Two-Dimensional Quality-Mesh Remodelling for Engineering Design Optimisation

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Abstract

Numeric simulation of physical phenomena of interest is currently a must in the development process of new products and systems, namely in the aerospace and automotive industries, among many others. However, greater simulation fidelity inevitably leads to computationally heavier simulators, strongly limiting the extent to which simulation-based numerical optimisation of physical systems may be employed in practice. In this work it is proposed a new type of re-meshing under domain boundary perturbations as a way of speeding up the simulation process in an optimisation context. The novelty of the new method, denominated mesh remodelling, is the ability to change the topology of the mesh, allowing the insertion and removal of its elements as necessary. A specialised version of mesh remodelling is presented and discussed for the specific application of airfoil shape optimisation.

Keywords: Unstructured Triangular Meshes; Delaunay Triangulations; Mesh Remodelling; Airfoil Parametrisation; Design Optimisation.

1 Introduction

Design optimisation and fluid dynamic simulators In many engineering domains, and especially in aeronautics, *fluid dynamic simulators* are commonly used to optimise various system designs. However, the complexity of such simulators is increasing rapidly and high-fidelity simulators are computationally heavy, leading to simulations that may take intolerable amounts of time to finish. In *engineering design optimisation*, these computer simulations are often driven by an optimisation process. The optimisation method, given a set of parameters concerning the shape of the model, repeatedly perturbs these parameters until a solution close to optimal is reached. Fluid dynamic simulators work with a partitioning of the space around the design in the form of *meshes*, also known as grids, to discretise the problem. The purpose of this work is to develop methods to adapt previous meshes to new designs, thereby preserving useful information between iterations. This should allow the simulator to perform more efficiently in an optimisation context, and thus decrease the overall computational cost of the optimisation process.

High quality meshes and Delaunay triangulations It is known that the quality of input meshes have a considerable influence over the quality and precision of the results of fluid dynamic simulators. Therefore, when it comes to be able to generate guaranteed good-quality meshes, using *Delaunay triangulations* is a natural choice. Among the optimal properties that they exhibit, the one that stands out the most is that the Delaunay triangulation maximizes the minimum angle among all possible triangulations of a fixed set of points. Moreover, Delaunay triangulations have been object of extensive study over the past years and good algorithms are widely available for their construction and refinement.

Mesh deformation Incremental approaches to re-meshing based on *mesh deformation* have been studied in the past. These operate by representing the edges and vertices of the mesh as tension and torsion springs, respectively, which, upon perturbations to the domain boundary, are moved so that the forces applied on them are in equilibrium. However, it is not possible to guarantee that the new mesh produced by these methods is still Delaunay, and thus that it retains its optimal properties, without a subsequent verification and correction steps for all the elements of the mesh that have been modified. Additionally, the number of elements in the mesh is not altered when using mesh deformation methods, which may not be suitable for large perturbations.

Mesh remodelling In this work it is proposed a new incremental approach to re-meshing, denominated *mesh remodelling*. Unlike mesh deformation, mesh remodelling has the possibility of removing and inserting elements to the mesh, and therefore vary their number as needed. Moreover, the new method has the benefit of maintaining a Delaunay triangulation, thus preserving its optimal properties and allowing the application of further refinement algorithms to guarantee high mesh quality in every iteration, crucial for the simulation process.

Document structure This document is organised as follows. Section 2 presents the concept of Delaunay triangulations, their properties, and methods for their refinement on a two-dimensional space. The mesh remodelling method is proposed in Section 3. Section 4 describes the context in which the new method was developed; both the models used in this work, a specialised version of mesh remodelling, and a practical application are addressed. Finally, Section 5 is dedicated to the experimentation and the analysis of its results.

2 Delaunay triangulations and Mesh generation

Properties of a Delaunay triangulation Given that the quality and precision of the results produced by fluid dynamic simulators are directly related to the quality of the mesh they use to perform their computations, it is advisable that the generated meshes are of high quality as well. The use of Delaunay triangulations as meshes becomes then a obvious choice. Their main advantage resides in their *optimal* features, these being the maximisation of the smallest angle and the minimisation of the largest circumcircle and the min.-containment circle, leaving the so-called "badly-shaped" or "skinny" triangles out of the triangulation, and also containing the range of influence of each triangle. Another — possibly useful — property of Delaunay triangulations is their *uniqueness*. Given a set of vertices there is only one possible Delaunay triangulation, the only exception being when four or more co-circular vertices are present and close enough for their triangles to be adjacent. Even so, the difference between meshes resides only within those permutable triangles.

Model discretisation and curved boundaries An important element of any mesh generation that uses models is the latter's *initial discretisation*, which will dictate the shape and size of the triangles, especially if any refinement methods are to be performed. That is more so if the models in question are defined by curves, where a perfect representation of them is impossible. A good curved model discretisation must have a *good resolution* — consistent presence of vertices throughout the whole model, even in straighter sections — and be an *accurate representation* of the model — higher presence of vertices in sections with higher curvature. Whenever refinement methods are considered, it is necessary to ensure that the initial discretisation is fine enough so that the sub-segments resulting from segment splitting do not intersect other segments of the model.

Mesh refinement When constructing a mesh based on models, as opposed to a pre-determined set of vertices, improving the quality of its triangles by applying refinement algorithms is common practice. The quality of a triangle is associated with its *minimum angle*; the higher the angle, the higher the quality. Another way to measure the quality of a triangle is through its *radius-edge* ratio, defined as its circumradius divided by its shortest edge. In a two-dimensional space the two measures are related by the formula

$$\frac{r}{e_{min}} = \frac{1}{2 \times \sin(\theta_{min})}$$

Triangles are considered to be of *poor quality* if the result of the formula above surpasses a pre-determined bound value, represented by B. Delaunay refinement algorithms operate by repeatedly inserting a vertex at the circumcenter of poor-quality triangles until the mesh contains none of the latter.

Mesh gradation For further control over the mesh gradation, i.e. the rate by which triangle size increases or decreases, one can use the concept of *length scale*, which is an attribute of vertices and represents the approximate distance from each vertex to the nearest boundary — shortest path through the edges. For vertices given as discretisation, the value of length scale is given by

$$LS_b(v) = \frac{lfs(v)}{R} = \min_{\text{neighbours } u_i} \left(\frac{\|u_i - v\|}{R} \right)$$

where *lfs* is the *local feature size*, defined as the radius of the smallest disk centred at v that touches two disjoint parts of the domain boundary — which, since v is a constituent of the boundary, can be simplified as being the distance to the nearest neighbour vertex — and R, denominated *resolution factor*, is a pre-defined value that controls the resolution of input features. For vertices inserted during the refinement process, denominated Steiner vertices, the value of length scale is computed as

$$LS_s(v) = \min_{\text{neighbours } u_i} \left(LS(u_i) + \frac{\|u_i - v\|}{G} \right)$$

where G, *gradation factor*, is also a pre-set value that controls the rate of triangle size increase as they get further from the boundaries. If a Steiner vertex also happens to be a boundary vertex — in the case of segment splitting —, the computation of its length scale takes into account an additional arc-length-based interpolation between its two boundary neighbours:

$$LS_{bs}(v) = \min\left(LS(w_1) \xrightarrow[\widehat{w_1 w_2}]{\widehat{w_1 v}} LS(w_2),\ LS_s(v) \right)$$

A triangle is considered to be of poor quality, or *poor gradation*, if its circumradius divided by the average length scale of its vertices is higher than a pre-determined bound value, represented by H.

3 Mesh remodelling

Note *The mesh remodelling method described in this chapter can be applied to any kind of mesh. Even so, given that the whole work (and this document) is based on Delaunay triangulations, please consider the mesh to be of such type for a better understanding of the method and the way it is applied.*

Motivation Mesh remodelling is a method which purpose is to replace one model with another in a given mesh. It was designed to be as *robust*, *simple* and *general* as possible; in other words, to not be prone to errors arising from special cases, to modify the mesh using only basic, well established algorithms, and to not make any assumptions regarding the type of mesh or models. The main goal of mesh remodelling, derived from the practical application addressed in this work, is to maximise element preservation, i.e. the percentage of elements from the old mesh that were maintained in the new mesh.

Methodology Since mesh remodelling relies solely on basic mesh modification procedures — vertex insertion and removal — the mesh is required either to be present at the locations of the new vertices or to allow their insertion outside its boundaries. The algorithm itself is relatively simple, almost trivial. It starts by inserting the initial discretisation vertices of the new model into the mesh. Then, a circular removal region is defined for each non-Steiner vertex of both models, inside of which all Steiner vertices are to be removed. The radius of such regions is determined by the distance between a vertex and the closest vertex of the other model and bounded from below by the maximum distance to its two current boundary neighbours. The computation of this distance can be easily accomplished using a quadtree. Additionally, the distance can be multiplied by a pre-defined value D, denominated *distance factor*. Finally, all boundary vertices of the old model are removed from the mesh.

Mesh quality, refinement and gradation Due to the algorithm's simplicity of operations, it is possible to maintain a Delaunay triangulation throughout the remodelling process simply by using the appropriate subroutines for vertex insertion and removal, which assures that the mesh retains its optimal properties and allows a subsequent Delaunay refinement process to be performed, thus guaranteeing that the quality of the new mesh is no worse than the last one. The mesh gradation however, cannot be the same as in a newly built mesh, in part because the length scale of the existing Steiner vertices was computed considering older models, at different distances. The value of D can be increased to improve the gradation component. The computation of length scale for the new model's discretisation vertices is a combination of the non-Steiner boundary vertex and Steiner vertex variants; LS_b and LS_s, respectively. The decision of which variant to employ is determined by the type of each neighbour.

$$LS_b(v) = \min_{\text{neighbours } u_i} \left(\begin{cases} \dfrac{\|u_i - v\|}{R} & , u_i \text{ is non-Steiner} \\ LS(u_i) + \dfrac{\|u_i - v\|}{G} & , \text{otherwise} \end{cases} \right)$$

Advantages and disadvantages The main advantage of mesh remodelling is the fact that it uses only subroutines that were needed to build the mesh in the first place, and therefore avoiding the inclusion of additional complexity to the implementation. When compared to mesh deformation methods, mesh remodelling has the convenience of actually removing elements — as opposed to just translating and reshaping —, thus producing a mesh with a number of elements more appropriate to the dimensions of new model. In the context of this work's practical application it also benefits from its ability to maintain a Delaunay triangulation, making it easier to guarantee mesh quality on every iteration. On the other hand, and despite being considerably faster, mesh remodelling cannot achieve the same value of mesh gradation as the mesh generation method, which results in a mesh with a different number of elements from the ideal — usually more.

4 Case study

Airfoils and design optimisation This work is carried out in the context of engineering design optimisation, being airfoils the design in question and the resulting meshes to be used by a fluid dynamics simulator. An airfoil is a cross-section of an airplane wing from a lateral standpoint. In the section that follows, the parametrisation used to create the airfoil models, the specialised version of the mesh remodelling method, and the practical application in which the method is to be used are described.

4.1 Model parametrisation

Class Shape Transformation The airfoil designs used in this study were created using a parametrisation called Class Shape Transformation. It was chosen due to being specifically designed to model the many components of an aircraft, such as fuselages, nacelles, winglets, airfoils, among others. Additionally, it is possible to achieve a great variety of shapes within a particular class of model by changing a single component of the parametrisation, simplifying the process of design optimisation. Also, the fact that one can predict the changes in shape from the parameters variation makes for a very intuitive and easy to work with parametrisation.

$$a(t) = \begin{cases} \eta_u(1-2t) & , 0 \le t < \frac{1}{2} \\ \eta_l(2t-1) & , \frac{1}{2} \le t < 1 \end{cases}$$

$$\begin{array}{l|l} \eta_u(\psi) = C_u(\psi) \times S_u(\psi) + T_u(\psi) & \eta_l(\psi) = C_l(\psi) \times S_l(\psi) + T_l(\psi) \\ C_u(\psi) = \psi^{e_1} \times (1-\psi)^{e_2} & C_l(\psi) = \psi^{e_1} \times (1-\psi)^{e_2} \\ S_u(\psi) = \sum_{i=0}^{n} A_{ui} \times B_{i,n}(\psi) & S_l(\psi) = \sum_{i=0}^{n} A_{li} \times B_{i,n}(\psi) \\ T_u(\psi) = \psi \times \Delta\eta_u & T_l(\psi) = \psi \times \Delta\eta_l \end{array}$$

$$\psi = \frac{x}{c} \qquad \eta = \frac{y}{c}$$

Formulation The shape of the model is given by the *airfoil function*[1], $a(t)$, which purpose is to merge the functions of the upper and lower surfaces of the airfoil into one, describing a counter-clockwise path that starts and ends at the trailing edge of the airfoil. In this formulation, y is a function of x, which in turn is a function of t. The upper surface functions and parameters contain the subscript u, while the lower surface functions and parameters contain the subscript l. The *Class function*, $C(\psi)$, defines the class of the model, giving it an initial shape. The coefficients e_1 and e_2 control the shape of the model at its leading and trailing edges, respectively, assuming the values 0.5 and 1 in the "NACA airfoil" class — the one used in this work. The *Shape function*, $S(\psi)$, adjusts the basic shape of the model with the help of A, a vector of *shape coefficients* — object of study in design optimisation. $B_{i,n}(\psi)$ represents the i^{th} Bernstein basis polynomial of degree n. Finally, the *Trailing edge function*[2], $T(\psi)$, is used to translate the trailing edge of the airfoil along the y axis, being $\Delta\eta$ the translation amount. The variable c denotes the length of the airfoil chord, i.e. the horizontal distance between the leading and trailing edges of the airfoil.

4.2 Mesh remodelling specialisation

Model characteristics and premises Given the specific characteristics of the CST parametrisation as well as some design choices, it is possible to optimise the more general mesh remodelling method into a faster and still as robust version, although more complex, so that it takes full advantage of the models in use. Some of these characteristics and choices are as follows:

- The model is defined by two functions, guaranteeing that for a given value of x there is a unique value of y;
- The chord length, c, is set to 1, therefore limiting the x-domain of all models to the interval $[0,1]$;
- The trailing edge translation value, $\Delta\eta$, is set to 0 on both surfaces, ensuring the continuous presence of a vertex at the coordinates $(1,0)$;
- The number of vertices in the initial discretisation is maintained throughout the optimisation process, allowing a one-to-one correlation between vertices of different models. Also, due to y being a function of x and x a function of t, these vertices differ only in their y-value.

Adjustment The first step of the specialised version of mesh remodelling aims to speed-up the method by adjusting the coordinates of some, if not all of the boundary vertices, thus reducing the number of circular region removals and vertex replacements to be performed in a latter stage. This is only possible due to the singular properties of the models addressed in this work, especially the first and fourth previously listed, which guarantee that no boundary edges of the same surface are going to intersect regardless of the magnitude of the adjustments. This process is applied to all boundary vertices, whether Steiner or non-Steiner. Let v be the vertex being currently checked and v_n its new position; let U be v's neighbours and U_n their new positions. Vertex v can only be adjusted if all the following prove true:

- v_n is not be beyond the current opposite surface;
- Considering v_n and U: v's surrounding triangles are valid;
- Considering v_n and U_n: v's surrounding triangles are valid, Delaunay, and respect the mesh's quality and gradation constraints.

[1] Not a part of the original scheme.
[2] Not presented as a separate function in the original scheme.

The first verification prevents the two surfaces from intersecting when one surface gets adjusted and the other does not. The second verification guarantees that, whichever combination of vertices is adjusted, the resulting mesh is still valid. The third and last verification covers the properties that any triangle in a fully refined Delaunay triangulation must possess, which is the reason why, in the case that every boundary vertex gets adjusted, not only the next step of mesh remodelling but also the subsequent refinement procedure can be skipped. If v passes all checks, then its coordinates are updated to v_n. If that is not the case and v happens to be non-Steiner, then the Steiner vertices between v and its two non-Steiner boundary neighbours are not even considered for adjustment. This is done to prevent the accumulation of vertices in regions that need to be rebuilt, which could lead to over-refinement.

Removal and replacement The second and final stage of the specialised mesh remodelling version is very similar to the original method, the only difference being the number of circular removal regions employed; one for each pair of old/new vertices, given the one-to-one correlation, instead of one for each new and old vertex. The centre of such removal regions is located at the outermost vertex — highest y-value for vertices belonging to the upper surface; the opposite for the lower surface. Regarding the radius of these regions, although it is recommended that it be determined in the same way as the original, one can use the vertical distance between the pair of vertices instead and still achieve good results. If the vertex was already adjusted, the use of a removal region for that particular pair of vertices can be avoided altogether.

