Algebraic Aspects of Designing Behavior Based Systems*

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Abstract. We address in this paper the design of behavior based systems from a bottom–up viewpoint. Although behavior is an observable property of a system, and therefore immediately causes a top–down model, the approach has to be inverted to support the learning of equivalence classes of the perception–action cycle. After introducing the paradigm in the frame of a socio–ecological theory of biological systems, we discuss the natural science problems to be solved for successful design of behavior based systems by a bootstrap of perception–action cycles. The necessary fusion of robotics with computer vision, neural computation, and signal theory needs a common theoretical framework. This framework consists of a global algebraic frame for embedding the perceptual and motor categories, a local algebraic framework for bottom–up construction of the necessary information, and a framework for learning and self–control, based on the equivalence of perception and action. Geometric algebra will be identified as the adequate global algebraic frame, and the Lie theory will be introduced as the local algebraic frame. We will demonstrate several applications of the frames in early visual processing. Finally, we will finish our discussion with the fusion of local approaches and the global algebraic frame with respect to both the formulation of an adequate multidimensional signal theory and the design of algebraic embedded neural processing. In both cases we will discuss the relation to the non–linear Volterra series approach, which, in our framework, will be reduced to a linear one.

1 Introduction

In this paper, we want to promote the consideration of algebraic aspects in the process of fusion of disciplines such as computer vision, robotics, neural computation, and signal theory, which have been developed separately until now. We conclude the necessity of following this line from two contradicting roots. In principle, the paradigmatic frame and the technical resources are available to develop vision based robots or autonomous self–navigating systems. However, the conceptions of handling phenomena of spatio–temporal geometry are very limited in the contributing disciplines and there exist, at least partly, deep gaps between the mathematical languages used. This judgement may surprise but

* This work was supported by DFG grant So 320–2–1.

will be substantiated later on. Another point of shortcoming is the following. The mathematical framework should support a constructive way of capturing spatio–temporal geometric phenomena of the world, while we often use a descriptive approach. Because each non–pathological algebraic structure can be approached either from top–down or from bottom–up we have the possibility at changing our viewpoint. That means that the problems to be solved have to be formulated from the systems point of view which has to gain structural concepts from seemingly non–coherent phenomena in the learning stage and has to match phenomena of the world to isomorphic representations of them in the matured stage. In other words, we have to support learning by experience by means of separation of variant from invariant phenomena and have to reduce the explicit formulation of categories by programming. In that way algebra will meet stochastics insofar as e.g. geometric entities as points, lines or planes are conceptions of the top–down approach which can only be approximated from bottom–up by recognizing the corresponding algebraic properties (symmetries) in the manifold of sensory data. From this follows that the bottom–up approach does not result in the so–called signal–symbol gap.

In the last decade great progress has been made in the conceptualization of computer vision (paradigmatic extension to active and purposive vision, use of projective geometry), signal theory (wavelet theory, local spectral features), and in the availability of dedicated and powerful computer systems. But we made only minor progress in adequate representation of multidimensional signals from a local approach, in the unification of spatio–temporal phenomena (patterns and motion), in recognition of projective geometric relations on signal level, in integration of neural computation and computer vision, in neural learning theory, in designing robust and adaptive systems, and e.g. in architectural design of visual systems. All these problems in a certain sense are related to the algebraic frames used to formulate the solutions.

The fusion of perception and action in the frame of the so–called perception action cycles (PAC) is the paradigmatic starting point for the design of behavior based technical systems. Behavior is the observable expression of the competence gained in realizing PAC. But while behavior corresponds the top–down approach, the design of PAC is our view of bottom–up approach. The hypothesis of this direction of research consists in a possible bootstrap of robust and adaptive systems by equipping systems with the ability to organize such cycles and the cooperation/competition between by themselves. In contrast to programmed solutions, the result of such self-organization may not be provably correct from a designers point of view, but the degree of success is observable and may be analyzed by statistical means.

Biological systems, if they are plants, ants or human beings, are successful behavior based systems, although most of them do not know about algebra. The question is, do we as system designers know enough on that topic or do the mathematicians have the right algebras in their desks. Nobody knows the answer. But we know the sluggishness of the human society and the preference of the technicians for simple, linear, and in consequence suboptimal solutions.

Therefore, linear algebra of vector spaces is the frame commonly used to embed almost all our problems. It is our opinion that we have indeed powerful mathematical languages for embedding PAC. But they are either not known to the community or are ignored because of the burden of both to make them useful or to pick them up.

D. Hestenes [34] for a period of thirty years has been promoting the use of a geometric interpreted version of Clifford algebra in physics. He calls it geometric algebra. We will follow him in using this algebra as a global frame of embedding PAC and even calling it geometric algebra. On the other hand the bottom–up approach of PAC necessitates a local algebraic embedding of perception and action to recognize and generate patterns of certain symmetry. This local frame is Lie theory [61].

It seems to us that the problems of putting real phenomena in space–time into a rich mathematical frame is rooted in the same manner in algebra as those of physics. May be, a special slot of scale (macroscopic phenomena) has to be considered and we have not to regard electrodynamics, quantum effects or relativity, although their metaphoric potentials are enormous. But our central questions are

- *In which manner the perceptible and experienced world can be structured most successfully?*
- *What has to be the functional architecture of such systems?*

To answer these questions, there is a need of more complex mathematics to formulate things more simply and to make them work with limited resources, taking into account both the complexity and the constraints of real world.

Indeed, geometric algebra and Lie theory are more complex than linear algebra of vector spaces. But our research group started two years ago successful work in overcoming shortcomings in disciplines contributing to the design of PAC systems by using extended possibilities of representing geometric relations in this frame. Nevertheless our work is in infancy. This may not wonder in comparison to the long lasting role of vector algebra in engineering and science.

The outline of the paper is as follows. In section two we will give a sketch of the behavior based systems paradigm. In section three we will outline our vision of the theoretical framework of embedding the design of such systems. Section four is dedicated to a short discussion of exemplary use of the framework of geometric algebra in early vision and learning of geometry. The paper will be accompanied by three special contributions of our group, dedicated to special problems and presented at the same workshop.

Our view will be biased by the aspects of visual sensory recognizing the world. This is based on the fact that we are predominantly interested in behavior based design of visual systems, and we have to make clear with this respect the role of action in building perceptual categories and their use in PAC. Another view may be perceptually supported autonomous systems, especially visual based robotics. Both viewpoints have to be understood as the two sides of one medal and are fused in the paradigm of behavior based systems.

2 Behavior Based Systems as Paradigm

In this section, we want to summarize the evolution and the essential features of the paradigm of behavior based systems. With these respects we will focus on the problems of computer vision and their overcoming by extension of the scope. On that base we will draw a sketch of natural science problems which in our opinion have to be coped with to develop technical systems based on that paradigm. The engineering science problems will not be dealt with.

2.1 Two Metaphors of Intelligence

The term behavior is borrowed from ethology and stands for the basic capabilities of biological systems which guarantee survival of the species and/or the individual. Famous ethologists as K. Lorenz [40] and N. Tinbergen [66] considerably contributed to the change of the metaphoric view of intelligence, respectively brain theory [57, 24]. Besides, results of molecular and evolutionary genetics on a completely other level of living systems brought into consideration that information processing is an inherent capability of all biological systems, on which level ever [25]. Not only from biology, ethology, and psychology [27] but also from the growing knowledge of physics on complex, non–linear, and dynamic self–organizing systems [32], the behavior based paradigm of "intelligent" systems is superseding the paradigm of knowledge based systems.

