Model-based Informative Path Planning

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Abstract

Surrogate model construction can enable autonomous agents to quantify the uncertainty of their beliefs about the world - to recognize what they do not know. This is valuable because it can allow those agents to plan to gain information. In this work, we study the problem of generating information-rich paths from a surrogate model, which is known as informative path planning. The earliest informative path planning systems were myopic, meaning that they optimized their planning over only a short time horizon. As general algorithms for non-myopic planning improve, their application stands to benefit the efficiency of informative path planning for the surveillance of large domains, dependent on the development of surrogate models that can take advantage of a structured environment to make useful non-myopic predictions. We present Model-based Informative Path Planning, an algorithm which uses active learning to rapidly estimate the parameters of linear-Gaussian structural relationships between intensive properties of a survey environment. We demonstrate that Model-based Informative Path Planning with passive parameter estimation constructs more accurate surrogate models than a baseline, Single-feature Informative Path Planning, in a benthic habitat environment where linear-Gaussian structural relationships are present. Our results indicate that actively learning the parameters of the knowledge model does not confer a significant benefit in our environment. Overall, we conclude that Model-based Informative Path Planning has strong potential for application to long-term planning techniques for informative path planning.

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1. Introduction

The efficient planning of surveys is critical for exploratory science missions in remote regions, such as the deep ocean or foreign planets, where the survey region is vast and resources are limited. A multi-institution team, consisting of researchers from Woods Hole Oceanographic Institution (WHOI), the Massachusetts Institute of Technology (MIT), the Austrailian Center for Field Research (ACFR), the University of Michigan, Teledyne-Webb Research, and others, embarked on one such exploratory science mission in December 2018 in Costa Rica, where multiple Autonomous Underwater Vehicles (AUVs) (Figure 1) and Remotely Operated Vehicles (ROVs) were deployed to sense oil seeps on the ocean floor (Schmidt Ocean Institute, 2019). Another such mission will take place in late 2019 in Santorini, Greece.

Figure 1. The Slocum Glider, an Autonomous Underwater Vehicle. These vehicles are used by the Woods Hole Oceanographic Institution and MIT to conduct science missions in remote regions of the deep ocean.

Past science missions carried out by WHOI and MIT have featured the deployment of autonomous information gathering systems which plan to explore their environment from world representations called surrogate models. The true state of a survey environment such as the ocean floor can be modeled as a function, mapping location coordinates to the value of the feature of interest. This function is black-box, meaning no closed-form expression is known for it, and is expensive to evaluate, meaning that observing the function consumes some limited resource such as energy or time. In this context, a survey plan is expressible as a sampling strategy for this expensive black-box function. When sampling an expensive black-box function, it is worthwhile to expend some computational effort to construct a surrogate model of the function, which represents our belief about the distribution of the true functions value, and to query that surrogate to decide where subsequent exploration should be made. Regions of high information entropy, where the value of a feature is predicted with the least certainty under the surrogate, can be expected to yield a greater increase in knowledge about the overall environment once they are explored, so an intelligent agent should recognize the value in visiting these regions. An algorithm that plans to gather information refer by computing high entropy paths through a surrogate model is known as an informative path planning (IPP) algorithm.

The informative path planning surrogate function is most commonly implemented as a Gaussian process (Rasmussen et al., 2009). Gaussian processes are adept at modeling spatial correlations, which are often an invariant property of physical features. They also provide an explicit representation of uncertainty, in the form of a probability distribution over output values. From this uncertainty representation, an acquisition function such as information entropy or the Upper Confidence Bound (UCB) can be computed at a set of points across a field (Krause et al., 2008). Such an acquisition function $a(x)$ quantifies the utility of collecting an observation at a location of interest x.

Once a predictive surrogate model has been constructed for a survey field, an autonomous agent must plan a sequence of observation locations to visit which cumulatively maximize the acquisition function while satisfying some constraints such as range, time, or energy. Previous work in informative path planning has commonly utilized greedy observation selection, known as myopic path planning (Binney et al., 2010; Martinez-Cantin et al., 2009). Recently, advances from the planning and search community have been applied to perform long-horizon, or nonmyopic, observation selection in an informative path planning context. Binney et al. (2010) used branch-and-bound search to speed up the selection of nonmyopic informative paths across a Gaussian process field. Nguyen et al. (2014) applied Monte Carlo Tree Search (MCTS) to perform nonmyopic informative path planning for a resource constrained-aerial glider.