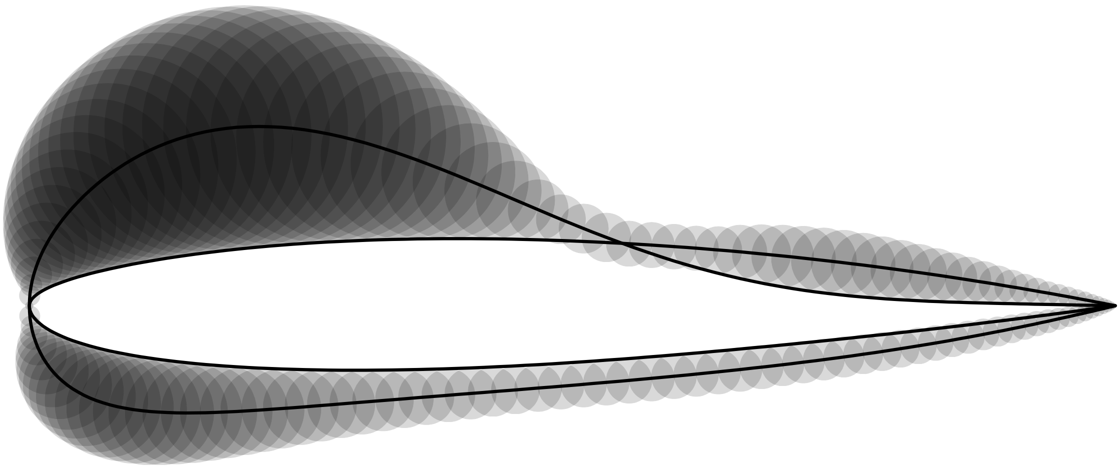


FIGURE 1: Circular removal regions (150 vertices, $D = 1$)
The set of circular removal regions is the same for each pair of models, regardless of which is the old and the new one.

4.3 Practical application

Describe how mesh remodelling is to be applied to the dynamics simulator, how the latter works (implicit vs explicit), in what ways remodelling can speed-up the process (element preservation), etc.. The time to perform simulation is 5 orders of magnitude above, so speed-up in remodelling is not that relevant.

5 Experimentation and Results

5.1 Experimental setup

Simulation of the optimisation process In order to reproduce the successive perturbations that are applied to the shape of the model in a real design optimisation scenario, a simple shape interpolation optimisation was employed. Starting with a standard airfoil design, the method modifies its parameters — namely the vector of shape coefficients, A — until it successfully interpolates a set of representative vertices of the target airfoil. The algorithm chosen to perform such task is the Covariance Matrix Adaptation - Evolution Strategy (CMA-ES), which is governed, among other things, by a standard deviation factor that controls the magnitude of the perturbations and whose initial value — represented by σ — can be controlled. Most real scenarios use population-based methods — like the CMA-ES — to generate the designs, which could be availed to maintain several meshes at the same time and thereby reduce the magnitude of the perturbations between consecutive models. Even so, in this work only one mesh was employed so that the impact of such high perturbations can be more noticeable and better comprehended.

Fixed parameters

- $B = 1$ (30°) minimum angle bound
- $H = \sqrt{2}/2$ triangle gradation bound
- $R = 1$ length scale resolution factor

The value chosen for B is considered a threshold value, above which most Delaunay refinement algorithms tend to not finish, and those who do, start showing signs of over-refinement — although it depends on the models used. The value of H is the same as the one adopted be the creators of the length scale concept. Finally, R was set so it has no impact on the refinement algorithm, entrusting the cardinality of the initial discretisation to provide the desired resolution.

Variable parameters

- $\sigma = \{0.01, 0.05, 0.25\}$ CMA-ES initial standard deviation factor
- $I = \{50, 100, 200, 300, 400, 700\}$ cardinality of the initial discretisation of the models
- $G = \{1, 2, 4, 6, 8, 14\}$ length scale gradation factor
- $D = \{0.5, 1, 2, 4\}$ mesh remodelling removal distance factor

The creators of the CMA-ES method recommend that σ be set to roughly one-fourth of the search space, which, by analysing the values of A of a diverse set of airfoils, was concluded to be around 0.05. Two other values — five times smaller and larger — were also used in order to assess the impact that the magnitude of the perturbations has on the mesh remodelling method. The values of I are paired with the values of G at the equivalent positions, producing only six combinations. Together, they control the precision/resolution/size of the mesh. Finally, the study of bad gradation produced by mesh remodelling is done by varying the value of D.

Methodology By employing the CMA-ES algorithm and setting the initial model to the NACA-0012 airfoil and the target model to the RAE-2822 airfoil, thirty design optimisation reproductions — with around nine thousand iterations each — are generated for each of the three values of σ. Surrounding the airfoil there is a circular model — often called *farfield* — with a radius of twenty-five, between which and the airfoil is situated the mesh. The mesh generation and mesh remodelling methods are then performed considering every combination of I, G and D — six and twenty-four, respectively.

5.2 Results analysis

Element preservation Being element preservation the most relevant metric given the practical application, the obtained results are quite satisfactory, consistently achieving values above 90% for any mesh resolution and any magnitude of model perturbations, even reaching values beyond 96% for finer meshes. As would be expected, the higher the value of D, the lower the element preservation, especially when the perturbations to the models are more significant. However, it can come as a surprise that on average, element preservation tends to be better for higher values of σ in coarser meshes.

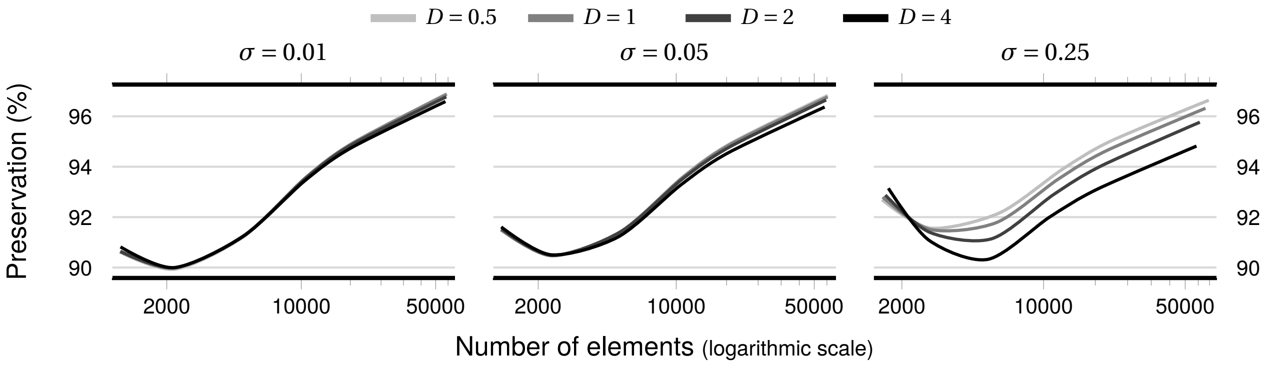


FIGURE 2: Element preservation results using mesh remodelling
Average of thirty optimisation scenarios - RAE-2822 airfoil

Element surplus Another metric of some significance is element surplus, defined as the percentage of elements(triangles) in excess in meshes produced by mesh remodelling relative to the mesh generation counterparts. Its significance is derived from the fact that the more elements the mesh has, the more computations the dynamic-fluid simulators have to perform. The first remark to be made is that mesh remodelling does not achieve good results for coarse meshes, reaching values up to 50% for larger magnitudes of model perturbations. Also note that the method achieves negative values of element surplus when performing smaller perturbations to the airfoil in very small meshes, a result derived from bad gradation. For medium-to-large meshes though, mesh remodelling achieves very good results, usually below 10%. The increase in removal distance also has a positive impact on the results, which is more noticeable on finer meshes, with a reduction of element surplus to below 5%, despite the also decrease in element preservation by 2%.

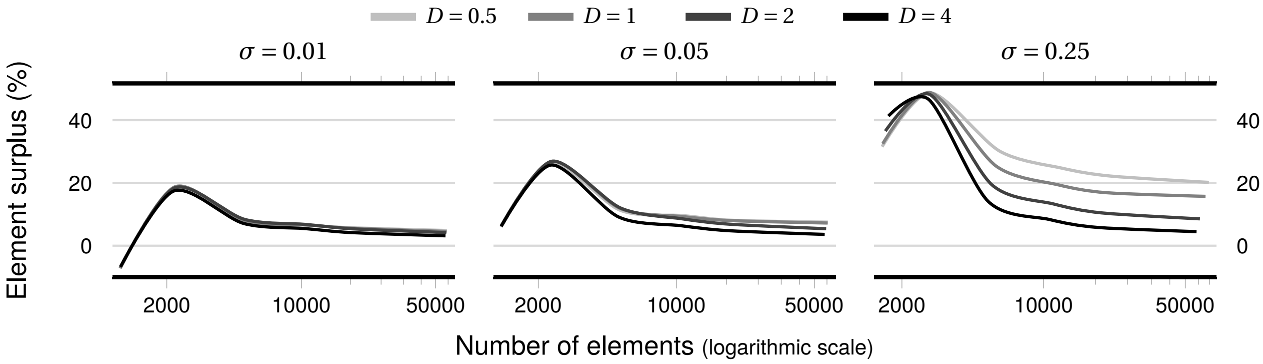


FIGURE 3: Element surplus results on meshes from mesh remodelling when compared to those from mesh generation
Average of thirty optimisation scenarios - RAE-2822 airfoil

Speed-up Perhaps the most impressive results, although the less important, are related to the time of execution. The speed-ups achieved by using mesh remodelling instead of mesh generation can vary greatly, from fifteen in larger perturbations to sixty in slighter ones. Even so, the results are as expected; the speed-up rises as mesh resolution increases and falls as either removal distance or the magnitude of the model perturbations increases.

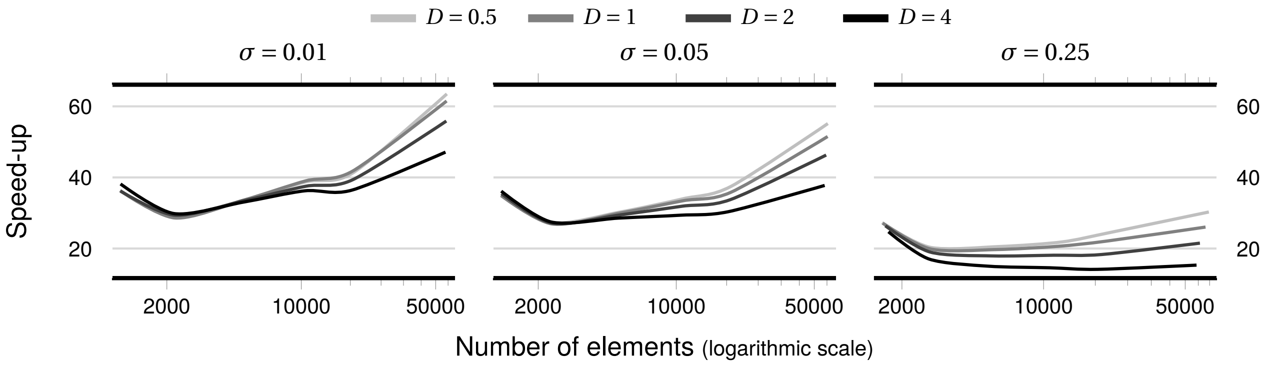
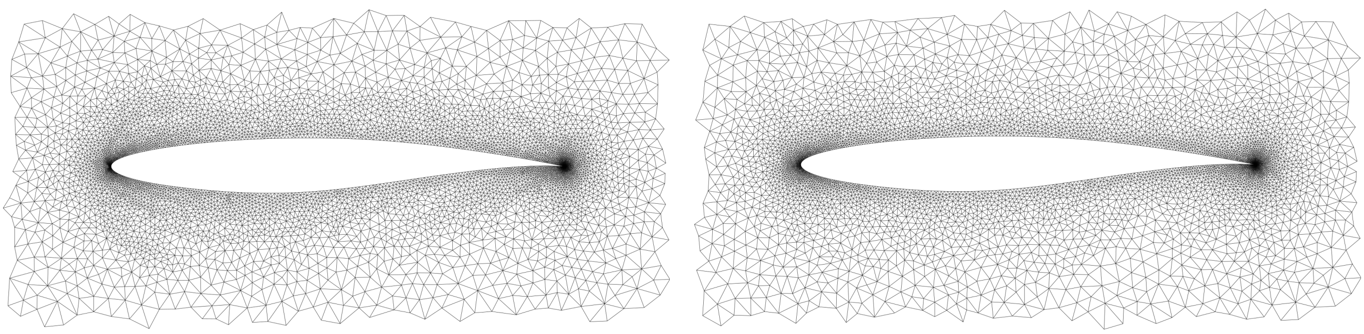


FIGURE 4: Speed-up results by mesh remodelling relative to mesh generation
Average of thirty optimisation scenarios - RAE-2822 airfoil



(A) $D = 1$

(B) $D = 4$

FIGURE 5: Illustration of element surplus and bad gradation
Mesh remodelling, $\sigma = 0.25, I = 400, G = 8$, RAE-2822 airfoil

Improvements Despite the good results, there is still room for improvement, especially concerning the problem of element surplus and bad gradation. One possible strategy, that follows from a previous work where its development and testing was carried out, involves choosing which method to perform — generation or remodelling — at a given iteration. The criteria for such decision can include the variation in the shape coefficient vector, A, the current value of element surplus — although difficult to control, given that there is no point of comparison and that the higher number of elements could just be a consequence of a thinner model — or even be a periodic event. One other alternative is as follows.

- In this work's particular case, due to the way the CMA-ES works, it is almost certain that at some point during the nine thousand iterations — depending on the value of σ — the mesh remodelling method will start performing only full adjustments (adjust all vertices) until the very end. One can take advantage of such behaviour and upon detecting, for example, ten successive full adjustments, instruct the program to perform a single mesh generation iteration. Since the remaining iterations are bound to be full adjustments, it is guaranteed that no elements will be either added or removed from the mesh, which results in an element surplus of 0% from that point onwards at the cost of a merely one-hundredth of a percent in element preservation in the end.

Additionally, it is perfectly possible to opt for mesh deformation rather than mesh remodelling if it turns out to be more efficient on such small model perturbations — although the only gain would be in speed-up. Ultimately, all possible improvements are to be considered from the fluid-dynamics simulator's point of view.

6 Conclusions and Further developments

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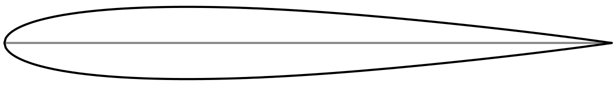
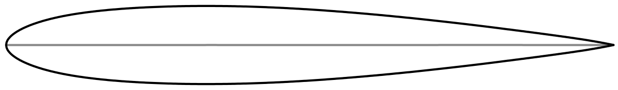
A Appendix

A.1 Airfoils



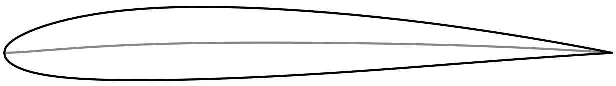
Max. Thickness: 12% at 30% of the chord
Max. Camber: 0

FIGURE A1: NACA 0012



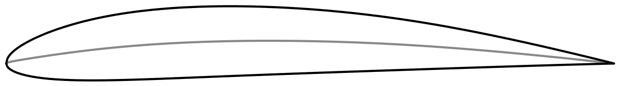
Max. Thickness: 12.7% at 30% of the chord
Max. Camber: 0

FIGURE A2: Gottingen 459



Max. Thickness: 12% at 30% of the chord
Max. Camber: 2% at 40% of the chord

FIGURE A3: NACA 2412

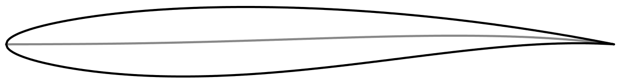


Max. Thickness: 11.7% at 28% of the chord
Max. Camber: 3.4% at 42% of the chord

FIGURE A4: Clark-Y

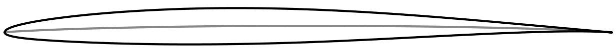
Max. Thickness: 10% at 39.9% of the chord
Max. Camber: 1.5% at 20.4% of the chord

FIGURE A5: Boeing 737



Max. Thickness: 12.1% at 37.9% of the chord
Max. Camber: 1.3% at 75.7% of the chord

FIGURE A6: RAE 2822

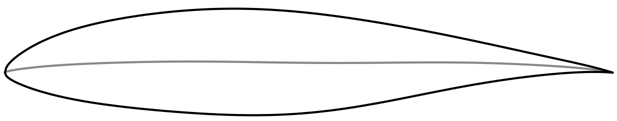


Max. Thickness: 6% at 35% of the chord
Max. Camber: 1.1% at 50% of the chord

FIGURE A7: NACA 63206

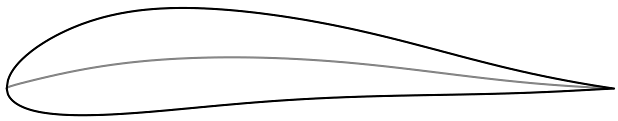
Max. Thickness: 2.5% at 4.3% of the chord
Max. Camber: 9% at 32.7% of the chord

FIGURE A8: Eppler 376



Max. Thickness: 17.4% at 39.3% of the chord
Max. Camber: 1.8% at 29.9% of the chord

FIGURE A9: Eppler 545



Max. Thickness: 16.6% at 19.4% of the chord
Max. Camber: 5.1% at 39.4% of the chord

FIGURE A10: Gottingen 702

A.2 Legend

- ***σ*** CMA-ES initial standard deviation factor
- ***I*** Cardinality of the initial discretisation
- ***G*** Length scale gradation factor
- ***D*** Mesh remodelling removal distance factor

- **# Tri.** Number of elements in the mesh
- **Time** Execution time
- **Preserv.** Element preservation
- **+ Tri.** Element surplus
- **Sp.-up** Speed-up

