20th JULY 2021

MECHANICS.

NEWTON'S LAWS OF MOTION.

1st law is called the law of Inertia.
Inertia – Is the ability of a body to resist change in linear motion.

2nd law.

$$\frac{d(mv)}{dt} \propto F.$$

$$m\frac{dv}{dt} + V\frac{dm}{dt} = KF \quad (K=1).$$

$$m\frac{dv}{dt} + V\frac{dm}{dt} = F$$ (limitation mass in 2nd law is constant

$$\frac{dm}{dt} = 0.$$

$$m\frac{dv}{dt} = F.$$

but $\frac{dv}{dt} = a$

$$F = ma.$$

$$\boxed{F = ma.}$$

3rd Law of Newton's of Motion.
It states that "To ~~an~~ every action there is an equal and opposite reaction".

Example: A bullet of mass 0.04kg moving at a speed of 90m/s. enters a heavy wooden block and is stopped at a distance of 60 cm. What is the resistive force exerted by the block?

Mass $= 0.04$ kgs.

$s = 60$ cm. $(60 \times 10^{-2} m)$.

$F = ?$

from 3rd law of motion

$$v^2 = u^2 + 2as.$$

$$a = \frac{(v^2 - u^2)}{2s}.$$

$$F = ma.$$

$$F = \frac{m(v^2 - u^2)}{2s}.$$

$$= \frac{0.04 \, (0^2 - 90^2)}{2 \times 60 \times 10^{-2}}.$$

$$= -2.7 \times 10^2 N.$$

∴ Resistive force exerted by the block will be 2.7×10^2 N.

Example 02.

Sand drops vertically at the rate of $100 gs^{-1}$ onto a horizontal convey or belt moving at a steady velocity of $5 cms^{-1}$. What is the force required to keep the belt moving?

Soln

Rate $= 100 gs^{-1}$

Velocity $= 5 cms^{-1}$

Force $= ?$

$$100 gs^{-1} = \left(\frac{100}{1000}\right) kgs^{-1} = 0.1 kgs^{-1}$$

$$5 cms^{-1} = \left(\frac{5}{100}\right) ms^{-1} = 0.05 ms^{-1}.$$

From 2nd Newton's laws of Motion.

$$F = ma.$$

$$F = \frac{V \Delta m}{\Delta t}$$

$= \text{Rate} \times \text{steady velocity}.$

$= 0.1\, kgs^{-1} \times 0.05\, ms^{-1}$

$\text{Force} = 5 \times 10^{-3}\, kgms^{-2} = 5 \times 10^{-3}\, N.$

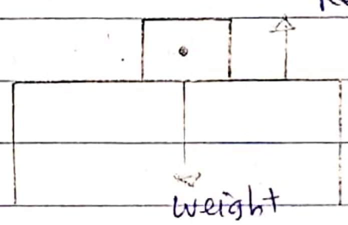
$$\frac{d(P)}{dt} = \frac{d\,mv}{dt} = \frac{m\,dv}{dt} + \frac{v\,dm}{dt}. \quad (v = u = 0).$$

$$F = \frac{v\,dm}{dt}.$$

Equilibrium forces.

These are forces which sets a system at equilibrium

e.g Action and reaction



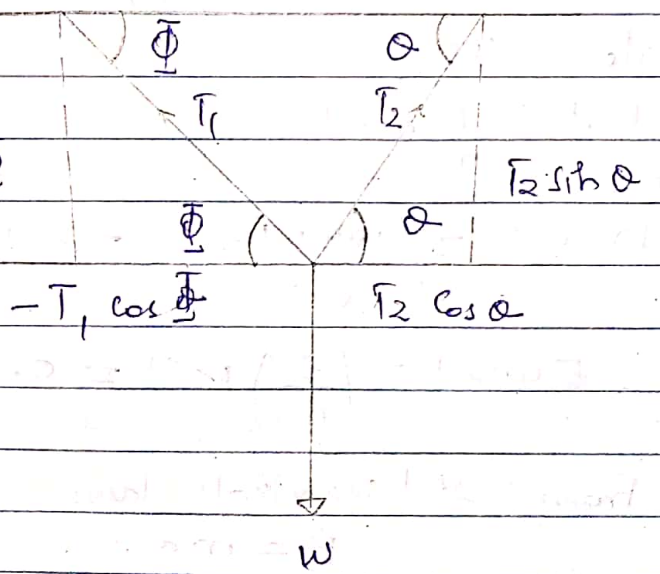
$\Sigma F = 0$

$W + R = 0$

$W = -R$

$Mg = -R.$

Consider a below example which shows three forces.



Horizontal motion.

$$-T_1 \cos \Phi = T_2 \cos \theta$$

$$T_2 \cos \theta + T_1 \cos \Phi = 0. \quad \text{——— (i)}$$

Vertical motion.

$$T_1 \sin \Phi + T_2 \sin \theta = W \quad \text{——— (ii)}$$

from equation (i) make subject T_1.

$$\frac{-T_1 \cos \Phi}{\cos \Phi} = \frac{T_2 \cos \theta}{\cos \Phi}$$

$$\boxed{T_1 = \frac{T_2 \cos \theta}{\cos \Phi}} \quad \text{——— (iii)}$$

Substitute eqn (iii) into eqn (ii).

If $T_1 \sin \Phi + T_2 \sin \theta = W$.

but $T_1 = \dfrac{T_2 \cos \theta}{\cos \Phi}$

$$\left(\frac{T_2 \cos \theta}{\cos \Phi}\right) \sin \Phi + T_2 \sin \theta = W.$$

$$(T_2 \cos \theta) \tan \Phi + T_2 \sin \theta = W.$$

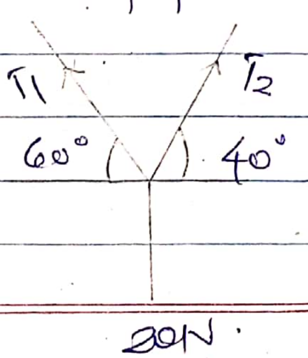
$$T_2 \left[(\cos \theta \tan \Phi) + \sin \theta\right] = W.$$

Divide by $(\cos \theta \tan \Phi + \sin \theta)$ both sides

$$\boxed{T_2 = \frac{W}{\cos \theta \tan \Phi + \sin \theta}}$$

Example.

Below is a system of forces at equilibrium Find T_1 and T_2.



Soln.

If $\Phi = 60°$, $\theta = 40°$ and $W = 20N$.

