.66 \times 10^{-1} m^3 kg^{-1}$. **Calculate:**

(a) The work done, and

(b) The increase in internal energy when 1.00 kg of the vapor is formed from the liquid under these conditions.

Problem 30

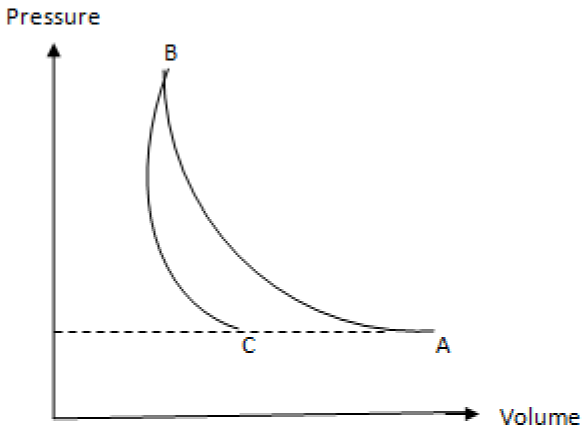
(a) Explain what is meant by a reversible change.

(b) A mass of 0.35 kg of ethanol is vaporized at its boiling point of 78 ^{o}C and a pressure of 10 $\times 10^5$ Pa. At this temperature. The specific latent heat of vaporization of ethanol is 0.95 $\times 10^{-6} Jkg^{-1}$ and the densities of the liquid and vapor are 790 kgm^{-3} and 1.6 kgm^{-3} respectively. Calculate:

(i) The work done by the system
(ii) The change in internal energy of the system
(iii) Explain in molecular terms what happens to the heat supplied to the system.

Problem 31

The graph below relates the pressure and volume of a fixed mass of an ideal gas which is first compressed isothermally from A to B and then allowed to expand adiabatically from B to C.



For each of the changes shown on the graph, state and explain whether:

(a) The temperature of the gas changes
(b) There is heat transfer to or from the gas
(c) Work is done on or by the gas

Problem 32

An ideal gas at 17 °C has a pressure of 760mmHg, and is compressed.

(a) Isothermally,
(b) Adiabatically until its volume is halved, in each case reversibly. Calculate in each case the final pressure and temperature of the gas, assuming C_p = 2100 $JKg^{-1}K^{-1}$ and C_V = 1500 $JKg^{-1}K^{-1}$.

Problem 33

(a) Show that for an ideal gas the curves relating pressure and volume for an adiabatic change have a greater slope than those for an isothermal change, at the same pressure.

(b) A gas in a cylinder initially at a temperature of 17 $^{\circ}C$ and a pressure of 1.01 $\times 10^5 Nm^{-2}$, is to be compressed to one-eighth of its volume. What would be the difference between the final pressures if the compression were done.

(i) Isothermally

(ii) Adiabatically?

Given that γ = 1.40

Problem 34

Given that the volume of a gas at S.T.P is 2.24 $\times 10^{-2}\ m^3 mol^{-1}$ and that standard pressure is 1.01$\times 10^5\ Nm^{-2}$, calculate the molar gas constant R and use it to find the difference between the quantities of heat required to raise the temperature of 0.01kg of oxygen from 0 $^{\circ}C$ to 10 $^{\circ}C$ when.

(a) The pressure is kept constant

(b) The volume is kept constant

(Given that relative molecular mass of oxygen = 32)

Problem 35

(a) By considering the expansion of an ideal gas contained in a cylinder and enclosed by a piston, show that the work done in a small expansion is equal to the pressure times the volume change.

(b) An ideal gas, at a temperature of 290 K and a pressure of 1.0 $\times 10^5\ Nm^{-2}$, occupies a volume of 1.0 $\times 10^{-3}\ m^3$. Its density conditions is 0.30 kgm^{-3}. It expands at constant pressure to a volume of 1.5 $\times 10^{-3}$. Calculate the energy added

(c) The gas is now compressed isothermally to its original volume. Calculate.

