# hw9

Graded

**Student**

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**Total Points**

28 / 42 pts

**Question 1**

countable or uncountable — **9.5** / 12 pts

1.1 **countable** — **5.5** / 6 pts

- ✔ **– 0.5 pts** Not specific enough stating that for bijection, mapping must start with length 1 strings, then length 2, ...

1.2 **uncountable** — **4** / 6 pts

- ✔ **– 1 pt** Possibility for number to still end up in set
- ✔ **– 1 pt** diagonalization not correct; cannot assume sets have an order
- If you're going to do something like this, watch out for corner cases, 2 is both even and prime, and there are multiple odd prime numbers. The point of this is to take a number not in that set without specifying an order to the set, which you do not do since you specify to take a certain number from a certain set.

**Question 2**

**cs420 fall 2022 undecidable** — **7.5** / 10 pts

- ✔ **– 1 pt** Did not construct/did not specify inner TM M1 (or equivalent) for accepting "CS420" and "Fall2022"
- ✔ **– 0.5 pts** Incomplete termination argument
- ✔ **– 1 pt** Issue with TM construction
- TM M1 should be checking for x equaling the strings, you shouldn't just assign x to be them or else you're negating all the other possibilities. Also, if R accepts, then that means the simulation with M on w already happened in M1 and M1 accepted.

**Question 3**

**closed op for decidable langs** — **7** / 9 pts

- ✔ **– 1 pt** decider input incorrect or didnt specify input; should just be a string w
- ✔ **– 1 pt** termination argument for decider: missing or incorrect

Question 4

## re-proving EQ_TM undecidable

**3** / 10 pts

- ✔ **− 4 pts** didnt construct decider for CS420_F22
- ✔ **− 1 pt** missing termination argument for decider
- ✔ **− 2 pts** didnt construct TM that accepts "cs420" and "fall2022", to compare with input M

Question 5

## readme

**1** / 1 pt

- ✔ **− 0 pts** Correct

Question assigned to the following page: 1.1

**1) Countable or Not Countable?**

1a) Infinite Monkey Theorem
"a monkey hitting keys at random on a typewriter keyboard for an infinite amount of time will almost surely type any given text, such as the complete works of William Shakespeare"

<table>
<tr><td>step</td><td>statement</td><td>justification</td></tr>
<tr><td>1</td><td>prove the set of possible monkey outputs is at least countable.</td><td>statement to prove</td></tr>
<tr><td>2</td><td>You may assume that a monkey output is a string drawn from the alphabet<br>$\Sigma = \{a, ..., z, A, \ldots, Z, \_\ \}$, where _ represents a space.</td><td>given</td></tr>
<tr><td>3</td><td>A set is countable if it is finite or there exists a bijection between the set and the set of natural numbers.</td><td>definition of countable from class</td></tr>
<tr><td>4</td><td>The monkey is hitting keys for an infinite amount of time.</td><td>given</td></tr>
<tr><td>5</td><td>Let $\beta = \mathit{element} \in \Sigma$<br>Let $\alpha(\beta) = \mathit{natural\ number\ element\ is\ paired\ to}$
<table>
<tr><td>β</td><td>α(β)</td></tr>
<tr><td>a</td><td>1</td></tr>
<tr><td>b</td><td>2</td></tr>
<tr><td>…</td><td>…</td></tr>
<tr><td>Z</td><td></td></tr>
<tr><td>_</td><td>53</td></tr>
</table>
a to z = 26<br>A to Z and space = 27<br>total = 53</td><td>alphabet is finite</td></tr>
<tr><td>6</td><td>Let f(x) be a function that maps all combinations of elements to their own unique natural numbers starting at 54.</td><td>computation</td></tr>
<tr><td>7</td><td>There is a bijection between the set of possible monkey outputs and the natural numbers</td><td>(5) and (6)</td></tr>
</table>

Question assigned to the following page: 1.1

| | | |
|---|---|---|
| 8 | The set of possible monkey outputs is at least countable. | (3) |

1b)