Recall.

$$T_2 = \frac{W}{\cos\theta \tan\Phi + \sin\theta}$$

$$= \frac{20}{(\cos 40°)(\tan 60°) + \sin 40}$$

$$= 10.1542 \approx 10.2N.$$

but.

$$T_1 = \frac{T_2 \cos\theta}{\cos\Phi}$$

$$= \frac{10.2 \times \cos 40°}{\cos 60°}.$$

$$T_1 = 15.62N.$$

$\therefore$ The value of $T_1 = 15.62N$ and $T_2 = 10.2N$.

03 AUGUST 2021.

PROJECTILE MOTION.

Projectile motion

Is a motion of a body under the influence of gravity.

Examples of Projectile motion.

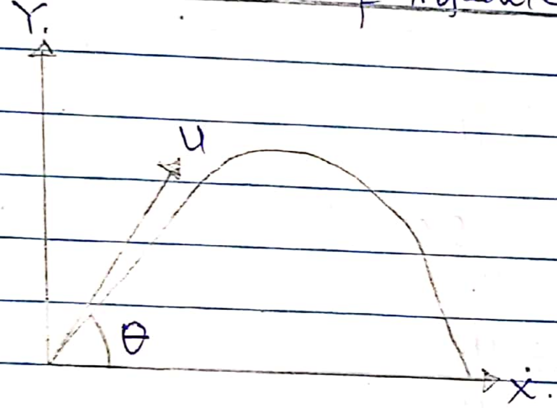
(i) Firing of the bullet from a gun.

(ii). A body droped from the window of moving train.

(iii). A ball when kicked at a certain distance.

Projectile Is the body which moves under the influence of force of gravity.

Illustration of Projectile Motion.



Where

U - Initial Velocity of Projectile.

θ - Angle of Projection ($0° < θ < 90°$).

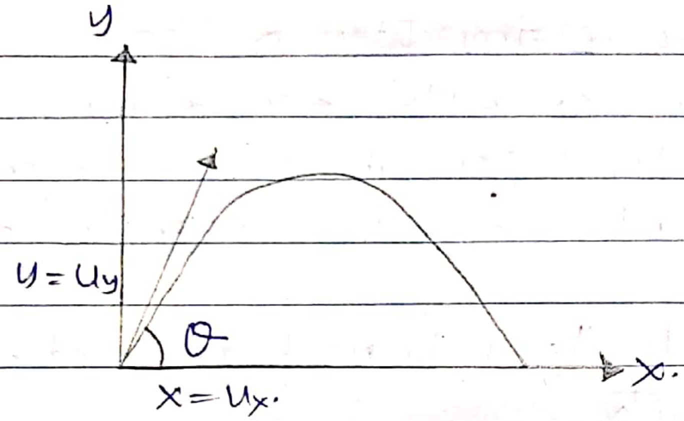
Projectile motion has two dimensional motion because projectile motion has both vertical and horizontal components

The path described by Projectile is Parabolic Path.

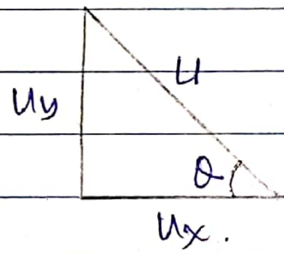
- Projectile motion can be treated as two rectilinear motion one in horizontal direction which experience zero acceleration and the other in the vertical direction which experience constant acceleration i.e (gravity).

Mathematical analysis of Projectile Motion

- Suppose projectile is projected from the origin with an initial velocity U at an angle θ above the horizontal as shown below.



From the figure above



$$\sin\theta = \frac{U_y}{U}.$$

$\boxed{U_y = U\sin\theta}$ Initial vertical Velocity of Projectile

$$\cos\theta = \frac{U_x}{U}.$$

$\boxed{U_x = U\cos\theta}$ Initial horizontal velocity of Projectile

Again from triangle above.

$$\tan\theta = \frac{U_y}{U_x}.$$

$$\boxed{\theta = \tan^{-1}\left(\frac{U_y}{U_x}\right)}.$$

Where θ - angle of Projection.

Also, Velocity of Projection.

$$U^2 = U_x^2 + U_y^2.$$

$$\boxed{U = U_x^2 + U_y^2}.$$

But also from.

$$S = ut + \tfrac{1}{2}at^2.$$

For, Horizontal motion of the Projectile.

$$x = U_x t + \tfrac{1}{2}\cancel{g}t^2$$

Remember $a_x = 0$ (There is constant velocity).

$$x = U_x t + \left(\frac{1}{2} \times 0 \times t^2\right)$$

$$x = U_x t \quad (U_x = U \cos \theta).$$

$$x = (U \cos \theta)\, t.$$

$\boxed{x = U \cos \theta t}$ Horizontal distance travelled by Projectile at any time (t).

Vertical motion.

If $S = ut + \tfrac{1}{2}at^2.$

$$y = U_y t + \tfrac{1}{2} a_y t^2 \quad (a = -g)$$

$$y = (U \sin \theta)t - \tfrac{1}{2}gt^2$$

$\boxed{y = U \sin \theta t - \tfrac{1}{2}gt^2}$ - Vertical distance travelled by projectile at any time (t)

Again

$V = U + at$ (V- Final velocity).

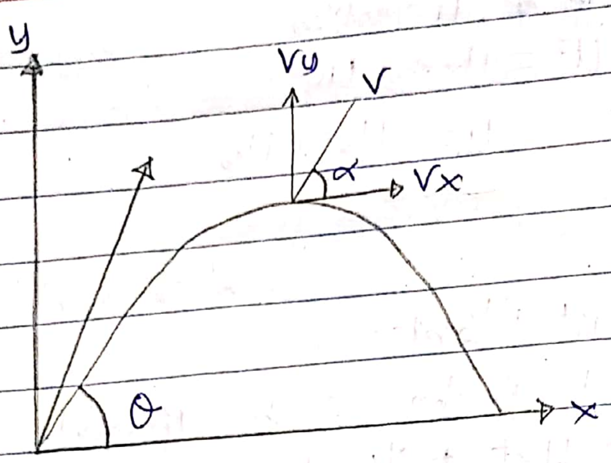
In horizontal Component

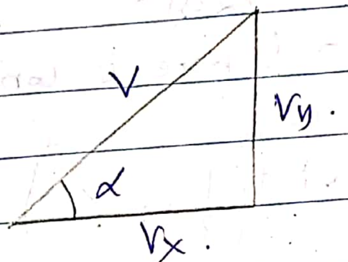
$$V_x = U_x + a_x t \quad (a_x = 0).$$

$$V_x = U \cos \theta$$

$$U_x = U \cos \theta$$

This implies that $\boxed{U_x = V_x = U \cos \theta}$.