(i) Its final pressure and temperature

(ii) The difference between its final and initial internal energies.

Given that specific heat capacity at constant volume of this gas =

7.1 $= 10^2 JKg^{-1}K^{-1}$.

Problem 36

A litre of air, initially at 20 $^{\circ}C$ and at 760mmHg pressure, is heated at constant pressure until its volume is doubled. Find

(a) The final temperature
(b) The external work done by the air in expanding
(c) The quantity of heat supplied. Assume that the density of air at S.T.P is 1.293 Kgm^{-3} and that the specific heat capacity of air at constant volume is 714 $JKg^{-1}K^{-1}$.

Problem 37

a) Deduce an expression for the difference between the specific heat capacities of an ideal gas.

(b) If the specific heat capacity of air at constant pressure is 1013 $JKg^{-1}K^{-1}$ and the density at S.T.P is 1.29 Kgm^{-3}, estimate a value for the specific heat capacity of air at constant volume.

Problem 36

(a) What is the importance of the ratio of the specific heat capacities of an ideal gas?

(b) A mass of air occupying initially a volume 2 $\times 10^{-3}m^3$ at a pressure of 760mmHg and a temperature 20 $^{\circ}C$ is expanded adiabatically and reversibly to twice its volume, and then compressed isothermally and reversibly to a volume of 3 $\times 10^{-3}m^3$. Find the final temperature and pressure, assuming the ratio of the specific heat capacities of air to be 1.40.

Problem 39

Air initially at 27 $^{\circ}C$ and at 750mmHg pressure is compressed isothermally until its volume is halved. It is then expanded adiabatically until its original volume is recovered. Assuming the changes to be reversible find the final pressure and temperature take γ = 1.40

Problem 40

When water at 100 $^{\circ}C$ and pressure of 101 kPa changes to steam under the same conditions, its volume increases by a factor of 1670 given the density of water is 960 Kgm^{-3} at 100 $^{\circ}C$ and 101 kPa, and its specific latent heat of vaporization is 2.26 $\times 10^{6}JKg^{-1}$,

Calculate

(a) The heat supplied to convert 1 kg of water at 100 $^{\circ}C$ to steam at the same temperature.

(b) The work done when 1 kg of water turns to steam at 101kPa pressure.

(c) The increase of internal energy.

Problem 41

A fixed mass of ideal gas is contained in a cylinder. The cylinder volume can be varied by moving a piston in or out. The gas has an initial volume 0.01m^3 at 100 kPa pressure and its temperature is initially 300K. The gas is cooled at constant pressure until its volume is 0.006m^3. Sketch a pressure against volume graph to show the change. **Calculate:**

(a) The final temperature of the gas.

(b) The work done on the gas.

(c) The number of moles of gas.

(d) The change of internal energy of the gas.

(e)The heat transfer from the gas

(Assume R =8.3 Jmol $^{-1}$k^{-1})

Problem 42

Two identical cylinders X and Y contain equal volumes of ideal gas at the same temperature and pressure. The volume of each cylinder can be varied by moving a piston in or out for the cylinder. The gas in each cylinder is then compressed to half its initial volume: X is compressed isothermally whereas Y is compressed adiabatically. Show the changes on a compressed adiabatically. Show the changes on a pressure against volume diagram and compare the energy changes for the two gases.

Problem 43

A motor car tyres has a pressure of four atmospheres at a room temperature of 27$^{\circ}C$. If the Tyre suddenly bursts, calculate the temperature of the escaping air. Value of γ for air is 1.4

Problem 44

A molecule of a gas at 27 $^{\circ}C$ expands isothermally until its volume is doubled. Find the amount of work done and heat absorbed.

