PRESIDENT'S OFFICE

REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

JOINT EXAMINATION SOUTHERN ZONE
(MTWARA AND LINDI)

142/1 **ADVANCED MATHEMATICS 1**
(For Both School and Private Candidates)

Time: 3:00 Hours **Tuesday, 05th, July,2022 a.m.**

Instructions

1. This paper consists of **ten(10)** questions each carrying **ten(10)** marks.
2. Answer **all** questions.
3. **All** necessary working and answers of each question must be shown clearly.
4. Mathematical tables and non-programable calculators may be used.
5. Cellular phones and **any** unauthorized materials are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

This paper consist of four printed pages

1. (a) Use a non programmable calculator to evaluate the following;

(i) $\sqrt[7]{\dfrac{\cot(e^2)+{}^5P_2\cosh(\ln 2)}{\sin 63°21' + \sum_{r=1}^{2}\ln(5r)}}$, correct to three decimal places.

(ii) $\ln\left[\dfrac{\sqrt{98.2}\times(0.0076)^{-1}\times 10^{-2}}{\tan\frac{\pi}{3}\times\cos^3\frac{\pi}{4}}\right]$, correct to four significant figures.

(b) By using a non-programable scientific calculator calculate

(i) $\sum x$

(ii) Mean $(\bar{x})$

(iii) Standard deviation.

(iv) variance of:

Values (x)	110	130	150	170	190
Frequency (f)	10	31	24	2	2

2. (a) Given that $\sinh^{-1}x = \text{sech}^{-1}x$, show that $x = \sqrt{\dfrac{\sqrt{5}-1}{2}}$

(b) Express $10\cosh x + 6\sinh x$ in terms of $R\cosh(x+\alpha)$ where $R > 0$, hence find the minimum value and corresponding value of x.

(c) Verify that $\lim_{a\to 0}\ln(\cosh a) = \dfrac{a^2}{2} - \dfrac{a^4}{12}$

3. A wheat flour company has factories at A and B which supply warehouses at C and D. The weekly factory capacities are 160 and 140 respectively and the warehouse requirements are 70 and 120 units respectively. The cost of transportation of one unit of wheat from A to C is sh 160 and from A to D is sh. 240. Similarly the cost of transportation of one unit of flour from B to C is sh.200 and from B to D is sh.260. Find how many units should be transported from each factory too each warehouse at minimum transportation cost.

4. (a) The mean and variance of 7 observations are 8 and 16 respectively. If five of the observations are 2, 4, 10, 12 and 14. Find the other two observations.

(b) The scores of 38 students in a basic mathematics test were grouped and tabulated in tables shown below;

Class interval	21-25	26-30	31-35	36-40	41-45	46-50
Frequency	5	4	12	6	3	5

Calculate

(i) Mean by coding method using class mark of modal class as assumed mean
(ii) Mode
(iii) Median

5. (a) (i) What is power set?
(ii) Given that A= $\{1,2,3,4\}$ and B= $\{1,4,5\}$ find the power set of AΔB.
(b) Use the laws of algebra of sets to simplify $(A' \cap B' \cap C) \cup (B \cap c) \cup (A \cap C)$
(c) There are 146 students at a certain school taking PCB (Physics, Chemistry and Biology). If 57 take physics, 91 take chemistry, 77 take biology, 35 take physics and chemistry only, 40 take chemistry and biology only, 16 take only chemistry and non take physics and biology only, how many students take
(i) All the three subjects.
(ii) Physics and chemistry
(iii) Chemistry or Biology but not Physics.
(iv) Biology, but not Physics or Chemistry.

6. (a) If $f(x) = 4x - 1$, $g(x) = \dfrac{1}{\mathbf{m}x - 2}$, find the value of **m** where f∘g(2)= 0
(b) If $f(x) = \dfrac{x^2 + x}{x^2 - 2x - 3}$, find
(i) The asymptotes
(ii) Sketch the graph of $f(x)$
(iii) State domain and range.

7. (a) Given the function defined by $f(1+x) = x^2 + 1$ find **h** if $f(2 - \mathbf{h}) = 10$
(b) Given that $f(x) = \sqrt{x-1}$ and $g(x) = \ln x$. State with reasons whether the composite function f∘g exists or not.
(c) Show that $f(x) = \ln(\sqrt{x^2+1} - x)$ is an odd unction.

8. (a) The line $y = \mathbf{m}x$ is tangent to curve $x^2 + y^2 - 2x + 6y + 5 = 0$ find the value of **m** and the possible point of contact.
(b) Find the value of p for the two circle $3x^2 + 3y^2 - 4px - 2py = 0$ and $x^2 + y^2 - 2x - py - 7 = 0$ are orthogonal.
(c) find the equation of the straight line which passes through the point (1,2) and makes an angle of 45° with x-axis.

9. (a) Show that $\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\mathrm{d}x = \ln[\mathrm{kf}(x)]$ where k is a constant.

   (b) $\int e^{\frac{x}{2}} \cos 2x \, \mathrm{d}x$

   (c) (i) Show that the area formed by a certain curve, $\mathrm{f}(x)$ under the x-axis from certain point x_1 to x_2 along the x-axis is given by Area$= \int_{x_1}^{x_2} \mathrm{f}(x) \, \mathrm{d}x$

   (ii) Use the formula in (i) above to find the re bounded by the curve $\mathrm{f}(x) = 2x - x^2$ and the x-axis.

10. (a) use the definition to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = \frac{1}{x+1}$

    (b) The parametric equation given by $x = \cos\mathrm{t} + \mathrm{t}\sin\mathrm{t}$ and $y = \sin\mathrm{t} - \mathrm{t}\cos\mathrm{t}$. Show that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\sec^3\mathrm{t}}{\mathrm{t}}$

    (c) Apply Taylor's theorem expand $\tan\left(x + \frac{\pi}{4}\right)$ as far as the term x^4 and evaluate $\tan 46.5°$ to four significant digits.