A.3 Results

A.3.1 Gottingen 459

σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Remodelling Time	Remodelling Preserv.	+ Tri.	Sp.-up
.01	50	1	1067	1.66 ms	.5	1036	0.05 ms	89.95 %	-2.88 %	33.13
					1	1036	0.05 ms	89.95 %	-2.88 %	33.11
					2	1036	0.05 ms	89.95 %	-2.88 %	33.13
					4	1036	0.05 ms	89.95 %	-2.88 %	33.06
	100	2	1866	3.47 ms	.5	1926	0.12 ms	88.54 %	3.21 %	30.09
					1	1926	0.12 ms	88.54 %	3.21 %	30.08
					2	1927	0.12 ms	88.54 %	3.24 %	30.09
					4	1917	0.11 ms	88.55 %	2.74 %	30.66
	200	4	4634	9.63 ms	.5	4681	0.24 ms	90.89 %	1.02 %	39.37
					1	4681	0.25 ms	90.88 %	1.01 %	39.30
					2	4687	0.24 ms	90.91 %	1.14 %	39.59
					4	4693	0.24 ms	90.98 %	1.28 %	40.29
	300	6	9759	20.67 ms	.5	9931	0.43 ms	93.40 %	1.77 %	48.33
					1	9937	0.42 ms	93.40 %	1.83 %	49.07
					2	9985	0.43 ms	93.43 %	2.32 %	48.43
					4	9996	0.42 ms	93.47 %	2.43 %	48.89
	400	8	17289	36.19 ms	.5	17403	0.67 ms	94.86 %	0.66 %	53.79
					1	17410	0.68 ms	94.86 %	0.70 %	53.44
					2	17499	0.69 ms	94.87 %	1.21 %	52.15
					4	17516	0.70 ms	94.89 %	1.31 %	51.80
	700	14	53560	154.08 ms	.5	53726	1.86 ms	96.88 %	0.31 %	82.78
					1	53782	1.87 ms	96.90 %	0.41 %	82.53
					2	54023	1.92 ms	96.88 %	0.86 %	80.33
					4	54124	2.05 ms	96.86 %	1.05 %	75.13

TABLE A1: Full results of mesh remodelling for $\sigma = 0.01$ - Gottingen 459 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Remodelling Time	Preserv.	+ Tri.	Sp.-up
.05	50	1	1069	1.67 ms	.5	1163	0.05 ms	90.83 %	8.82 %	32.95
					1	1164	0.05 ms	90.83 %	8.85 %	32.89
					2	1173	0.05 ms	90.91 %	9.69 %	33.02
					4	1185	0.05 ms	91.16 %	10.81 %	34.36
	100	2	1871	3.47 ms	.5	2177	0.12 ms	89.94 %	16.34 %	29.59
					1	2178	0.12 ms	89.92 %	16.38 %	29.24
					2	2195	0.12 ms	90.02 %	17.27 %	29.57
					4	2197	0.12 ms	90.16 %	17.42 %	30.17
	200	4	4632	9.60 ms	.5	4885	0.29 ms	91.03 %	5.46 %	33.54
					1	4931	0.29 ms	91.09 %	6.45 %	33.21
					2	5003	0.30 ms	91.17 %	8.00 %	32.50
					4	4945	0.30 ms	91.10 %	6.76 %	31.79
	300	6	9759	20.43 ms	.5	10297	0.54 ms	93.38 %	5.51 %	37.79
					1	10320	0.54 ms	93.42 %	5.75 %	38.01
					2	10387	0.57 ms	93.39 %	6.44 %	36.15
					4	10277	0.61 ms	93.30 %	5.30 %	33.48
	400	8	17280	36.11 ms	.5	17953	0.90 ms	94.78 %	3.89 %	40.06
					1	18010	0.93 ms	94.80 %	4.22 %	38.94
					2	18072	0.99 ms	94.75 %	4.58 %	36.53
					4	17926	1.07 ms	94.62 %	3.74 %	33.84
	700	14	53571	153.94 ms	.5	55333	2.58 ms	96.78 %	3.29 %	59.58
					1	55424	2.72 ms	96.76 %	3.46 %	56.60
					2	55455	3.00 ms	96.68 %	3.52 %	51.31
					4	55178	3.53 ms	96.50 %	3.00 %	43.65

TABLE A2: Full results of mesh remodelling for $\sigma = 0.05$ - Gottingen 459 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation		D	Remodelling				
			# Tri.	Time		# Tri.	Time	Preserv.	+ Tri.	Sp.-up
.25	50	1	1093	1.69 ms	.5	1556	0.07 ms	92.60 %	42.37 %	25.62
					1	1564	0.07 ms	92.59 %	43.09 %	25.53
					2	1623	0.07 ms	92.89 %	48.51 %	25.62
					4	1657	0.07 ms	93.01 %	51.56 %	24.06
	100	2	1867	3.43 ms	.5	2658	0.16 ms	91.35 %	42.36 %	21.53
					1	2667	0.16 ms	91.33 %	42.85 %	21.02
					2	2673	0.17 ms	91.30 %	43.20 %	20.14
					4	2657	0.19 ms	91.07 %	42.34 %	18.10
	200	4	4624	9.08 ms	.5	5946	0.43 ms	92.10 %	28.58 %	21.10
					1	5705	0.45 ms	91.70 %	23.36 %	20.38
					2	5490	0.49 ms	91.15 %	18.72 %	18.59
					4	5227	0.57 ms	90.36 %	13.03 %	15.87
	300	6	9755	19.32 ms	.5	12148	0.88 ms	93.85 %	24.53 %	21.99
					1	11595	0.92 ms	93.48 %	18.85 %	20.99
					2	11039	1.03 ms	92.90 %	13.16 %	18.73
					4	10538	1.27 ms	92.07 %	8.02 %	15.19
	400	8	17250	35.52 ms	.5	21002	1.44 ms	94.98 %	21.75 %	24.70
					1	20114	1.57 ms	94.65 %	16.61 %	22.65
					2	19017	1.89 ms	94.07 %	10.25 %	18.79
					4	18230	2.41 ms	93.22 %	5.68 %	14.71
	700	14	53523	152.32 ms	.5	64168	4.95 ms	96.65 %	19.89 %	30.75
					1	61973	5.78 ms	96.35 %	15.79 %	26.37
					2	58137	6.94 ms	95.83 %	8.62 %	21.95
					4	55999	9.58 ms	94.95 %	4.63 %	15.90

TABLE A3: Full results of mesh remodelling for $\sigma = 0.25$ - Gottingen 459 airfoil
Average of thirty optimisation scenarios

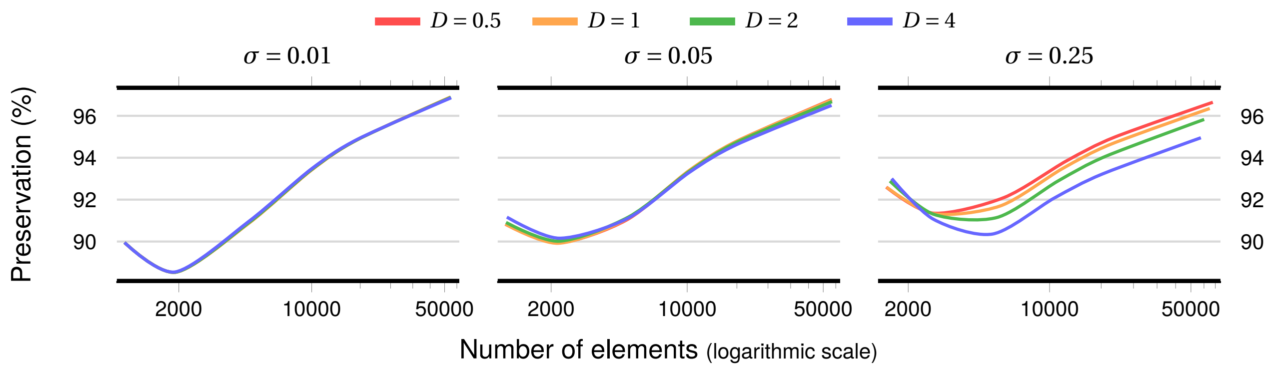


FIGURE A11: Element preservation results using mesh remodelling - Gottingen 459 airfoil
Average of thirty optimisation scenarios

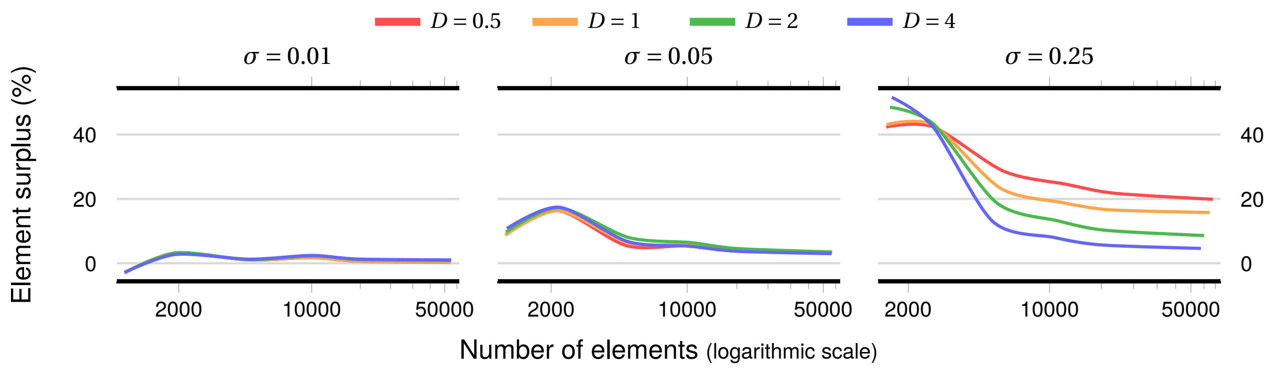


FIGURE A12: Element surplus results on meshes from mesh remodelling - Gottingen 459 airfoil
Average of thirty optimisation scenarios

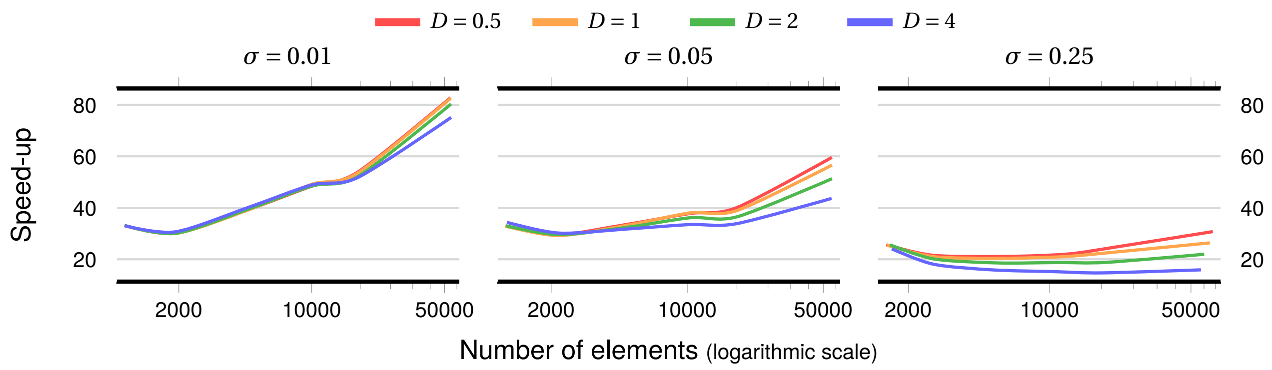


FIGURE A13: Speed-up results by mesh remodelling - Gottingen 459 airfoil
Average of thirty optimisation scenarios

A.3.2 NACA 2412

σ	I	G	Generation		D	Remodelling				
			# Tri.	Time		# Tri.	Time	Preserv.	+ Tri.	Sp.-up
.01	50	1	1048	1.64 ms	.5	1115	0.05 ms	90.49 %	6.45 %	32.23
					1	1115	0.05 ms	90.49 %	6.45 %	32.23
					2	1115	0.05 ms	90.52 %	6.43 %	32.31
					4	1114	0.05 ms	90.58 %	6.37 %	33.25
	100	2	1823	3.39 ms	.5	2154	0.12 ms	89.75 %	18.16 %	28.42
					1	2153	0.12 ms	89.77 %	18.14 %	28.68
					2	2147	0.12 ms	89.76 %	17.80 %	28.75
					4	2130	0.11 ms	89.87 %	16.86 %	29.74
	200	4	4618	9.49 ms	.5	4981	0.29 ms	91.19 %	7.86 %	32.36
					1	4967	0.29 ms	91.17 %	7.54 %	32.64
					2	4982	0.30 ms	91.18 %	7.88 %	32.02
					4	4907	0.29 ms	91.12 %	6.26 %	32.18
	300	6	9781	20.52 ms	.5	10377	0.55 ms	93.47 %	6.08 %	37.12
					1	10369	0.56 ms	93.48 %	6.01 %	36.85
					2	10390	0.58 ms	93.46 %	6.23 %	35.13
					4	10286	0.59 ms	93.37 %	5.16 %	34.81
	400	8	17288	35.91 ms	.5	18148	0.96 ms	94.86 %	4.97 %	37.22
					1	18134	0.98 ms	94.86 %	4.89 %	36.78
					2	18107	1.02 ms	94.80 %	4.74 %	35.30
					4	17957	1.07 ms	94.68 %	3.87 %	33.61
	700	14	53632	153.47 ms	.5	55929	2.85 ms	96.84 %	4.28 %	53.91
					1	55791	2.97 ms	96.81 %	4.03 %	51.60
					2	55631	3.26 ms	96.71 %	3.73 %	47.03
					4	55294	3.75 ms	96.53 %	3.10 %	40.91

TABLE A4: Full results of mesh remodelling for $\sigma = 0.01$ - NACA 2412 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation		D	Remodelling				
			# Tri.	Time		# Tri.	Time	Preserv.	+ Tri.	Sp.-up
.05	50	1	1062	1.65 ms	.5	1295	0.05 ms	91.44 %	21.95 %	31.17
					1	1303	0.05 ms	91.48 %	22.73 %	31.25
					2	1298	0.05 ms	91.52 %	22.18 %	31.21
					4	1299	0.05 ms	91.62 %	22.29 %	32.14
	100	2	1818	3.37 ms	.5	2320	0.13 ms	90.38 %	27.60 %	25.82
					1	2326	0.13 ms	90.42 %	27.94 %	25.94
					2	2346	0.13 ms	90.51 %	29.07 %	25.98
					4	2311	0.13 ms	90.49 %	27.14 %	26.42
	200	4	4617	9.38 ms	.5	5139	0.33 ms	91.28 %	11.29 %	28.84
					1	5118	0.32 ms	91.27 %	10.84 %	28.89
					2	5136	0.34 ms	91.24 %	11.24 %	27.89
					4	5030	0.35 ms	91.06 %	8.94 %	26.97
	300	6	9783	19.84 ms	.5	10653	0.63 ms	93.48 %	8.89 %	31.27
					1	10642	0.64 ms	93.46 %	8.78 %	30.86
					2	10592	0.70 ms	93.36 %	8.27 %	28.38
					4	10376	0.73 ms	93.12 %	6.07 %	27.23
	400	8	17288	35.80 ms	.5	18583	1.11 ms	94.83 %	7.49 %	32.13
					1	18521	1.13 ms	94.79 %	7.13 %	31.60
					2	18418	1.22 ms	94.66 %	6.54 %	29.31
					4	18060	1.34 ms	94.40 %	4.46 %	26.72
	700	14	53646	153.45 ms	.5	57211	3.39 ms	96.75 %	6.65 %	45.32
					1	56982	3.59 ms	96.69 %	6.22 %	42.78
					2	56504	4.08 ms	96.54 %	5.33 %	37.61
					4	55566	4.90 ms	96.23 %	3.58 %	31.33

TABLE A5: Full results of mesh remodelling for $\sigma = 0.05$ - NACA 2412 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Remodelling Time	Remodelling Preserv.	Remodelling + Tri.	Remodelling Sp.-up
.25	50	1	1084	1.67 ms	.5	1570	0.07 ms	92.66 %	44.88 %	25.39
					1	1592	0.07 ms	92.72 %	46.85 %	25.20
					2	1610	0.07 ms	92.78 %	48.57 %	24.96
					4	1664	0.07 ms	93.04 %	53.51 %	23.36
	100	2	1822	3.34 ms	.5	2669	0.16 ms	91.37 %	46.44 %	20.34
					1	2668	0.17 ms	91.29 %	46.42 %	19.96
					2	2668	0.17 ms	91.25 %	46.39 %	19.17
					4	2645	0.19 ms	90.99 %	45.11 %	17.19
	200	4	4610	8.98 ms	.5	5943	0.44 ms	92.09 %	28.91 %	20.40
					1	5717	0.46 ms	91.68 %	24.01 %	19.67
					2	5463	0.51 ms	91.06 %	18.50 %	17.76
					4	5240	0.60 ms	90.29 %	13.67 %	15.07
	300	6	9778	19.19 ms	.5	12118	0.91 ms	93.84 %	23.94 %	21.20
					1	11591	0.95 ms	93.45 %	18.55 %	20.17
					2	10997	1.08 ms	92.86 %	12.47 %	17.79
					4	10558	1.33 ms	91.99 %	7.99 %	14.46
	400	8	17273	35.35 ms	.5	20974	1.51 ms	94.97 %	21.43 %	23.36
					1	20113	1.69 ms	94.61 %	16.45 %	20.90
					2	18937	1.99 ms	94.02 %	9.64 %	17.81
					4	18201	2.55 ms	93.12 %	5.37 %	13.86
	700	14	53607	151.91 ms	.5	64172	5.29 ms	96.62 %	19.71 %	28.70
					1	61833	6.15 ms	96.32 %	15.34 %	24.69
					2	57864	7.40 ms	95.76 %	7.94 %	20.54
					4	55940	10.16 ms	94.86 %	4.35 %	14.95

