hw10

Graded

Student

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Total Points

30 / 42 pts

Question 1

mapping red class problems: iff — **11** / 11 pts

1.1 **EQ_TM undecidable** — **5** / 5 pts

✔ **– 0 pts** Correct

1.2 **E_TM undecidable** — **6** / 6 pts

✔ **– 0 pts** Correct

Question 2

A_TM undecidable — **7** / 10 pts

✔ **– 1 pt** computable fn TM: constructed $\langle M_1 \rangle$ behavior incorrect, should do something like: run input $\langle M \rangle$ with both "cs420" and "fall2022" and then $\langle M_1 \rangle$ accepts its input if $\langle M \rangle$ accepts both

✔ **– 1 pt** iff forward direction proof missing or incorrect: doesnt explain why M_1 accepts some string w when M accepts both "cs420" and "fall2022"

✔ **– 1 pt** iff reverse direction proof missing or incorrect: doesnt explain why M_1 does not accepts some string w when M does not accepts both "cs420" and "fall2022"

Question 3

no mapping red — **6** / 10 pts

✔ **– 2 pts** Did not mention that there would be a contradiction with 'ETM

✔ **– 2 pts** Used ETM definition to justify proof

You did recognize that the theorem is a part of this problem, but failed to recognize that there will be a contradiction because of 'ETM being recognizable.

Question 4

more iff practice

5 / 10 pts

- **– 4 pts** Did not do both directions, (If L Turing recog. then L <=m ATM and if L <=m ATM then L Turing recog.)
- **– 1 pt** Issues with computable function

Did not do the reverse direction, and your computable function should output <M,w> and is not correct (should check if w in L, and if so then f(w) in ATM, etc.)

Question 5

readme

1 / 1 pt

- **– 0 pts** Correct

Question assigned to the following page: 1.1

1) Mapping reducibility step 2: if and only if

1.a.1)

$< M > \ \in E_{TM} \Leftrightarrow \ < M, M_1 > \ \in EQ_{TM}$

Show $< M > \ \in E_{TM}$ if and only if $< M, M_1 > \ \in EQ_{TM}$

1.a.2)

step	statement	justification
1	f: <M> → <M_1, M_2>	given computable function
2	$< M > \ \in E_{TM}$ if and only if $< M, M_1 > \ \in EQ_{TM}$	if and only if statement
3	⇒ if M accepts the empty input, then M and M_1 are equal ● M_1 rejects all input, which means it accepts only the empty input. ○ accepting the empty input = rejects all input ● EQ_{TM} accepts	forward direction, (2)
4	⇐ if M does not accept the empty input, then M and M_1 are not equal ● when M loops: EQ_{TM} rejects ○ M_1 ≠ loop ■ reject all ≠ loop ● M rejects: EQ_{TM} rejects ○ M_1 ≠ accept all except empty input ■ reject all ≠ accept not empty	contrapositive reverse direction, (2)
5	Therefore, the computable function satisfies the if and only if statement.	(1), (2), (3), (4)

⬛

Question assigned to the following page: 1.2

1.b.1)

$< M, w > \ \in A_{TM} \Leftrightarrow < M_1 > \ \in \overline{E_{TM}}$

Show $< M, w > \ \in A_{TM}$ if and only if $< M_1 > \ \in \overline{E_{TM}}$

1.b.2)

step	statement	justification
1	f: <M, w> → <M'>	given computable function
2	$< M, w > \ \in A_{TM}$ if and only if $< M_1 > \ \in \overline{E_{TM}}$	if and only if statement
3	⇒ if M accepts w, then M_1 accepts all TM that are not the empty input ● M_1 accepts if M accepts	forward direction, (2)
4	⇐ if M does not accept w, then M_1 does not accepts all TM that are not the empty input ● if M loops: M_1 loops ● if M rejects: M_1 rejects ● does not accept all TM that are not the empty input = accepts no input, not even empty input.	contrapositive reverse direction, (2)
5	Therefore, the computable function satisfies the if and only if statement.	(1), (2), (3), (4)

⬛

Question assigned to the following page: 1.2

1.b.3)

step	statement	justification
1	proof by contradiction	method of proof
2	Assume: E_{TM} is undecidable and $\overline{E_{TM}}$ is decidable	(1) and given
3	$\overline{E_{TM}}$ has a decider R, use it to create an E_{TM} decider called S. The decider S can be made from R	creating a decider
4	S = “on input <M>, where M is a TM: 1. Run R on input <M> 2. if R accepts, reject, 3. if R rejects, accept”	(3)
5	E_{TM} was assumed to be undecidable, therefore it cannot have a decider S. Also $\overline{E_{TM}}$ is undecidable so there is no decider R, which means there is no decider S.	(2), (3) and contradiction
6	This means that E_{TM} is undecidable.	(5)