Problem 45

A cylinder fitted with a movable piston contains 3 moles of hydrogen at standard temperature are made and pressure. The walls of the cylinder are late by having a pile of sand on it. By what factor does the pressure of the gas increase if the gas is compressed to half its original volume? Given γ = 1.4

Problem 46

A quantity of air $(\gamma = 1.4)$ at 27$^{\circ}C$ is compressed

(i) Slowly and

(ii) Suddenly to one third of its volume. Find the change in temperature in each case.

Problem 47

A Tyre pumped to a pressure of 6 a.t.m suddenly burst. The room temperature is 15 $°C$. Calculate the temperature of escaping air.

Take γ = 1.4

Problem 48

A litre of air, initially at 20 $°C$ and at 760mmHg pressure, is heated at constant pressure until its volume is doubled.

Find:

(i) The final temperature

(ii) The external work done by the air in expanding

(iii)The quantity of heat supplied.Assume that the density of air at N.T.P is 1.293 kgm^{-3} and C_V = 714 $Jkg^{-1}K^{-1}$

Problem 49

A gas is suddenly compressed to one-half of its volume. Calculate the rise in temperature, the original temperature being 27 $°C$. Take γ = 1.5

Problem 50

A certain volume of dry air at N.T.P is allowed to expand four times its original volume under.

i) Isothermal conditions

(ii) Adiabatic conditions

Calculate the final pressure and temperature in each case. Take γ = 1.4

Problem 51

10 moles of hydrogen gas at NTP are compressed adiabatically so that its temperature becomes 400 $°C$. How much work is done by the gas? Also find the increase in internal energy of the gas.

Given R = 8.4 $Jmol^{-1}K^{-1}$ and γ = 1.4

Problem 52

Calculate the work done when one mole of a perfect gas is compressed diabatically. The initial pressure and volume of the gas are $10^5 Nm^{-2}$ and 6 litres respectively. The final volume of the gas is 2 litres. Molar specific heat of the gas at constant volume is $3R/_2$

Problem 53

A cylinder contains 1 mole of oxygen at a temperature of 27 $^{\circ}C$. The cylinder is provided with a frictionless piston maintains a constant pressure of 1 a.t.m on the gas.

The gas is heated until its temperature rises to 127 $^{\circ}C$.

(i) How much work is done by the gas in the process?

(ii)What is the increase in internal energy of the gas?

(iii) How much heat was supplied to the gas?

Given that C_P = 7.03 cal $mol^{-1}C^{-1}$ and

R = 1.99 cal $mol^{-1}C^{-1}$

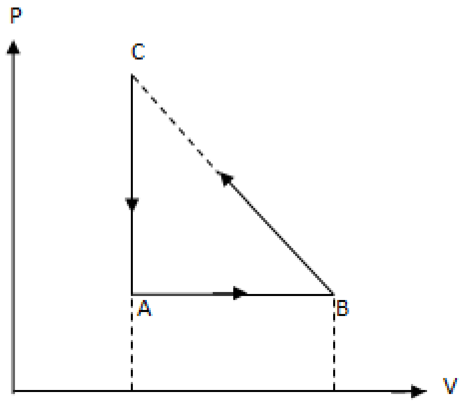
Problem 54

Two moles of helium gas $\gamma = \frac{5}{3}$ are initially at a temperature 27 $^{\circ}C$ and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled then it undergoes adiabatic change until its temperature returns to its original value.

(i) Sketch the process the P – V diagram

(ii)What are the final volume and pressure of a gas?

(iii) What is the work done by the gas? Gas constant R = 8.3 $Jmol^{-1}K^{-1}$



Problem 55

Consider the cyclic process ABC on a sample of 2.0 mole of an ideal gas as shown in the figure below

The temperature of the gas at A and B are 300 K and 500 K respectively. A total of 1200 J of heat is withdrawn from the sample. Find the work done by the gas in part BC.

Given that R = 8.3J $mol^{-1}K^{-1}$

Problem 56

A cylinder contains 3 moles of oxygen at a temperature of 27$^{\circ}C$. The cylinder is provided with a frictionless piston which maintains a constant pressure of 1 atmosphere on the gas. The gas is heated unless its temperature rises to 127$^{\circ}C$.

