COLLISION.

When a body in motion interacts with body (either at rest or in motion) a collision is said to have taken place

TYPES OF COLLISION.

Collision can be either elastic or inelastic collision.

1. ELASTIC COLLISION

Is a type of collision in which the total kinetic energy of the colliding bodies is conserved.

or.

Is the collision in which both linear momentum and kinetic energy are conserved.

2. INELASTIC COLLISION.

Is the type of collision where by the kinetic energy of colliding bodies are not conserved but the momentum is conserved.

- Kinetic energy is not conserved because, some of the mechanical energy is lost in the collision in the form of heat or used in deforming the bodies.

Hence the body stick together after collision on moves with common velocity.

Before Collision. After collision.

from.

Linear Conservation of momentum.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2.$$

$$v_1 = v_2 = v_c \quad \text{(Common velocity).}$$

$$M_1 U_1 + m_2 U_2 = m_1 V_c + m_2 V_c.$$

$$m_1 u_1 + m_2 u_2 = V_c (m_1 + m_2).$$

$$\boxed{V_c = \frac{m_1 u_1 + m_2 u_2}{(m_1 + m_2)}}$$

Where

V_c - Is common velocity of two bodies after collision.

OBLIQUE COLLISION.

Is the collision where by both colliding or one of them being at a certain angle.

This is mainly applicable to pool table.

- Consider a particle of mass m_1 colliding elastically with a particle of mass m_2 which is initially at rest.

Let U_1 be the initial velocity of mass m_2 move along X-axis.

After collision, the two particles move with velocities V_1 and V_2 making angle θ_1 and θ_2 with X-axis.

In horizontal component.

from conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1u_1 + (m_2 \times 0) = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2.$$

$$m_1u_1 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2.$$

In Vertical component

from principle of conservation of linear momentum.

$$(m_1u_1 + m_2v_2) = m_1v_1 + m_2v_2.$$

$$0 = m_1v_1\sin\theta_1 + m_2(-v_2\sin\theta_2).$$

$$m_1v_1\sin\theta_1 = m_2v_2\sin\theta_2.$$

BALLASTIC PENDULUM.

This is the device used to measure the speed of very light bodies e.g Bullet.

A bullet must collide with a heavy stationary body suspended by a string.

- Hence to derive the expression of velocity of the bullet we use the principle of conservation of linear momentum.

- Consider a bullet of mass m and the block of mass M hanging freely from the point of attachment when the bullet was fired from the gun hit the block and imbended itself in it and they move to

Consider

from linear conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$

$$mu + m(0) = mv + mv$$

$$mu = mv + mv$$

$$mu = v(m + M).$$

$$V = \frac{mu}{(m+M)}.$$

Again

from linear conservation of energy

Total energy before = Total momentum after

Initial total energy = Final total energy.

$$(P.E + K.E)_i = (P.E + K.E)_f$$

$$0 + \frac{1}{2}\left((m+M)V^2\right) = (m+M)gh + \frac{1}{2}(m+M_2)0^2$$

$$\frac{1}{2}(m+M)V^2 = (M+M)gh + 0.$$

$$\frac{1}{2}V^2 = gh.$$

$$V^2 = 2gh.$$

$$V = \sqrt{2gh}.$$

but

You are given angle

from $\cos\theta = \frac{L-h}{L}$

$$L - h = L\cos\theta.$$

$$h = L - L\cos\theta$$

$$\boxed{h = L(1 - \cos\theta).}$$

Also.

$$V = \sqrt{2gh}.$$

$$\boxed{V = \sqrt{2g(L(1-\cos\theta))}}$$

If $V = \frac{mu}{M+m}$.

Since $V^2 = 2gh$.

$$\left(\frac{mu}{M+m}\right)^2 = 2gh.$$

$$h = \frac{1}{2g}\left(\frac{mu}{M+m}\right)^2.$$

but

$$\cos\theta = \frac{L-h}{L}.$$

$$\cos\theta = \frac{L}{L} - \frac{h}{L} = 1 - \frac{h}{L}$$

$$\cos\theta = 1 - \frac{\left(\frac{1}{2g}\left(\frac{mu}{M+M}\right)^2\right)}{L}.$$

$$\boxed{\theta = \cos^{-1}\left(1 - \frac{1}{2gL}\left(\frac{mu}{M+M}\right)^2\right)}$$

COEFFICIENT OF RESTITUTION (e).