TABLE A6: Full results of mesh remodelling for $\sigma = 0.25$ - NACA 2412 airfoil
Average of thirty optimisation scenarios

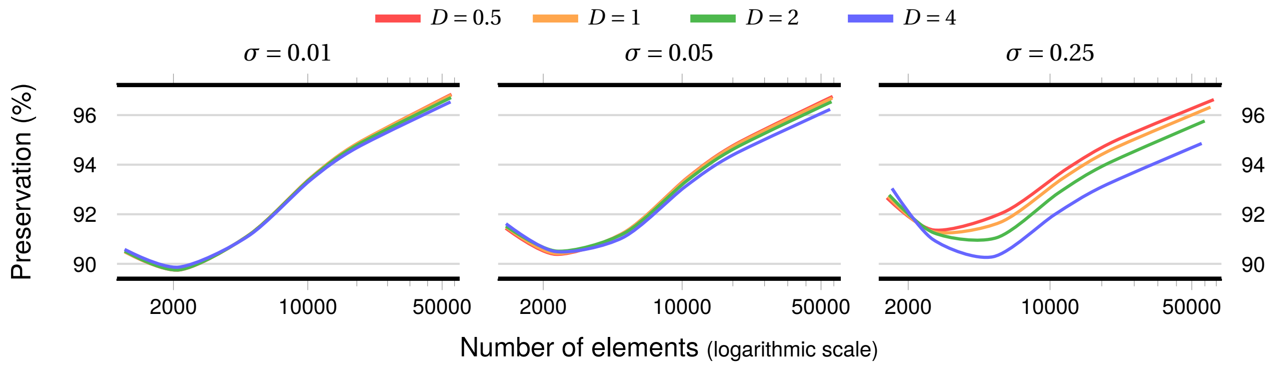


FIGURE A14: Element preservation results using mesh remodelling - NACA 2412 airfoil
Average of thirty optimisation scenarios

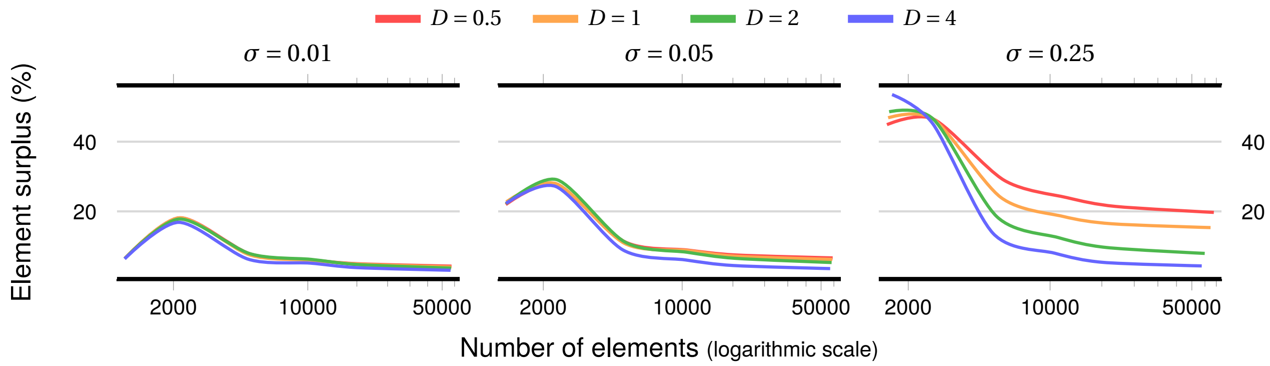


FIGURE A15: Element surplus results on meshes from mesh remodelling - NACA 2412 airfoil
Average of thirty optimisation scenarios

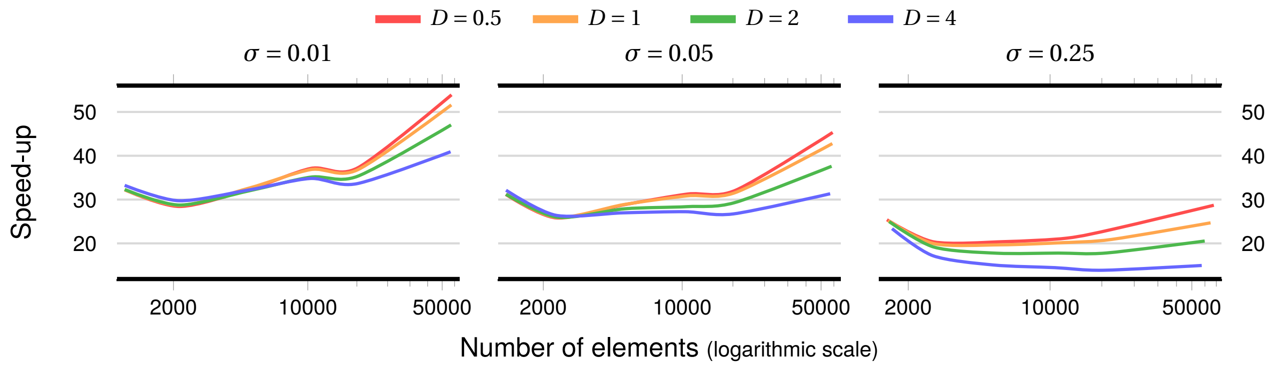


FIGURE A16: Speed-up results by mesh remodelling - NACA 2412 airfoil
Average of thirty optimisation scenarios

A.3.3 Clark-Y

σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Time	Preserv.	+ Tri.	Sp.-up
.01	50	1	1052	1.68 ms	.5	1269	0.05 ms	91.31 %	20.65 %	31.69
					1	1264	0.05 ms	91.27 %	20.14 %	31.61
					2	1268	0.05 ms	91.35 %	20.56 %	31.97
					4	1271	0.05 ms	91.50 %	20.87 %	32.95
	100	2	1850	3.52 ms	.5	2276	0.13 ms	90.20 %	23.05 %	26.87
					1	2292	0.13 ms	90.29 %	23.87 %	26.88
					2	2317	0.13 ms	90.39 %	25.25 %	26.82
					4	2295	0.13 ms	90.40 %	24.07 %	27.41
	200	4	4627	9.81 ms	.5	5182	0.34 ms	91.36 %	12.00 %	29.25
					1	5161	0.33 ms	91.33 %	11.54 %	29.44
					2	5159	0.35 ms	91.24 %	11.50 %	28.17
					4	5047	0.36 ms	91.05 %	9.08 %	27.11
	300	6	9833	20.79 ms	.5	10781	0.66 ms	93.53 %	9.64 %	31.71
					1	10724	0.67 ms	93.48 %	9.06 %	31.07
					2	10662	0.70 ms	93.37 %	8.43 %	29.90
					4	10421	0.76 ms	93.10 %	5.97 %	27.42
	400	8	17337	37.37 ms	.5	18828	1.13 ms	94.86 %	8.60 %	32.94
					1	18701	1.16 ms	94.80 %	7.86 %	32.28
					2	18531	1.26 ms	94.66 %	6.88 %	29.72
					4	18138	1.38 ms	94.37 %	4.62 %	27.00
	700	14	53738	157.90 ms	.5	57984	3.49 ms	96.77 %	7.90 %	45.20
					1	57488	3.72 ms	96.69 %	6.98 %	42.39
					2	56804	4.23 ms	96.52 %	5.71 %	37.35
					4	55824	5.07 ms	96.18 %	3.88 %	31.14

TABLE A7: Full results of mesh remodelling for $\sigma = 0.01$ - Clark-Y airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Remodelling Time	Preserv.	+ Tri.	Sp.-up
.05	50	1	1057	1.68 ms	.5	1386	0.06 ms	91.85 %	31.09 %	29.81
					1	1384	0.06 ms	91.85 %	30.87 %	29.60
					2	1397	0.06 ms	91.97 %	32.13 %	30.38
					4	1381	0.05 ms	91.99 %	30.63 %	31.08
	100	2	1844	3.49 ms	.5	2377	0.14 ms	90.54 %	28.93 %	25.43
					1	2384	0.13 ms	90.59 %	29.33 %	26.00
					2	2420	0.14 ms	90.75 %	31.26 %	25.39
					4	2398	0.14 ms	90.74 %	30.05 %	25.37
	200	4	4627	9.55 ms	.5	5279	0.35 ms	91.44 %	14.09 %	27.40
					1	5254	0.35 ms	91.40 %	13.56 %	27.21
					2	5228	0.37 ms	91.28 %	13.01 %	25.98
					4	5081	0.39 ms	90.96 %	9.83 %	24.26
	300	6	9834	20.30 ms	.5	10966	0.69 ms	93.56 %	11.51 %	29.28
					1	10885	0.71 ms	93.49 %	10.69 %	28.75
					2	10717	0.75 ms	93.29 %	8.98 %	27.12
					4	10439	0.83 ms	92.94 %	6.16 %	24.37
	400	8	17336	37.06 ms	.5	19098	1.19 ms	94.86 %	10.16 %	31.21
					1	18943	1.24 ms	94.77 %	9.27 %	29.79
					2	18604	1.35 ms	94.57 %	7.31 %	27.46
					4	18175	1.55 ms	94.20 %	4.84 %	23.86
	700	14	53740	157.14 ms	.5	58814	3.72 ms	96.74 %	9.44 %	42.21
					1	58424	4.09 ms	96.63 %	8.72 %	38.46
					2	56995	4.65 ms	96.41 %	6.06 %	33.81
					4	55852	5.81 ms	96.00 %	3.93 %	27.05

TABLE A8: Full results of mesh remodelling for $\sigma = 0.05$ - Clark-Y airfoil
Average of thirty optimisation scenarios

σ	I	G	**Generation** # Tri.	**Generation** Time	D	**Remodelling** # Tri.	**Remodelling** Time	**Remodelling** Preserv.	**Remodelling** + Tri.	**Remodelling** Sp.-up
.25	50	1	1061	1.67 ms	.5	1568	0.07 ms	92.55 %	47.84 %	24.36
					1	1588	0.07 ms	92.64 %	49.75 %	24.67
					2	1607	0.07 ms	92.77 %	51.58 %	24.35
					4	1694	0.07 ms	93.12 %	59.69 %	22.85
	100	2	1860	3.48 ms	.5	2702	0.17 ms	91.46 %	45.28 %	20.74
					1	2687	0.17 ms	91.38 %	44.47 %	20.69
					2	2684	0.18 ms	91.29 %	44.30 %	19.69
					4	2676	0.20 ms	91.04 %	43.88 %	17.37
	200	4	4617	9.28 ms	.5	6014	0.45 ms	92.14 %	30.27 %	20.67
					1	5772	0.47 ms	91.72 %	25.01 %	19.70
					2	5507	0.52 ms	91.09 %	19.28 %	17.84
					4	5274	0.61 ms	90.27 %	14.24 %	15.23
	300	6	9822	19.84 ms	.5	12278	0.92 ms	93.88 %	25.00 %	21.51
					1	11711	0.99 ms	93.47 %	19.23 %	20.11
					2	11073	1.11 ms	92.82 %	12.73 %	17.82
					4	10604	1.37 ms	91.95 %	7.96 %	14.47
	400	8	17313	36.42 ms	.5	21270	1.54 ms	95.00 %	22.86 %	23.60
					1	20286	1.70 ms	94.63 %	17.17 %	21.43
					2	19096	2.07 ms	93.99 %	10.30 %	17.58
					4	18295	2.63 ms	93.09 %	5.67 %	13.86
	700	14	53661	154.67 ms	.5	64968	5.46 ms	96.64 %	21.07 %	28.32
					1	62456	6.32 ms	96.31 %	16.39 %	24.47
					2	58286	7.58 ms	95.73 %	8.62 %	20.41
					4	56173	10.48 ms	94.80 %	4.68 %	14.76

TABLE A9: Full results of mesh remodelling for $\sigma = 0.25$ - Clark-Y airfoil
Average of thirty optimisation scenarios

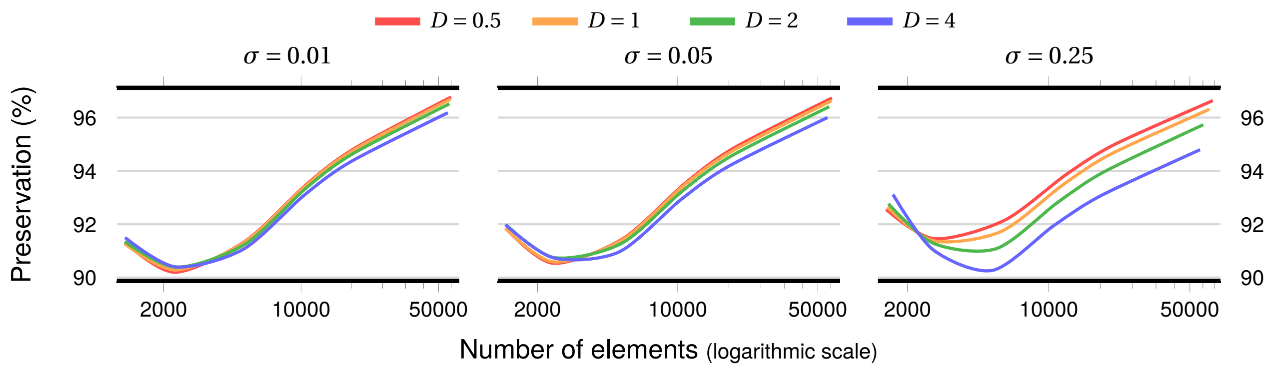


FIGURE A17: Element preservation results using mesh remodelling - Clark-Y airfoil
Average of thirty optimisation scenarios

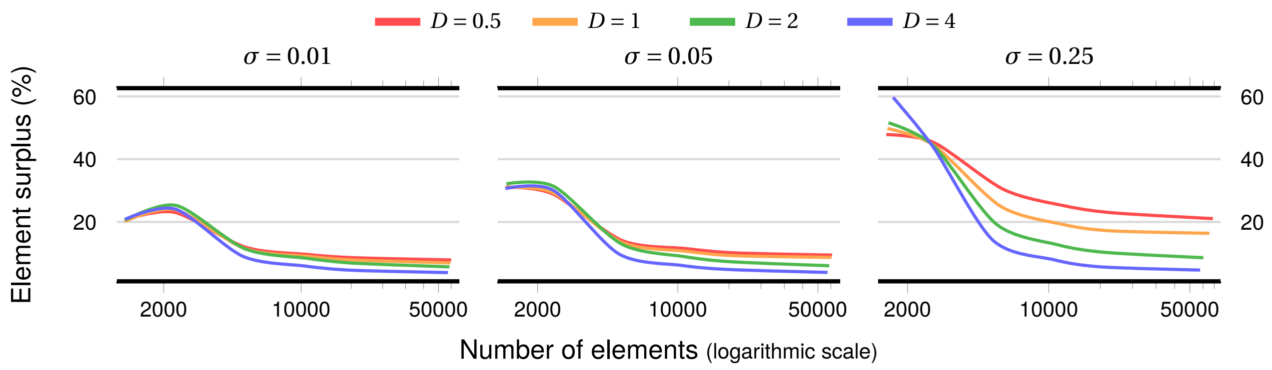


FIGURE A18: Element surplus results on meshes from mesh remodelling - Clark-Y airfoil
Average of thirty optimisation scenarios

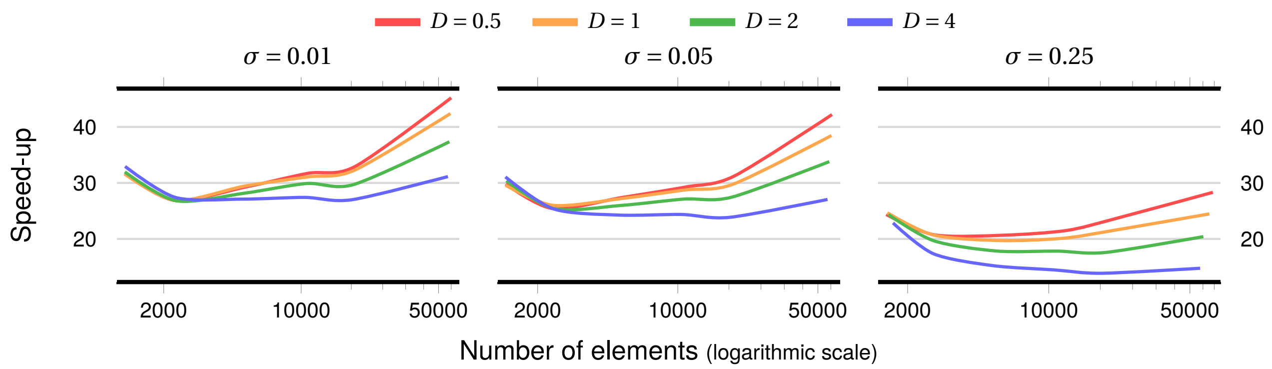


FIGURE A19: Speed-up results by mesh remodelling - Clark-Y airfoil
Average of thirty optimisation scenarios