Both paradigms of system design are outcomes of quite different metaphors on understanding intelligent achievements or intelligence by themselves (see table 1). Common to both metaphors is only that there are biological systems (man), which are interpreting what they observe and are trying to use the gained models for construction of technical systems with comparable performance to that of living systems. All other aspects are fundamentally different.

The computational theory of mind is rooted in the results of logics, linguistics, and the brilliant von Neumann architecture of computers. Their advocates viewpoint is a top–down one and expresses the dominance of description of observed phenomena.

To oppose that metaphor to the socio–ecological theory of biological systems, the following summary of the implicit assumptions of the computational theory of mind will be given:

1. *The world can be formally modeled using terms of our language or categories as representations (symbols) of equivalence classes.*
2. *Information is an intrinsic entity of the world as energy or matter. Information processing is a process as conversion of energy in physical processes. It can be done on an abstract level of symbols and independently of the material base of the system.*
3. *Intelligence is that of human beings and those may use it for top–down design of tasks. The problem solutions should be provably correct and themselves they can be interpreted as achievement of intelligence of the computing system.*

4. *On that base, any partial aspect of the world can be considered in isolation to construct a complete solution from a set of partial ones of any problem the designer may formulate.*

Metaphor	Computational Theory of Mind	Socio–Ecological Theory of Biological Systems
basic roots	logics, linguistics von Neumann computer architecture	cybernetics molecular and evolut. genetics ethology evolutionary theory of knowledge synergetics
basic paradigms	information processing (symbol processing, connectionism)	information selection and construction
empiric aim	understanding intelligence of human beings	understanding competence of biological systems
disciplines	computer science artificial intelligence cognitive science (computational neuroscience)	artificial life neural Darwinism vision based robotics synthetic psychology
engineering roots	computer engineering computer vision robotics	mechatronics computer vision (active, animate, qualitative, purposive) reactive robotics
engineering paradigms	knowledge based system design	behavior based system design
normative aim	construction of intelligent machines	construction of autonomous systems

Table 1. The change of metaphors

Although the power of facts created by universal computer machines is impressive, on that base the hopes of computer vision and robotics regarding the development of engines which are robust and adaptive in their performance could not be realized.

The socio–ecological theory of biological systems stands in very contrast to these conceptions. Although their advocates also start with the observation of phenomena of living systems (behavior), they have to invert their viewpoint into a bottom–up approach by asking what principles are running so that living systems can show stabile behavior.

The implicit assumptions of the socio–ecological theory of biological systems are:

1. *The formalized models of the world are idealized conceptions of the reality and insofar they are not useful, neither to understand living systems nor to realize comparable performance.*
2. *Instead, any behavior is based on a sufficient approximation of categories by equivalence classes with respect to the resources of the system and its purpose in dependence of the situation.*
3. *Information is a prerequisite for behavior. It is a result of an active process of interaction of the system with its environment, and it is the result of purposive construction, gathering and selection. If we call this process also information processing, it happens on real (sensoric mapped) data from the total complexity of the world.*
4. *Equivalence classes stand for the gained order of the system's internal degrees of freedom and without having a language there is no need to conceptualize categories.*
5. *Competence should be understood of having equivalence classes as a prerequisite of behavior. Insofar, competence is a kind of real intelligence and much more general than this because each living system on each level of organization, reaching from phages to vertebrates and from macromolecules to the body, will need it.*
6. *Competence cannot be programmed but has to develop by purposively constrained self–organization of the internalized representations of phenomena of the environment.*
7. *Competence is only provable by the success of behavior. It may be based on knowledge which has been acquired by the species during phylogenesis, has to be transmitted to the individual by genes, or/and may be learned by individuals during ontogenesis.*
8. *As emergent property of a behavior based system, competence is robust and adaptive if there are no dramatical changes of individual resources or environment.*
9. *Behavior based systems are open systems with respect to their environment, non–linear ones with respect to the situation dependence of their responses to sensory stimuli, and dynamic ones as they change their behavior as answer on the same signals in different situations.*

If we want to follow that metaphor [52] to design behavior based systems, the result would be in any case a kind of autonomous system. It seems to become visible that we not only have to make systems running but have to work out fundamental new principles of design. It is far from being sufficient to change from the symbolic to the subsymbolic level of information processing. This change of the computational paradigm remains in the computational theory of mind metaphor if not the essence of behavior based systems will be considered. This essence is to close the loop between system and environment, the perception–action cycle, by using the afferent and efferent channels [35].

2.2 Evolution of the Paradigms of Visual Information Processing

Remarkable stimuli for the yet ongoing process of redefinition of the paradigmatic base of artificial intelligence came from deep conflicts recognized in computer vision one decade ago. The proceedings of computer vision conferences of the last years make obvious important contributions to shape the conception of behavior based visual architectures. Progress has also been reached in robotics.

D. Marr [43] formulated the computational theory of vision on the base of the information processing paradigm of cognitive science using the symbol processing paradigm of Newell and Simon [48]. Assuming that vision is an inherent task in the world and resting on a general theory, he postulated the famous three–step procedure for the top–down design of a visual system:

1. **computational theory**: formulation of the general task and the way to find the solution by considering necessary constraints and boundary conditions,
2. **representation and algorithm**: specification of a formal procedure,
3. **implementation**: assumption of independence of the procedure with respect to the hardware at hand.

This approach resulted in insights into important relations between the spatio–temporal macroscopic structures of the world and hypothetical vision tasks. Examples are the role of projection operator and shape–from–X tasks. A dramatical consequence of Marr's theory has been the thesis that vision mainly could be understood as reconstruction of the world from sensory data. The sensory part of the visual system together with geometry and illumination would constitute a general operator of sensory imaging. Perception would be defined by application of the inverse operator. Visual perception as ill–posed inverse problem should be regularized to become well–posed [65] by adding to the sensory data constraints regarding geometry and physics of imaging, and knowledge with respect to the imaged scene. This conception fitted very well to the knowledge based approach of artificial intelligence. Nevertheless, it failed with respect to realize recognition and to construct general useful and robust systems.

To give a summary of characterizations of knowledge based vision the following drawbacks will be noticeable:

1. *The mysterious gap between signals and symbols cannot be closed in the frame of the paradigm.*
2. *The dominant role of symbolic processing versus signal processing totally underestimates the role of early visual processes.*
3. *The top–down control of bottom–up directed data flow allows no return from symbols to signals in case of erroneous interpretations.*
4. *Recognition has to be done by matching prototypes as world models because equivalence classes cannot be adequate modeled.*
5. *The explicit formal representation of models is limited to a simple world and therefore restricts application fields. Other restrictions follow from a time–consuming search–based matching process.*
6. *The use of a maximum of knowledge to solve a visual task contradicts the*

The contemporary spectrum of alternative paradigms was initiated by R. Bajcsy [5], Y. Aloimonos [1], and D. Ballard [6] who remembered that a visual system is a sensory, perceptual, and a motor one.