Is the ratio of relative velocity of separation to the relative velocity of approach.

$$e = \frac{V_2 - V_1}{U_1 - U_2}.$$

When

$e = 1$, such collision is Elastic collision.

so $$1 = \frac{V_2 - V_1}{U_1 - U_2}$$

$$\boxed{u_1 - u_2 = v_2 - v_1}$$ or

$$\boxed{u_1 + v_1 = u_2 + v_2.}$$

For perfect elastic collision, both linear momentum and K.E are conserved.

Proof.

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$$

$$m_1 u_1^2 + m_2 u_2^2 = m_1 v_1^2 + m_2 v_2^2.$$

$$m_1 u_1^2 - m_1 v_1^2 = m_2 v_2^2 - m_2 u_2^2.$$

$$m_1 (u_1^2 - v_1^2) = m_2 (v_2^2 - u_2^2).$$

$$m_1 ((u_1 - v_1)(u_1 + v_1)) = m_2 ((v_2 + u_2)(v_2 - u_2))$$

but for perfect elastic collision.

$$u_1 + v_1 = u_2 + v_2.$$

then

$$m_1 ((u_1 - v_1)(v_2 + u_2)) = m_2 ((v_2 + u_2)(v_2 - u_2)).$$

$$m_1 (u_1 - v_1) = m_2 (v_2 - u_2).$$

$$m_1 u_1 - m_1 v_1 = m_2 v_2 - m_2 u_2$$

$$\boxed{m_1 u_1 + m_2 u_2 = m_2 v_2 + m_1 v_1}$$

~~that~~

That both K.E and linear momentum is conserved.

When $e = 0$, It means perfect inelastic collision.

from $e = \dfrac{v_2 - v_1}{u_1 - u_2}.$

$$0 \times (u_1 - u_2) = v_2 - v_1$$

$$\boxed{v_1 = v_2.}$$

For inelastic collision, velocity of separation is less than velocity of Approach.

$$\frac{V_2 - V_1}{U_1 - U_2} < 1.$$

$$e < 1.$$

This results, show that after collision the colliding object moves with common velocity for this to happen the objects must stic together and move as a single unit.

MOMENTUM OF SYSTEM OF VARYING MASSES AND VELOCITY.

There are numerous cases where momentum changes are produced by reaction or explosive forces.

Example of System of Varying masses and velocity are rocket and Jet propulsion Sand on convey or belts and hose pipe.

Case 01. Rocket Propulsion.

Consider diagram below.

Initially at time t a system of mass m is moving with velocity v. Later at time $(t + \Delta t)$ the mass $(m - \Delta m)$ and move the speed $(v + \Delta v)$.

But at the sametime the rocket ejects mass Δm which moves in the opposite direction with the speed u as the time, mass and velocity change or become very small. Then.

$$\Delta t = dt, \quad \Delta m = dm, \quad \Delta v = dv$$

So also these obey linear conservation of momentum from that principle

$P_{initially} = P_{final}$.

$MV = (m - \Delta m)(V + \Delta V) + dm(U)$

as U – Relative velocity but in opposite direction as V_r

$MV = (m - \Delta m)(V + \Delta V) + dm(-U)$.

$MV = (m - \Delta m)(V + \Delta V) - Udm$.

$= mV + m\Delta V - V\Delta m - \Delta m \Delta V + Udm$.

$MV = mV + mdV - Vdm + dmdV + Udm$.

so for small change equal to zero

$dmdV = 0$

$0 = + mdV - Vdm + dmdV + Udm$.

$0 = + mdV - Vdm + Udm$.

$mdV = Vdm - Udm$

$mdV = -Udm + Vdm$.

$mdV = -dm(U - V)$.

$dV = -\frac{dm}{m}(U - V)$.

Note

quantity $U - V$ give the relative velocity in which the ~~objects~~ ejected mass move with respect to the system as V_r

If $dV = -\frac{dm}{m}(U - V)$.

$dV = -\frac{dm}{m}(V_r)$.

$dV = -V_r \frac{dm}{m}$

This is applicable for all system of changing masses.

then, To get the velocity of the rocket in out space (Neglect gravitational pull of the earth).