One shortcoming of the Gaussian process as a surrogate model is that for a field in which observations are only available in a small region of space, the predicted distribution returns to a homogeneous prior distribution at points not physically close to the observed region. This is prone to produce relatively simple planning behavior, akin to lawnmower patterns for an explorative acquisition function such as information entropy, or to gradient-following for an exploitative acquisition function such as UCB (Poloczek et al., 2016). In constrast, constructing a heterogeneous prior distribution on our Gaussian process surrogate can elicit more interesting and effective behavior from a path planner. However, fixing such a heterogeneous prior using purely expert knowledge and careful pre-engineering would dampen the adaptive, data-driven nature of informative path planning.

To autonomously learn a useful heterogeneous Gaussian process prior for large survey domains, we propose Model-based Informative Path Planning (M-IPP). An agent performing traditional informative path planning actively learns a surrogate function; An agent performing M-IPP actively learns both a surrogate function and a knowledge model describing the relationship between intensive properties in the survey region. We implement our agents knowledge model as a Gaussian Graphical Model with known structure and actively learned parameters. We demonstrate that in an environment where some side information expert information about features correlated with the feature of interest is provided, M-IPP correctly estimates a knowledge model and achieves lower predictive error than its model-free alternative.

2. Background

2.1. Gaussian Processes

A Gaussian process (GP) is a random process in which the correlation between two input points x_1 and x_2 is defined by a kernel function $k(x_1, x_2)$ (Rasmussen et al., 2009). Given a set of observations in a Gaussian process, we can predict the mean and variance for any unobserved input, conditioned on those observations (Figure 2). Gaussian processes are particularly effective surrogate models for geospatial mapping, since the value of a continuous-valued feature across physical space commonly exhibits consistent spatial correlation that are modeled well by a kernel covariance.

Typically, a Gaussian process model with multiple output variables assumes that those output variables are independent of one another. However, the Linear Model of Coregionalization (LMC) (Williams et al., 2008) represents linear correlative relationships between outputs of a multi-output Gaussian process. The predictive mean and variance of each output variable under LMC is conditioned on the training data collected about all output and input variables at all locations, according to a kernel covariance in the inputs and a linear-Gaussian conditional dependence in the outputs. Typically, learning a multi-output Gaussian process under the LMC is an expensive maximum likelihood optimization problem. However, given an external estimate of the feature covariance matrix, we can fix the hyperparameters of the LMC and avoid this optimization step.

2.2. Gaussian Graphical Models

A Gaussian Graphical Model (GGM) (Picka, 2006) is an undirected graph defined by a set of vertices and edges, $G = \langle V, E\rangle$. Each vertex in a GGM represents a different

Gaussian-distributed variable, and each edge represents a conditional dependence between the variables it connects (Figure 3). A GGMs edges may be defined by its precision matrix. The precision matrix of a fully connected GGM is the inverse of the covariance matrix of the joint Gaussian distribution over all variables. Employing a sparse approximation of the precision matrix approximates independence relations between variables in the graph to reduce the size of its edge set.

The estimation of Gaussian Graphical Model parameters can be achieved by maximum-likelihood estimation or Bayesian estimation techniques (Picka, 2006). Maximum-likelihood techniques offer greater speed and flexibility than Bayesian estimation, formulating parameter estimation as an optimization problem. However, Bayesian estimation produces distributions over parameter values which explicitly represent uncertainty, facilitating integration of GGMs with active learning and informative path planning approaches. Tong & Koller (2001a) presented a theoretical framework for the active learning of Bayesian Network parameters and structure and demonstrated that this approach improved the sample-efficiency of learning in both cases.

3. Related Work

3.1. Informative Path Planning

The selection of observations which maximize information gain, also known as adaptive sampling, has been an extensively studied problem in the contexts of optimal experimental design, algorithm optimization, and robotics. Lindley (1956) proposed the use of entropy reduction as a utility or acquisition function for optimal experimental design. Shewry & Wynn (1987) built upon this work to define an algorithm for the optimal placement of sensors to maximize entropy reduction in a survey field. More recently, Krause et al. (2008) proposed a Gaussian process-based sensor placement approach which used mutual information as its information gain function and exploits the submodularity property of Gaussian processes for significant speed-up. This approach has been widely applied in sensor placement and remains among the state-of-the-art approaches to the sensor placement problem.

