ADVANCED MATHEMATICS PROBLEM SOLVERS' SERIES EXAMS (AMPSSE)

FORM FIVE - SERIES 04 2021/2022

ADVANCED MATHEMATICS

ARUSHA SCIENCE	MOROGOGORO SEC	KWEMARAMBA	FEZA GIRLS
YEMEN S.S	WAJA BOYS	TUSIIME S.S	KIGONSERA S.S
MAZINDE JUU	TANGA TECH.	KWIRO BOYS	MWANZA S.S
ST. MARRY'S DULUTI	CHAMWINO S.S	KAHORORO S.S	LUKOLE S.S
BIHARAMLO S.S	MWENGE S.S	BUNGE GIRLS	FEZA BOYS
MSOLWA S.S	CHANG'OMBE S.S	ARUSHA GIRLS	TABORA BOYS
MTWARA TECH.	BENHAMIN MKAPA	IHUNGO S.S	ILEJE S.S
MRINGA GIRLS	SOKOINE MEMORIAL	LINDI S.S	PUGU S.S
NYAKATO S.S	WAJA GIRLS	BARBRO JOHANSSON	JOSIAH GIRLS
MASASI GIRLS	KIBOSHO GIRLS	NSIMBO S.S	LULUMBA S.S
MARANGU S.S	BUMANGI S.S	ILONGERO S.S	MARIAN GIRLS
ST. MAURUSCHEMCHEMI	SHAABAN ROBERT	SOYA S.S	

TIME: 3HRS **DATE: 26/01/2022 (08:00 AM)**

INSTRUCTIONS

i) This paper consists of **Ten (10)** Questions

ii) Answer **All** questions

iii) All work done in answering questions must be shown clearly

iv) Mathematical tables and Non-programmable Calculators may be used

v) Cellular Phones are not allowed in examination room

1. By using non-programmable calculator:

   (a) Calculate $\dfrac{\left(\dfrac{22}{7}+\pi\sin\dfrac{\pi}{3}\right)^{\pi}+\sqrt{\pi^{\pi}+\pi\log\pi+\sqrt[3]{\pi}\ln\pi}}{\sqrt[\pi]{\cot 89^{0}}+3.14}$ to four (4) significant figures.

   (b) Evaluates $\dfrac{dy}{dx}$ at $x=2.1$ given that $y=\left(\dfrac{e^{x^2}\tan x}{(1+x^2)\ln x}\right)^2$ to five (5) decimal places.

   (c) Find A^{-1} if matrix $A=\begin{pmatrix}2 & -1 & 3\\ 2 & 1 & -5\\ -3 & 0 & -2\end{pmatrix}$ (write the entries in fraction form).

2. A wheat flour company has factories at A and B which supply warehouses at C and D. The weekly factory capacities are 160 and 140 units respectively and the warehouse requirements are 70 and 120 units respectively. The cost of transportation of one unit of wheat flour from A to C is Sh.160 and from A to D is Sh.240. Similarly the cost of transportation of one unit of flour from B to C is Sh.200 and from B to D is Sh.260. Find how many units should be transported from each factory to each warehouse at a minimum transportation cost.

3. (a) Define the following terms:
   (i) Set
   (ii) Sub set
   (iii)Power set

   (b) Use algebraic laws of sets to simplify $\left(A^{c}\cap B\right)^{c}-\left(A^{c}\cup(A\cap B)\right)$

   (c) Of 260 animals in Mikumi National park, 50 animals eat all types of food in The National park i.e. grass, meat and bones. 60 animals eat grass and meat only, 20 animals eat grass and bones only and 40 animals eat meat and Bones only. The number of animals eating one type of food only is divided equally between the three types of food. Use Venn diagram to find the number of animals eating
   (i) Grass
   (ii) Exactly one type of food
   (iii) Bone, but not grass or meat
   (iv) Grass and bones, but not meat

4. (a) (i) What is the truth value of the statement "$3+4=7$, if 3 is an odd number and 4 is a prime number"

(ii) By using laws of algebra of propositions prove that $p \vee (p \wedge q) = p$.