(i) How much heat is supplied to the gas?

(ii) What is the change in internal energy of the gas?

(iii) How much work is done by the gas in the process? Given that C_P = 7.03 cal $mol^{-1}C^{-1}$

Problem 57

An ideal gas having initial pressure P, volume V and temperature T is allowed to expand adiabatically until its volume becomes 5.66 V while its temperature falls to $T/_2$

i) What is the value of γ for the gas?

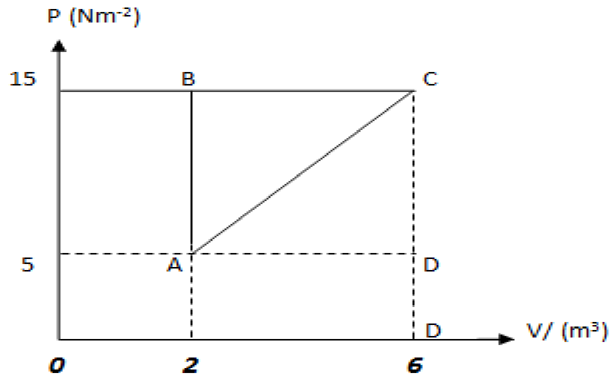
ii) Obtain the work done by the gas during expansion as a function of initial pressure P and volume.

Problem 58

What amount of heat is to be transferred to nitrogen in an isobaric heating so that the gas may perform 2 J of work?

Problem 59

In the figure below an ideal gas changes its state from state A to C by two paths ABC and AC



(i) Find the path along which work done is less

(ii)The internal energy of gas at A is 10 J and the amount of heat supplied to change its state to C through the path AC is 200J . Calculate the internal energy at C.

(iii) The internal energy of gas at state B is 20J. Find the amount of heat supplied to the gas to go from A to B.

Problem 60

As a result of isobaric heating by ΔT = 72 K, one mole of a certain ideal gas obtains an amount of heat Q = 1.60 KJ.

Find:

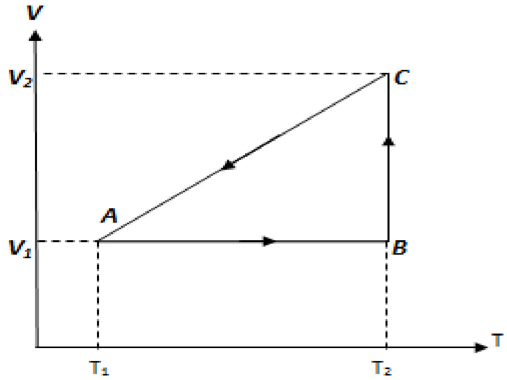
(i) The work done by the gas

(ii) The increment in its internal energy

(iii) The value of γ

Problem 61

The figure below shows a process ABCA performed on an ideal gas. Find the net heat given to the system during the process.

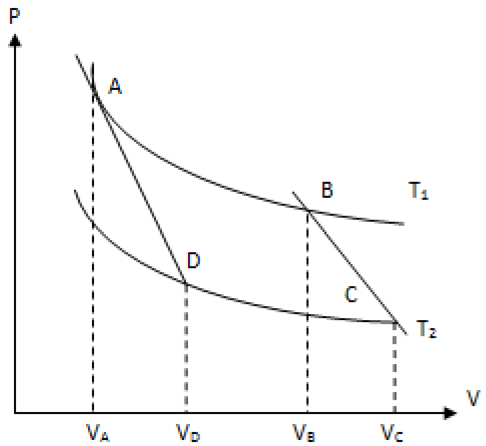


Problem 62

In a thermodynamic process the pressure of a fixed mass of a gas is changed in such a manner that the gas releases 20 J of heat and 8 J of work is done on the gas. If the initial energy of the gas was 30 J. What will be its final internal energy?

