Technique for Order Preference by Similarity to Ideal Solution

(TOPSIS)

Origin and History

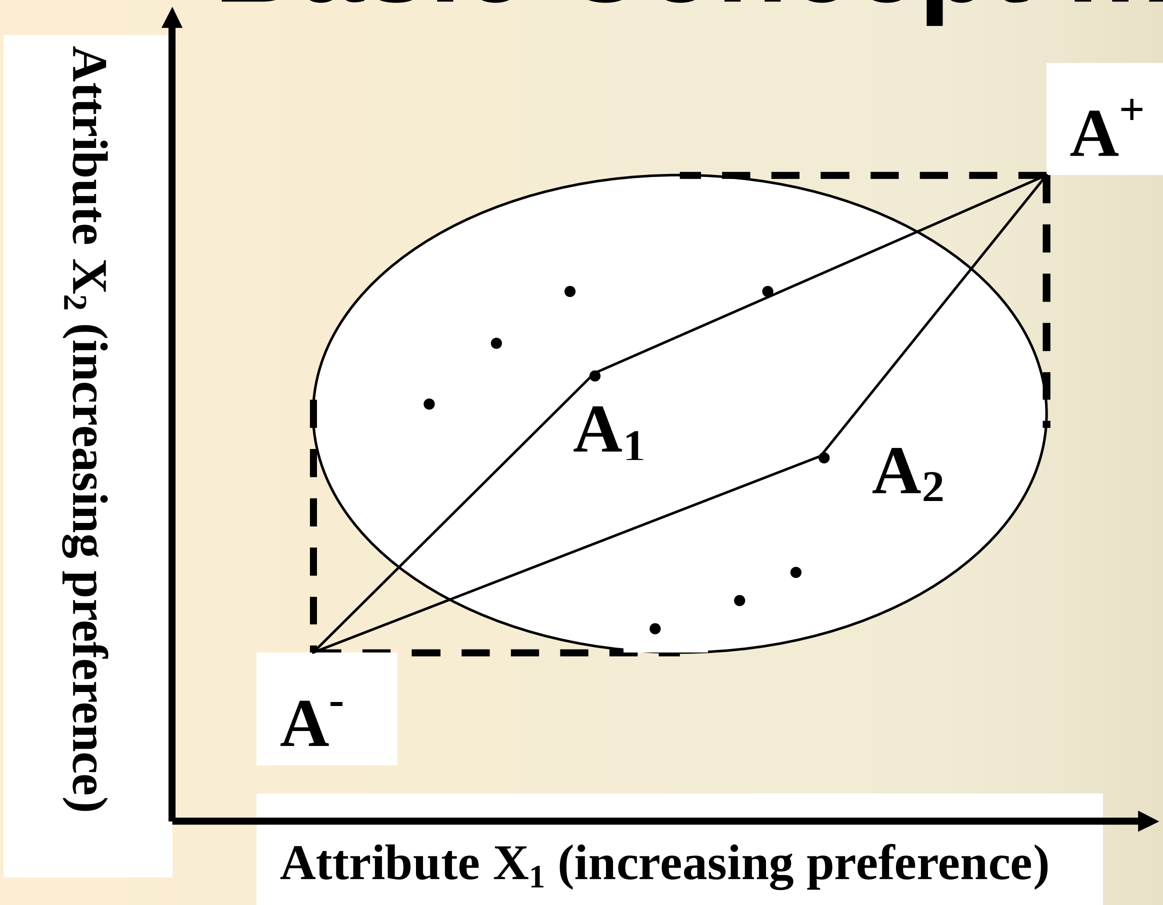
1980: development by Kwangsun Yoon and Hwang Ching-Lai

- **Yoon, K., "System Selection by Multiple Attribute Decision Making," Ph. D. Dissertation, Kansas State University, Manhattan, Kansas, 1980.**
- **Yoon, K. and C. L. Hwang, "TOPSIS (Technique for Order Preference by Similarity to Ideal Solution)- A Multiple Attribute Decision Making," a paper to be published, 1980.**

Basic Concept

- The chosen Alternative should have the shortest distance from the ideal solution and the farthest from the negative-ideal solution.

Basic Concept ...