1. **Active Vision**: [2, 39] The active control of the outer degrees of freedom of the oculomotor system enables the vision system to break down the complexity of the task. If vision tasks are coupled to an oculomotor behavior, the coupling of algorithmic solutions to behavioral strategies will result.
2. **Animate Vision**: [6] Similar to Active Vision also animate vision supports the division of the vision process into two phases: the preattentive and the attentive stages. While preattentive vision is interpreted as fast and data driven bottom–up processing, attentive vision is top–down controlled and related to the next term.
3. **Purposive Vision**: [3] The active system is controlled by the purpose of vision and that is action. Purpose is understood as the driving power of the system to interact with the environment. This indeed is in almost agreement with the behavior based approach.
4. **Qualitative Vision**: [4] The opportunism of purposive vision calls for using minimal efforts in realizing any visual task. That means gathering of sufficient hints in minimum time with respect to the task. That also means the use of minimal models of the world in very contrast to knowledge based vision.

Interesting questions of research are of such kind:

> *Which knowledge of the structure of the world is necessary to perform purposive vision in a limited range of time using oculomotor strategies.*

But two fundamental differences to the behavior based paradigm, projected to visual systems, remain. These are the unsolved recognition problem and the problem of the origin of categories of behavior. Although the coupling of visual tasks with oculomotor behavior and purpose introduced an important strategic component, the recognition problem only gained some redefinition but no general solution. Now, indeed, recognition is decomposed into partial tasks connected with the oculomotor behavior and defined by the mismatch between the task and the fusion of partial solutions. Learning and acquisition of competence until now only in exceptional cases is integrated with active vision [67].

Another aspect, not yet well understood, is the mutual support of visual and motor categories. A reasonable hypothesis, drawn from cognitive psychology and ethology, leads to the conjecture that vision as isolated process is a too hard task. Visual used categories cannot be learned and are not defined by vision alone, but can be interpreted as projections of multiple defined (and learned) categories onto the visual system.

2.3 Natural Science Problems

We have to our disposal now stereo–camera heads, miniaturized robots, radio Ethernet, and powerful computers. This allows to think of designing new systems, we never had before. We may be encouraged simply to do it and we should.

But there are a lot of serious and fundamental problems to be solved in advance if we want to classify such systems as behavioral ones.

Replying a debate on the paradigmatic changes in computer vision, Aloimonos concludes [2] that the task to be solved may be summarized by:

Find a general solution for special problems.

Indeed, the great challenge will be to understand the general principles underlaining the success of living systems in performing their perception/action tasks. Only if we find some sufficient approximation to the answers, we will be able to equip technical systems with the resources to develop the wanted competences.

As nature brought forth rather different levels of behavior, each with different amount of directness and indirectness of behavior, we should start with the simplest categories of systems. That means, although we want to have systems with human like competence, this should not be our goal in the moment. Cognitive processes as indirect behavior can be our concern if we sufficiently understand the functionality of more direct behaviors as e.g. signal–motor mappings.

If also nature is constrained by the principles of evolution, which leads some people to state that nature is a tinker, nature within these constraints most effectively uses those general principles.

The most important features of behavior based systems are:

1. **situatedness**: The system is managing its tasks in the total complexity of concrete situations of the environment.
2. **corporeality**: The system can only gain experiences with respect to the environment by means of the specific resources of its body, including mind.
3. **competence**: The system's order of inner degrees of freedom is an expression of the captured order of the environment and causes order of the system's outer parameters and actions.
4. **emergence**: The system's competence emerges from disorder if the purposive rooted and physical constrained actions are fitting well the general and invariant aspects of the situations in the environment.

With respect to the purpose of the system, any behavior has the properties of usefulness, efficiacy, effectiveness, and suitability. It corresponds the equilibrium with respect to the purpose between the system and its environment. Its robustness with respect to minor distortions of this equilibrium has to be completed by adaptivity with respect to more heavy distortions. A general natural science theory of the principles underlaining behavior based systems will be related to the theory of non–linear dynamic systems. This is a theory of the dynamics of large (physical) systems and far from being the theory of vision, which Marr asked for. Although all the features of behavior based systems are well fitted by the growing up theory of non–linear dynamics, their metaphorical use in practice is limited yet [49].

As the most important conclusion from situatedness with respect to the limited resources of a system, not the knowledge of detailed models of the world but of useful relations between the system and its environment has to be considered. The set of situative important relations and the amount of knowledge has to be minimal because actions should be suitable and effective.

Information cannot be simply gathered and used as Gibsonian invariants [28]. A perceptual system is no pure reactive system (not only controlled by data and instincts as proposed by Brooks [11]), just as it is no pure cognitive system (not only controlled by expectations or knowledge). Instead, perception is bottom–up driven within the limits of corporeality and top–down controlled [19] by the purpose, projected to the situations. This implies that the pragmatic aspect of information is mainly determined by the purpose, the semantic aspect is constructed, respectively selected, by physical experience, and the syntactic aspect is mainly matter of perception.

Recognition in this frame is a purposive constraint matching to equivalence classes whose meaning is based on using corporeality. Consequently, there is in the stage of competence no problem of heaving too less invariants but selecting the right ones. This principle has to be supported by an architecture with sufficient purposive constraint dynamics.

In the stage of incompetence, learning will be the process, which will result in the mentioned equilibrium. Actual learning paradigms hardly can be understood in the frame of emergent systems. The used least mean square minimization as linear approach lacks the attractor properties of non–linear dynamic systems. The most promising approach is reinforcement learning because it supports evaluation of interpretation of sensory data by action most naturally [12]. Using this approach, the important contribution of the knowledge bias for fast learning will become obvious [31]. The resulting question is, how to partition the necessary knowledge base to learned and acquired or programmed contributions.

Our contemporary understanding of multiple supported categories is in agreement with implicit representations as a result of optimization of the perception–action cycle by self–supervised learning. Insofar, the mapping of the non–linear spatio–temporal relations between sensory and motor signals, respectively vice versa, by means of the paradigm of artificial neural nets, is a promising way of semantically based coupling of perception and action. But semantics is based on pragmatics and pragmatics is strongly related to purpose. In the frame of reinforcement strategy of learning, the pragmatics is submitted to the system as cost function or confidence measure. Appearance based vision by self–supervised learning [53] is starting to become useful for the design of bottom–up constructed perception–action cycles.

Such PAC not necessarily must be an elementary one. A more complex behavior is not a linear sum of a set of primitive ones. Therefore, the top–down partitioning of behaviors is of limited value. Moreover, any complex behavior should emerge from a set of primitive ones as a result of adaption to new situations. Although some experiments could be interpreted as such emergence [18], the construction of relations between primitive cycles is not well understood yet [10]. This is also the case for the design of hierarchically structured behavior based systems. Inverse relations of dependence between primitive behaviors (necessary for survival) and higher–order behaviors (necessary for the task) cause different total behaviors [11, 44].

3 The Theoretical Framework

The realization of behavior based systems by bootstrap of perception–action cycles necessitates the fusion of robotics, computer vision, neural computation, and signal theory in a unified, respectively compatible framework. That framework should allow to embed all the tasks constituting the PAC system with sufficient flexibility, dynamics, and completeness.

The behavior based system has to experience any perception–action cycle and not to report or to reason on it. Insofar, the explicit symbolic level is restricted to the programming of the dynamic frame of PAC and to the interface between system and user. Within PAC implicit representation of knowledge is dominating a certain amount of explicitly formulated basic rules as instincts.