Problem 63

Two different adiabatic paths for the same gas intersect two isothermal at T_1 and T_2 as shown in the P – V diagram below. How does V_A/V_A compare with V_B/V_C?



Problem 64

At 27 $^{\circ}C$ two moles of an ideal mono atomic gas occupies a volume V. The gas expands adiabatically to a volume 2V. Find:

(i) The final temperature of the gas

(ii)The change in its internal energy

(iii) The work done by the gas during the process.

Given that R = 8.31 J $mol^{-1}K^{-1}$

Problem 65

A gas $(\gamma = 1.4)$ of 2m^3 volume and at a pressure of 4 $\times 10^5 Nm^2$ is compressed adiabatically to a volume 0.5m^3 . Find its new pressure. Compare it with the pressure obtained if compression were isothermal. Calculate the work done in each process.

Problem 66

(a) Explain what is meant by temperature gradient.

(b) An ideally lagged compound bar 25 cm long consists of a copper bar 15 cm long joined to an aluminum bar 10 cm long and of equal cross-sectional area. The free end of the copper is maintained at 100 $^{\circ}C$and the free end of the aluminium at 0 $^{\circ}C$. Calculate the temperature gradient in each bar when steady state conditions have been reached. (Thermal conductivity of copper = 390 W $m^{-10}C^{-1}$. Thermal conductivity of aluminium = 210 W $m^{-1\circ}C^{-1}$.)

Problem 67

(a) If a copper kettle has a base of thickness 2.0mm and area3.0 x $10^{-2}m^2$, estimate the steady difference in temperature between inner and outer surfaces of the base which must be maintained to enable enough heat to through so that the temperature of 1.00 kg of water rises at the rate of 0.25 Ks^{-1} . Assume that there are no heat losses, the thermal conductivity of copper = 3.8 $\times 10^2$ W $m^{-1}K^{-1}$ and the specific heat capacity of water = 4.2×10^3J/kgK .

(b) After reaching the temperature of 373 K the water in (a) above is allowed to boil under the same conditions for 120 seconds and the mass of water remaining in the kettle is 0.948 kg. Deduce a value for the specific latent heat of vaporization of water (neglecting condensation of the steam in the kettle

Problem68

A cubical container full of hot water at a temperature of 90 $^{\circ}C$ is completely lagged with an insulating material of thermal conductivity 6.4 $\times 10^2$ W $m^{-1\circ}C^{-1}$. The edges of the container are 1.0m. Estimate the rate of flow of heat through the lagging if the external temperature of the lagging is 40 $^{\circ}C$. Mention any assumptions you make in deriving your result.

Problem69

A thin-walled hot-water tank having a total surface area 5 m^2, contains 0.8 m^3 of water at a temperature of 350 K. It is lagged with a 50mm thick layer of material of thermal conductivity 4 $\times 10^{-2}$ W $m^{-1}K^{-1}$. The temperature of the outside surface of the lagging is 290 K. What electrical power must be supplied to an immersion heater to maintain the temperature of the water at 350 K? Assume the thickness of the copper walls of the tank to be negligible) What is the justification for the assumption that the thickness of the copper walls of the tank may be neglected? (Thermal conductivity of copper = 400 W $m^{-1}K^{-1}$.

If the heater were switched off, how long would it take for the temperature of the hot water to fall 1 K?

(Density of water $=$1000 kgm^{-3}; specific heat capacity of water = 4170 J $kg^{-1}K^{-1}$.)

Problem 70

(a) Sketch graphs to illustrate the temperature distribution along a metal bar heated to one end when the bar is (a) lagged, and (b) unlagged. In each case assume the temperature equilibrium has been reached. Explain the difference between the two graphs.

(b) A window pane consists of a sheet of glass of area 2.0 m^2 and thickness 5.0mm. if the surface temperatures are maintained at 0 $^{\circ}C$ and 20 $^{\circ}C$, calculate the rate of flow of heat through the pane assuming a steady state is maintained. The window is now double glazed by adding a similar sheet of glass so that a layer of air 10mm thick is trapped between the two panes. Assuming that the air is still calculate the ratio of the rate of flow of heat through the window in the first case to that in the second.