A.3.4 Boeing 737

σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Remodelling Time	Preserv.	+ Tri.	Sp.-up
.01	50	1	1167	1.78 ms	.5	1182	0.05 ms	90.97 %	1.24 %	34.92
					1	1182	0.05 ms	90.97 %	1.24 %	34.99
					2	1184	0.05 ms	90.99 %	1.47 %	35.11
					4	1192	0.05 ms	91.15 %	2.11 %	36.38
	100	2	1864	3.42 ms	.5	2215	0.12 ms	89.98 %	18.86 %	27.65
					1	2217	0.12 ms	90.00 %	18.95 %	27.64
					2	2229	0.12 ms	90.07 %	19.60 %	27.86
					4	2228	0.12 ms	90.19 %	19.53 %	28.44
	200	4	4716	9.56 ms	.5	5182	0.32 ms	91.46 %	9.87 %	30.26
					1	5156	0.31 ms	91.43 %	9.32 %	30.80
					2	5150	0.32 ms	91.38 %	9.21 %	29.66
					4	5054	0.31 ms	91.30 %	7.16 %	30.63
	300	6	9982	20.06 ms	.5	10894	0.62 ms	93.71 %	9.14 %	32.25
					1	10845	0.63 ms	93.66 %	8.65 %	32.01
					2	10788	0.66 ms	93.60 %	8.08 %	30.42
					4	10584	0.68 ms	93.45 %	6.04 %	29.38
	400	8	17626	36.61 ms	.5	19120	1.09 ms	95.04 %	8.47 %	33.47
					1	18986	1.11 ms	94.99 %	7.71 %	32.94
					2	18838	1.17 ms	94.90 %	6.88 %	31.31
					4	18490	1.25 ms	94.72 %	4.90 %	29.31
	700	14	54768	158.33 ms	.5	59055	3.31 ms	96.89 %	7.83 %	47.84
					1	58578	3.44 ms	96.84 %	6.96 %	45.97
					2	57903	3.71 ms	96.73 %	5.72 %	42.68
					4	56903	4.32 ms	96.51 %	3.90 %	36.68

TABLE A10: Full results of mesh remodelling for $\sigma = 0.01$ - Boeing 737 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Remodelling Time	Preserv.	+ Tri.	Sp.-up
.05	50	1	1165	1.78 ms	.5	1280	0.05 ms	91.48 %	9.81 %	33.55
					1	1279	0.05 ms	91.47 %	9.73 %	33.46
					2	1317	0.05 ms	91.71 %	12.99 %	34.04
					4	1310	0.05 ms	91.78 %	12.44 %	34.50
	100	2	1864	3.41 ms	.5	2390	0.13 ms	90.59 %	28.20 %	25.95
					1	2411	0.13 ms	90.67 %	29.31 %	26.13
					2	2404	0.13 ms	90.69 %	28.93 %	26.03
					4	2416	0.13 ms	90.83 %	29.58 %	26.55
	200	4	4717	9.20 ms	.5	5291	0.34 ms	91.50 %	12.16 %	27.31
					1	5293	0.34 ms	91.50 %	12.20 %	26.80
					2	5296	0.35 ms	91.46 %	12.27 %	26.09
					4	5161	0.36 ms	91.27 %	9.41 %	25.91
	300	6	9982	19.73 ms	.5	11085	0.69 ms	93.68 %	11.06 %	28.43
					1	11011	0.70 ms	93.63 %	10.31 %	28.25
					2	10928	0.73 ms	93.54 %	9.48 %	27.21
					4	10652	0.77 ms	93.27 %	6.71 %	25.47
	400	8	17627	36.40 ms	.5	19376	1.22 ms	94.97 %	9.92 %	29.74
					1	19272	1.25 ms	94.92 %	9.33 %	29.20
					2	18999	1.32 ms	94.79 %	7.78 %	27.49
					4	18521	1.46 ms	94.50 %	5.07 %	24.99
	700	14	54755	157.51 ms	.5	59893	3.69 ms	96.82 %	9.38 %	42.66
					1	59538	3.96 ms	96.74 %	8.74 %	39.81
					2	58279	4.30 ms	96.60 %	6.44 %	36.62
					4	56986	5.19 ms	96.27 %	4.08 %	30.35

TABLE A11: Full results of mesh remodelling for $\sigma = 0.05$ - Boeing 737 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation		D	Remodelling				
			# Tri.	Time		# Tri.	Time	Preserv.	+ Tri.	Sp.-up
.25	50	1	1172	1.78 ms	.5	1646	0.07 ms	92.82 %	40.50 %	26.26
					1	1663	0.07 ms	92.94 %	41.94 %	25.52
					2	1689	0.07 ms	92.99 %	44.13 %	25.79
					4	1801	0.08 ms	93.38 %	53.73 %	23.76
	100	2	1859	3.39 ms	.5	2810	0.17 ms	91.71 %	51.15 %	20.31
					1	2790	0.17 ms	91.62 %	50.07 %	19.89
					2	2773	0.18 ms	91.48 %	49.11 %	19.13
					4	2773	0.20 ms	91.23 %	49.14 %	16.82
	200	4	4715	9.06 ms	.5	6177	0.45 ms	92.32 %	31.02 %	19.92
					1	5894	0.47 ms	91.88 %	25.02 %	19.37
					2	5655	0.52 ms	91.28 %	19.94 %	17.57
					4	5387	0.60 ms	90.45 %	14.26 %	15.04
	300	6	9974	19.53 ms	.5	12514	0.92 ms	94.00 %	25.47 %	21.22
					1	11932	0.99 ms	93.59 %	19.63 %	19.82
					2	11316	1.11 ms	92.97 %	13.45 %	17.57
					4	10820	1.36 ms	92.10 %	8.48 %	14.34
	400	8	17610	36.04 ms	.5	21685	1.57 ms	95.09 %	23.14 %	22.99
					1	20686	1.72 ms	94.71 %	17.47 %	20.99
					2	19525	2.03 ms	94.11 %	10.87 %	17.78
					4	18649	2.62 ms	93.21 %	5.90 %	13.78
	700	14	54727	156.47 ms	.5	66406	5.46 ms	96.69 %	21.34 %	28.66
					1	63703	6.31 ms	96.35 %	16.40 %	24.80
					2	59598	7.52 ms	95.81 %	8.90 %	20.82
					4	57326	10.38 ms	94.88 %	4.75 %	15.08

TABLE A12: Full results of mesh remodelling for $\sigma = 0.25$ - Boeing 737 airfoil
Average of thirty optimisation scenarios

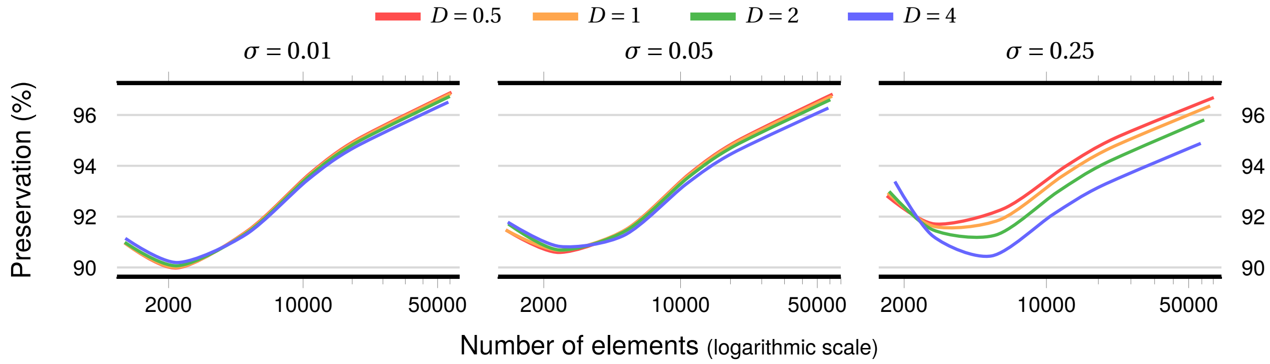


FIGURE A20: Element preservation results using mesh remodelling - Boeing 737 airfoil
Average of thirty optimisation scenarios

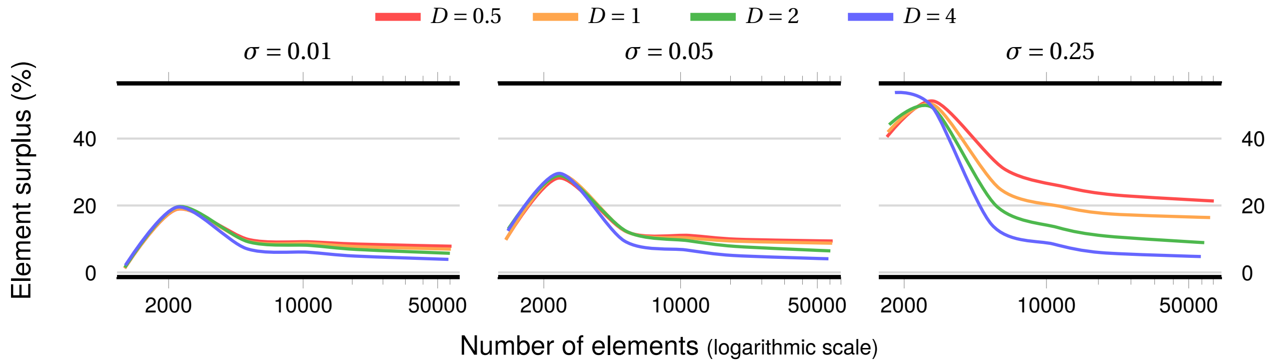


FIGURE A21: Element surplus results on meshes from mesh remodelling - Boeing 737 airfoil
Average of thirty optimisation scenarios

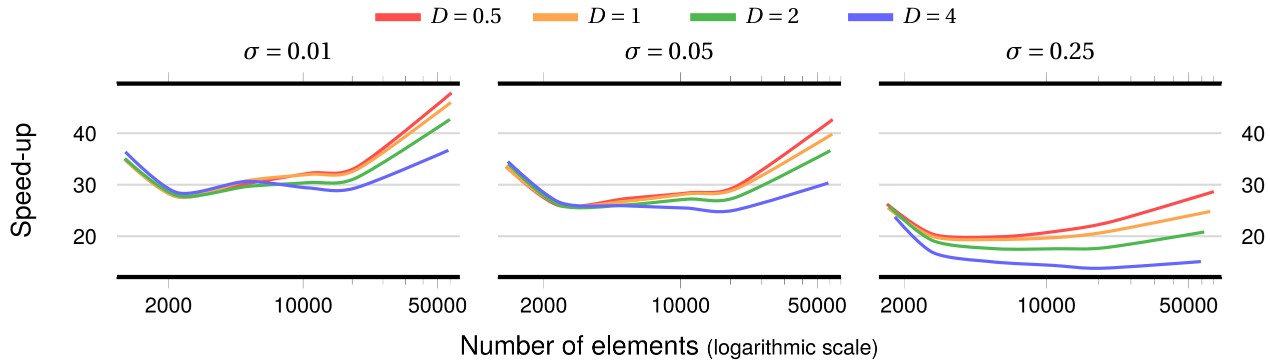


FIGURE A22: Speed-up results by mesh remodelling - Boeing 737 airfoil
Average of thirty optimisation scenarios

A.3.5 RAE 2822

σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Remodelling Time	Remodelling Preserv.	+ Tri.	Sp.-up
.01	50	1	1231	1.87 ms	.5	1143	0.05 ms	90.62 %	-7.19 %	36.29
					1	1143	0.05 ms	90.62 %	-7.19 %	36.29
					2	1147	0.05 ms	90.65 %	-6.81 %	36.17
					4	1149	0.05 ms	90.82 %	-6.72 %	38.17
	100	2	1856	3.47 ms	.5	2190	0.12 ms	89.93 %	18.03 %	28.77
					1	2204	0.12 ms	89.98 %	18.76 %	28.60
					2	2198	0.12 ms	89.99 %	18.44 %	29.39
					4	2180	0.12 ms	90.00 %	17.46 %	29.83
	200	4	4656	9.76 ms	.5	5024	0.29 ms	91.23 %	7.90 %	33.61
					1	5020	0.29 ms	91.24 %	7.81 %	33.61
					2	5049	0.29 ms	91.28 %	8.43 %	33.47
					4	4988	0.30 ms	91.24 %	7.13 %	33.04
	300	6	9904	20.91 ms	.5	10557	0.54 ms	93.56 %	6.59 %	38.76
					1	10559	0.54 ms	93.57 %	6.61 %	39.04
					2	10575	0.56 ms	93.55 %	6.77 %	37.53
					4	10445	0.58 ms	93.44 %	5.46 %	36.24
	400	8	17522	37.35 ms	.5	18518	0.91 ms	94.95 %	5.69 %	41.25
					1	18496	0.89 ms	94.93 %	5.56 %	41.78
					2	18463	0.95 ms	94.89 %	5.37 %	39.32
					4	18253	1.03 ms	94.75 %	4.18 %	36.39
	700	14	54517	161.17 ms	.5	57160	2.54 ms	96.89 %	4.85 %	63.50
					1	56994	2.62 ms	96.87 %	4.54 %	61.50
					2	56796	2.88 ms	96.77 %	4.18 %	55.86
					4	56235	3.42 ms	96.59 %	3.15 %	47.12

TABLE A13: Full results of mesh remodelling for $\sigma = 0.01$ - RAE 2822 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation		D	Remodelling				
			# Tri.	Time		# Tri.	Time	Preserv.	+ Tri.	Sp.-up
.05	50	1	1224	1.86 ms	.5	1298	0.05 ms	91.49 %	6.06 %	34.92
					1	1298	0.05 ms	91.50 %	6.02 %	34.90
					2	1303	0.05 ms	91.53 %	6.45 %	35.43
					4	1300	0.05 ms	91.61 %	6.19 %	36.15
	100	2	1852	3.46 ms	.5	2341	0.13 ms	90.46 %	26.41 %	27.25
					1	2350	0.13 ms	90.48 %	26.91 %	26.92
					2	2350	0.13 ms	90.48 %	26.89 %	27.13
					4	2328	0.13 ms	90.51 %	25.71 %	27.46
	200	4	4659	9.60 ms	.5	5187	0.32 ms	91.39 %	11.34 %	30.25
					1	5212	0.32 ms	91.42 %	11.87 %	30.00
					2	5219	0.33 ms	91.41 %	12.03 %	29.47
					4	5087	0.34 ms	91.22 %	9.20 %	28.55
	300	6	9904	20.25 ms	.5	10853	0.60 ms	93.60 %	9.58 %	34.00
					1	10822	0.61 ms	93.58 %	9.27 %	33.44
					2	10752	0.63 ms	93.51 %	8.56 %	31.92
					4	10537	0.69 ms	93.27 %	6.38 %	29.37
	400	8	17522	37.31 ms	.5	18944	1.00 ms	94.93 %	8.11 %	37.48
					1	18923	1.04 ms	94.90 %	7.99 %	35.82
					2	18722	1.11 ms	94.79 %	6.84 %	33.71
					4	18361	1.23 ms	94.55 %	4.79 %	30.36
	700	14	54516	160.95 ms	.5	58605	2.92 ms	96.83 %	7.50 %	55.16
					1	58453	3.12 ms	96.77 %	7.22 %	51.51
					2	57464	3.48 ms	96.65 %	5.41 %	46.29
					4	56479	4.26 ms	96.37 %	3.60 %	37.75

TABLE A14: Full results of mesh remodelling for $\sigma = 0.05$ - RAE 2822 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Remodelling Time	Preserv.	+ Tri.	Sp.-up
.25	50	1	1214	1.84 ms	.5	1596	0.07 ms	92.69 %	31.53 %	27.27
					1	1608	0.07 ms	92.79 %	32.47 %	27.14
					2	1657	0.07 ms	92.88 %	36.52 %	26.39
					4	1715	0.07 ms	93.15 %	41.31 %	24.72
	100	2	1854	3.42 ms	.5	2757	0.17 ms	91.56 %	48.72 %	20.33
					1	2758	0.17 ms	91.51 %	48.77 %	19.90
					2	2746	0.18 ms	91.40 %	48.17 %	19.02
					4	2716	0.20 ms	91.09 %	46.55 %	17.06
	200	4	4656	9.14 ms	.5	6060	0.44 ms	92.17 %	30.16 %	20.56
					1	5836	0.46 ms	91.78 %	25.35 %	19.71
					2	5559	0.51 ms	91.14 %	19.40 %	17.92
					4	5316	0.61 ms	90.33 %	14.18 %	15.04
	300	6	9895	19.76 ms	.5	12338	0.90 ms	93.89 %	24.70 %	21.85
					1	11813	0.96 ms	93.51 %	19.39 %	20.63
					2	11210	1.09 ms	92.90 %	13.29 %	18.11
					4	10719	1.36 ms	92.00 %	8.33 %	14.58
	400	8	17501	36.46 ms	.5	21384	1.49 ms	95.01 %	22.19 %	24.53
					1	20455	1.65 ms	94.64 %	16.88 %	22.08
					2	19330	1.99 ms	94.04 %	10.46 %	18.31
					4	18519	2.58 ms	93.12 %	5.82 %	14.14
	700	14	54443	158.33 ms	.5	65427	5.23 ms	96.64 %	20.18 %	30.28
					1	63005	6.08 ms	96.32 %	15.73 %	26.03
					2	59103	7.36 ms	95.77 %	8.56 %	21.52
					4	56886	10.33 ms	94.82 %	4.49 %	15.33