The level of signal processing with respect to afferents and efferents, including representation of equivalence classes, has to be able to realize all aspects of PAC. Within such frame no signal–symbol gap will exist. Symbols are not necessary as representation of equivalence classes for PAC systems.

The central problem of the existence of a behavior based system will be to cope with all spatio–temporal phenomena of the environment which are of relevance with respect to the objective of the system. Concerning visual information processing, these phenomena are spatio–temporal structures, including those which are caused by the actions of the system.

Within such scenario, situations are embedded in the Euclidean space–time. Spatio–temporal phenomena are fused but may be projected to spatial or temporal ones. This will be supported by the use of oculomotor behavior. With respect to the coupling of perception and action, the most important task will be to recognize and to generate patterns of spatio–temporal geometry with a certain degree of symmetry. These patterns represent equivalence classes of that property and, as expression of the competence of the system, support a statement of mathematical equivalence of perception and action:

- *similar visual patterns cause similar motor actions*
- *similar motor actions cause similar visual patterns.*

This unity of perceptual and motor equivalence classes enables the system to self–control and to learn from actions by using oculomotor behaviors.

The theoretical embedding of the perception–action cycle will be constituted by an algebraic framework and by the frame of learning and using the knowledge on implicit (neural) representations. The algebraic framework will be a dual one because it has to support both the forming and representation of experience from global phenomena of the environment and the process of local generation or verification of global pattern concepts. In this section, we will refrain from presentation of the frame of learning and neural information processing. With this respect, we developed a type of neural net, called Dynamic Cell Structure – DCS [14], which in the context of behavior based system design could be successfully proved [15]. Its main idea is the optimally topology preserving adaption to the manifold by self–supervised vector quantization.

3.1 The Global Algebraic Frame

The global frame has to represent the perceptual relevant phenomena of the Euclidean space–time E_4. These are resting or moving objects of any dimension less than or equal three and their relations between. The classical mathematical framework enables modeling of either resting objects by means of either analytical geometry using entities of dimension zero (points), one (lines), two (planes), respectively three (cubes), or differential geometry using entities as curves (1D) and surfaces (2D), or modeling of objects in motion within the frame of kinematics as rigid body movement in E_3 using these entities. Normally, the movement of rigid bodies is described by rigid displacement of a set of points. The entities of motion concepts are geometric transformations as translation, rotation, and scaling. From these entities complex patterns of motion are constructed. This decoupling of space–time can be done by a competent system using the oculomotor behaviors (e.g. fixation and tracking). But fixation to infinity while moving is also useful and will result in patterns from E_4.

Both object and motion concepts are determined by the correlations in the data of their patterns. These correlations have certain aggregated global symmetry properties which are important to define and which represent the equivalence classes. But the construction of global symmetry from local correlations is a bottom–up process and therefore, it is matter of the local algebraic frame. Nevertheless, the global symmetries have to be represented to form isomorphic representations to the observed phenomena. If Pellionisz wrote [56] "*the brain is for geometrical representation of the external world*" or if Koenderink [36] and von der Malsburg [42] stated that the brain is a geometry engine, the importance of representation of the environment is underlined. But this representation is implicitly constructed and has not to be complete in a mathematical sense but complete with respect to purpose, situatedness, and corporeality of the system. The algebraic frame should allow to support this flexibility.

The global symmetries and the metric properties of objects in Euclidean space E_3 are distorted in a systematic way by global projective transformations between objects and the observer's visual sensory system. Nevertheless, the stratification of the space [26] in connection with the transformation group of each shell (projective, affine, Euclidean) [41] allows recognition of the corresponding invariants. The necessary amount of effort to use the invariants of different shells can be strongly modified by oculomotor behavior. In this way, oculomotor constraint recognition from stratified space is in accordance with the situation dependent use of minimal resources [21].

The global algebraic frame should support all these aspects of geometric mappings and should also support the effective control of actions. This will be possible by realizing geometric transformations in the same framework. But even as invariance is important for recognition, this is valid also for actions. One aspect of invariance with this respect is the independence of transformations from a fixed coordinate system. This aspect is often neglected in the design of behaviors.

The linear vector space of real and complex numbers is not satisfactory to represent all the mentioned phenomena in the requested quality. This is known for a long time and therefore, tensor algebra has been intensively studied in neural science [56], robotics [20], and computer vision [30]. Pellionicz [56] argues that neural coupling of sensory receptions with motor actions has to consider the special transformation properties of these signals (contravariance of motor signals and covariance of sensory signals) to gain a metric internalization of the world. The arguments of using tensor algebra in the frame of signal processing [30] mainly refer to the poorness of the representation power of vectors and scalars with respect to multidimensional signals. While the first argument seems not so obvious, the second one corresponds also our experience and is of greatest importance for the bottom–up design of the PAC.

Indeed, the Hilbert vector space is representing only sets of points (using vectors) with the result that all correlations which specify symmetries of higher dimensional entities with great effort have to be reconstructed by the analysis of the occupation of the vector space and by construction of subspaces. In the linear frame, besides addition and multiplication of vectors, there is only the poor operation of the scalar product, which even shrinks vectors to a scalar, and which in addition is defined only as bilinear operation. What we want to have are more rich structures in the vector space, which represent in any way higher order entities as planes or volumes as expression of their correlation. In tensor algebra and in vector algebra, there are constructs of such type as outer product tensor or cross product of vectors to extend the algebraic limitations.

Vector Space Structuring by Geometric Algebra In our opinion, the most systematic way to endow a vector space with such entities is based on Clifford algebra [58] and can be most intuitive related to geometric conceptions in the version promoted by Hestenes [33]. This is the geometric algebra. Only to give an impression of the richness of defining structure in vector spaces by definition of partial product spaces from the contributing vectors, we will give a short summary of the subspace conception of the algebra [64].

The geometric algebra $G(\mathbf{A}) = G_n$ is the algebra of multivectors $\mathbf{A}$,

$$\mathbf{A} = \mathbf{A}_o + \mathbf{A}_1 + \ldots + \mathbf{A}_n$$

with $\mathbf{A}_k$, $k \leq n$, as homogeneous k–vectors, i.e. multivectors of grade k. The geometric algebra G_n results from a vector space V_n by means of endowing it with a so–called geometric product. This geometric product causes a mapping of V_n onto G_n, which themselves is a linear multivector space of dimension 2^n. Any n linear independent vectors $\mathbf{a}_1, \ldots, \mathbf{a}_n \in V_n$ are therefore transformed to the multivector $\mathbf{A}$. The basis of this multivector space is constituted by $n+1$ sets of $\binom{n}{k}$ linear independent k–blades $\mathbf{M}_k$ which themselves constitute the basis of linear subspaces $G_k(\mathbf{A})$ of dimension $\binom{n}{k}$ of all k–vectors in G_n.

The geometric product of 1–vectors (i.e. normal vectors) $\mathbf{a}, \mathbf{b} \in V_n$,

$$\mathbf{ab} = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \wedge \mathbf{b}$$

has the important property that it maps vectors $\mathbf{a}, \mathbf{b}$ into both a scalar as a result of the symmetric inner product $\alpha_0 = \mathbf{a} \cdot \mathbf{b}$ as well as a bivector as a result of the antisymmetric outer product $\mathbf{A}_2 = \mathbf{a} \wedge \mathbf{b}$, $\alpha_0, \mathbf{A}_2 \in G_n$.