(Conductivity of glass = 0.80 W $m^{-1}K^{-1}$, conductivity of air = 0.025 W $m^{-1}K^{-1}$.)

Problem 71

An iron pan containing water boiling steadily at 100 $^{\circ}C$ stands on a hot-plate and heat conducted through the base of the pan evaporates 0.090 kg of water per minute. If the base of the pan has an area of 0.04m^2 and a uniform thickness of 2.0 $\times 10^{-3}$ m, calculate the surface temperature of the pan.

(Thermal conductivity of iron = 66 W $m^{-1}K^{-1}$. Specific latent heat of vaporization of water at 100$^{\circ}C$ = 2.2 $\times 10^6$ J kg^{-1})

Problem 72

(a) A sheet a glass has an area of 2.0 m^2 and a thickness 8.0 x 0^{-3} m. The glass has a thermal conductivity of 0.80 W $m^{-1}K^{-1}$. Calculate the rate of heat transfer through the glass when there is a temperature difference of 20 K between its faces.

(b) A room in a house is heated to a temperature 20 K above that outside. The room has 2 m^2 of windows of glass similar to the type used in (a) above. Suggest why the rate of heat transfer through glass is much less than the value calculated above.

Problem 73

(a) Explain why two sheets of similar glass each 4mm thick separated by a 10mm layer of air. Assuming the thermal conductivity of glass to be 50 times greater than that of air calculate the ratio.

(b) A double-glazed window consists of two panes of glass each 4mm thick separated by a 10mm layer of air. Assuming the thermal conductivity of glass to be 50 times greater than that of air calculate the ratio.

(i) Temperature gradient in the glass to temperature gradient in the air gap.

(ii)Temperature difference across one pane of the glass to temperature difference across the air gap.

Problem 74

(a) Outline an experiment to measure the thermal conductivity of a solid which is a poor conductor, showing how the result is calculated from the measurements.

(b) Calculate the theoretical percentage change in heat loose by conduction achieved by replacing a single glass window by a double window consisting of two sheets of glass separated by 10mm of air.

Problem 75

The silica cylinder of a radiant wall heater is 0.6m long and has a radius of 5mm. If it is rated at 1.5 kW estimates its temperature when operating. State two assumptions you have made in making your estimate.

(The Stefan constant, σ = 6 $\times 10^{-8}$ W $m^{-2}K^{-4}$).

Problem 76

(a) Explain what is meant by black body radiation

(b) A blackened metal sphere of diameter 10mm is placed at the focus of a concave mirror of diameter 0.5m directed towards the sun. If the solar power incident on the mirror is 1600 W m^{-2}. Calculate the maximum temperature in which the sphere can attain. State the assumptions you have estimated.

(The Stefan's constant, = $6 \times 10^{-8} Wm^{-2}K^{-4}$ $\times 10^{-8}$

Problem 176

If the mean equilibrium temperature of the Earth's surface is T and the total rate of energy emission by the sun is E Show that

$$T^4 = \frac{E}{16\sigma\pi R^2}$$

Where δ is the Stephan constant and R is the radius of the Earth's orbit around the sun.

(Assume that the Earth behaves like a black body)

Problem 78

An unlagged thin-walled copper pipe of diameter 2.0 cm carries water at a temperature of 40 K above that the surrounding air. Estimate the power loss per unit length of the pipe if the temperature of the surroundings is 300K and the Stefan constant, σ, is 5.67 $\times 10^{-8}$ W $m^{-2}K^{-4}$).

State two important assumptions you have made.

Problem 79

The solar radiation falling normally on the surface of the Earth has an intensity 1.40 k Wm^{-2}. If this radiation fell normally on one side of a thin, freely suspended blackened metal plate and the temperature of the surroundings was 300 K, calculate the equilibrium temperature of the plate. Assume that all heat interchange is by radiation.