$$\tan\alpha = \frac{V_y}{V_x}$$

$$\alpha = \tan^{-1}(V_y/V_x).$$

but $V = U + at.$

At vertical component

$$V_y = U_y + at \quad (a = -g).$$

$$V_y = U\sin\theta - gt.$$

$$\boxed{V_y = U\sin\theta - gt}$$ Final velocity in vertical component.

but $V = \sqrt{V_x^2 + V_y^2}$

$$V = \sqrt{(U\cos\theta)^2 + (U\sin\theta - gt)^2}.$$

$$V = \sqrt{U^2\cos^2\theta + U^2\sin^2\theta - 2gt\,U\sin\theta + g^2t^2}$$

$$V = \sqrt{U^2(\cos^2\theta + \sin^2\theta) - 2gt\,U\sin\theta + g^2t^2}$$

but $\cos^2\theta + \sin^2\theta = 1.$

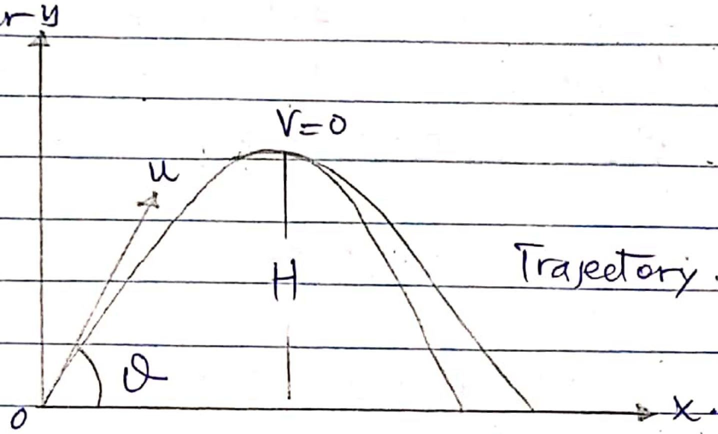
$$\boxed{V = \sqrt{U^2 - 2gt\,U\sin\theta + g^2t^2}}$$ Final velocity at any time (t).

PARAMETERS OF PROJECTILE MOTION.

(a) Trajectory Is the path described by projectile during projectile motion.

TRAJECTORY EQUATION.

Consider



from.

$$y = u\sin\theta t - \tfrac{1}{2}gt^2$$

but $x = u\cos\theta t$.

Make t subject of the formula.

$$t = \frac{x}{u\cos\theta}$$

then.

$$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$y = \frac{u\sin\theta}{u\cos\theta}x - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = \frac{-gx^2}{2u^2\cos^2\theta} + x\tan\theta$$

let $a = \dfrac{g}{2u^2\cos\theta}$

$b = \tan\theta$

$y = -ax^2 + bx$. hence shown.

(b). Time of flight: Is a time taken by a projectile to complete its flight.

from.

$$y = u \sin\theta t - \frac{1}{2}gt^2.$$

but $y = 0$ (Vertical distance be zero since our time we measure found at horizontal distance).

$$0 = u \sin\theta t - \frac{1}{2}gt^2.$$

$$\frac{gt^2}{gt} = \frac{2u\sin\theta t}{gt}.$$

$$t = \frac{2u\sin\theta}{g}.$$

but $t = T = \frac{2u\sin\theta}{g}.$

$$\boxed{T = \frac{2u\sin\theta}{g}.}$$

(c) Maximum height: Is the maximum vertical distance travelled by the projectile.

from.

$$v^2 = u^2 + 2as.$$

then $a = -g,\ s = h.$

$$v_y = u_y - 2gh.$$

($v_y = 0$) (because we assume a body when stop few time and then again return to the ground)-

$$0 = u_y^2 - 2gh.$$

$$2gh = (u\sin\theta)^2.$$

$$\frac{2gh}{2g} = \frac{u^2\sin 2\theta}{2g}.$$

$$\boxed{h = \frac{u^2\sin 2\theta}{2g}.}$$

For Maximum height (H_{max}).

$$H_{max} = \frac{U^2 \sin^2\theta}{2g}.$$

d. Horizontal range (R) is the horizontal distance covered by a projectile during projectile motion.

from.

$$x = U\cos\theta t \quad (t = T).$$

$$x = U\cos\theta T.$$

$$x = U\cos\theta\left(\frac{2U\sin\theta}{g}\right).$$

$$R = \frac{2U^2\sin\theta\cos\theta}{g}.$$

$$R = \frac{U^2}{g}(2\sin\theta\cos\theta) \quad (\sin 2\theta = 2\sin\theta\cos\theta).$$

$$R = \frac{U^2\sin 2\theta}{g}.$$

$$R = \frac{U^2\sin 2\theta}{g}.$$

Note.

For maximum range.

$$\sin 2\theta = 1$$

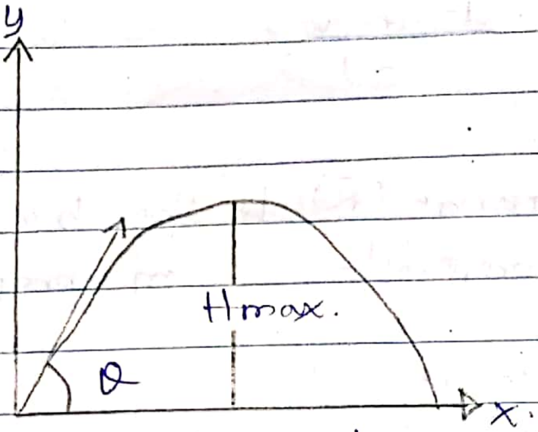
$$2\theta = \sin^{-1}(1).$$

$$\frac{2\theta}{2} = \frac{90}{2}.$$

$$\theta = 45.$$

∴ Inorder to Obtain maximum range, the angle of projection must be 45°.

EXPRESSION FOR TIME TO REACH MAXIMUM HEIGHT



from $v = u + at$

$V_y = U_y + at \quad (a = -g)$

$V_y = u\sin\theta - gt$

But

$V_y = 0$

$0 = u\sin\theta - gt$

$gt = u\sin\theta$

$$\boxed{t_H = \frac{u\sin\theta}{g}}$$

RELATIONSHIP OF SOME PARAMETERS USED IN PROJECTILE

(a) Relationship between time to reach the maximum height and time of flight.

from.