Any two homogeneous multivectors $\mathbf{A}_r, \mathbf{B}_s \in G_n$ are mapped by the geometric product into a spectrum of multivectors of different grade, ranging from grade $|r-s|$ as a result of the pure inner product $\mathbf{A}_r \cdot \mathbf{B}_s = \mathbf{C}_{|r-s|}$ until grade $r+s$ as a result of the pure outer product $\mathbf{A}_r \wedge \mathbf{B}_s = \mathbf{C}_{r+s}$. This corresponds the partitioning of G_n into the subspaces $G_k(\mathbf{A})$. Thus, any k–blade $\mathbf{M}_k \in G_k(\mathbf{A})$ can be geometrically interpreted as the uniquely oriented k–dimensional vector space $V_k = G_1(\mathbf{A}_k)$ of all vectors $\mathbf{a}$, satisfying $\mathbf{a} \wedge \mathbf{A}_k = 0$, respectively of all k linear independent vectors $\mathbf{a}_1, ..., \mathbf{a}_k$, spanning V_k and constituting a factorization of the k–blade $\mathbf{M}_k = \mathbf{a}_1 \mathbf{a}_2 ... \mathbf{a}_k$. Therefore, any $\mathbf{A}_k$ is understood as projection of $\mathbf{A}$ onto G_k and, on the other hand, can be formulated as a linear superposition of all k–blades, constituting the basis of G_k.

Because $\mathbf{A}_n = \lambda I$ with I as unit pseudoscalar or direction of V_n, there follows an intrinsic duality principle of geometric algebra. This duality is based on $I_k I_{n-k} = I$ for any k–blade, respectively $(n-k)$–blade. From the dual $\mathbf{A}^* = \mathbf{A} I^{-1}$ of any multivector $\mathbf{A}$ follows that there is a simple change of the base of any multivector with respect to the dual blade.

The property of orientation of G_k obviously transmits to the multivectors $\mathbf{A}_k \in G_k$. Therefore, the multivectors of the Euclidean spaces E_2 or E_3 result in directed numbers with the algebraic properties of complex numbers, respectively quaternions. Additionally, certain algebraic restrictions or extensions will result in other number conceptions as dual or double numbers [60]. They altogether possess nice algebraic properties which can be either interpreted in the frame of geometry or kinematics. Besides, each multivector has a quantitative or an operational interpretation. The reason for that duality of interpretation of multivectors lies in the fact that the Clifford algebra unifies Grassmann algebra with quaternion algebra of Hamilton. This makes the geometric algebra so attractive for our fusion of disciplines because the same number may be operand or operator.

Moreover, geometric algebra does not only subsume the mathematics of metric vector spaces but also that of projective geometry. For instance the qualitative incidence operations meet and join of entities, homogeneous coordinates, and the operation called projective split which relates vector spaces of different dimension in a simple way can be used.

3.2 The Local Algebraic Frame

With the local algebraic frame we will understand several aspects of supporting the bottom–up approach of PAC. That means with respect to the conception that the system uses (visual) behavior to reduce the amount of data, to reduce the complexity of the task, and to control the gathering of hints in a purposive manner. All these strategic aspects can be subsumed by attention to a local patch of signals to get a partial contribution to resolve conflicts or to solve the actual task. Global interpretations result from local contributions by fusion.

Even the same problem the system will have with the control of actions. Of course, these constructive processes are covered in biological systems by high parallel processing and are accelerated by special equipment as the sensory geometry in the human retina. Besides, in the highly trained stage, there is a process of shortcutting to global interpretations. These special effects will be disregarded here.

In the following, we will discuss some problems which are related to the local algebraic frame, with respect to irregular sampling using extended sampling functions and to the choice of basis functions for signals and operators. We will propose an attentive, purposive, and progressive visual architecture, and the use of the canonical local basis generated by Lie group invariants.

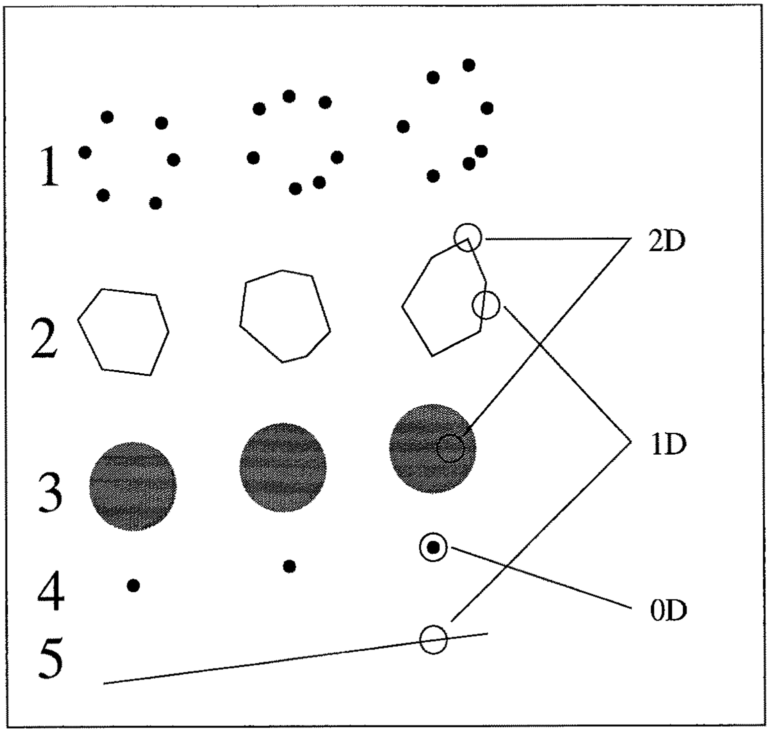

Fig. 1. Several conceptions of structure, dependend on purpose and/or aperture of the sampling operator.

Sampling with Finite Aperture The first problem concerns the term "local". In mathematics exists the conception of the entity point as interpretation of an ideal location, defined by its coordinates. With respect to our problem of designing a real technical system by understanding something of living systems, we have to define a locus as a finite extended patch and the extension depends on both situation and task. Thus, a local patch of (visual) attention has to be scalable. The scaling function of wavelet theory or blob hierarchy [37] plays the role of a regularization operator by integration on a finite set of signals. But we have to consider two aspects. Each structure has at least one intrinsic scale. Because of the hierarchy of concepts of structure, there are normally some. The observer should be able to adapt the scale of his local patch of interest with the chance of getting one or several unambiguous interpretations. In figure 1, there are several levels of interpretation. In dependence of the purpose of the observer and of the aperture function used, rather different interpretations can be found. The canonical coordinate frame of pointwise regular sampling in linear signal theory (sampling theorem) does no longer fit such strategy of sampling with extended operators. Now, estimation theory has to be considered to get high significance using bloblike sampling without loosing resolution as metric property or failing in estimation of dimension as topologic property.