If $dV = -\frac{V_r dm}{m}$.

$$\int dv = \int -V_r \frac{dm}{m}$$

$$\int_{v_i}^{v_f} dv = -V_r \int_{m_i}^{m_f} \frac{dm}{m}$$

$$\left[V\right]_{v_i}^{v_f} = -V_r \left[\ln m\right]_{m_i}^{m_f}$$

$$V_f - V_i = -V_r (\ln m_f - \ln m_i)$$

$$V_f - V_i = -V_r \left[\ln \left(\frac{m_f}{m_i}\right)\right]$$

$$V_f - V_i = V_r \left(\ln \left(\frac{m_i}{m_f}\right)\right)$$

$$V_f = V_i + V_r \ln \left(\frac{m_i}{m_f}\right)$$

The Truth on a rocket is a recoil force exerted on the rocket by exahust gases. Thrust can be obtained as follows.

from

$$dV = -V_r \frac{dm}{m}$$

$$\frac{m\,dv}{dt} = -V_r \frac{dm}{dt}$$

divide by dt.

$$m\left(\frac{dv}{dt}\right) = -V_r \frac{dm}{dt}$$

$$ma = -V_r \frac{dm}{dt} \quad (\text{Since } F = ma)$$

$$F = -V_r \frac{dm}{dt} \quad \text{Thrust}$$

If the rocket is under the influence of gravity.

$$m\frac{dv}{dt} = -V_r\frac{dm}{dt} - mg.$$

$$m\frac{dV}{dt} = -V_r\frac{dm}{dt} - mg.$$

$$\frac{dv}{dt} = \frac{-V_r}{m}\frac{dm}{dt} - \frac{mg}{m}.$$

$$\frac{dv}{dt} = \frac{-V_r}{dt}\frac{dm}{m} - g.$$

$$\int_{V_i}^{V_f} dV = \int_{m_i}^{m_f} -V_r\frac{dm}{m} - \int_0^t g\,dt$$

$$V_f - V_i = V_r\left[\ln\frac{m_f}{m_i}\right] - gt$$

$$V_f = V_i - V_r\ln\left(\frac{m_f}{m_i}\right) - gt.$$

$$V_f = V_i + V_r\ln\left(m_i/m_f\right) - gt.$$

JET PROPULSION.

A jet engine uses surrounding oxygen for space travel - The compressor draws air at the front compresses it fuel is injected and the mixture burns to produce hot exhaust gases which escape at high speed from the reer end of the engine. This inturn cause forward propulsion and drive the turbine which rotates the turbines hence the jet Takes off.

Suppose air of mass ma enter front with the velocity V_i and it mixes with fuel of mass mf in the Combustion chamber. The mixture of air and fuel burns and the exhaust (burnt) gases will be ejected with velocity V_0 through the rear end.

Initial linear momentum P_i of coming air is given by.

$$P_i = m_a v_i$$

. Find linear momentum.

$$P_f = (m_a + m_f) V_0.$$

Change in momentum $= P_f - P_i$

$$\Delta P = P_f - P_i$$

$$\Delta P = (m_a + m_f) V_0 - m_a v_i$$

from.

Thrust = Force = Rate of change in momentum.

$$F = \frac{dP}{dt}$$

$$F = \frac{(m_a + m_f) V_0 - m_a v_i}{t}$$

$$F = \left(\frac{m_a}{t} + \frac{m_f}{t}\right) V_0 - \frac{m_a v_i}{t}$$

$$= \left(\frac{m_a}{t}\right) V_0 + \left(\frac{m_f}{t}\right) V_0 - \frac{m_a v_i}{t}$$

hence.

$$\text{Thrust} = F = \left(\frac{m_a}{t}\right) V_0 + \left(\frac{m_f}{t}\right) V_0 - \left(\frac{m_a}{t}\right) v_i$$

REACTION ON A HOSE PIPE

This is mainly used to spread water on a tall building. It operates on the basis of Newton third and second law. Consider diagram below.

water hits the walls at right angle which moves from the hose pipe with velocity V, where by the hosepipe has cross-section area A and the distance of the end of the hose pipe to the wall is x.

Expression of rate of mass of water that strikes on wall

from

$$mass = Density \times volume$$

$$m = \rho V$$

but $V = Ax$, $m = \rho Ax$

$$\text{Rate of mass} = \frac{dm}{dt}$$

$$\frac{dm}{dt} = \frac{d}{dt}(\rho Ax).$$

$$Rate = \rho . A \frac{dx}{dt}$$

$$\frac{dm}{dt} = \rho . A . V$$

but Also

Force exerted on the wall $F = \frac{dP}{dt}$.

$$F = \frac{d(mv)}{dt}$$

$$F = v\frac{dm}{dt}$$ but $dm/dt = \rho AV$

$$F = (V . \rho . A . V)V = V^2 \rho . A .$$

$$F = \rho . A . V^2$$

Note: When water strike at a certain angle.

$$F_x = \rho . A . V^2 \cos\theta$$

$$F_y = \rho . A . V^2 \sin\theta$$

Assumptions to arrive at the expression.

(a) velocity of water is constant after it strikes the wall it loses its velocity.

(b) water does not rebounce after the impact with water.