$t_H = \frac{u\sin\theta}{g}$ ——— (i).

$T = \frac{2u\sin\theta}{g}$ ——— (ii).

Divide (i) by (ii).

$\frac{t_H}{T} = \frac{u\sin\theta}{g} \times \frac{g}{2u\sin\theta}$

$\frac{t_H}{T} = \frac{1}{2}$.

$$\boxed{T = 2t_H}$$

(b) Relationship between θ, R and T.

from.

$$x = u\cos\theta t \quad (x = R,\ t = T).$$

$$R = u\cos\theta T \text{ ——— (i)}.$$

$$T = \frac{2u\sin\theta}{g} \text{ ——— (ii)}.$$

from equation (i).

$$\cos\theta = R/uT \text{ ——— (iii)}.$$

$$\sin\theta = \frac{gT}{2u} \text{ ——— (iv)}$$

Square eqn (iii) and (iv) both side and add.

$$\sin^2\theta + \cos^2\theta = \frac{R^2}{u^2T^2} + \frac{g^2T^2}{4u^2}.$$

$$\boxed{1 = \frac{R^2}{u^2T^2} + \frac{g^2T^2}{4u^2}}$$

Also

eqn (iv) $\div$ eqn (iii).

$$\frac{\sin\theta}{\cos\theta} = \frac{gT}{2u} \times \frac{uT}{R}$$

$$\tan\theta = \frac{gT^2}{R}.$$

$$\boxed{\theta = \tan^{-1}\left(\frac{gT^2}{2R}\right)}$$

(c) Relationship between Hmax and R.

$$H_{max} = \frac{u^2\sin^2\theta}{2g}. \text{ ——— (i)}.$$

$$R = \frac{u^2\sin 2\theta}{g} \text{ ——— (ii)}.$$

or.

$$R = \frac{2u^2\sin\theta\cos\theta}{g}.$$

Take eqn (ii) divide by (i).

$$\frac{R}{H_{max}} = \frac{2u^2 \sin\theta \cos\theta}{g} \times \frac{2g}{u^2 \sin^2\theta}$$

$$\frac{R}{H_{max}} = \frac{(2\cos\theta)2}{\sin\theta}.$$

$$\frac{R}{H_{max}} = \frac{4\cos\theta}{\sin\theta}$$

$$\frac{H_{max}}{R} = \frac{\sin\theta}{4\cos\theta}$$

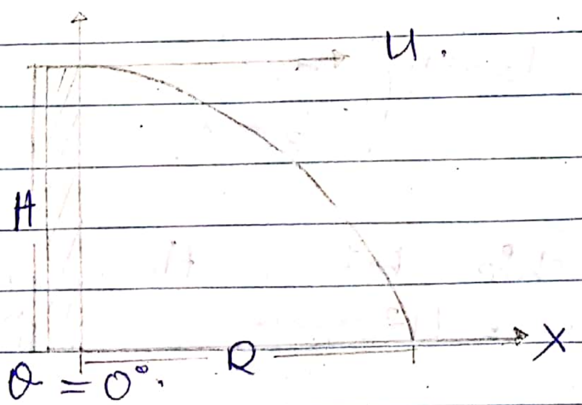
$$\frac{H_{max}}{R} = \frac{1}{4}\tan\theta$$

$$\tan\theta = \frac{4H_{max}}{R}.$$

$$\boxed{R = \frac{4H_{max}}{\tan\theta}}$$

Projectile motion of a body projected from a given high

Consider.



$\theta = 0^\circ$.

Initial vertical velocity $(u_y) = 0$.

Initial horizontal velocity (v_x) = velocity of Projection.

EXPRESSION FOR TIME OF FLIGHT.

· from.

$$S_y = ut + \frac{1}{2}at^2.$$

$$-H = (u\sin 0)t - \frac{1}{2}gt^2.$$ (-ves ath because goes towards -ve direct

$$-H = -\frac{1}{2}gt^2 \quad (t = T).$$

$$2H = gT^2.$$

$$\frac{2H}{g} = T^2.$$

$$T = \sqrt{\frac{2H}{g}}.$$

EXPRESSION FOR RANGE

$$R = u\cos\theta T \quad (\theta = 0).$$

$$R = (u\cos 0^\circ)T \quad (\cos^\circ = 1)$$

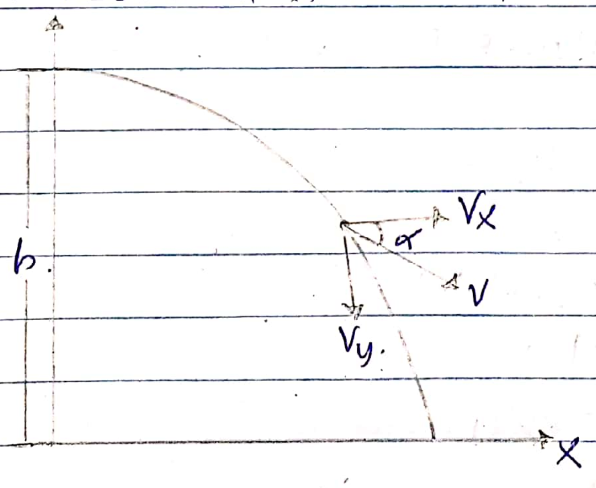
$$R = uT.$$

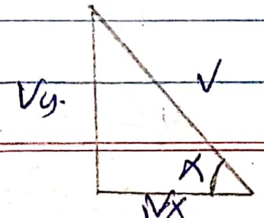
$$R = u.\sqrt{\frac{2H}{g}}.$$

Note

H - given hight

EXPRESSION OF A VELOCITY AT ANY TIME.





$$v = \sqrt{(V_x)^2 + (V_y)^2}.$$

But

$$V_x = U_x = \cos\theta = (U\cos 0) = U.$$

$$V_y = U\sin\theta - gt.$$

$$= (U\sin 0) - gt.$$

$$V_y = -gt.$$

hence.

$$V = \sqrt{U^2 + (-gt)^2} = \sqrt{U^2 + g^2 + t^2}.$$

$$V = \sqrt{U^2 + g^2t^2}.$$

DIRECTION OF VELOCITY AT ANY TIME

- It means to find angle.

from.

$$\tan\alpha = \frac{V_y}{V_x}.$$

$$\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right).$$

THE PARABOLIC EQUATION.

from.