Local Intrinsic Dimensionality of Data In the example of figure 1, also the local dimension of the structure dramatically changes if both different apertures or different conceptions are used. A visual sensory system will interpret the data of local patches with respect to their local dimension because it strongly correlates with local symmetry. A set of measured signals (1), e.g. as a result of corner detection, may be interpreted as examples, taken from entities as in (2). By assuming that these entities are disturbed by noise, a more reasonable interpretation may be given by (3). From step (2) to (3), a change of the dimensionality conception happened. While in (2) a sequential process of small aperture induces an one-dimensional contour, in (3) a two-dimensional patch is assumed. In (3) all locations, within the patches may have the same meaning. Therefore, as a result of vector quantization, the points of (4) may suffice to represent the manifold. In (5), on a global level these representations again may be fused by an one-dimensional conception. As an implicit assumption of this discussion, we used the definition of the local intrinsic dimension of an entity as the number of degrees of freedom which suffice the chosen criterion. While in (2) the sequential process induces an one-dimensional contour, its embedding in the plane as a polygon necessitates the assumption of maximal intrinsic dimension two (see also chapter 4). In (2), the contour is constituted by fusion of multiple one–dimensional components. Therefore, any patch covering a corner can only represent the detected local geometry in a two–dimensional base, as the outer product of two one–dimensional bases. The interpretation of (noised) data with respect to their intrinsic local dimension has to separate between the intrinsic dimension of the manifold and the dimensional aspect, induced by noise. Both are dependent of the used aperture. By vector quantization, the dimensionality orthogonal to that of the manifold may be surpressed. In [13] an interesting approach of the estimation of the local intrinsic dimensionality is presented which is based on the KLT of vector quantized optimally topology representing sets of sampling points. Interestingly, this approach relates pattern recognition with signal theory.

Global Partial Reconstruction using Wavelet Nets The third problem is related to the global reconstruction from local hints. The linear signal theory can from algebraic reasons only support regular sampling in a compact way. Wavelet theory elegantly relates the distance of regular sampling with the scaling factor of the wavelet functions. In the wavelet transform at each sampling position, the signal will be mapped to a complete set of functions. From these projection coefficients, the original signal may be reconstructed. But with respect to the economy of resources, this strategy is very dumb. In contrast to that, irregular sampling makes possible the adaption of sampling positions to interesting structures. The choice of positions of interest may be based on the signal structure and/or on the interest of the observer. In our group, a drastic modification of wavelet transform in direction of a wavelet net has been developed [55]. The wavelet basis functions are coupled to irregular sampling points of the wavelet net. Scale and orientation are continuous adaptively controlled. In very contrast to the wavelet transform, there is a need for a minimum of irregular sampling positions and a minimum of basis functions at the sampling points. Indeed, only one wavelet per sampling point is used. This one has to be globally optimized with respect to the wavelets at the other points. As result of global optimization, from a purposive controlled set of sampling points an image of partial reconstruction raises up from an extremely sparse code. The purpose of the task not only influences the locations of the sampling points, but also the scheme of regularization in the optimization process.

In figure 2, we demonstrate the result of putting about 30 sampling points on the right eye (top left) or 90 sampling points on both eyes and on the mouth (top right) as most prominent regions of a face. The results of reconstruction with the same sets of wavelets, coupled to these points, is shown in the bottom part of the figure. The left code needs 98 bytes and the right one needs 196 bytes for representation on a level of 256×256 pixels.

This example of global fusion from irregular distributed local hints demonstrates a philosophy of purposive minimal decomposition of the visual signal by a limited set of basis functions. Obviously, the goal is not complete global reconstruction but in the sense of qualitative vision the reconstruction of the necessary structures to suffice the task.

Optimal Filter Control by Steerability In the fourth problem we consider the filtering process in the attentive stage, i.e. if the operator is fixated at a keypoint of interest. In traditional signal processing, if the translation DOF is frozen, there is no other possibility than projecting the signal energy to the operator. But remembering the remaining degrees of freedom, that means rotation and scale, there should be the possibility of a dense sampling with respect to these DOF. Because coarse sampling may seriously mismatch the structure,

Fig. 2. Partial global reconstruction from local hints as purposive and progressive vision task.

dense sampling would be wanted. As a way to prevent the infinite amount of effort with respect to such processing, the steering filter scheme can be used. This consists in (now in contrast to the last problem) decomposition of the wanted filter into a small set of basis functions. These basis functions have the property of exact or approximate reconstruction of the wanted filter. Instead of filtering with an infinite set of (e.g. oriented) filters, only the projection of the local signal to the small set of basis functions (e.g. 10) will be done.

From that very limited set of projection coefficients, the response of the optimal adapted filter will be reconstructed by interpolation. Figure 3 shows a set of ten basis functions of an edge detection filter. Both the basis functions and the interpolation functions can be computed either from Lie group theory [46] or using SVD [47]. In the Lie group approach, the eigenfunctions of the

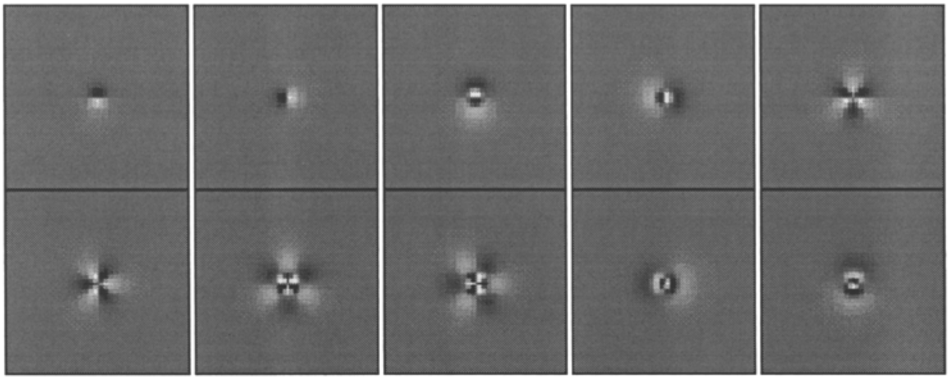

Fig. 3. Ten basis functions of an edge detection filter with orientation as DOF.

generating operator of the considered Lie group are the basis functions and the eigenvalues are the interpolation functions of the steering filter problem. In the SVD approach, the basis functions will be given by the right singular values, whereas the interpolation functions will be given by the left singular values.

Steering filters is a very powerful principle for local analysis of multidimensional signals. Coupled with the quadrature filter concept, the steering of filters results in signatures of the local energy and of the local phase as functions of the considered DOF.

Attentive, Purposive and Progressive Visual Architecture In problem number five, we want to propose a visual architecture for attentive, purposive, and progressive recognition.

This architecture has to follow the general needs of visual perception in a PAC system:

1. **fast** with respect to the time scale of the special PAC,
2. **flexible** with respect to the change of intention or purpose,
3. **complete** with respect to the task,
4. **unambiguous** with respect to the internalized conceptions or categories,
5. **selective** with respect to the importance of data,
6. **constructive** with respect to learning and recognition of equivalence classes.

From an algebraic point of view the Lie group approach is the canonical way for exactly local signal analysis (in multiple parameters of deformation). Because the generating operator of the considered Lie group generates local symmetry, this will be an invariance criterion for a bottom–up approach of grouping aggregated patterns from irreducible invariants. A scheme of an attentive visual system architecture will be shown in figure 4.

