# hw3

Graded

17 Hours, 47 Minutes Late

**Student**

Giancarlos Marte

**Total Points**

**22.5 / 30 pts**

**Question 1**

## Fun With Regular Expressions — 3 / 5 pts

- ✔ **− 0.5 pts** part 1 incorrect: should be $\mathtt{aab} \cup \mathtt{abb}$ or $\mathtt{a}\Sigma\mathtt{b}$
- ✔ **− 1 pt** part 2 incorrect: should be $\Sigma^*\mathtt{aaa}\Sigma^*$
- ✔ **− 0.5 pts** part 3: missing single $\mathtt{a}$ case
- answer uses union in many places when it should be concat

**Question 2**

## Reg Lang Proving Closed Operation Practice — 5 / 6 pts

- ✔ **− 1 pt** FSM is incorrect
- Conceptually you understand this, but your N is wrong, it should accept every string that N1 does not accept

**Question 3**

## Converting Strings to a Different Alphabet — 6 / 8 pts

### 3.1 String Def — 3 / 4 pts

- ✔ **− 1 pt** recursive case should use concat, not union or other operation

### 3.2 Convert Function — 3 / 4 pts

- ✔ **− 1 pt** recursive case should use concat, not union
- close!

Question 4

## Caesar Cipher Closed?

**7.5** / 10 pts

- **− 1 pt** Missing assumptions
- **− 1.5 pts** Skipping steps

Seems like you understand the concept, but are not taking all the steps necessary to prove the different induction cases.

Question 5

## readme

**1** / 1 pt

- **− 0 pts** Correct

Question assigned to the following page: 1

**1. Fun with regular expressions**

$\Sigma = any\ char\ in\ the\ alphabet\ of\ the\ language.$

$\Sigma^* = zero\ or\ more\ of\ any\ char\ in\ the\ alphabet\ of\ the\ language.$

1) Let R represent a regular expression for the following language:
{aab,abb}
$R = (aa \cup b) \cup (a \cup bb)$

2) Let R represent a regular expression for the following language,
$\{x | x$ contains aaa somewhere in the string $\}$

$R = (aaa) \cup (\Sigma^*)$

3) Let R represent a regular expression for the following language,
$\{x | x$ is a string that both starts and ends with a$\}$

$R = a \cup (\Sigma^*) \cup a$

4) Let R represent a regular expression for the following language,
$\{\varepsilon\}$
$R = \varepsilon$

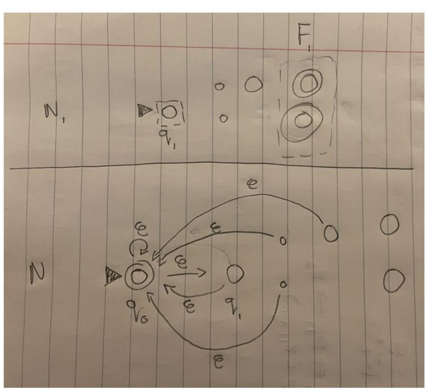
5) Let R represent a regular expression for the following language,
$\varnothing$
$R = \emptyset$

Question assigned to the following page: 2

## 2. Practice: proving an operation closed for regular languages

$OP2(L) = \{w \mid L \text{ does not contain } w\}$

| step | statement | justification |
|---|---|---|
| 1 | L is a regular language | given |
| 2 | proof:<br>Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize L<br>Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize OP2(L). | definition of an NFA and (1) |
| 3 | 1. $Q = Q_1 \cup \{q_0\}$<br>2. $q_0$ *is the start state*<br>3. $F = \{q_0\}$<br>4. *define* $\delta$ *so that for any* $q \in Q$ *and any* $a \in \Sigma_\varepsilon$,<br>$\delta(q, a) =$<br>$\{$<br>$q = q_0 \text{ and } a = \varepsilon$<br>$q \in Q_1 \text{ and } q \notin F_1 \text{ and } a = \varepsilon$<br>$q = q_0 \text{ and } a = \varepsilon$<br>$\}$ | NFA computation and (2) |
| 4 | A set is closed under an operation if applying the operation to elements in the set produces a result in the same set. | definition of a closed language under an operation |
| 5 | There is an NFA that recognizes OP2(L),which means that its result is a regular language, just like the input L. Therefore it is a closed operation. | (4) |



Questions assigned to the following page: 3.1 and 3.2

**3. Converting strings to a different alphabet**

R is a string if R is

ε,

{c} for some character c in an alphabet Σ,

or $(R_1 \cup R_2)$ , where $R_1$ *and* $R_2$ are strings.