In the informative path planning scenario, a mobile sensing agent must autonomously select its next observation site based on the observations it has made previously. As in optimal sensor placement, the agent seeks to maximize the cumulative information gain achieved by its observations. However, unlike in optimal sensor placement, IPP considers that a vehicle must expend time and energy as a function of the distance it travels between observation locations. An agent performing IPP also makes its observation decisions sequentially, so an online learning approach can be adopted which improves the agents plan at each step. Binney et al. (2010) published an IPP algorithm based on greedy mutual information maximization and demonstrated that it gathers information with considerably better efficiency than exhaustive search. The algorithm we present in this paper is a type of IPP, and similarly to the work of Binney, we employ a strategy of mutual information maximization.

3.2. Nonmyopic Planning

Early work on informative path planning (Binney et al., 2010; Martinez-Cantin et al., 2009) focused on deciding sampling actions one at a time by selecting the action which would immediately return the greatest reduction in entropy. This style of planning is known as greedy or myopic planning.

Recent developments on state space search within the planning community have brought nonmyopic path planning closer to practicality. Binney & Sukhatme (2012) applied branch and bound search, a heuristic search algorithm, to generate optimal nonmyopic plans in the informative path planning scenario. Nguyen et al. (2014) applied Monte Carlo Tree Search to perform nonmyopic informative path planning for a resource constrained-aerial glider. Our work presents a system for making informed long-range predictions, which bears potential to inform effective long-term planning in the IPP setting.

4. Method

4.1. Method Overview

We propose a method, Model-based Informative Path Planning, for an autonomous agent to gather information about a survey field in a resource-efficient manner. We achieve this improved resource efficiency by maintaining an informative surrogate function computed from a knowledge model which is informed by an expert. In contrast to previous work on IPP (Binney et al., 2010; Binney & Sukhatme, 2012; Ayton, 2017; Ma et al., 2017; Martinez-Cantin et al., 2009; Nguyen et al., 2014), in which an agent seeks to reduce uncertainty about a single features value at all locations in a survey field, our algorithm directs an agent to additionally reduce uncertainty about its knowledge model, which encodes the conditional relationships between multiple intensive features in a survey field. When adequate expert knowledge is initially supplied, our algorithm enables an agent to quickly infer the distribution of a new feature in locations where it has not yet been observed, leading the agent to plan a more effective survey. Our algorithm is demonstrated to correctly estimate the conditional relationships between features in a synthetic survey environment, resulting in a surrogate function with improved predictive accuracy.

4.2. Belief Modeling

We represent the knowledge model of our agent as a Gaussian Graphical Model, which represents the joint Gaussian distribution of features at any given location in the survey region. We learn a GGM from our past observations by finding a precision matrix that describes those observations with maximum likelihood. In addition, we determine the agents confidence in its GGM representation of the world by tracking the variance of each element of the precision matrix based on the number of times the relevant features have been observed together (Browne et al., 2011; Picka, 2006).

4.3. Inference

We infer a prior distribution over our feature of interest as the output of a multi-output Gaussian process with a Linear Model of Coregionalization. The feature covariance hyperparameters in the LMC are fixed as the inverse of the precision matrix estimated in our GGM belief model. Each feature in our belief model is thus modeled as an output of the multi-output Gaussian process. The multi-output Gaussian process is trained on all available expert-provided observations as well as observations made by the agent. The input covariance kernel of each output is modeled as a Gaussian kernel with fixed lengthscale and variance.

4.4. Planning

We demonstrate the impact of our improved prior on agent inference capability when performing myopic planning. Since our improved inference allows us to make better predictions at a distance, we expect that improved predictive accuracy in a myopic example will translate to improved long-term planning when a non-myopic planning algorithm such as MCTS is applied. In the simplest problem we consider, we restrict our agent to take one of four actions, which consist of moving either North, South, East, or West by a fixed distance and then making a sensor observation of the feature of interest.

Our agent selects its action to greedily minimize the total model entropy achieved after the action is taken. If the uncertainty in the agents belief model is taken to be fixed, this is the same mutual information acquisition function used in Binney et al. (2010) With nonzero uncertainty in the belief model, the acquisition function is equal to the mutual information acquisition function, plus the knowledge model information gain scaled by some tunable constant.

4.5. Agent Behavior

After taking an action and collecting an observation, our agent first updates its belief model by estimating its parameters given the new data it has received. It also decrements the uncertainty of any correlative relationships involving exclusively the features known at the newly sampled location. Next, it updates the feature covariance parameters of the LMC to match those estimated in its belief model, augments the LMC training data with the new observation, and infers the new prior distribution over the feature of interest. An entropy-minimizing action and observation pair are selected and taken, and the process repeats.