TABLE A15: Full results of mesh remodelling for $\sigma = 0.25$ - RAE 2822 airfoil
Average of thirty optimisation scenarios

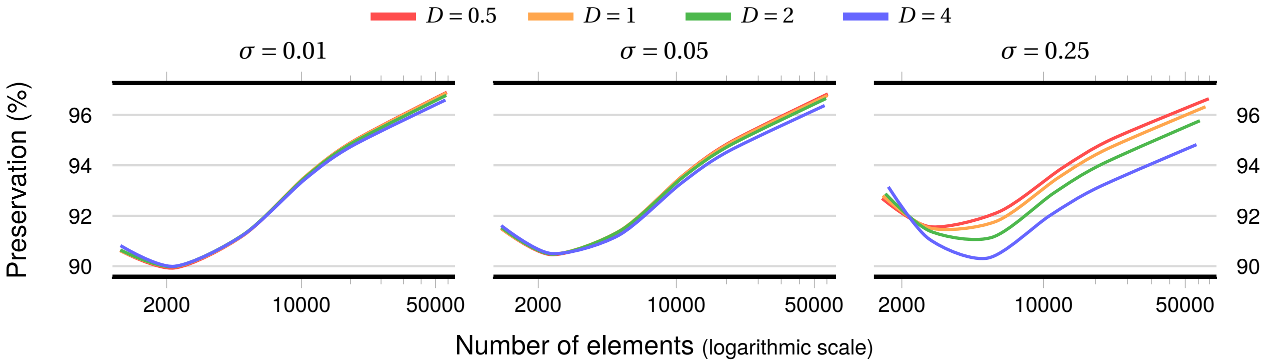


FIGURE A23: Element preservation results using mesh remodelling - RAE 2822 airfoil
Average of thirty optimisation scenarios

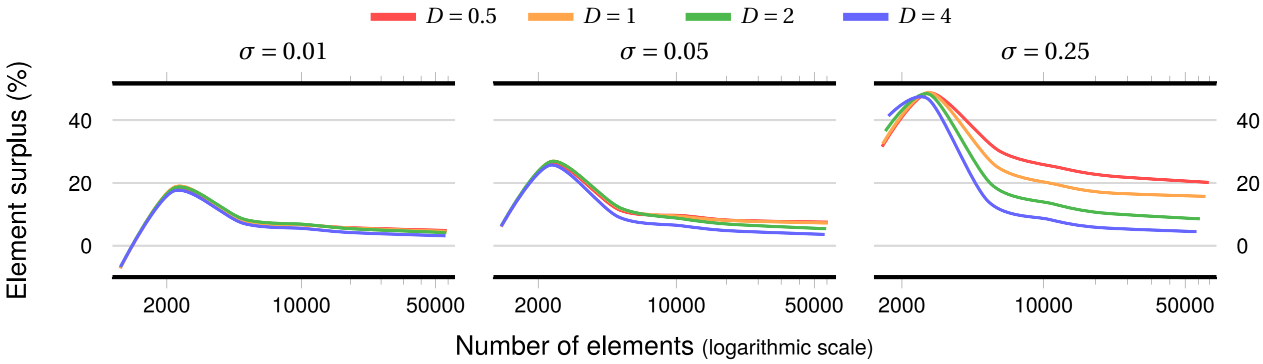


FIGURE A24: Element surplus results on meshes from mesh remodelling - RAE 2822 airfoil
Average of thirty optimisation scenarios

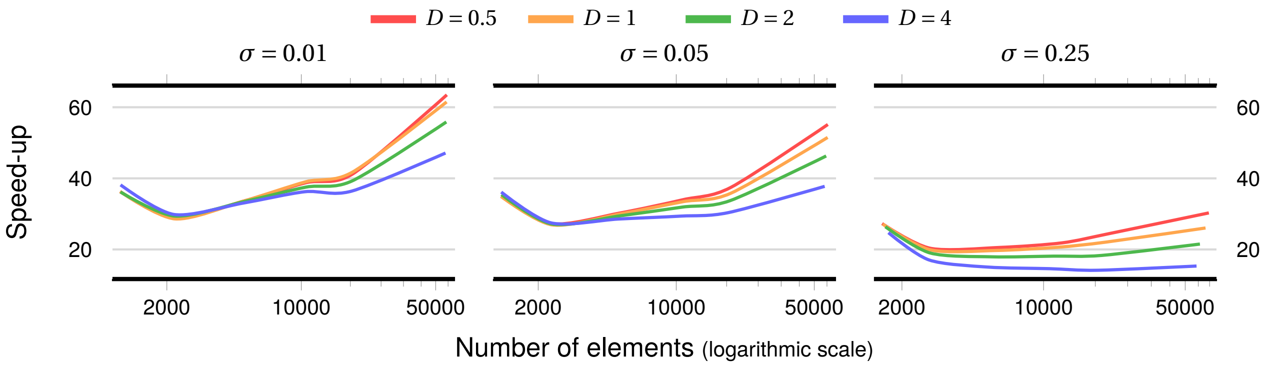


FIGURE A25: Speed-up results by mesh remodelling - RAE 2822 airfoil
Average of thirty optimisation scenarios

A.3.6 NACA 63206

σ	I	G	Generation		D	Remodelling				
			# Tri.	Time		# Tri.	Time	Preserv.	+ Tri.	Sp.-up
.01	50	1	1218	1.89 ms	.5	1340	0.05 ms	91.71 %	10.01 %	34.31
					1	1340	0.06 ms	91.71 %	10.01 %	34.25
					2	1353	0.05 ms	91.77 %	11.12 %	34.46
					4	1350	0.05 ms	91.81 %	10.89 %	34.35
	100	2	1913	3.53 ms	.5	2421	0.14 ms	90.63 %	26.54 %	26.01
					1	2437	0.14 ms	90.68 %	27.39 %	25.98
					2	2476	0.14 ms	90.84 %	29.42 %	25.97
					4	2465	0.14 ms	90.86 %	28.85 %	25.64
	200	4	4824	9.46 ms	.5	5621	0.34 ms	91.96 %	16.52 %	28.13
					1	5562	0.34 ms	91.87 %	15.31 %	28.06
					2	5481	0.35 ms	91.70 %	13.62 %	26.69
					4	5298	0.37 ms	91.38 %	9.83 %	25.46
	300	6	10184	20.50 ms	.5	11779	0.65 ms	94.03 %	15.66 %	31.56
					1	11592	0.66 ms	93.92 %	13.83 %	31.25
					2	11371	0.70 ms	93.72 %	11.66 %	29.46
					4	10943	0.77 ms	93.33 %	7.45 %	26.49
	400	8	17993	37.82 ms	.5	20727	1.06 ms	95.28 %	15.19 %	35.70
					1	20333	1.08 ms	95.16 %	13.00 %	34.89
					2	19843	1.17 ms	94.94 %	10.28 %	32.27
					4	19079	1.39 ms	94.54 %	6.04 %	27.26
	700	14	55800	164.74 ms	.5	64357	3.31 ms	97.01 %	15.34 %	49.70
					1	63204	3.59 ms	96.90 %	13.27 %	45.88
					2	60989	4.05 ms	96.68 %	9.30 %	40.69
					4	58687	5.10 ms	96.28 %	5.17 %	32.28

TABLE A16: Full results of mesh remodelling for $\sigma = 0.01$ - NACA 63206 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Remodelling Time	Preserv.	+ Tri.	Sp.-up
.05	50	1	1234	1.90 ms	.5	1443	0.06 ms	92.05 %	16.93 %	32.73
					1	1444	0.06 ms	92.07 %	17.01 %	32.94
					2	1453	0.06 ms	92.17 %	17.72 %	33.38
					4	1456	0.06 ms	92.20 %	18.03 %	32.83
	100	2	1918	3.54 ms	.5	2533	0.14 ms	91.00 %	32.09 %	25.42
					1	2542	0.14 ms	90.99 %	32.54 %	25.11
					2	2570	0.14 ms	91.12 %	34.01 %	25.15
					4	2594	0.15 ms	91.20 %	35.25 %	24.21
	200	4	4825	9.40 ms	.5	5669	0.35 ms	91.98 %	17.49 %	26.82
					1	5601	0.36 ms	91.86 %	16.07 %	26.46
					2	5537	0.37 ms	91.67 %	14.74 %	25.10
					4	5339	0.41 ms	91.27 %	10.65 %	23.20
	300	6	10189	20.36 ms	.5	11837	0.69 ms	93.98 %	16.17 %	29.47
					1	11628	0.70 ms	93.86 %	14.12 %	29.25
					2	11369	0.75 ms	93.61 %	11.58 %	27.07
					4	10950	0.86 ms	93.19 %	7.46 %	23.80
	400	8	17999	37.58 ms	.5	20681	1.11 ms	95.22 %	14.90 %	33.88
					1	20296	1.17 ms	95.07 %	12.76 %	32.16
					2	19780	1.29 ms	94.82 %	9.90 %	29.09
					4	19063	1.55 ms	94.39 %	5.91 %	24.30
	700	14	55826	164.24 ms	.5	63874	3.54 ms	96.94 %	14.41 %	46.38
					1	62901	3.91 ms	96.80 %	12.67 %	42.01
					2	60739	4.47 ms	96.56 %	8.80 %	36.73
					4	58617	5.71 ms	96.11 %	5.00 %	28.77

TABLE A17: Full results of mesh remodelling for $\sigma = 0.05$ - NACA 63206 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Remodelling Time	Remodelling Preserv.	Remodelling + Tri.	Remodelling Sp.-up
.25	50	1	1234	1.89 ms	.5	1691	0.07 ms	92.97 %	37.02 %	27.05
					1	1677	0.07 ms	92.95 %	35.87 %	27.28
					2	1735	0.07 ms	93.11 %	40.57 %	26.39
					4	1805	0.08 ms	93.32 %	46.22 %	24.49
	100	2	1923	3.50 ms	.5	2904	0.17 ms	91.89 %	51.05 %	20.39
					1	2901	0.17 ms	91.86 %	50.88 %	20.12
					2	2861	0.18 ms	91.64 %	48.78 %	19.11
					4	2875	0.21 ms	91.45 %	49.54 %	16.89
	200	4	4815	9.20 ms	.5	6401	0.45 ms	92.54 %	32.94 %	20.32
					1	6100	0.46 ms	92.09 %	26.70 %	19.88
					2	5784	0.51 ms	91.42 %	20.12 %	17.96
					4	5506	0.62 ms	90.53 %	14.34 %	14.93
	300	6	10166	19.89 ms	.5	13004	0.90 ms	94.19 %	27.92 %	22.00
					1	12252	0.96 ms	93.72 %	20.52 %	20.72
					2	11598	1.08 ms	93.09 %	14.09 %	18.37
					4	11007	1.37 ms	92.13 %	8.27 %	14.52
	400	8	17955	36.72 ms	.5	22482	1.49 ms	95.24 %	25.21 %	24.63
					1	21241	1.64 ms	94.83 %	18.30 %	22.45
					2	20042	1.94 ms	94.19 %	11.62 %	18.89
					4	19061	2.59 ms	93.25 %	6.16 %	14.20
	700	14	55701	160.93 ms	.5	68776	5.16 ms	96.78 %	23.47 %	31.19
					1	65460	5.98 ms	96.44 %	17.52 %	26.92
					2	61283	7.33 ms	95.86 %	10.02 %	21.94
					4	58546	10.33 ms	94.91 %	5.11 %	15.58

TABLE A18: Full results of mesh remodelling for $\sigma = 0.25$ - NACA 63206 airfoil
Average of thirty optimisation scenarios

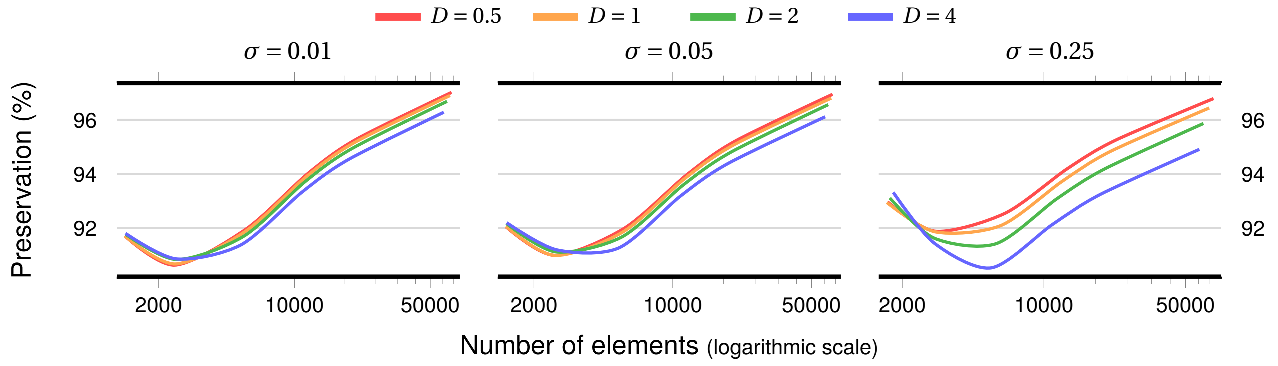


FIGURE A26: Element preservation results using mesh remodelling - NACA 63206 airfoil
Average of thirty optimisation scenarios

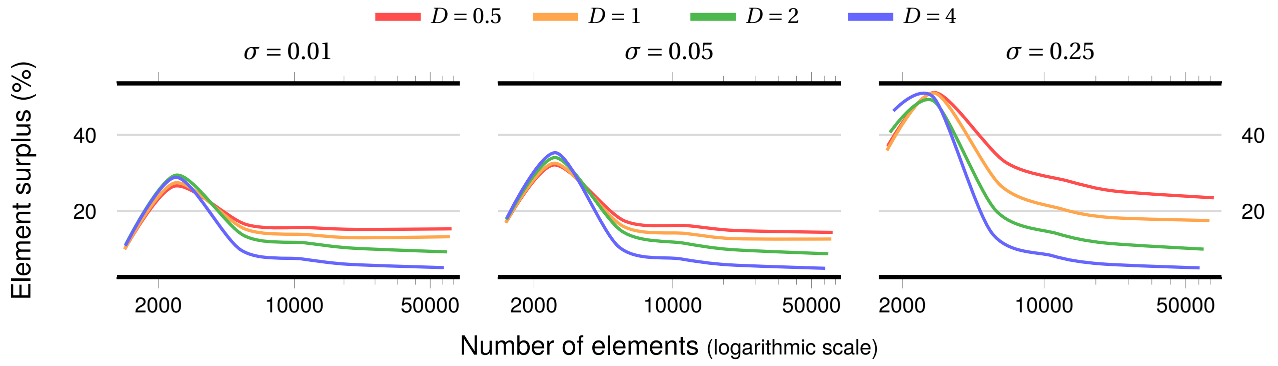


FIGURE A27: Element surplus results on meshes from mesh remodelling - NACA 63206 airfoil
Average of thirty optimisation scenarios

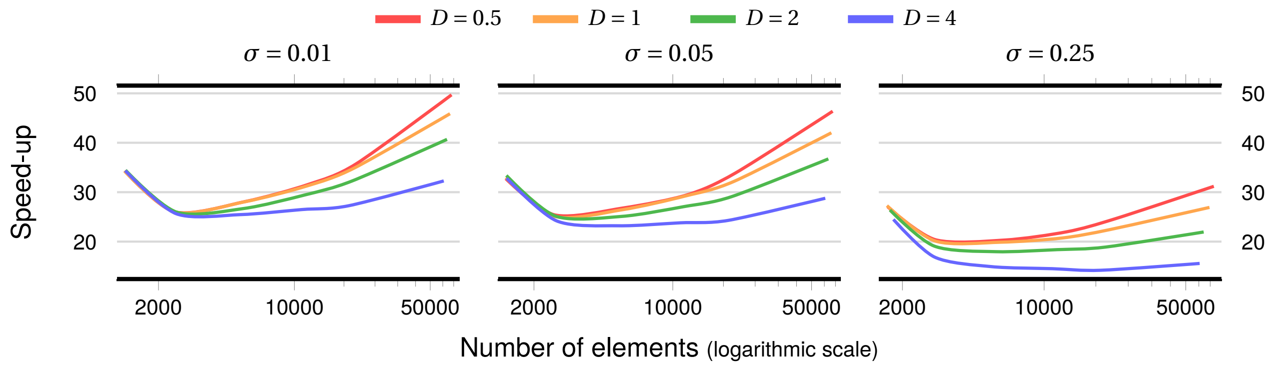


FIGURE A28: Speed-up results by mesh remodelling - NACA 63206 airfoil
Average of thirty optimisation scenarios