Shorthand pseudocode:

Let S be a string, $\Sigma_1$ *

$map_{str}(S)$

{

$\{if\ (S\ equals\ to\ \varepsilon)\ then\ return\ \varepsilon\}$

$\{else$

$let\ first\ =\ map\ (first\ character\ of\ S)$

$let\ rest\ =\ delete\ first\ character\ of\ S$

$first\ \cup\ map_{str}(rest)$

}

}

In words:

$map_{str}$ takes a string ($\Sigma_1$ *).

If this string equals to the empty string, then the function returns the empty string.

If it does not then the first character of the string is mapped to a character from the $\Sigma_2$ alphabet.

This converted character will be represented by “first”.

Then the first character of the string is deleted and will be represented by “rest”.

“first” will then be unioned with the the results of $map_{str}$ with “rest” as input.

Question assigned to the following page: 4

**4. Caesar Cipher-closed?**

| step | statement | justification |
|---|---|---|
| 1 | $map_{lang}(L) = \{map_{str}(w) \mid w \in L)$ | given |
| 2 | The Caesar Cipher encryption can be represented by $map: \Sigma_1 \rightarrow \Sigma_2$ and $map_{str}: \Sigma_1 * \rightarrow \Sigma_2 *$ functions. | given |
| 3 | Let S be a string, $\Sigma_1 *$<br>$map_{str}(S)$<br>$\{$<br>$\{if\ (S\ equals\ to\ \varepsilon)\ then\ return\ \varepsilon\}$<br>$\{else$<br>$let\ first\ =\ map\ (first\ character\ of\ S)$<br>$let\ rest\ =\ delete\ first\ character\ of\ S$<br>$first\ \cup\ map_{str}(rest)$<br>$\}$<br>$\}$ | recursive function of $map_{str}$ |
| 4 | In the recursive function, the base case is:<br>$if\ (S\ equals\ to\ \varepsilon)\ then\ return\ \varepsilon\}$ | definition of recursive function; base case. |
| 5 | ε can be represented by an NFA, therefore it is a regular language | NFA can recognize regular languages theorem |
| 6 | In the recursive function, the first part of the recursive case is:<br>$let\ first\ =\ map\ (first\ character\ of\ S)$<br>$let\ rest\ =\ delete\ first\ character\ of\ S$<br><br>both first and rest can each be represented by an NFA, therefore they are both regular languages. | (5) |
| 7 | In the recursive function, the second part of the recursive case is:<br>$first\ \cup\ map_{str}(rest)$ | definition of recursive function; recursive case. |
| 8 | The union of two regular languages is closed. | union of regular languages is closed theorem |

Question assigned to the following page: 4

| | | |
|---|---|---|
| 9 | $Let\ N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)\ recognize\ L_1$ | Definition of an NFA |
| 10 | $Let\ N = (Q, \Sigma, \delta, q_0, F)\ recognize\ map_{lang}(L)$ | (9) |
| 11 | 1. $Q = map_{str}(Q_1)$<br>2. $q_0 = map_{str}(q_1)$<br>3. $F = map_{str}(F_1)$<br>4. $\delta(q, a) = \{q \in map_{str}(Q_1)\}$ | (9) and NFA description/computation |
| 12 | Since $map_{str}$ is closed that means the outputs of the description of the NFA that recognizes $map_{lang}(L)$ must be regular languages. This means it must be a closed operation | definition of a closed operation |

∎

Question assigned to the following page: 5

README
time spent: 4 hours
other students: none
books/website used: class slide