hw4

Graded

23 Hours, 50 Minutes Late

Student

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Total Points

22.5 / 25 pts

Question 1

induction

6 / 6 pts

- **– 0 pts** Correct

Question 2

induction with regexps

9 / 10 pts

- **– 1 pt** base case: did not use single char map fn

Question 3

non regular language

6.5 / 8 pts

- **– 1.5 pts** Missing 2 cases

You did not split y in the middle and the right

Question 4

readme

1 / 1 pt

- **– 0 pts** Correct

Question assigned to the following page: 1

1) Prove the following statement is true:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

step	statement	justification
1	A natural number is: 0 or k+1, where k is a natural number	definition of a natural number
2	Proof by induction on the natural number n.	method of proving because the definition of a natural number is recursive
3	base case: n=1 goal: show both sides of the statement are equal	given, (1) and (2)
4	proof of goal: $\sum_{i=1}^{n} 1^2 = \frac{1(1+1)(2(1)+1)}{6}$ $1 = \frac{2(3)}{6}$ 1=1 both sides are equal	plugging in and mathematical computation
5	The statement is true for the base case.	(4)
6	Inductive case: n = k+1 for some natural number k	(1)
7	Inductive hypothesis: $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$	(2)
8	goal: prove that for n = k + 1, $\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2$	(2) and (7)
9	proof of goal (left hand side): $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$ $\frac{(k+1)(k+2)(2k+2+1)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$ $\frac{(k^2+3k+2)(2k+3)}{6} = \frac{2k^3+3k^2+6k^2+9k+4k+6}{6}$ $\frac{2k^3+9k^2+13k+6}{6}$	plugging in and mathematical computation

Question assigned to the following page: 1

10	proof of goal (right hand side): $\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2$ $\frac{2k^3+9k^2+13k+6}{6} = \frac{k(k+1)(2k+1)}{6} + (k^2 + 2k + 1)$ $\frac{2k^3+9k^2+13k+6}{6} = \frac{k(k+1)(2k+1)}{6} + \frac{6k^2+12k+6}{6}$ $\frac{2k^3+9k^2+13k+6}{6} = \frac{(k^2+k)(2k+1)}{6} + \frac{6k^2+12k+6}{6}$ $\frac{2k^3+9k^2+13k+6}{6} = \frac{(2k^3+k^2+2k^2+k)}{6} + \frac{6k^2+12k+6}{6}$ $\frac{2k^3+9k^2+13k+6}{6} = \frac{2k^3+k^2+2k^2+k+6k^2+12k+6}{6}$ $\frac{2k^3+9k^2+13k+6}{6} = \frac{2k^3+9k^2+13k+6}{6}$ both sides are equal	plugging in, (7) and mathematical computation
11	The statement is true for the base case and the inductive case, therefore the statement is true.	(2), (5) and (10)

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Question assigned to the following page: 2

2) prove map_{lang} is closed

$map_{lang}(L) = \{map_{str}(w) \mid w \in L\}$

step	statement	justification
1	Theorem: if L is regular, so is $map_{lang}(L)$	given
2	Proof by induction on the regular expression of L	method of proof
3	Definition of a regular expression: R is a regular expression if R is 1. a for some a in the alphabet Σ, 2. ε, 3. ∅, 4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions 5. $(R_1 \circ R_2)$, where R_1 and R_2 are regular expressions, 6. $(R_1 *)$, where R_1 is a regular expression	definition of a regular expression
4	Base cases: 1. a for some a in the alphabet - Same regular expression represents $map_{lang}(L)$ 2. ε - Same regular expression represents $map_{lang}(L)$ 3. ∅ - Same regular expression represents $map_{lang}(L)$	(1), (2) and (3)
5	Language L is regular with regular expression, $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions	(3)
6	R_1 and R_2 describe languages L_1 and L_2	regular expression ⇔ regular language theorem
7	If L_1 is a regular language, then $map_{lang}(L_1)$ is a regular language	inductive hypothesis
8	If L_2 is a regular language, then $map_{lang}(L_2)$ is a regular language	inductive hypothesis