Algorithm 1 Single-feature Informative Path Planning

Load previous observations of the sensed feature $\rightarrow$ obs
Initialize agent.
while time_remaining > 0 **do**
    surrogate $=$ GP(obs)
    prior $=$ InferPrior(surrogate)
    action $= argmin_{actions}$(Entropy(prior, action))
    agent.Move(action)
    obs $=$ obs + agent.MakeObservation()
    time_remaining $=$ time_remaining - action.Duration
end while

Algorithm 2 Model-based Informative Path Planning

Load previous observations of all features $\rightarrow$ obs
Load expert knowledge model structure $\rightarrow$ M
Initialize agent.
while time_remaining > 0 **do**
    K $=$ GMM(obs, M)
    surrogate $=$ LMC-GP(obs, K)
    prior $=$ InferPrior(surrogate)
    action $= argmin_{actions}$(Entropy(prior, K, action))
    agent.Move(action) $= 1$
    obs $=$ obs + agent.MakeObservation()
    time_remaining $=$ time_remaining - action.Duration
end while

5. Results

We evaluate the performance of our approach, Model-based Informative Path Planning (Algorithm 2), by simulating its ability to facilitate an agents exploration of a simulated underwater environment over three experiments. In our first simulated environment, only a single sensor is available to the agent for use, and no expert knowledge is available to the agent during planning. In this domain, we demonstrate that our algorithm performs equivalently to the Single-feature Informative Path Planning formulation, referred to as S-IPP (Algorithm 1), which is well-established in prior work (Binney et al., 2010) (Martinez-Cantin et al., 2009) (Nguyen et al., 2014). This is because our algorithm is identical to S-IPP in the single-sensor case with no expert knowledge. We demonstrate that both our algorithm and S-IPP outperform a random walk policy on the metric of predictive accuracy in this domain.

We introduce expert knowledge in our second environment in the form of multiple side information features and a linear-Gaussian Bayesian Network knowledge model with known structure, but unknown parameters. Here, we compare M-IPP so S-IPP as well as to a best-case expert who knows the true parameters of the knowledge model from the start. We observe a large improvement in survey efficiency

Our third environment illustrates the impact of active parameter learning in M-IPP, where information gain in the parameters of the linear-Gaussian Bayesian Network knowledge model is incorporated into the acquisition function in addition to the typical IPP information gain term. The effect of incentivizing the knowledge model information gain is most evident in an environment where expert measurements of side information state variables are sparsely available over disjoint spatial regions of the explored domain, and an agent must decide which of these regions to visit in which order to learn an accurate knowledge model more quickly. In our environment, where information is quite dense and the agent is likely to learn the knowledge model without additional incentive, we do not observe a significant benefit from active parameter learning.

5.1. Single-sensor Environment

The physical range of our simulated single-sensor environment is a two-dimensional rectangular region 600 meters across. In this environment, we model temperature as our only observed variable, with no expert knowledge about the domain encoded. The true temperature distribution is a random sample drawn from a Gaussian process prior, with a Gaussian kernel having a variance of 1.0 and a lengthscale of 225 meters.

Our agent is represented as an AUV restricted to move at 0.5 m/s, and for simplicity we assume full observability of its location and velocity as well as perfect control over its velocity. Our agent has four actions available to it: motions for one second to the North, South, East, or West. The agent employs a policy to select one of these four available actions at any given minute, take that action, update its model, and replan at each step.

We compare three types of agent in this environment: an agent employing S-IPP to greedily maximize information gain, an agent employing M-IPP to greedily maximize information gain, and an agent employing S-IPP but exploring the map with a random walk. We observe that the greedy M-IPP and greedy S-IPP models exhibit nearly identical learning, and both outperform the random-walking agent (Figure 2). Since M-IPP and S-IPP models are mathematically identical in the single-variable case, the result that they produce nearly identical learning curves is to be expected. In addition, the greedily information-maximizing agents are shown here to explore more effectively than random-