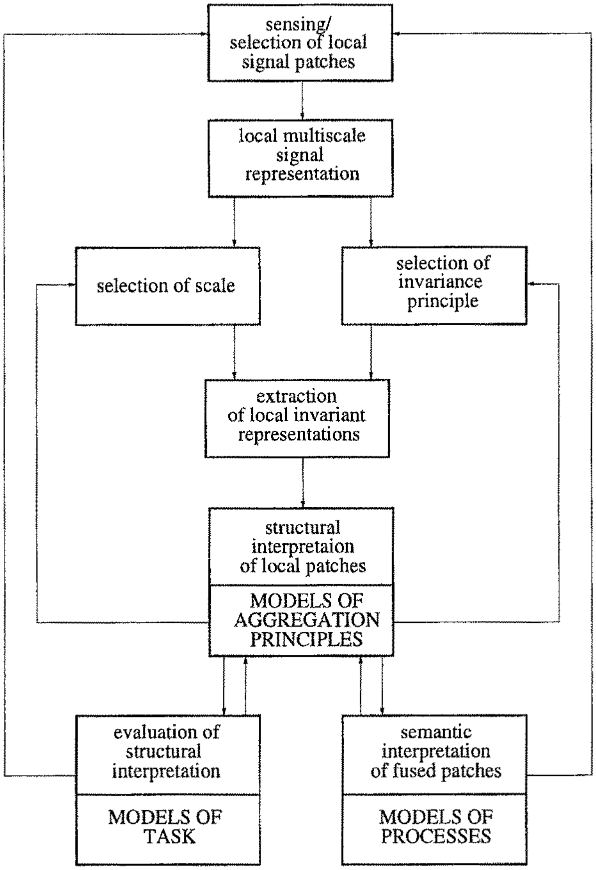

Fig.4. The cycles of an attentive processing scheme.

Noticeable is the organization of several cycles of processing. While the outer cycle indicates to the gaze control aspect, the inner cycle allows the consideration of several invariance principles for grouping of aggregated invariants from irreducible ones in dependence of the evaluation process with respect to the known aggregation results (coherent patterns of e.g. faces) and with respect to the purpose. This bottom–up process is top–down modulated or controlled. This is purposiveness. The recognition should also be progressive in the sense that the process should be stopped if a measure of confidence or evidence is sufficient high. This principle of economy of time requires support by the matching process. The template in this frame is the equivalence class which is only a purposively constrained approximation of the ideal one. With other words, the template should be decomposable to allow a gradual increase of matching results. The principle of progressive recognition is inherent both to the wavelet net approach of reconstruction and to the steering filter approach of recognition. In the first case, the sampling points are weighted by a factor of importance. Therefore, the reconstruction can be stopped if wanted. In the case of steering filters, a large set of basis functions may be computed offline. This ordered set will be sequentially used. In figure 5, the proposed and partially realized visual architecture of purposive progressive recognition will be shown.

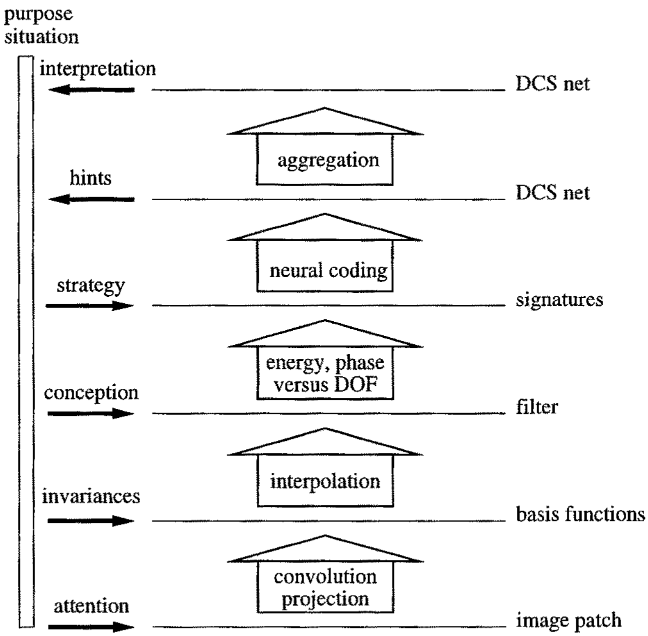

Fig.5. Layered architecture for purpose, progressive visual recognition.

On the left side, we see the communication with the control unit of purpose and task. To the right, we see a hierarchy of structure levels which will be passed by bottom–up driven recognition. The procedure starts with a mapping of the signal to irreducible invariants which correspond to Lie group eigenvectors as symmetry primitives and which are the basis functions of filters. These filters may be any complex templates as aggregates of the irreducible invariants, and indeed they are steerable filters [47]. Their output can be evaluated or/and associated in a cascade of DCS nets [14] for recognizing more and more complex patterns. This process of aggregation is modulated from top–down.

This bottom–up scheme, embedded in the cyclic scheme of fig. 4, seems to be too slowly with respect to the summarized needs. But first, the projection to the basis functions and several other steps of processing can be done in a parallel procedure. Second, in the non–competent phase, respectively in the attentive phase, this sequential scheme may be adequate. In the competent phase, respectively in the preattentive phase, the pathway of recognizing the grandmother should be engraved to shortcut quickly the grandmother cells with her visual pattern.

Lie Group Approach The Lie group approach is the general method to design the perception–action cycle as bottom–up process. It may constitute the local algebraic frame for both recognition and forming patterns in space–time from a differential viewpoint. Because local patterns have local symmetry of low dimension, the task of recognizing or forming of smooth patterns by a sequential process will be locally supported. Within that frame, the equivalence between recognition and action can be seen most obviously. To follow a chosen conception means selection of the corresponding Lie group and the change of conception in case of any events means change of the Lie group, or at least their parameters. As an example, in fig. 1, case 2, the polygons result from a sequence of the translation group and the rotation group.

Although Lie algebra and Lie group theory for a long time is known in the community, its breakthrough is missed. There is a lot of relevant papers in computer vision [29, 63]. But in robotics [17, 51] and neural computation [54] the attention is very limited yet.

Both the local and the global algebraic frames can be fused with success [23] because every Lie algebra corresponds a bivector algebra and every Lie group can be represented as a spin group.

4 PAC and Geometric Algebra

The special problems of PAC with respect to the geometric algebra are

- multi–dimensionality of visual signals from space–time,
- need of fast multilink actions in an invariant manner,
- nonlinearity of perception–action mapping,
- nonlinearity of recognition of higher–order correlations,
- need of a switch between strategies, conceptions, qualitative and quantitative aspects.

After working for two years on that field of geometric algebra, we can summarize the momentary expected benefits from geometric algebra:

- flexible change of interpretation frames of different dimension for multi–dimensional local signals [16],
- flexible change of interpretation frames between entities of different dimension of the Euclidean space [7],
- flexible change between projective, Euclidian, and motor space (of kinematics) [9],
- transformation of nonlinear to linear problems [22],
- enrichment of representation and learning capabilities of neural nets [8],
- effective recognition of higher–order correlations.

We often observe a reduced complexity of symbolic coding of a problem. Of course, the numeric complexity often expands on computers which do not know the type of quaternions or so. Nevertheless, the answer depends on the problem, e.g. in conversion of nonlinear iterative solutions to linear straightforward ones.

In the following, we want to introduce into a special problem, which is of central importance for the design of PAC systems. This problem concerns the complete recovery of local geometry and the capturing of geometric relations in neural nets. We will show, how nonlinear processing of signals may be reduced to a linear one, without loss of information.