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Question assigned to the following page: 2

2) Reproving A_{TM} undecidable

step	statement	justification
1	Prove A_{TM} is undecidable	statement to prove
2	$CS420_{F22} = \{< M > \mid M\ is\ a\ TM$ $where\ CS420 \in L(M)\ and\ Fall2022 \in L(M)\}$ $A_{TM} = \{< M,\ w > \mid M\ is\ a\ TM\ and\ M\ accepts\ w\}$	given
3	Language A is mapping reducible to language B if there is a computable function f: $\Sigma^* \to \Sigma^*$, where for every w, $w \in A \Leftrightarrow f(w) \in B \ldots$	definition of mapping reducibility from lecture22 slide 13
4	show: $CS420_{F22} \le_m A_{TM}$	(2) and (3)
5	step 1: create computable function f: <M> → $<M', w>$, computed by the turing machine S	(3)
6	S = "On input <M>, where M is a TM: 1. use the description of M to construct TM M_1 and string w a. M_1 accepts if M accepts b. M_1 rejects if M rejects c. M_1 loops of M loops d. w is any string 2. output: $<M_1, w>$"	creating a turing machine
7	step 2: show <M> ∈ $CS420_{F22}$ if and only if $< M', w > \in A_{TM}$	if and only if function and (3)
8	⇒ if M accepts CS420 and Fall2022, then M_1 accepts w ● M_1 accepts if M accepts	forwards direction
9	⇐ if M does not accept CS420 and Fall2022, then M_1 does not accepts w ● M loops: M_1 loops ● M rejects: M_1 rejects and does not accept w	contrapositive reverse direction

Question assigned to the following page: 2

10	If $CS420_{F22} \leq_m A_{TM}$ and $CS420_{F22}$ is undecidable, then A_{TM} is undecidable.	corollary theorem from class
11	Therefore, A_{TM} is undecidable	(10)

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Question assigned to the following page: 3

3) No mapping red

step	statement	justification
1	$A_{TM} = \{< M, w > \mid M \textit{ is a TM and M accepts w}\}$ $E_{TM} = \{< M > \mid M \textit{ is a TM and } L(M) = \emptyset\}$	given
2	Language A is mapping reducible to language B if there is a computable function f: $\Sigma^* \to \Sigma^*$, where for every w, $w \in A \Leftrightarrow f(w) \in B \ldots$	definition of mapping reducibility from lecture22 slide 13
3	let A_{TM} = A Let E_{TM} = B Assume A $\leq_m$ B f: $\Sigma^* \to \Sigma^*$, where for every w, $w \in A \Leftrightarrow f(w) \in B$	renaming and (2)
4	if w is in A then f(w) is in B = True In this case all TM in A would be paired to the single element $\emptyset$ in B.	if and only if forward, logic and (3)
5	if w is not in A then f(w) is not in B = False The only element that is not in A is $\emptyset$, but it is in B. All other TMs are in A, but not in B. This means you cannot pair a TM not in A that is not in B. Therefore, the computable function does not satisfy the if and only if statement that would be made to make these languages mapping reducible.	if and only if contrapositive reverse, logic and (3)
6	Therefore, there cannot exist a mapping reduction from A_{TM} to E_{TM}	(5)

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Question assigned to the following page: 4

4) More if and only if practice

step	statement	justification
1	L is Turing-Recognizable if-and-only-if L is mapping reducible to A_{TM}	given statement to prove
2	A language is turing recognizable if there is a turing machine that recognizes it.	definition of turing-recognizable from class.
3	Language A is mapping reducible to language B if there is a computable function f: $\Sigma^* \rightarrow \Sigma^*$, where for every w, $w \in A \Leftrightarrow f(w) \in B$ …	definition of mapping reducibility from lecture22 slide 13
4	A_{TM} is turing recognizable	proved in class lecture
5	Let L be mapping reducible to A_{TM} $L \leq_m A_{TM}$	assumption
6	If $L \leq_m A_{TM}$ and A_{TM} is turing recognizable then L is turing recognizable Let A = L Let B = A_{TM} If $A \leq_m B$ and B is turing recognizable then A is turing recognizable M = Turing Machine for B f = reduction form A to B Describe Turing Machine N: N = “On input w: 1. Compute f(w). 2. Run M on input f(w) and output whatever M outputs.”	theorem proof, creating a turing machine and (5)
7	Therefore, L is turing recognizable	(6) and (1)

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Question assigned to the following page: 5

README
other students: none
websites/books used: class lectures and slides
time spent: 5 hours