$$y = U\sin\theta t - \tfrac{1}{2}gt^2. \qquad \theta = 0°.$$

$$-y = (U\sin 0)t - \tfrac{1}{2}gt^2$$

$$-y = -\tfrac{1}{2}gt^2.$$

$$y = \tfrac{1}{2}gt^2.$$

But $x = U\cos\theta t.$

$$x = (U\cos 0°)t.$$

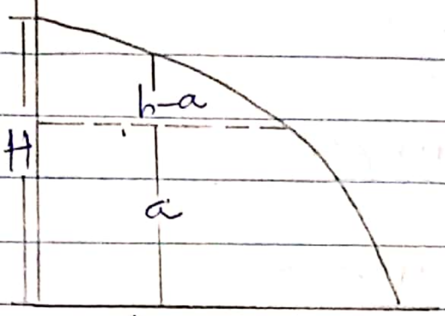
$$x = Ut.$$

$$t = x/U.$$

$$y = \tfrac{1}{2}g\frac{x^2}{U^2}.$$

$$y = \frac{gx^2}{2U^2}.$$

Consider.



$$-(h-a) = u_y t - \tfrac{1}{2}gt^2.$$

$$-(h-a) = u\sin\theta - \tfrac{1}{2}gt^2.$$

$$-(h-a) = (u\sin 0^\circ) - \tfrac{1}{2}gt^2.$$

$$h-a = \tfrac{1}{2}gt^2.$$

but $t = x/u.$

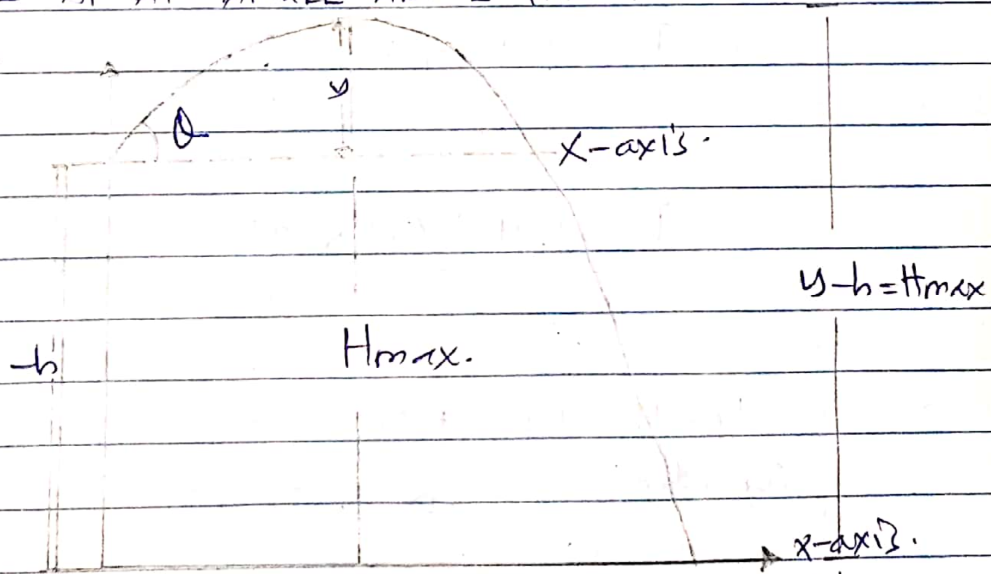
$$a = -\frac{1}{2}\frac{gx^2}{u^2} + h.$$

$$a = h - \frac{1}{2}\frac{gx^2}{u^2}.$$

$$a = h - \frac{gx^2}{2u^2}.$$

PROJECTILE AT AN ANGLE ABOVE THE HORIZONTAL.

Consider.



(a) Expression for time to reach maximum height.

from.

$$V_y = U_y - gt$$

$$V_y = U\sin\theta - gt$$

$$0 = U\sin\theta - gt_H \quad (t = \text{Time to reach maximum height})$$

$$\frac{g t_H}{g} = \frac{u \sin\theta}{g}$$

$$t_H = t = \frac{u \sin\theta}{g}$$

$$t_H = \frac{u \sin\theta}{g}$$

(b). Time of flight.

from.

$S_y = u_y t - \frac{1}{2} g t^2$.

but $H_{max} = (y + -h)$.

$y - h = u \sin\theta T - \frac{1}{2} g T^2$.

$0 - h = u \sin\theta T - \frac{1}{2} g T^2$.

$\frac{1}{2} g T^2 - u \sin\theta T - h = 0$

$g T^2 - 2u \sin\theta T - 2h = 0$.

from.

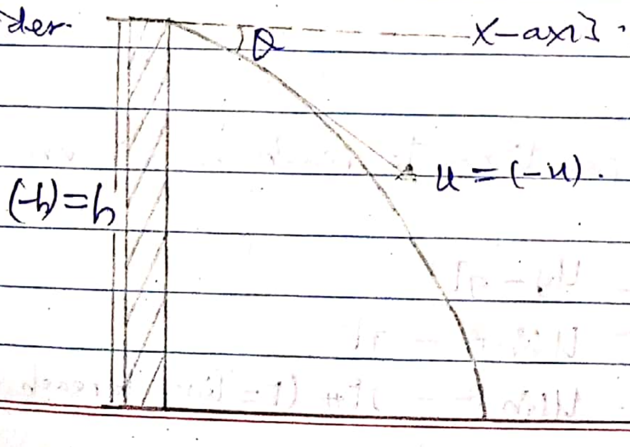
$$x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$T = \frac{2u \sin\theta \pm \sqrt{4u^2 \sin^2\theta - 4(g)(-2h)}}{2(g)}.$$

$$T = \frac{2u \sin\theta \pm \sqrt{4u^2 \sin^2\theta + 8gh}}{2g}.$$

PROJECTILE MOTION OF THE BODY BELOW THE HORIZONTAL

Consider



(a) Expression of time of flight.

from.

$$S = ut + \frac{1}{2}at^2.$$

$$-h = U_y t - \frac{1}{2}gt^2.$$

$$-h = -U\sin\theta t - \frac{1}{2}gt^2 \quad (t=T).$$

$$-h = -U\sin\theta T - \frac{1}{2}gT^2$$

$$h = U\sin\theta T + \frac{1}{2}gT^2.$$

$$2h = 2U\sin\theta T + gT^2.$$

$$gT^2 + 2U\sin\theta T - 2h = 0.$$

$$T = \frac{-2U\sin\theta \pm \sqrt{4U^2\sin^2\theta - 4g(-2h)}}{2g}.$$

$$T = \frac{-U\sin\theta \pm \sqrt{U^2\sin^2\theta + 2gh}}{g}.$$

So negative sign can be neglected due there is no negative time.