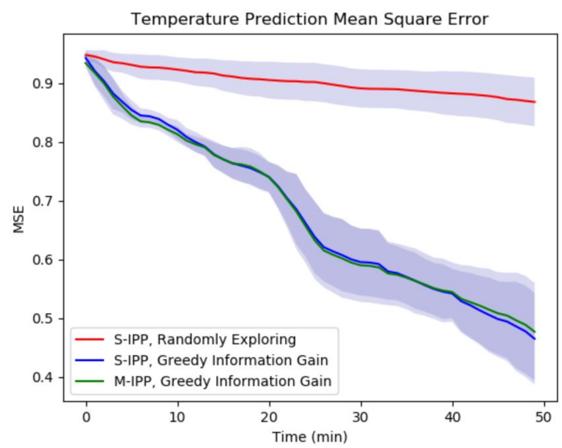

Figure 2. Mean square predictive error over the temperature field in the single-sensor environment. M-IPP exhibits the same behavor as single-variable path planning in this environment. The mean of 10 trials is plotted with shading within 1 standard deviation of the mean.

walking agents in the single-sensor environment, a result commonly supported by previous work on S-IPP type algorithms (Binney et al., 2010; Binney & Sukhatme, 2012; Ayton, 2017).

5.2. Side-information Environment

Our simulated side-information environment is the same as our single-sensor baseline, but with three additional side information state variables which are known over the entire field before exploration begins (Figure 3). Two of these variables, sea floor depth and luminosity, are strongly linearly correlated with the temperature signal field, while the third, ocean current intensity, is uncorrelated with the temperature signal field. Temperature and ocean current are independent draws from a Gaussian process prior, with a Gaussian kernel having a variance of 1.0 and a lengthscale of 225 meters.

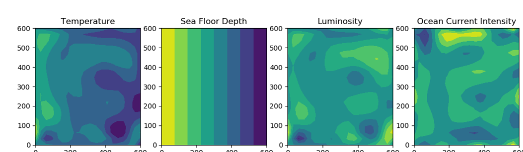

Figure 3. Contour maps of the four state variables in a sample from the posterior distribution of our side-information environment.

In M-IPP, the training inputs for our Gaussian process models are (x, y) coordinate locations. Temperature, sea floor depth, luminosity, and ocean current intensity are modeled as GP outputs. The knowledge model, or relationship between these outputs, is modeled as a linear-Gaussian Bayesian Network, which is the main innovation of our approach. This parametric model allows extrapolative inference on locations with partial data and new values of the output variables in a way that is appropriate for the output variables but not for the coordinate input variables, the coordinates not being physical properties of the system.

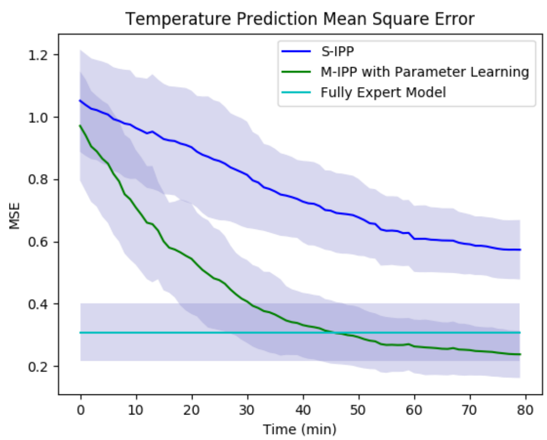

Figure 4. Mean square predictive error over the temperature field in the side information environment. M-IPP is shown to approach a low predictive error much faster than single-variable IPP in this domain. The mean of 10 trials is plotted with shading within 1 standard deviation of the mean.

We compare this M-IPP model with Bayesian Network parameter learning to the baseline S-IPP, which assumes all output variables to be independent, as well as to an M-IPP model where the Bayesian Network parameters are provided by an expert. M-IPP initially matches the predictive accuracy of S-IPP, since its prior on the Bayesian Network parameters is independence between the outputs. As the agent explores and improves its estimate of the covariance matrix, the predictive accuracy of M-IPP with parameter learning quickly surpasses the predictive accuracy of the model with parameters fixed to their true expert-provided value. It is possible to achieve better accuracy than the expert model because The predictive accuracy of S-IPP will also eventually approach that of the fully expert model, but is shown here to do so at a much slower rate than M-IPP with passive parameter estimation (Figure 4).

5.3. Sparse Side-information Environment

Our simulated sparse side-information environment is constructed in the same way as our side-information environment in Section 5.2, but instead of being provided with measurements of the side variables (sea floor depth, luminosity, ocean current intensity) over the entire field before exploration begins, our agent is provided with the values of these variables in sparsely distributed disjoint spatial regions. Specifically, sea floor depth is available in only the Northeast quadrant of the survey space, while luminosity and ocean current intensity are available everywhere.