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σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Remodelling Time	Remodelling Preserv.	Remodelling + Tri.	Remodelling Sp.-up
.01	50	1	1210	1.78 ms	.5	1598	0.07 ms	92.70 %	32.00 %	25.53
					1	1585	0.07 ms	92.66 %	30.99 %	25.95
					2	1616	0.07 ms	92.77 %	33.51 %	25.71
					4	1614	0.07 ms	92.70 %	33.39 %	24.76
	100	2	1934	3.27 ms	.5	2740	0.17 ms	91.52 %	41.65 %	18.78
					1	2714	0.17 ms	91.44 %	40.30 %	18.72
					2	2715	0.18 ms	91.42 %	40.35 %	18.41
					4	2738	0.19 ms	91.33 %	41.56 %	17.38
	200	4	4768	8.41 ms	.5	6220	0.48 ms	92.43 %	30.44 %	17.58
					1	5987	0.48 ms	92.11 %	25.55 %	17.63
					2	5687	0.51 ms	91.54 %	19.26 %	16.48
					4	5399	0.58 ms	90.73 %	13.22 %	14.58
	300	6	10103	18.17 ms	.5	12841	0.97 ms	94.19 %	27.10 %	18.68
					1	12307	1.00 ms	93.88 %	21.82 %	18.09
					2	11575	1.10 ms	93.29 %	14.57 %	16.47
					4	11013	1.31 ms	92.48 %	9.01 %	13.91
	400	8	17867	33.71 ms	.5	22453	1.62 ms	95.31 %	25.67 %	20.76
					1	21450	1.72 ms	94.98 %	20.05 %	19.56
					2	20057	1.93 ms	94.42 %	12.26 %	17.50
					4	19064	2.45 ms	93.58 %	6.70 %	13.78
	700	14	55533	151.14 ms	.5	69114	5.52 ms	96.85 %	24.46 %	27.37
					1	66056	6.13 ms	96.55 %	18.95 %	24.66
					2	61394	7.01 ms	96.06 %	10.55 %	21.56
					4	58530	9.56 ms	95.19 %	5.40 %	15.82

TABLE A19: Full results of mesh remodelling for $\sigma = 0.01$ - Eppler 376 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation		D	Remodelling				
			# Tri.	Time		# Tri.	Time	Preserv.	+ Tri.	Sp.-up
.05	50	1	1210	1.77 ms	.5	1608	0.07 ms	92.70 %	32.95 %	24.47
					1	1594	0.07 ms	92.61 %	31.80 %	24.49
					2	1605	0.07 ms	92.63 %	32.66 %	24.61
					4	1647	0.07 ms	92.82 %	36.17 %	23.77
	100	2	1936	3.27 ms	.5	2816	0.18 ms	91.71 %	45.45 %	18.20
					1	2784	0.18 ms	91.63 %	43.83 %	18.02
					2	2771	0.18 ms	91.55 %	43.15 %	17.83
					4	2748	0.20 ms	91.28 %	41.97 %	16.31
	200	4	4767	8.37 ms	.5	6255	0.49 ms	92.41 %	31.20 %	16.99
					1	6009	0.49 ms	92.06 %	26.04 %	16.95
					2	5749	0.54 ms	91.52 %	20.60 %	15.56
					4	5426	0.61 ms	90.63 %	13.83 %	13.79
	300	6	10101	18.10 ms	.5	12940	1.01 ms	94.17 %	28.10 %	17.90
					1	12319	1.05 ms	93.80 %	21.95 %	17.31
					2	11598	1.15 ms	93.19 %	14.82 %	15.76
					4	11030	1.38 ms	92.33 %	9.20 %	13.11
	400	8	17863	33.60 ms	.5	22515	1.67 ms	95.25 %	26.04 %	20.14
					1	21445	1.78 ms	94.90 %	20.05 %	18.86
					2	20081	2.02 ms	94.31 %	12.42 %	16.66
					4	19092	2.60 ms	93.42 %	6.88 %	12.93
	700	14	55526	151.14 ms	.5	69029	5.74 ms	96.79 %	24.32 %	26.34
					1	65971	6.41 ms	96.47 %	18.81 %	23.57
					2	61432	7.47 ms	95.94 %	10.64 %	20.24
					4	58570	10.22 ms	95.01 %	5.48 %	14.79

TABLE A20: Full results of mesh remodelling for $\sigma = 0.05$ - Eppler 376 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Remodelling Time	Remodelling Preserv.	Remodelling + Tri.	Remodelling Sp.-up
.25	50	1	1207	1.77 ms	.5	1730	0.08 ms	93.15 %	43.27 %	23.10
					1	1730	0.08 ms	93.15 %	43.31 %	22.96
					2	1773	0.08 ms	93.24 %	46.83 %	22.07
					4	1825	0.09 ms	93.32 %	51.15 %	20.51
	100	2	1938	3.25 ms	.5	2962	0.19 ms	92.05 %	52.85 %	17.55
					1	2923	0.19 ms	91.90 %	50.82 %	17.34
					2	2923	0.20 ms	91.78 %	50.84 %	16.22
					4	2863	0.23 ms	91.27 %	47.74 %	13.87
	200	4	4764	8.34 ms	.5	6693	0.52 ms	92.77 %	40.50 %	16.11
					1	6279	0.53 ms	92.19 %	31.81 %	15.75
					2	5860	0.59 ms	91.38 %	23.01 %	14.14
					4	5562	0.71 ms	90.32 %	16.76 %	11.76
	300	6	10089	18.02 ms	.5	13606	1.07 ms	94.31 %	34.86 %	16.90
					1	12649	1.12 ms	93.77 %	25.38 %	16.05
					2	11707	1.28 ms	92.95 %	16.04 %	14.10
					4	11066	1.61 ms	91.80 %	9.69 %	11.21
	400	8	17845	33.46 ms	.5	23499	1.77 ms	95.31 %	31.68 %	18.87
					1	21894	1.94 ms	94.81 %	22.69 %	17.26
					2	20180	2.31 ms	94.03 %	13.09 %	14.50
					4	19062	3.09 ms	92.85 %	6.82 %	10.83
	700	14	55458	150.16 ms	.5	71718	6.26 ms	96.77 %	29.32 %	23.98
					1	67278	7.18 ms	96.35 %	21.31 %	20.93
					2	61629	8.65 ms	95.64 %	11.13 %	17.36
					4	58557	12.39 ms	94.44 %	5.59 %	12.12

TABLE A21: Full results of mesh remodelling for $\sigma = 0.25$ - Eppler 376 airfoil
Average of thirty optimisation scenarios

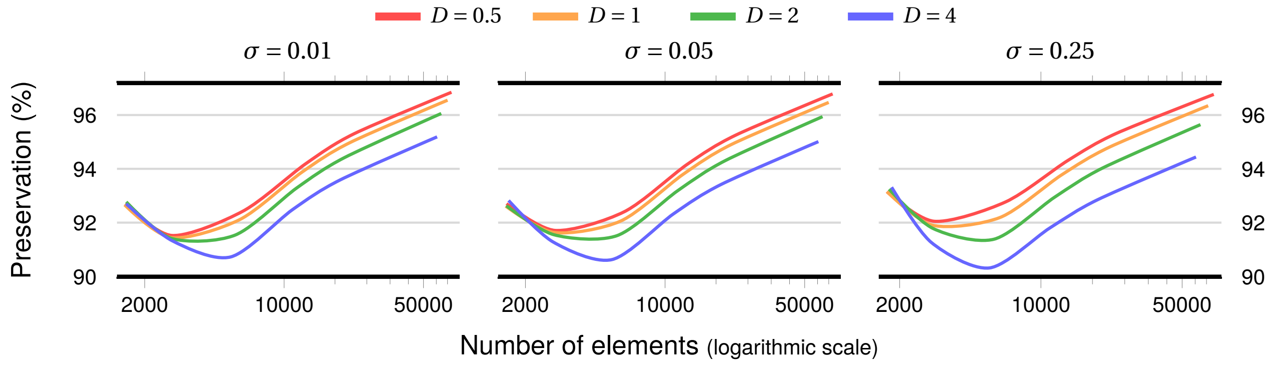

FIGURE A29: Element preservation results using mesh remodelling - Eppler 376 airfoil
Average of thirty optimisation scenarios

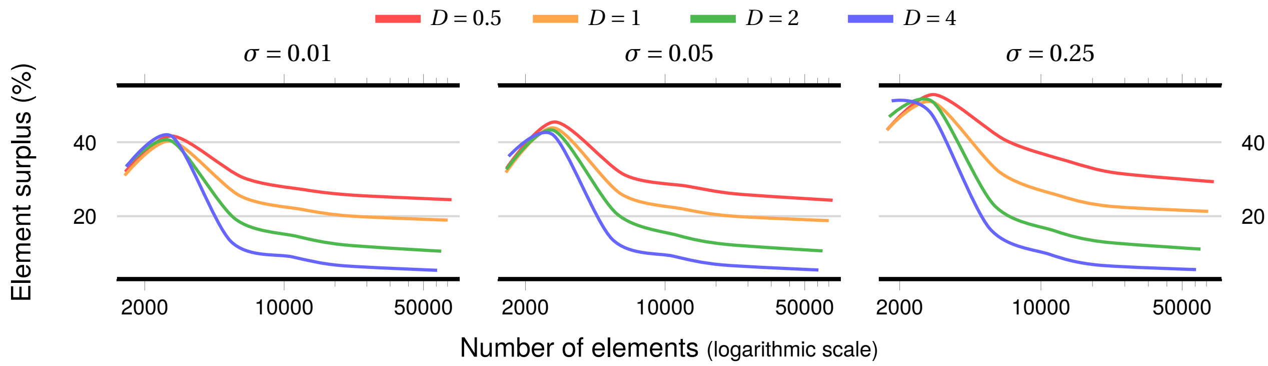

FIGURE A30: Element surplus results on meshes from mesh remodelling - Eppler 376 airfoil
Average of thirty optimisation scenarios

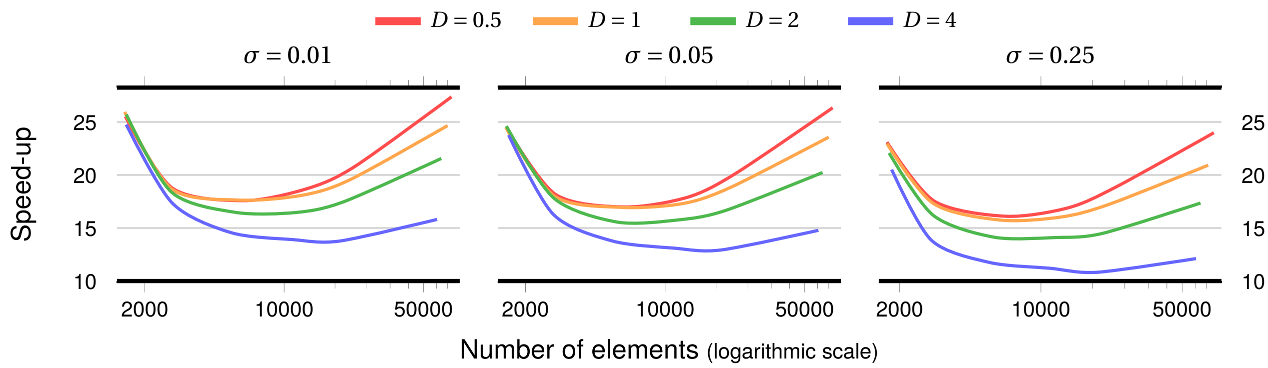

FIGURE A31: Speed-up results by mesh remodelling - Eppler 376 airfoil
Average of thirty optimisation scenarios

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σ	I	G	Generation		D	Remodelling				
			# Tri.	Time		# Tri.	Time	Preserv.	+ Tri.	Sp.-up
.01	50	1	1175	1.79 ms	.5	1286	0.05 ms	91.41 %	9.46 %	32.61
					1	1288	0.05 ms	91.42 %	9.61 %	32.55
					2	1299	0.05 ms	91.50 %	10.59 %	33.19
					4	1298	0.05 ms	91.59 %	10.47 %	34.17
	100	2	1832	3.38 ms	.5	2338	0.14 ms	90.42 %	27.63 %	24.88
					1	2340	0.13 ms	90.46 %	27.72 %	25.31
					2	2327	0.13 ms	90.39 %	27.04 %	25.33
					4	2329	0.13 ms	90.52 %	27.16 %	25.86
	200	4	4635	9.43 ms	.5	5061	0.35 ms	91.13 %	9.19 %	26.68
					1	5079	0.35 ms	91.16 %	9.59 %	26.70
					2	5118	0.37 ms	91.19 %	10.42 %	25.70
					4	5019	0.37 ms	90.99 %	8.30 %	25.39
	300	6	9838	20.09 ms	.5	10481	0.73 ms	93.31 %	6.54 %	27.60
					1	10513	0.73 ms	93.33 %	6.86 %	27.41
					2	10530	0.78 ms	93.26 %	7.04 %	25.86
					4	10373	0.82 ms	93.03 %	5.45 %	24.53
	400	8	17344	36.17 ms	.5	18260	1.28 ms	94.67 %	5.28 %	28.27
					1	18295	1.32 ms	94.65 %	5.48 %	27.32
					2	18304	1.40 ms	94.55 %	5.54 %	25.76
					4	18062	1.55 ms	94.27 %	4.14 %	23.27
	700	14	53884	155.74 ms	.5	56146	3.83 ms	96.61 %	4.20 %	40.65
					1	56228	4.08 ms	96.56 %	4.35 %	38.16
					2	56051	4.59 ms	96.41 %	4.02 %	33.91
					4	55541	5.57 ms	96.08 %	3.07 %	27.96

TABLE A22: Full results of mesh remodelling for $\sigma = 0.01$ - Eppler 545 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation		D	Remodelling				
			# Tri.	Time		# Tri.	Time	Preserv.	+ Tri.	Sp.-up
.05	50	1	1170	1.78 ms	.5	1403	0.06 ms	91.95 %	19.90 %	30.78
					1	1403	0.06 ms	91.94 %	19.89 %	30.31
					2	1401	0.06 ms	91.98 %	19.75 %	31.16
					4	1387	0.05 ms	91.97 %	18.52 %	32.31
	100	2	1832	3.37 ms	.5	2398	0.14 ms	90.62 %	30.87 %	24.20
					1	2419	0.14 ms	90.71 %	32.02 %	23.99
					2	2444	0.14 ms	90.77 %	33.37 %	23.74
					4	2413	0.14 ms	90.75 %	31.66 %	23.83
	200	4	4635	9.29 ms	.5	5191	0.37 ms	91.26 %	11.98 %	24.87
					1	5204	0.38 ms	91.30 %	12.27 %	24.71
					2	5219	0.39 ms	91.21 %	12.59 %	23.80
					4	5078	0.42 ms	90.88 %	9.55 %	22.28
	300	6	9838	19.59 ms	.5	10684	0.77 ms	93.36 %	8.60 %	25.32
					1	10707	0.77 ms	93.35 %	8.84 %	25.30
					2	10668	0.83 ms	93.20 %	8.44 %	23.58
					4	10451	0.92 ms	92.85 %	6.23 %	21.33
	400	8	17342	35.98 ms	.5	18661	1.35 ms	94.68 %	7.61 %	26.56
					1	18670	1.44 ms	94.63 %	7.66 %	25.05
					2	18483	1.54 ms	94.46 %	6.58 %	23.31
					4	18153	1.72 ms	94.08 %	4.68 %	20.90
	700	14	53880	155.27 ms	.5	57379	4.16 ms	96.58 %	6.49 %	37.33
					1	57488	4.55 ms	96.49 %	6.70 %	34.13
					2	56548	5.19 ms	96.29 %	4.95 %	29.94
					4	55799	6.40 ms	95.87 %	3.56 %	24.25

TABLE A23: Full results of mesh remodelling for $\sigma = 0.05$ - Eppler 545 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation		D	Remodelling				
			# Tri.	Time		# Tri.	Time	Preserv.	+ Tri.	Sp.-up
.25	50	1	1168	1.77 ms	.5	1571	0.07 ms	92.58 %	34.53 %	26.10
					1	1577	0.07 ms	92.63 %	35.07 %	26.03
					2	1631	0.07 ms	92.81 %	39.68 %	25.06
					4	1680	0.07 ms	93.01 %	43.92 %	23.97
	100	2	1832	3.35 ms	.5	2706	0.17 ms	91.48 %	47.69 %	19.77
					1	2695	0.17 ms	91.37 %	47.06 %	19.76
					2	2694	0.18 ms	91.32 %	47.01 %	18.79
					4	2707	0.20 ms	91.12 %	47.75 %	16.76
	200	4	4635	9.01 ms	.5	6004	0.46 ms	92.11 %	29.53 %	19.52
					1	5761	0.49 ms	91.69 %	24.30 %	18.45
					2	5526	0.53 ms	91.10 %	19.21 %	17.07
					4	5279	0.61 ms	90.28 %	13.90 %	14.69
	300	6	9829	19.33 ms	.5	12189	0.97 ms	93.82 %	24.01 %	19.88
					1	11659	1.04 ms	93.42 %	18.62 %	18.63
					2	11109	1.15 ms	92.84 %	13.02 %	16.85
					4	10617	1.40 ms	91.94 %	8.01 %	13.80
	400	8	17325	35.58 ms	.5	21086	1.64 ms	94.93 %	21.71 %	21.75
					1	20235	1.82 ms	94.58 %	16.80 %	19.57
					2	19145	2.14 ms	93.98 %	10.51 %	16.61
					4	18310	2.68 ms	93.07 %	5.68 %	13.27
	700	14	53838	153.88 ms	.5	64501	5.75 ms	96.57 %	19.81 %	26.75
					1	62186	6.62 ms	96.26 %	15.51 %	23.24
					2	58572	7.93 ms	95.70 %	8.79 %	19.41
					4	56245	10.76 ms	94.77 %	4.47 %	14.30

