

# hw8

● Graded

## Student

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## Total Points

35.5 / 40 pts

## Question 1

### Chomsky Normal Form

9 / 9 pts

✓ - 0 pts Correct

## Question 2

### Algorithm About DFAs

6 / 10 pts

✓ - 2 pts decider behavior incorrect; didnt construct DFA that accepts all strings with a single B char (from hw1)

✓ - 2 pts decider behavior incorrect; should compare input DFA to DFA that accepts all strings with a single B char (from hw1) using EQ\_DFA decider

## Question 3

### Algorithm About PDAs

10 / 10 pts

✓ - 0 pts Correct

## Question 4

### Regular and Decidable?

■ 9.5 / 10 pts

✓ - 0.5 pts Incomplete termination argument

💬 Did not state that decider halts

## Question 5

### readme

1 / 1 pt

✓ - 0 pts Correct

Question assigned to the following page: [1](#)

# 1) Chomsky Normal Form

Given CFG:

- $E \rightarrow T \mid E-T \mid -T$
- $T \rightarrow T \times F \mid F$
- $F \rightarrow L \mid F \wedge L$
- $L \rightarrow x \mid \mid (E)$

Step 1: new start var

- $S_0 \rightarrow E$
- $E \rightarrow T \mid E-T \mid -T$
- $T \rightarrow T \times F \mid F$
- $F \rightarrow L \mid F \wedge L$
- $L \rightarrow x \mid \mid (E)$

Step 2: remove  $\epsilon$

- $S_0 \rightarrow E$
- $E \rightarrow T \mid E-T \mid -T$
- $T \rightarrow T \times F \mid F$
- $F \rightarrow L \mid F \wedge L$
- $L \rightarrow x \mid \mid (E)$

Step 3: remove  $A \rightarrow B$

- $S_0 \rightarrow E$
- $E \rightarrow T \mid E-T \mid -T$
- $T \rightarrow T \times F \mid F$
- $F \rightarrow L \mid F \wedge L$
- $L \rightarrow x \mid \mid (E)$

→

- $S_0 \rightarrow T \times F \mid x \mid \mid (E) \mid F \wedge L \mid E-T \mid -T$
- $E \rightarrow T \times F \mid x \mid \mid (E) \mid F \wedge L \mid E-T \mid -T$
- $T \rightarrow T \times F \mid x \mid \mid (E) \mid F \wedge L$
- $F \rightarrow x \mid \mid (E) \mid F \wedge L$
- $L \rightarrow x \mid \mid (E)$

Step 4: split up RHS > 2

- $S_0 \rightarrow T \times F \mid x \mid \mid (E) \mid F \wedge L \mid E-T \mid -T$
- $E \rightarrow T \times F \mid x \mid \mid (E) \mid F \wedge L \mid E-T \mid -T$
- $T \rightarrow T \times F \mid x \mid \mid (E) \mid F \wedge L$
- $F \rightarrow x \mid \mid (E) \mid F \wedge L$
- $L \rightarrow x \mid \mid (E)$

→

- $A \rightarrow xF$
- $B \rightarrow E$
- $C \rightarrow \wedge L$
- $D \rightarrow E-$
- $S_0 \rightarrow TA \mid x \mid \mid (B \mid FC \mid DT) \mid -T$
- $E \rightarrow TA \mid x \mid \mid (B \mid FC \mid DT) \mid -T$
- $T \rightarrow TA \mid x \mid \mid (B \mid FC$
- $F \rightarrow x \mid \mid (B \mid FC$
- $L \rightarrow x \mid \mid (B$

Question assigned to the following page: [1](#)

Step 5: Replace all terminals

- $A \rightarrow xF$
- $B \rightarrow E)$
- $C \rightarrow ^1L$
- $D \rightarrow E-$
- $S_0 \rightarrow TA|x|1|(B|FC|DT|-T$
- $E \rightarrow TA|x|1|(B|FC|DT|-T$
- $T \rightarrow TA|x|1|(B|FC$
- $F \rightarrow x|1|(B|FC$
- $L \rightarrow x|1|(B$

→

- $J \rightarrow ($
- $K \rightarrow -$
- $G \rightarrow x$
- $H \rightarrow )$
- $I \rightarrow ^1$
- $A \rightarrow GF$
- $B \rightarrow EH$
- $C \rightarrow IL$
- $D \rightarrow EK$

- $S_0 \rightarrow TA|x|1|JB|FC|DT|KT$
- $E \rightarrow TA|x|1|JB|FC|DT|KT$
- $T \rightarrow TA|x|1|JB|FC$
- $F \rightarrow x|1|JB|FC$
- $L \rightarrow x|1|JB$

Chomsky Normal Form:

- $J \rightarrow ($
- $K \rightarrow -$
- $G \rightarrow x$
- $H \rightarrow )$
- $I \rightarrow ^1$
- $A \rightarrow GF$
- $B \rightarrow EH$
- $C \rightarrow IL$
- $D \rightarrow EK$
- $S_0 \rightarrow TA|x|1|JB|FC|DT|KT$
- $E \rightarrow TA|x|1|JB|FC|DT|KT$
- $T \rightarrow TA|x|1|JB|FC$
- $F \rightarrow x|1|JB|FC$
- $L \rightarrow x|1|JB$

Question assigned to the following page: [2](#)

## 2) An algorithm about DFA's?

Prove that the following language is decidable:

$B_{DFA} = \{ \langle M \rangle \mid M \text{ is a DFA that accepts all strings containing a single } B \text{ character} \}$

step	statement	justification
1	A language is decidable if there is a decider that recognizes it.	definition of a decidable language
2	Let $w$ be a string with a single $B$ character in it.	given
3	A decider for $B_{DFA}$ : X = "On input $\langle M \rangle$ , where $M$ is DFA: 1. Simulate $M$ on input string $w$ . 2. If the simulation ends in an accept state, accept. If it ends in a nonaccepting state, reject."	Creating a decider for $B_{DFA}$ and (1)
4	Where "simulate" = <ul style="list-style-type: none"><li>• Define "current" state <math>q_{current} = \text{start state } q_0</math></li><li>• For each input char <math>x</math> in <math>w</math>...<ul style="list-style-type: none"><li>◦ Define <math>q_{next} = \delta(q_{current}, x)</math></li><li>◦ Set <math>q_{current} = q_{next}</math></li></ul></li></ul>	A detailed explanation of what is meant by "simulate" in step 3
5	Termination argument: X from step 3 is a decider because the input $w$ is a finite string, so the loop in the simulation has a finite amount of iterations and will always halt.	termination argument and (1)
6	$B_{DFA}$ has a decider X that recognizes it, which means that it is a decidable language.	(1) and (3)

Question assigned to the following page: [3](#)



### 3) An algorithm about PDA's?

Prove the following language is decidable:

$PW = \{ \langle P, w \rangle \mid P \text{ is a PDA where } w \in \text{Lang}(P) \}$

step	statement	justification
1	A language is decidable if there is a decider that recognizes it.	definition of a decidable language
2	$A_{CFG}$ is a decidable language, where S represents its decider.	theorem from class
3	Decider for $A_{CFG}$ : X = "On input $\langle P, w \rangle$ , where P is a PDA and w is a string in the language of P: 1. Convert PDA P to an equivalent CFG C, using the procedure $CFG \rightarrow PDA$ from the theorem $CFG \leftrightarrow PDA$ . 2. Run the turing machine S on input $\langle C, w \rangle$ 3. If M accepts, accept; otherwise, reject."	creating a decider for $A_{CFG}$ , $CFG \leftrightarrow PDA$ theorem, (1) and (2)
4	Termination argument: X from step 3 is a decider because: <ul style="list-style-type: none"><li>• It's step one (converting) always halts because there's a finite number of states in a PDA.</li><li>• It's step two always halts because the turing machine S is a decider.</li></ul>	termination argument and (1)
5	$A_{CFG}$ has a decider X that recognizes it, which means that it is a decidable language.	(1) and (3)

Question assigned to the following page: [4](#)

#### 4) Regular and Decidable?

If a language  $L$  is a regular language, then  $L$  is decidable.

DFA

step	statement	justification
1	A language is regular if there is a DFA that recognizes it.	definition of a regular language from class
2	A language is decidable if there is a decider that recognizes it.	definition of a decidable language from class
3	Let $X$ be a DFA, $(Q, \Sigma, \delta, q_0, F)$ that recognizes the language $L$	(1) and given
4	$A_{DFA}$ is a decidable language, where $M$ represents a decider that recognizes it.	theorem from class
5	A decider for $L$ : $Y =$ "On input $\langle X, w \rangle$ , where $X$ is a DFA and $w$ is a string from the language $L$ : 1. Run the turing machine $M$ on input $\langle X, w \rangle$ . 2. If $M$ accepts, accept; otherwise reject."	creating a decider
6	termination argument: $Y$ from step 5 is a decider because: <ul style="list-style-type: none"><li>• it's step one is finite because <math>M</math> is a decider</li></ul>	termination argument
7	There is a DFA $X$ and a decider $Y$ that recognize the language $L$ . Therefore, $L$ is a regular language and it is decidable.	(1), (2) and (5)

Question assigned to the following page: [4](#)

# NFA

step	statement	justification
1	A language is regular if there is a NFA that recognizes it.	definition of a regular language from class
2	A language is decidable if there is a decider that recognizes it.	definition of a decidable language from class
3	Let X be an NFA, $(Q, \Sigma, \delta, q_0, F)$ that recognizes the language L	(1) and given
4	$A_{NFA}$ is a decidable language, where N represents a decider that recognizes it.	theorem from class
5	A decider for L: Y = "On input $\langle X, w \rangle$ , where X is an NFA and w is a string from the language L: 1. Run the turing machine N on input $\langle X, w \rangle$ . 2. If N accepts, accept; otherwise reject."	creating a decider
6	termination argument: Y from step 5 is a decider because: • it's step one is finite because N is a decider	termination argument
7	There is a NFA X and a decider Y that recognize the language L. Therefore, L is a regular language and it is decidable.	(1), (2) and (5)

Question assigned to the following page: [4](#)

# Regex

step	statement	justification
1	A language is regular if there is a regular expression that recognizes it.	definition of a regular language from class
2	A language is decidable if there is a decider that recognizes it.	definition of a decidable language from class
3	Let X be regular expression that recognizes L	(1) and given
4	$A_{\text{REG}}$ is a decidable language, where P represents a decider that recognizes it.	theorem from class
5	A decider for L: Y = "On input $\langle X, w \rangle$ , where X is a regular expression and w is a string from the language L: 1. Run the turing machine P on input $\langle X, w \rangle$ . 2. If P accepts, accept; otherwise reject."	creating a decider
6	termination argument: Y from step 5 is a decider because: • it's step one is finite because P is a decider	termination argument
7	There is a regular expression X and a decider Y that recognize the language L. Therefore, L is a regular language and it is decidable.	(1), (2) and (5)

Question assigned to the following page: [5](#)



**README**

name of other students: none

books or websites used: class slides and lecture

time spent: 3 hours