Thus.

$$T = \frac{U\sin\theta \pm \sqrt{U^2\sin^2\theta + 2gh}}{g}$$

Example.

1. A projectile is projected with velocity of 30m/s at an angle of elevation of 30° Find.

(a) Maximum height.

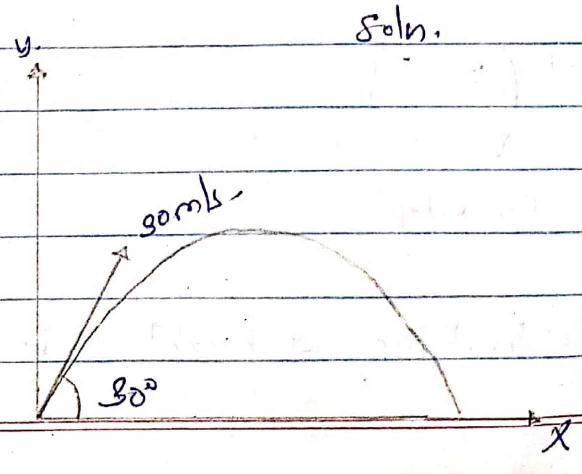
(b). The range.

(c). The time of flight.

(d). The velocity and direction at a height of 4m.

Soln.

Consider.



(a) $H_{max} = \frac{u^2 \sin^2\theta}{2g}$

$= \frac{30^2 \times \sin^2 30}{2 \times 9.8}$

$= 11.5\ M.$

∴ The maximum height 11.5m.

(b). Range $= \frac{u^2 \sin 2\theta}{g}$.

$R = \frac{(30)^2 \sin(2 \times 30)}{9.8}$.

$R = 79.5m.$

∴ The Range is 79.5m.

(c) $T = \frac{2u \sin\theta}{g}$.

$T = \frac{(2 \times 30) \sin 30}{9.8}$

$T = 3.06 sec$

~~(d) velocity $v =$~~ ∴ Time of flight is 3.06sec.

(d) velocity $v = \sqrt{(V_x)^2 + (V_y)^2}$.

$V_y = u \sin\theta t - \frac{1}{2} g t^2$.

$V_y = (30 \sin 30)\ 306 - \frac{1}{2} \times 9.8 \times (3.06)^2$.

$V_y = 0.02.$

$V = \sqrt{(0.02)^2 + (79.5)^2}$.

$V = 79.5.$

∴ The velocity is 79.5m/s.

$\theta = \tan^{-1}\left(\frac{0.02}{79.5}\right)$.

$\theta = 0.014°$

∴ The direction at height of 4m is 0.014°.

PROJECTILE MOTION OF TWO BODIES THROWN SIMULTANEOUSLY

- The following are the problems solving techniques for projectile motion of two bodies thrown simultaneously.

Hints.

i) Identify the common parameter (variable for both projectile) example Distance, time etc.

ii) Formulate equations for the common parameter for both sides.

iii) Equate the two equations and solve for unknown.

iv). The bodies collide initial vertically velocities are the same horizontal plane.

v). If the bodies collide their initial horizontal velocities are the same, if they are projected from the same vertical plane.

Example.

A stone A is projected horizontally from the top of the Tower 54M high at a velocity of 15m/s. At the same time instant stone B is projected from the bottom of the tower with a velocity of 30m/s at an angle of 60° to the horizontal. Find

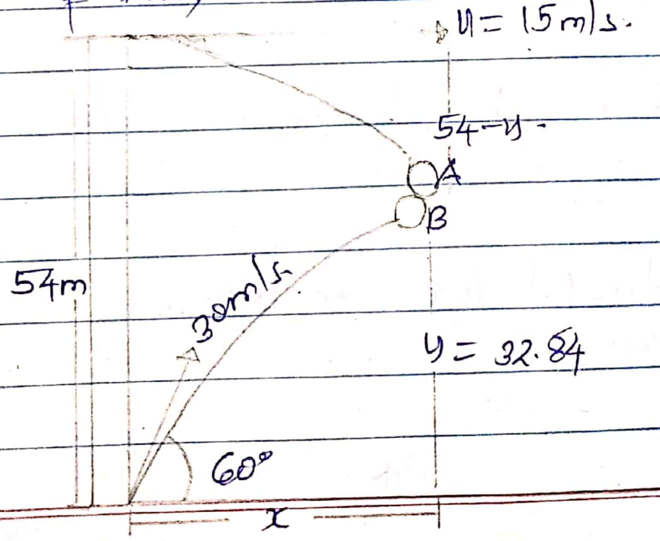
(i) The height above the ground where stone collide.

(ii) The horizontal distance from the tower where the collision occurs.

Soln.

Consider

θ of body A = 0°



For stone A

from $S_y = U_y t - \frac{1}{2}gt^2$.

$S_A = U_A t - \frac{1}{2}gt^2$.

$-(54-y) = U \sin\theta t - \frac{1}{2}gt^2$ (height will be -ve).

but $\theta = 0$

$-(54-y) = (U \sin 0)t - \frac{1}{2}gt^2$.

$-(54-y) = -\frac{1}{2}gt^2$.

$54 - y = -\frac{1}{2}gt^2$

$y_A = 54 - \frac{1}{2}gt^2$ ———— (i).

For stone B.

$y = U \sin\theta t - \frac{1}{2}gt^2$

$y_B = U_B \sin\theta t - \frac{1}{2}gt^2$

$y_B = (30 \sin 60°)t - \frac{1}{2} \times 9.8 t^2$.

$y_B = (30 \sin 60°)t - \frac{1}{2}gt^2$ ———— (ii)

Equate eqns two above ($y_A = y_B = y$).

$54 - \frac{1}{2}gt^2 = 30 \sin 60 t - \frac{1}{2}gt^2$.

$54 = 30 \sin 60° t$.

$t = \frac{54}{30 \sin 60}$

$t = 2.078$ sec

from.

$y = (54 - \frac{1}{2} \times 9.8 \times (2.078)^2)$

$y = 54 - 21.15$

$y = 32.84$m.

$\therefore$ The height above the ground where the stone collide be 32.84 m.

(ii) $x = U \cos\theta t$ (Consider body B)

$= 30 \cos 60 \times 2.078$

$= 31.17$

OR

$X = U\cos\theta t$ (For body A, $\theta = 0°$).

$x = (U\cos 0°)t$.