TABLE A24: Full results of mesh remodelling for $\sigma = 0.25$ - Eppler 545 airfoil
Average of thirty optimisation scenarios

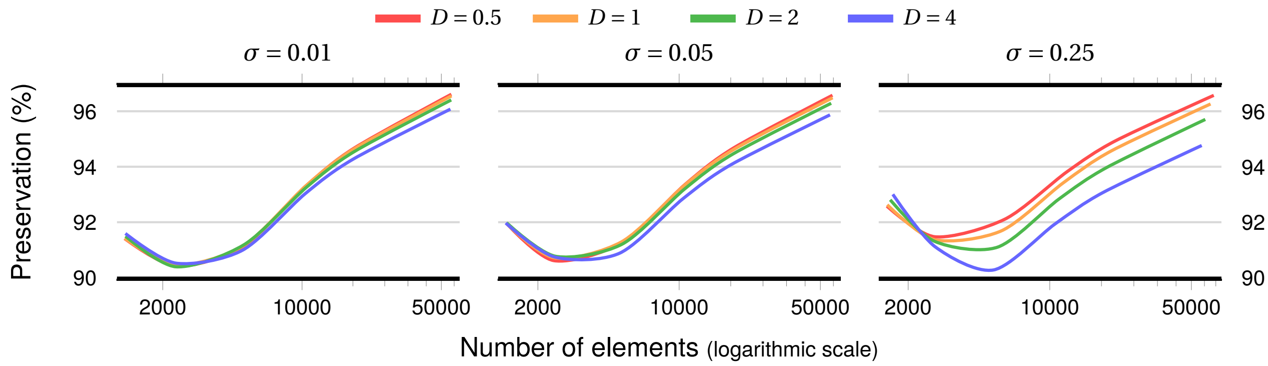

FIGURE A32: Element preservation results using mesh remodelling - Eppler 545 airfoil
Average of thirty optimisation scenarios

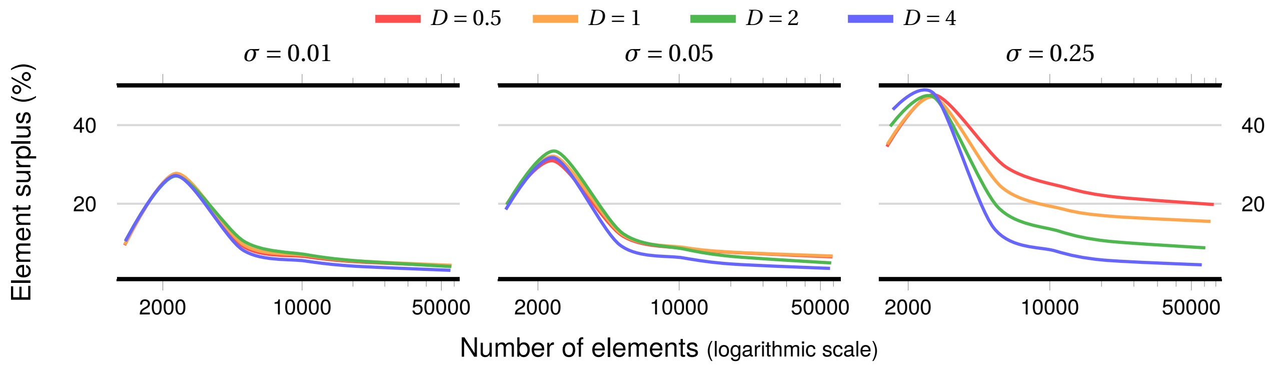

FIGURE A33: Element surplus results on meshes from mesh remodelling - Eppler 545 airfoil
Average of thirty optimisation scenarios

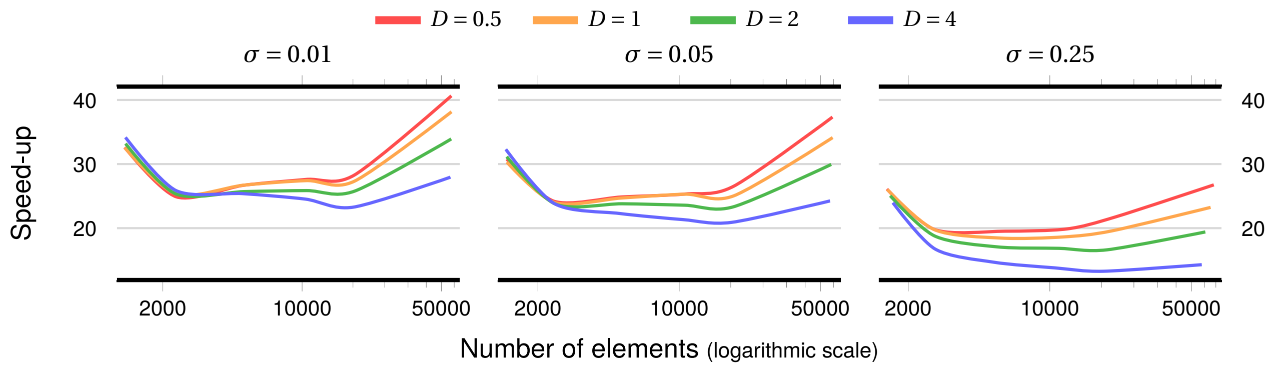

FIGURE A34: Speed-up results by mesh remodelling - Eppler 545 airfoil
Average of thirty optimisation scenarios

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σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Remodelling Time	Preserv.	+ Tri.	Sp.-up
.01	50	1	1122	1.72 ms	.5	1338	0.06 ms	91.61 %	19.24 %	30.37
					1	1343	0.06 ms	91.64 %	19.64 %	30.38
					2	1342	0.06 ms	91.67 %	19.54 %	31.04
					4	1326	0.05 ms	91.74 %	18.14 %	32.30
	100	2	1891	3.51 ms	.5	2354	0.14 ms	90.46 %	24.52 %	24.44
					1	2336	0.14 ms	90.40 %	23.54 %	24.51
					2	2364	0.14 ms	90.54 %	25.05 %	24.64
					4	2351	0.14 ms	90.55 %	24.35 %	24.59
	200	4	4598	9.63 ms	.5	5224	0.39 ms	91.33 %	13.61 %	24.89
					1	5220	0.39 ms	91.31 %	13.52 %	24.71
					2	5178	0.41 ms	91.15 %	12.61 %	23.72
					4	5045	0.43 ms	90.83 %	9.72 %	22.46
	300	6	9724	20.76 ms	.5	10833	0.83 ms	93.44 %	11.41 %	24.86
					1	10744	0.83 ms	93.37 %	10.49 %	24.99
					2	10623	0.89 ms	93.18 %	9.24 %	23.37
					4	10365	0.95 ms	92.81 %	6.59 %	21.84
	400	8	17197	36.76 ms	.5	18899	1.52 ms	94.76 %	9.90 %	24.24
					1	18702	1.58 ms	94.66 %	8.75 %	23.31
					2	18427	1.70 ms	94.44 %	7.15 %	21.61
					4	18010	1.85 ms	94.03 %	4.73 %	19.89
	700	14	53404	157.23 ms	.5	57909	4.97 ms	96.63 %	8.43 %	31.61
					1	57364	5.35 ms	96.51 %	7.41 %	29.38
					2	56478	5.99 ms	96.26 %	5.76 %	26.23
					4	55401	7.20 ms	95.82 %	3.74 %	21.85

TABLE A25: Full results of mesh remodelling for $\sigma = 0.01$ - Gottingen 702 airfoil
Average of thirty optimisation scenarios

σ	I	G	Generation # Tri.	Generation Time	D	Remodelling # Tri.	Remodelling Time	Preserv.	+ Tri.	Sp.-up
.05	50	1	1122	1.72 ms	.5	1390	0.06 ms	91.92 %	23.89 %	29.21
					1	1390	0.06 ms	91.96 %	23.90 %	29.65
					2	1404	0.06 ms	92.09 %	25.15 %	30.10
					4	1367	0.06 ms	91.93 %	21.91 %	30.80
	100	2	1892	3.51 ms	.5	2433	0.15 ms	90.75 %	28.64 %	23.93
					1	2448	0.15 ms	90.82 %	29.43 %	24.14
					2	2451	0.15 ms	90.82 %	29.58 %	23.97
					4	2382	0.15 ms	90.60 %	25.91 %	23.40
	200	4	4598	9.59 ms	.5	5326	0.40 ms	91.40 %	15.83 %	23.79
					1	5287	0.41 ms	91.32 %	14.98 %	23.46
					2	5244	0.42 ms	91.16 %	14.06 %	22.76
					4	5067	0.45 ms	90.74 %	10.20 %	21.22
	300	6	9724	20.47 ms	.5	11021	0.85 ms	93.49 %	13.34 %	23.95
					1	10911	0.87 ms	93.39 %	12.21 %	23.59
					2	10719	0.93 ms	93.14 %	10.23 %	21.95
					4	10410	1.01 ms	92.70 %	7.06 %	20.35
	400	8	17197	36.71 ms	.5	19167	1.57 ms	94.78 %	11.46 %	23.36
					1	18933	1.64 ms	94.65 %	10.10 %	22.35
					2	18579	1.76 ms	94.38 %	8.04 %	20.80
					4	18073	1.98 ms	93.92 %	5.10 %	18.50
	700	14	53397	156.93 ms	.5	58808	5.18 ms	96.62 %	10.13 %	30.29
					1	58120	5.66 ms	96.47 %	8.84 %	27.73
					2	56917	6.32 ms	96.20 %	6.59 %	24.82
					4	55583	7.72 ms	95.70 %	4.09 %	20.33

TABLE A26: Full results of mesh remodelling for $\sigma = 0.05$ - Gottingen 702 airfoil
Average of thirty optimisation scenarios

σ	I	G	**Generation**		D	**Remodelling**				
			# Tri.	**Time**		**# Tri.**	**Time**	**Preserv.**	**+ Tri.**	**Sp.-up**
.25	50	1	1121	1.72 ms	.5	1543	0.07 ms	92.55 %	37.72 %	25.41
					1	1541	0.07 ms	92.55 %	37.55 %	25.34
					2	1600	0.07 ms	92.75 %	42.79 %	25.10
					4	1619	0.07 ms	92.83 %	44.47 %	23.83
	100	2	1884	3.48 ms	.5	2673	0.17 ms	91.39 %	41.86 %	20.62
					1	2655	0.17 ms	91.28 %	40.93 %	20.22
					2	2644	0.18 ms	91.19 %	40.36 %	19.40
					4	2621	0.20 ms	90.88 %	39.13 %	17.48
	200	4	4593	9.31 ms	.5	5964	0.49 ms	92.06 %	29.85 %	19.16
					1	5720	0.49 ms	91.65 %	24.52 %	18.84
					2	5442	0.54 ms	91.02 %	18.46 %	17.35
					4	5228	0.62 ms	90.21 %	13.81 %	15.01
	300	6	9715	19.76 ms	.5	12115	1.02 ms	93.81 %	24.70 %	19.45
					1	11573	1.09 ms	93.39 %	19.13 %	18.06
					2	10962	1.20 ms	92.78 %	12.84 %	16.42
					4	10509	1.43 ms	91.89 %	8.17 %	13.82
	400	8	17179	36.42 ms	.5	20964	1.80 ms	94.92 %	22.03 %	20.23
					1	20060	2.01 ms	94.54 %	16.77 %	18.09
					2	18906	2.28 ms	93.95 %	10.05 %	16.01
					4	18156	2.81 ms	93.04 %	5.68 %	12.95
	700	14	53353	156.36 ms	.5	64139	6.59 ms	96.58 %	20.22 %	23.74
					1	61691	7.51 ms	96.25 %	15.63 %	20.83
					2	57769	8.64 ms	95.69 %	8.28 %	18.09
					4	55778	11.45 ms	94.75 %	4.55 %	13.65

TABLE A27: Full results of mesh remodelling for $\sigma = 0.25$ - Gottingen 702 airfoil
Average of thirty optimisation scenarios

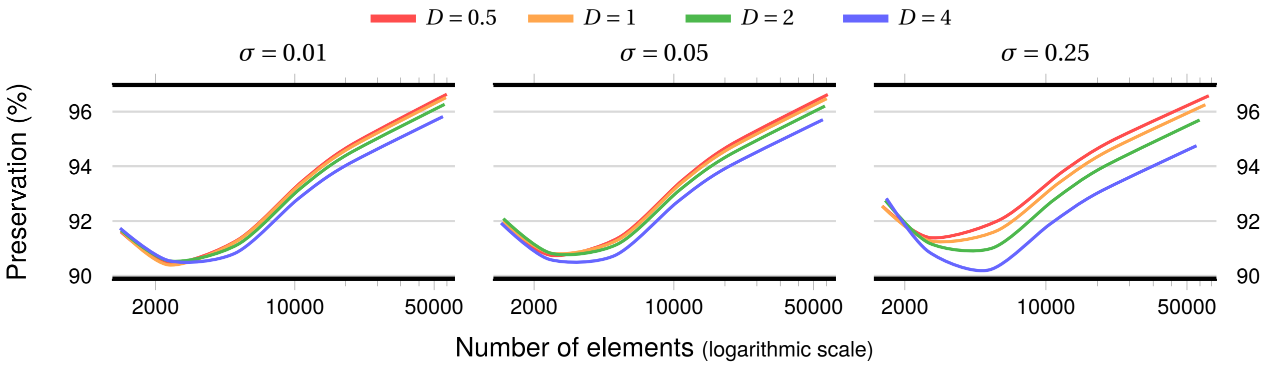

FIGURE A35: Element preservation results using mesh remodelling - Gottingen 702 airfoil
Average of thirty optimisation scenarios

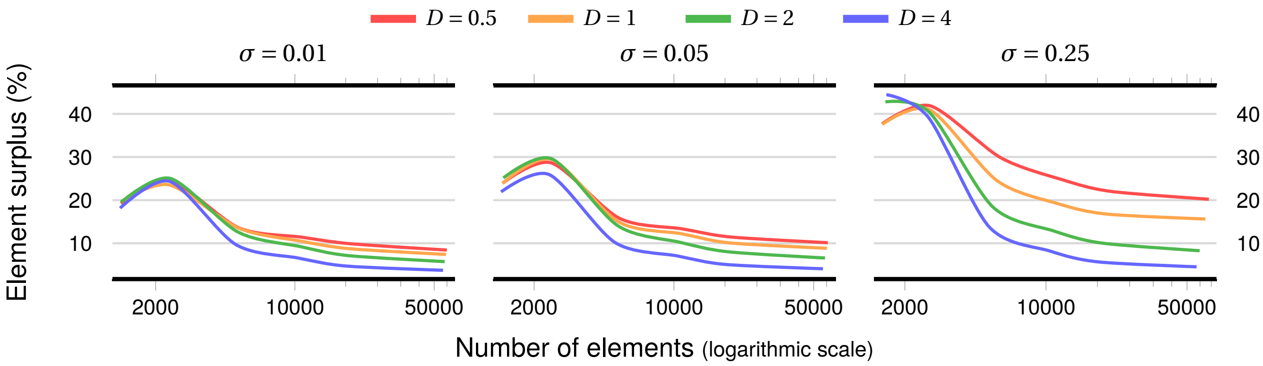

FIGURE A36: Element surplus results on meshes from mesh remodelling - Gottingen 702 airfoil
Average of thirty optimisation scenarios

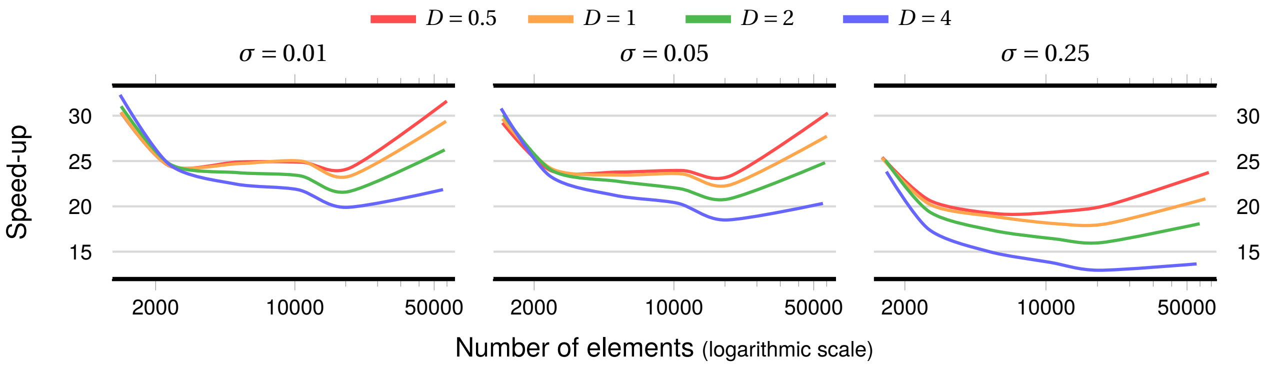

FIGURE A37: Speed-up results by mesh remodelling - Gottingen 702 airfoil
Average of thirty optimisation scenarios