(The Stefan constant $5.67 \times 10^{-8} Wm^{-2}K^{-4}$

Problem 80

A steel rod has length 1.5m and radius 1 cm. One end of the rod is maintained at 100 °C and the other end is at 0°C. Find the quantity of heat conducted through the rod in 2 minutes. The thermal conductivity of steel is 50.4 W/m K.

Problem 81

A glass window pane of a room has dimensions 2m × 0.5 m x 0.002m. The temperature on its two sides are 300 K and 295 K respectively. Find the quantity of heat conducted out of the room in 10 minutes if the room has two windows, each having two such panes. $(K_{glass} = 0.84 w/m\,K)$

Problem 82

In Searle's method. A metal rod of length 50cm and area of cross-section 8 cm^2 is used. The flow of water through the tube is adjusted at 20 grams per minute. The stead temperature of 65°C and 55°C respectively are shown by the two thermometers instead in the rod. The separation between the thermometers is 4 cm. The out flowing water shows a rise of 6°C. Find the thermal conductivity of the metal.

Problem 83

In Searle's experiment for the measurement of thermal conductivity of a metal, a road having a cross-sectional area of 10 cm^2 is used. The flow of water through the cooling tube is adjusted at 150gm/minute. When a steady state is reached, two thermometers, inserted in the road at a distance of 5cm from each other, record temperature of 60°C and 50°C respectively. If the rise in temperature of the water flowing through the cooling tube is 5°C, find the thermal conductivity of metal.

Problem 84

The temperature inside an air-conditioned room is maintained at 20°C when the outside temperature is 30°C. Calculate the quantity of heat conducted per minute through a glass window pane of area 0.25cm^2 and thickness 5mm if the thermal conductivity of glass is 0.84 W/mK.

Problem 85

The temperature inside the room is 15°C and that of outside is 5°C. How much heat will be lost by conduction per hour through one square meter of the wall if its thickness is 25cm [K for the material of the wall = 2.5 w/mK]

Problem 86

Calculate the amount of heat conducted per minute through a glass window pane of length 50cm, breadth 20cm and thickness 0.5 cm, if there is a steady temperature difference of 10°C on its two sides (Thermal conductivity of glass = 0.002 CGS units)

Problem 87

One end of a copper rod 20 cm long and 5 cm in diameter is maintained at 50°C while the other end is kept at a constant temperature of 20°C. Calculate the quantity of heat conducted through the rod in 10 seconds if the thermal conductivity of copper is 0.92 CGS units

Problem 88

A large glass window has an area of $10cm^2$ and thickness of 3mm. If the temperature in side and outside the room is 20°C and -10 °C respectively, calculate the quantity of heat flowing per second through the window. Thermal conductivity of glass = 1.5 $\times 10^{-4}$ MKS units)

Problem 89

In Searle's method, rod of length 30cm and cross-sectional area $5cm^2$ is used and flow of water is adjusted at 60 grams per minute. Steady temperature of 60°C and 50°C respectively are shown by two thermometers inserted in the rod 8cm apart. If the water coming out of the spiral shows 5°C rise in temperature, calculate the thermal conductivity of the metal.

Problem 90

The thermal conductivity of brass is 0.26 cal/s cm °C. In Searle's experiment a brass rod having a cross-sectional area of 10 cm^2 is used. When the steady state is reached, the temperature recorded by the two thermometers, inserted in the rod at a distance of 4 cm from each other, differ by 5°C. If the rate of flow of water through the cooling tube is 0.5 gm/s. find the rise in temperature of water.

Problem 91

A hollow cube of metal having mean length 10cm and thickness 0.25cm is filled with ice at 0°C and is surrounded by water at 80 °C. How much ice will melt in ten minutes?

Latent heat of ice = 80 kCal/g.

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