$= ut$

$= 15 \times 2.078$

$= 31.17m$. Is the horizontal distance

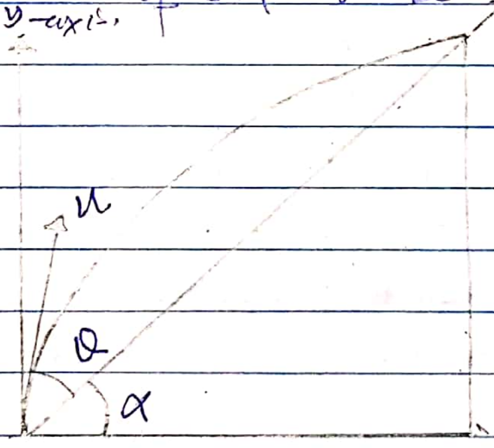
PROJECTILE ON AN INCLINED PLANE.

(a) MOTION UP THE PLANE

Projectile motion on an inclined plane is the one of the various types of projectile motion.

The main distinguish aspect is that points of projections and point of striking the ground are not on the same plane

- Consider a projectile projected at an angle θ up the plane inclined at an angle θ to the horizontal with velocity U. Assume the line of slope to be x-axis.



from diagram

$U_x = U\cos\theta$

$U_y = U\sin\theta$

$U_x = +U_x$

$U_y = +U_y$.

$S_x = X = +X$

$g_x = -g$.

but $g_x = g\sin\alpha = -g\sin\alpha$ (α - made by components of g)

$g_y = g\cos\alpha = -g\cos\alpha$.

EQUATION OF TIME OF FLIGHT

from.

$S_y = U_y t - \frac{1}{2}gt^2$

$S_y = U\sin\theta T - \frac{1}{2}gT^2$

$$0 = U\sin\theta T - \tfrac{1}{2} g \cos\alpha T^2.$$

$$U\sin\theta T = \tfrac{1}{2} g \cos\alpha T^2.$$

$$\frac{2U\sin\theta}{g\cos\alpha} = T$$

hence

$$T = \frac{2U\sin\theta}{g\cos\alpha}$$

Range

from $S_x = U_x T - \tfrac{1}{2} g T^2.$

$$x = U\cos\theta T - \tfrac{1}{2} g_x T^2$$

$$x = U\cos\theta T - \tfrac{1}{2} g \sin\alpha T^2.$$

$$R = U\cos\theta T - \tfrac{1}{2} g \sin\alpha \left(\frac{2U\sin\theta}{g\cos\alpha}\right)^2$$

$$R = U\cos\theta T - \frac{(2U\sin\theta)^2}{g}$$

$$= U\cos\theta\left(\frac{2U\sin\theta}{g\cos\alpha}\right) - \frac{g}{2}\sin\alpha\left(\frac{4U^2\sin^2\theta}{g^2\cos^2\alpha}\right)$$

$$= \frac{2U^2\cos\theta \cdot \sin\theta}{g\cos\alpha} - \frac{2U^2\sin\alpha\sin^2\theta}{g\cos^2\alpha}$$

$$= \frac{2U^2\sin\theta}{g\cos\alpha}\left(\cos\theta - \frac{\sin\alpha\sin\theta}{\cos\alpha}\right)$$

$$= \frac{2U^2\sin\theta}{g\cos\alpha} \; \frac{\cos\theta\cos\alpha - \sin\alpha\sin\theta}{\cos\alpha}.$$

$$R = \frac{2U^2\sin\theta}{g\cos\alpha}\left(\frac{\cos(\theta+\alpha)}{\cos\alpha}\right)$$

When

$\alpha = 0$

$$R = \frac{2U^2\sin\theta}{g\cos\alpha}\left[\frac{\cos(\theta+\alpha)}{\cos\alpha}\right]$$

$$= \frac{2u^2 \sin\theta}{g\cos 0}\left[\frac{\cos(\theta+0)}{\cos 0}\right]$$

$$= \frac{2u^2 \sin\theta\cos\theta}{g}$$

$$R = \frac{u^2 \sin 2\theta}{g}$$ Previous formula to find Range, but include α angle due to be in inclined plane

MAXIMUM POSSIBLE RANGE.

$$R = \frac{2u^2 \sin\theta}{g\cos\alpha}\left(\frac{\cos(\theta+\alpha)}{\cos\alpha}\right).$$

$$R = \frac{u^2}{g\cos\alpha}\left(2\sin\theta\cos(\theta+\alpha)\right).$$

$$R = \frac{u^2}{g\cos\alpha}\left(2\sin\theta\cos(\theta+\alpha)\right).$$

$$R = \frac{u^2}{g\cos\alpha}\left(2\sin\theta\cdot\cos(\theta+\alpha)\right).$$

$$R = \frac{u^2}{g\cos\alpha}\left(\sin(\theta+\theta+\alpha) - \sin(\theta-\alpha-\theta)\right).$$

for maximum range.

$$\sin(\theta+\theta+\alpha) = 1.$$

$$\sin(2\theta+\alpha) = 1.$$

$$2\theta+\alpha = \sin^{-1}(1).$$

$$2\theta+\alpha = 90$$

$$\frac{\pi}{2} = 90.$$

$$2\theta = \frac{\pi}{2} - \alpha.$$

$$\theta = \pi/4 - \alpha/2.$$

$$R_{max} = \frac{u^2}{g\cos^2\alpha}(1-\sin\alpha)$$

$$R_{max} = \frac{U^2}{g\cos^2\alpha}\left(\underbrace{\sin(\theta+\theta+\alpha)}_{1} - \sin(\theta+\alpha-\theta)\right)$$

$$R_{max} = \frac{U^2}{g\cos^2\alpha}\left(1 - \sin\alpha\right)$$

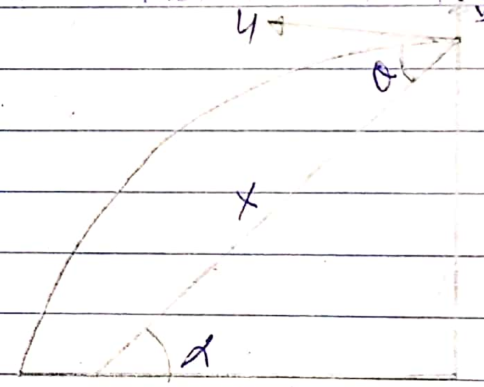
When

$\alpha = 0$

$$R_{max} = \frac{U^2}{g\cos^2 0}\left(1 - \sin 0\right)$$

$$R_{max} = \frac{U^2}{g}$$

PROJECTILE MOTION DOWN THE INCLINED PLANE



θ - angle of Projection.