(b) Use a truth table to determine whether the following argument is valid or not:

"If I work hard, then I will get praised. If I get praised then I will not have to get a second job. I got a second Job. Therefore, I did not work hard"

(c) Consider the truth table below:

p	q	r	Z
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

(i) Write the compound statement equivalent to the truth table of statement Z

(ii) Simplify the compound statement in 3(c)(i)

(iii) Draw the corresponding network of 3(c)(ii)

5. (a) Given the statement S: $Q \rightarrow P$, write down:

(i) the contrapositive of the converse of S

(ii) the inverse of the contrapositive of S

(b) Draw a simplified network for the statement $(p \rightarrow q) \wedge (p \vee q)$

(c) Symbolize statements S_1 and S_2. Hence use truth table to show that $S_1 \equiv S_2$.

S_1: If it is Saturday and it is a holiday, then we can sleep late

S_2: If it is Saturday, then we can sleep late or it is not a holiday

6. (a) Show that $f(x) = \ln\left(\sqrt{x^2+1} - x\right)$ is an odd function.

(b) If $f(x) = x^2 - 2x + 1$ and $g(x) = x^2$, find the coefficient of x^2 in $f \circ g(x)$.

(c) Given $f(x) = \dfrac{x-1}{x^2 - x - 6}$

(i) Find the asymptotes and intercepts of $f(x)$

(ii) Sketch the graph of $f(x)$

(iii) State the domain and range of $f(x)$

7. (a) Find the shortest distance from point $(-2,7)$ to the circle $x^2+y^2-2x+6y+6=0$

   (b) Find the equation of a straight line that pass though the origin and divide the line segment joining $(4,-2)$ and $(1,10)$ internally into the ratio $2:1$.

   (c) Find the equation of the circle passing through the points (1,1), (2,4) and (5,3).

8. (a) Calculate the area of the polygon with vertices $A(0,0)$, $B(-2,8)$, $C(-5,-5)$, $D(0,-1)$, $E(5,-8)$ and $F(7,-1)$.

   (b) A point $P(x,y)$ moves so that its distance from $A(7,0)$ is equal to its distance from the y-axis. Find the locus of the point P.

   (c) Find the equation of the circle orthogonal to both the circles $x^2+y^2+4x-4y-2=0$ and $x^2+y^2+2x-2y-1=0$, and whose centre lies on the line $2x-3y-2=0$.

Section B (20 Marks)

Answer any two (2) questions

9. (a) Given that α and β are acute angles such that $\sin\alpha=\frac{7}{25}$ and $\cos\beta=\frac{5}{13}$. Find, without using calculating devices, the value of $\tan(\alpha+\beta)$.

   (b) Write $\sin(\sin^{-1}x+\cos^{-1}x)$ in its most simplified form.

   (c) Find the general solution of the equation $\sin 2x+\sin 3x+\sin 5x=0$.

10. (a) Find an approximation for the expression $\frac{\sin 3\theta}{1+\cos 2\theta}$ when θ (in radians) is small.

    (b) Solve the equation $\tan^{-1}\left(\frac{x-1}{x+2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{2}$

    (c) Find the greatest and least values of $\frac{1}{4\sin\theta-3\cos\theta+6}$.

11. (a) Show that the first four terms of the expansion of the expression $\frac{1}{(2+3x)^2}$ are

$$\frac{1}{4}-\frac{3x}{4}+\frac{27x^2}{16}-\frac{27x^3}{8}$$

    (b) Solve the simultaneous equations $\log(x+y)=0$ and $2\log x=\log(y+1)$.

(c) Given that matrix $A=\begin{pmatrix} a & -4 & -6 \\ -8 & 5 & 7 \\ -5 & 3 & 4 \end{pmatrix}$ is the inverse of matrix $B=\begin{pmatrix} 1 & 2 & -2 \\ 3 & b & 1 \\ -1 & 1 & -3 \end{pmatrix}$, find the values of a and b. Hence, solve the system of equations:

$$\begin{cases} x+2y-2z=5 \\ 3x+by+z=0 \\ -x+y-3z=a-1 \end{cases}$$

12. (a) What are the partial fractions of $\dfrac{x^2+2}{x^3+x^2}$

(b) (i) Show that $x-1$ is a factor of the expression $2x^3+5x^2-4x-3$ and then factorize the polynomial $P(x)=2x^3+5x^2-4x-3$.

(ii) Hence, or otherwise solve the inequality $2x^3+5x^2-4x-3<0$

(c) Use the formula for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that $\sum_{r=1}^{n} r^2(4r-3)=kn(n+1)(2n^2-1)$, where k is a constant. Hence, evaluate $\sum_{r=1}^{40} r^2(4r-3)$.