<table>
<tr><td>step</td><td>statement</td><td>justification</td></tr>
<tr><td>1</td><td>Now show that the set of all possible such sets, i.e., the set of all possible infinite subsets of the natural numbers, is not countable</td><td>statement to prove</td></tr>
<tr><td>2</td><td>proof by contradiction</td><td>proof strategy</td></tr>
<tr><td>3</td><td>assume a bijection between the natural and the set of all possible infinite subsets of the natural numbers exists.</td><td>(2)</td></tr>
<tr><td>4</td><td><table>
<tr><td>n</td><td>f(n)</td></tr>
<tr><td>1</td><td>{even, odd, prime…}</td></tr>
<tr><td>2</td><td>{odd, prime, even</td></tr>
<tr><td>…</td><td></td></tr>
</table>
<br>This shows that some infinite subset of the natural number is not mapped to.<br><br>x = {prime, odd, even, …}<br>This set is not in the mapping</td><td>diagonalization</td></tr>
<tr><td>5</td><td>the set of all possible infinite subsets of the natural numbers, is not countable</td><td>(4)</td></tr>
</table>

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Question assigned to the following page: 2

## 2) Trying to decide about CS420, Fall 2022

| step | statement | justification |
|---|---|---|
| 1 | prove the following language is undecidable:<br>$CS420_{F22} = \{< M > \mid M\ is\ a\ TM$<br>$where\ CS420 \in L(M)\ and\ Fall2022 \in L(M)\}$ | statement to prove |
| 2 | proof by contradiction | method of proof |
| 3 | Assume $CS420_{F22}$ has a decider R; use it to create a decider for $A_{TM}$ | assumption and (2) |
| 4 | Let x be a string from L(M), where<br>x = CS420 = Fall2022<br>S = "On input <M>, an encoding of a TM M:<br>1. run TM R on input <M><br>2. If R rejects, reject.<br>3. If R accepts, simulate M on x until it halts.<br>4. If M has accepted, accept; if M has rejected, reject." | creating a decider |
| 5 | Step 1: R is a decider so it always halts<br>Step 3: M always halts because R said it does. | termination argument |
| 6 | $A_{TM}$ is undecidable, which means this decider does not exist. | using previous theorem |
| 7 | $CS420_{F22}$ is undecidable. | (6) |

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Question assigned to the following page: 3

**3) A closed operation for decidable languages**

OV($L_1$, $L_2$) = The largest set of all strings X such that X ⊆ $L_1$ and X ⊆ $L_2$

prove OV is closed for decidable languages

3.1) If $L_1$, $L_2$ and the resulting set of OV($L_1$, $L_2$) are decidable languages, then OV is closed for decidable languages.

3.2)

| step | statement | justification |
|---|---|---|
| 1 | Let $L_1$ and $L_2$ be decidable languages, with A and B being their corresponding deciders. | given |
| 2 | Let F = OV($L_1$, $L_2$), where<br>$F$ = {<A, X> \| A is a decider and X is a string} | renaming and given |
| 3 | $A_F$ has a decider R; use it to create a decider for $A_{TM}$:<br>S = “On input <A, X>, where A is a TM and X is a string:<br>1. run TM R on input <A, X><br>2. If R rejects, reject.<br>3. If R accepts, simulate A on x until it halts.<br>4. If A has accepted, accept; if A has rejected, reject.” | computation |
| 4 | If $L_1$, $L_2$ and the resulting set of OV($L_1$, $L_2$) are decidable languages, then OV is closed for decidable languages. | (3) |

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Question assigned to the following page: 4

**4) Re-proving that EQ_TM is undecidable**

| step | statement | justification |
| --- | --- | --- |
| 1 | prove the following language is undecidable:<br>EQ_TM | statement to prove |
| 2 | proof by contradiction | method of proof |
| 3 | EQ_TM has decider R; use it to create decider for $CS420_{F22}$ | assumption |
| | S= “On input <M> where M is a TM:<br>1. Run R on input… | |

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Question assigned to the following page: 5

**README**
other students: none
websites/books used: lecture slides
time spent: 1hr (did not have time to finish or do it correctly)