The designer of PAC systems has to guarantee that in principle all structures could be recognized, if necessary. This was not possible till now, without loosing the nice properties of the linear signal theory. The reason is based on the fact that until now we had no adequate signal theory for multidimensional signals. Textbooks refer to that problem with the statement that a two–dimensional local phase is not defined. Indeed, the deep reason is that, within the frame of complex numbers (the Fourier domain), there is no possibility to express all the symmetries of a two–dimensional signal. While one–dimensional signals with respect to a fixed position can be decomposed into an even and an odd component, two–dimensional signals can have any combination of both in each dimension.

Using a linear integral transformation as the Fourier transform, in the domain of complex numbers only even and odd symmetries can be represented. Thus, the Fourier transform in complex domain is only adequate to one–dimensional signals, although, as we know, the two–dimensional Fourier transform in complex domain is well defined.

In the case of using energy and phase as local spectral features, this limitation becomes obvious. To give an example, the recognition of an L–junction with steering filters [45] results in two peaks of the signature of energy. These have to be identified in a second step of non–linear processing, that means by application of a threshold to the energy. In contrast to that, complete linear processing would result in only one peak if the steering filter could represent all the symmetries of two–dimensional signals.

The geometric algebra offers the right frame for embedding the considered problem of representation of the Fourier transform of multidimensional signals in a higher–order domain of numbers without loosing the property of linearity. Indeed, a line as one–dimensional ideal structure (case 4 in fig. 1) may be represented as a vector, while any two–dimensional structure as e.g. the L–junction and any constant 2D–path (cases 2, respectively 3 in fig. 1) are represented by bivectors as the result of the outer product of vectors. The even subalgebra G_3^+ of G_3 equals the algebra of quaternions. Therefore, quaternions constitute the domain of embedding a two–dimensional Fourier transform without loss of information.

We show in [16] that even the analytic signal, the Hilbert transform, and the Gabor filter can systematically be embedded in this domain. Of course, the two–dimensional phase is now constituted by three components, each resulting from the combination of the real part with one of three separate imaginary parts. In figure 6, middle row, we show the four components of the quaternionic Gabor filter with (from left to right) real, i–imaginary, j–imaginary, and k–imaginary parts. These quaternionic Gabor filter components are estimators of symmetry conceptions, adequate to differential geometric curvatures, from raw data. Insofar, they are adequate to the bottom–up approach, embedded into the global algebraic frame. By combination of these components, the well known complex Gabor filter results and, in addition, several components, which are able to respond to the other symmetries, which are missed else.

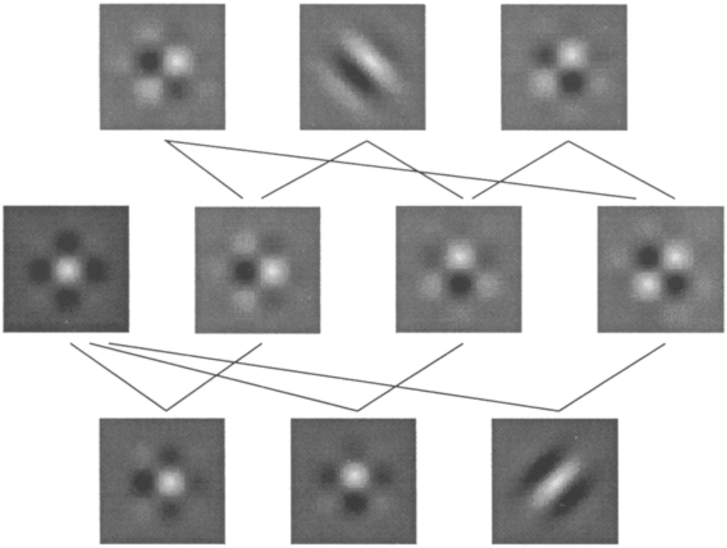

Fig.6. The quaternionic Gabor function (middle row) and their compositions to the symmetrie concepts of the plane.

The problem of recognition of structure in spatial domain is based on the recognition of the correlations with the corresponding symmetry. The Volterra series approach of filter design [62] recently was used to formulate non–linear filters for responding to higher–order correlations. In [38] the higher–order statistics of two–dimensional signals was estimated using a second–order Volterra filter in frequency domain to estimate the local intrinsic dimensionality of two–dimensional signals. In their approach, the authors had to extend the Fourier transform to 4D to respond the second–order Volterra approach.

Of course, the quaternionic quadrature filters are linear ones in contrast to the Volterra filters.

To summarize, the embedding of the analysis of multidimensional signals remains the linearity of operators, respectively remains the use of Fourier transform, but needs higher–order numbers. Any switch to lower dimensions is possible by combination of higher order numbers to lower order ones, just as we requested for a flexible vision system, embedded into the global algebraic frame.

We will finish our journey on linear signal processing by considering the functionality of neural nets. Of central importance in neural nets is the so–called linear associator, that means the functionality of a neuron, which linearly associates its input with the weights. This simple scalar product is based on the conception of linear vector algebra just as other linear methods of signal processing or recognition. The idea is to substitute the linear associator by a so–called geometric associator. That means to design a geometric algebra based neuron [8].

There have been several trials in the past, to algebraically extend neurons to complex numbers or quaternions. In [8], the general frame of geometric algebra with its multivector entities has been used with respect to MLP and RBF nets. In MLP nets, the central problem is to define an activation function which remains the locality of the backpropagated error function. This problem could be solved. Another aspect is the adequate representation of the input data to the used algebra. With this respect, we chose an outer product polynomial extension in accordance with the multiplication rules of the geometric algebra. Not completely to our surprise, both the convergence of learning as well as the generalization capability of the nets gained profit from that extension. Indeed, the used polynomial preprocessing is with some respect comparable to the well known design of higher–order nets (HONN, [50]), respectively to the Volterra connectionist model (VCM, [59]). But in these approaches, both the processing within the neurons and the polynomial extension were not algebraically correctly embedded, although they handle the polynomial extension also for multiple linear processing. The proposed so–called geometric neuron operates as multi–linear neuron on multivector entities. We hope to demonstrate in nearest future not only its capability of capturing correlations, but its useful application in real geometric problems.

5 Summary and Conclusions

In this paper, we discussed the socio–ecological metaphor as a base of the design of behavior based systems. But in contrast to the well known methodology of top–down designing a system from the model which we develop in our mind, this direct mapping between the functional behavior of the system and its model does not work in the context of behavior based systems. Instead, the model of functionality which we gained by observation of the appearance of the behavior has to be inverted into a bottom–up approach for the design of the PAC. This bottom–up approach implies both the importance of signal processing and learning of the competence.

Therefore, we concluded that both a global frame and a local one for algebraic embedding are requested to fulfill all the needs of the fusion of computer vision, neural computation, signal theory, and robotics. These global frames have been motivated and identified as geometric algebra and Lie theory. Finally, we discussed several aspects of the application of both frames. We demonstrated the usefulness with respect to recovery of geometry from signals by filtering and neural net processing.

As a result of this discussion of the task of designing behavior based systems, both the chances for general scientific progress as well as the amount of work, which has to be done, will become obvious. Only by concentration of the power and the experience of different disciplines, and only by considering the theoretical roots of the task, and growing support of basic research, we will successfully proceed in next future. This is in contrast to the actual interests of politics and industry, which want to support short–term development instead of long–term research.

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