We compare two different types of agent in this environment, both of which use M-IPP as specified in Section 5.2 and maximize information gain. The difference between these agents lies in their information gain formulae. Agent 1 measures information gain as the reduction of entropy in a Gaussian process model of the sensed variable (temperature). This is the type of M-IPP agent demonstrated in Sections 5.1 and 5.2. Agent 2 measures information gain as the sum of two terms: the first being the reduction of entropy in a Gaussian process model of the sensed variable, and the second term being the reduction of entropy in the linear-Gaussian Bayesian Network knowledge model of the M-IPP agent. Since this knowledge acquisition term of the information gain is increased by observing the sensed variable in a location where a side variable is known, the second agent is incentivized to follow a plan which visits those locations where the value of a side variable has been provided, while the first agent is not incentivized to follow such a plan.

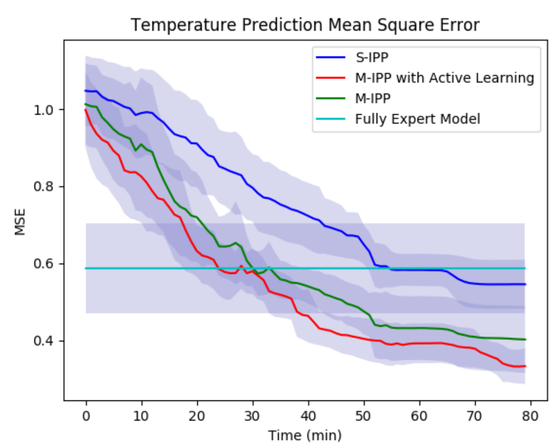

Figure 5. Mean square predictive error over the temperature field in the sparse side information environment. M-IPP with active parameter learning does not achieve a significantly lower MSE than M-IPP without active parameter learning. The mean of 10 trials is plotted with shading within 1 standard deviation of the mean.

As shown in Figure 5, active model learning in M-IPP provides an insignificant improvement in survey efficiency over passive parameter estimation. We believe that active model learning will be particularly useful when performing long-term planning in a large environment, where Monte Carlo

Tree Search or another non-myopic planning algorithm is used to make long-term decisions and and information reward is less dense. However, our result indicates that active model learning does not provide a significant improvement in an environment of dense reward like ours, where there is a high probability of correctly estimating the model parameters without focused planning.

6. Conclusion and Future Work

In this work, we addressed the problem of producing a descriptive prior for informative path planning from expert-provided knowledge. Our proposed solution, M-IPP, actively estimates a probabilistic map of a survey field based on expert knowledge of the GGM dependence structure between known and surveyed features. We demonstrated that M-IPP learns the true parameters of the feature GGM in a survey region exhibiting strong correlation between features, and that this results in improved predictive performance for an autonomous surveillance agent under a fixed time budget.

Our result has implications for improving the resource-effectiveness of science surveys conducted with autonomous vehicles. We hope that this will enable such surveys to gather more meaningful data in remote environments where limited expert knowledge is available. The deep ocean and deep space are two exciting and large domains which humanity has yet to map in significant detail.

In the future, we expect to apply M-IPP with knowledge model learning in a real-world AUV deployment, potentially in Santorini 2019. We hope to evaluate whether the efficiency improvement obtained in 4 will translate to the noisy, complex real world.

Future work should apply active parameter estimation in a larger simulation environment and incorporate a non-myopic planning algorithm such as Monte Carlo Tree Search. Although active parameter estimation was not observed to significantly improve survey efficiency in our simulation environment, we believe that an agent exploring a larger environment with greater information sparsity might benefit more from active parameter learning by formulating long-term plans to collect observations that inform the knowledge model.

Several extensions in knowledge model learning may be possible beyond the algorithms presented in this paper. In our formulation of M-IPP, we assume that the structure of our GGM is provided by an expert, and actively learn the GGM parameters. The estimation of GGM structure would be a valuable extension to our method, reducing the degree of expert knowledge necessary to apply M-IPP. Tong & Koller (2001b) demonstrated active learning in a structural dependence model. Furthermore, a GGM is an undirected dependence graph, meaning it does not encode causality but rather correlation. An causality learning extension to M-IPP might enable an agent with interventional experiment-running capability to actively learn the direction of causal relationships between features in its environment.

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