Time of flight (T)

$S_y = u_y t + \frac{1}{2} g_y t^2$

$S_y = 0$ (objects starts from $y=0$ to x-axis)

$u_y = + u\sin\theta$ (+ve sign due to towards x direction)

but $g_y = -g = -g\cos\alpha$

$0 = u\sin\theta\, t - \frac{1}{2} g\cos\alpha\, t^2$

$0 = u\sin\theta\, T - \frac{1}{2} g\cos\alpha\, T^2$

$\frac{1}{2} g\cos\alpha\, T^2 = u\sin\theta\, T$

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

Range

$S_x = \cancel{U}t + \frac{1}{2}\cancel{g}t^2$ ($U_x = -U_x$ - axis highest to lowest part towards -ve direction).

$-S_x = -U\cos\theta t - \frac{1}{2} g\sin\alpha t^2$

$-S_x = -U\cos T \ \frac{1}{2} g\sin\alpha T^2.$

$-R = -U\cos\theta T - \frac{1}{2} g\sin\alpha T^2.$

$R = U\cos\theta T + \frac{1}{2} g\sin\alpha T^2.$

but $T = \frac{2U\sin\theta}{g\cos\alpha}$

$$R = U\cos\theta\left(\frac{2U\sin\theta}{g\cos\alpha}\right) + \frac{1}{2} g\sin\alpha\left(\frac{2U\sin\theta}{g\cos\alpha}\right)^2$$

$$R = \frac{U^2(2\sin\theta\cos\theta)}{g\cos\alpha} + \frac{1}{2} g\sin\alpha\left(\frac{4U^2\sin^2\theta}{g^2\cos^2\alpha}\right).$$

$$R = \frac{U^2(2\sin\theta\cos\theta)}{g\cos\alpha} + \frac{g\sin\alpha(2U^2\cancel{\cos}\sin^2\theta)}{g^2\cos^2\alpha}.$$

$$R = \frac{2U^2\sin\theta\cos\theta}{g\cos\alpha} + \frac{\sin\alpha(2U^2\sin^2\theta)}{g\cos^2\alpha}.$$

$$R = \frac{2U^2\sin\theta}{g\cos\alpha}\left(\cos\theta + \frac{\sin\alpha\sin\theta}{\cos\alpha}\right)$$

$$R = \frac{2U^2\sin\theta}{g\cos\alpha}\left(\frac{\cos\theta\cos\alpha + \sin\alpha\sin\theta}{\cos\alpha}\right)$$

$$R = \frac{2U^2\sin\theta}{g\cos\alpha}\left(\frac{\cos(\theta-\alpha)}{\cos\alpha}\right)$$

$$\boxed{R = \frac{2U^2\sin\theta}{g\cos\alpha}\left(\frac{\cos(\theta-\alpha)}{\cos\alpha}\right)}$$

from.

$$R = \frac{2U^2}{g\cos\alpha}\left(\frac{\sin\theta\cos(\theta-\alpha)}{\cos\alpha}\right)$$

$\sin\theta\cos(\theta-\alpha) = ?$

from

$\sin(\theta-(\theta-\alpha)) = \sin\theta\cos(\theta-\alpha) - \cos\theta\sin(\theta-\alpha)$ — (i).

$\sin(\theta+(\theta-\alpha)) = \sin\theta\cos(\theta-\alpha) + \cos\theta\sin(\theta-\alpha)$ — (ii)

Add two equations.

$\sin(\theta-(\theta-\alpha)) + \sin(\theta+(\theta-\alpha)) = 2\sin\theta\cos(\theta-\alpha)$

but

$$R_{max} = \frac{u^2}{g\cos\alpha}\left(\frac{2\sin\theta\cos(\theta-\alpha)}{\cos\alpha}\right)$$

$$R_{max} = \frac{u^2}{g\cos\alpha}\left[\frac{\sin(\theta-(\theta-\alpha)) + \sin(\theta+(\theta-\alpha))}{\cos\alpha}\right]$$

For maximum range.

$$\sin(\theta+(\theta-\alpha)) = 1.$$

$$\theta+(\theta-\alpha) = \sin^{-1}(1)$$

$$2\theta-\alpha = 90 \quad (90 = \pi/2).$$

$$2\theta-\alpha = \pi/2$$

$$\theta = \pi/4 + \alpha/2.$$

$\therefore$ For maximum range $\theta = \pi/4 + \alpha/2$.

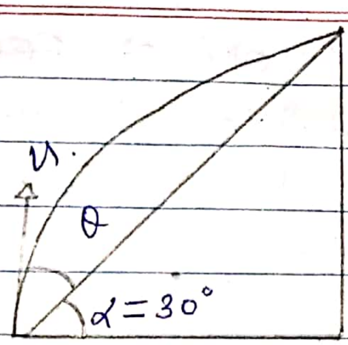
$$R_{max} = \frac{u^2}{g\cos 0}\left(\frac{\sin 0 + \sin 2\theta}{\cos 0}\right)$$

$$R_{max} = \frac{u^2\sin 2\theta}{g}.$$

Example.

A projectile is thrown from the base of incline of angle. What should be the angle of projection, as measured from the horizontal direction so that the range on the inclined plane is maximum.

soln.



from.

$$R = \frac{u^2}{g\cos^2\alpha}(\sin(2\theta - \alpha) - \sin\alpha).$$

but $R_{max} = \sin(2\theta + \alpha) = 1.$

$$(2\theta + \alpha) = 1.$$

$$2\theta + \alpha = 90 = \pi/2.$$

$$2\theta = \pi/2 - \alpha$$

$$\theta = \pi/4 - \alpha/2.$$

$$\theta = (180°/4 - 30/2).$$

$$\theta = 30°.$$

∴ Angle of Projection is 30°.

ASSUMPTION OF PROJECTILE MOTION.

01. Air resistance to the motion is negligible.
02. Acceleration due to gravity is constant
03. The curvature and rotation of the earth are not considered.

LIMITATION OF PROJECTILE MOTION.

01. Variation of acceleration due to gravity
02. Air resistance
03. Curvature and rotation of the earth

CHARACTERISTICS OF PROJECTILE MOTION.

01. Projectile Motion has two dimensional.
02. The path described by projectile is parabolic in nature.

APPLICATION OF PROJECTILE MOTION.

01. Projectile motions are used in distributing relief supplies from flying planes to areas isolated by flo[illegible] fairly long time.

02. Projectile motion used in distributing relief supplies from plane in ground heavy shelling during wars.

03. It is used by hunter
04. Used in sport
05. Used in setting satellites into orbits.