Question assigned to the following page: 2

9	$map_{lang}(L_1)$ and $map_{lang}(L_2)$ are regular	(6), (7), (8)
10	$map_{lang}(L_1) \cup map_{lang}(L_2)$ is regular	union closed for regular languages theorem
11	$map_{lang}(L_1) \cup map_{lang}(L_2) = map_{lang}(L_1 \cup L_2)$	union and map_{lang} are commutative
12	$L = (L_1 \cup L_2)$	(1), (5) and (6)
13	$map_{lang}(L)$ is regular	(10), (11), (12)
14	Language L is regular with regular expression, $(R_1 \circ R_2)$, where R_1 and R_2 are regular expressions	definition of a regular expression
15	R_1 and R_2 describe languages L_1 and L_2	regular expression ⇔ regular language theorem
16	If L_1 is a regular language, then $map_{lang}(L_1)$ is a regular language	inductive hypothesis
17	If L_2 is a regular language, then $map_{lang}(L_2)$ is a regular language	inductive hypothesis
18	$map_{lang}(L_1)$ and $map_{lang}(L_2)$ are regular	(15), (16), (17)
19	$map_{lang}(L_1) \circ map_{lang}(L_2)$ is regular	concatenation is closed for regular languages theorem
20	$map_{lang}(L_1) \circ map_{lang}(L_2) = map_{lang}(L_1 \circ L_2)$	concatenation and map_{lang} are commutative
21	$L = (L_1 \circ L_2)$	(14) and (15)
22	$map_{lang}(L)$ is regular	(19), (20), and (21)
23	Language L is regular with regular expression, $(R_1 *)$, where R_1 is a regular expression	definition of a regular expression

Question assigned to the following page: 2

24	R_1 describes languages L_1	regular expression ⇔ regular language theorem
25	If L_1 is a regular language, then $map_{lang}(L_1)$ is a regular language	inductive hypothesis
26	$map_{lang}(L_1)$ is regular	(23), (24), (25)
27	$(map_{lang}(L_1))*$ is regular	kleene star is closed for regular languages theorem
28	$(map_{lang}(L_1))*$ = $map_{lang}(L_1*)$	kleene star and map_{lang} are commutative
29	$L = (L_1*)$	(23) and (24)
30	$map_{lang}(L)$ is regular	(27), (28), (29)

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Question assigned to the following page: 3

3) show that the following language is not a regular language:

$L_3 = \{x == y \mid where\ x\ and\ y\ are\ equal\ binary\ numbers\}$

$\Sigma = \{0, 1, =\}$

step	statement	justification
1	proof by contradiction on language L_3	method of proof
2	If a language B is regular then there exists a pumping constant p where if s is a string in the language and is at least length p, then it can be divided into three pieces, s=xyz, satisfying these conditions: 1) for each $i \geq 0$, $xy^iz \in B$, 2) |y| > 0 and 3) |xy| ≤ p.	definition of pumping lemma
3	assume $L_3 = \{x == y\}$ is a regular language	definition of pumping lemma, (1) and assumption
4	There exists a pumping constant for L_3	definition of a regular language and (2)
5	choose string, $w = \{1^p == 1^p\}$ as a counter example	(1) and (2)
6	possible split: all one's before the two equal signs (==) is y let p = 3 $w = \{111 == 111\}$ x: 1 y: 11 z: ==111 |y| > 0 2>0 |xy| ≤ p 2*1 ≤ 3 xy^iz let i = 2 then w = {11111==111} w is not in the language anymore because the binary number 11111 is not equal to 111	pumping lemma computation, (2) and counter example
7	Since w was not in the language, that means it is not pumpable.	(2) and (6)

Question assigned to the following page: 3

8	If w is not pumpable then that means it is a counterexample to the pumping lemma.	(4), (5) and (7)
9	There are no other ways to split the string w so that the condition |xy| is satisfied.	(2)
10	Since $w = \{1^p == 1^p\}$ is not pumpable that means $\{x == y\}$ is not a regular language.	(7) and (8)
11	This is a contradiction to the assumption, therefore it cannot be a regular language.	(1), (2), (3), (5) and (10)

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Question assigned to the following page: 4

README
names of other students: none
books/websites: class slides
time: 4 hours