It is very difficult to justify the selection of A_1 or A_2

Decision Matrix

- **m Alternative, n Attributes (or criteria)**

$$\begin{array}{cc} & \begin{array}{ccccccc} X_1 & X_2 & X_3 & \cdot & \cdot & \cdot & X_n \end{array} \\ D = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{array} & \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdot & \cdot & \cdot & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdot & \cdot & \cdot & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdot & \cdot & \cdot & x_{3n} \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ x_{m1} & x_{m2} & x_{m3} & \cdot & \cdot & \cdot & x_{mn} \end{bmatrix} \end{array}$$

Hypothesis-1

- **Each Attribute in the Decision Matrix takes either monotonically increasing or monotonically decreasing utility**

Hypothesis-2

- **A Set of Weights for the Attributes is required**

Hypothesis-3

- **Any Outcome which is expressed in a non-numerical way, should be quantified through the appropriate scaling technique**

Steps-1

- **Construct the Normalized Decision Matrix**
  - **To transform the various attribute dimensions into non-dimensional attributes, which allows comparison across the attributes**

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$$

Steps-2

- **Construct the Weighted Normalized Decision Matrix**

$$V = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1j} & \cdots & v_{1n} \\ \vdots & \vdots & & \vdots & & \vdots \\ v_{i1} & v_{i2} & & v_{ij} & & v_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mj} & \cdots & v_{mn} \end{bmatrix} = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \cdots & w_j r_{ij} & \cdots & w_n r_{1n} \\ \vdots & \vdots & & \vdots & & \vdots \\ w_1 r_{i1} & w_2 r_{i2} & & w_j r_{ij} & & w_n r_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ w_1 r_{m1} & w_2 r_{m2} & \cdots & w_j r_{mj} & \cdots & w_n r_{mn} \end{bmatrix}$$

Steps-3

- **Determine Ideal and Negative-Ideal Solutions**

$$A^{+} = \{(\max_{i} v_{ij} \mid j \in J), (\min_{i} v_{ij} \mid j \in J^{'}) \mid i = 1,2,\ldots m\}$$
$$= \{v_1^{+}, v_2^{+}, \ldots, v_j^{+}, \ldots, v_n^{+}\}$$

$$A^{-} = \{(\min_{i} v_{ij} \mid j \in J), (\max_{i} v_{ij} \mid j \in J^{'}) \mid i = 1,2,\ldots m\}$$
$$= \{v_1^{-}, v_2^{-}, \ldots, v_j^{-}, \ldots, v_n^{-}\}$$

$$where \quad J = \left\{ j = 1,2,\ldots,n \middle| j \quad associated \quad with \quad benefit \quad criteria \right\}$$
$$J^{'} = \left\{ j = 1,2,\ldots,n \middle| j \quad associated \quad with \quad \cos t \quad criteria \right\}$$

Steps-4

- **Calculate the Separation Measure:**
  - **Ideal Separation**

$$s_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2} \qquad i = 1,2,\ldots,m$$

  - **Negative-Ideal Separation**

$$s_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2} \qquad i = 1,2,\ldots,m$$

Steps-5

- **Calculate the Relative Closeness to the Ideal Solution**

$$c_i^* = \frac{S_i^-}{(S_i^+ + S_i^-)}, \quad 0 < c_i^+ < 1, \quad i = 1,2,\dots,m$$

$$c_i^* = 1 \quad if \quad A_i = A^+$$

$$c_i^* = 0 \quad if \quad A_i = A^-$$

Steps-6

- **Rank the preference order**
  - **A set of alternatives can now be preference ranked according to the descending order of c_i^***

An example of using TOPSIS Method

	Style	Reliability	Fuel Eco.	Cost
Weight	0.1	0.4	0.3	0.2
Civic	7	9	9	8
Saturn	8	7	8	7
Ford	9	6	8	9
Mazda	6	7	8	6

Steps of TOPSIS

- Step 1: Calculate $(\Sigma x^2_{ij})^{1/2}$ for each column and divide each column by that to get r_{ij}

	Style	Rel.	Fuel	Cost
Civic	**0.46**	**0.61**	**0.54**	**0.53**
Saturn	**0.53**	**0.48**	**0.48**	**0.46**
Ford	**0.59**	**0.41**	**0.48**	**0.59**
Mazda	**0.40**	**0.48**	**0.48**	**0.40**

Steps of TOPSIS

- Step 2 : Multiply each Column by w_j to get v_{ij}.

	Style	Rel.	Fuel	Cost
Civic	**0.046**	**0.244**	**0.162**	**0.106**
Saturn	**0.053**	**0.192**	**0.144**	**0.092**
Ford	**0.059**	**0.164**	**0.144**	**0.118**
Mazda	**0.040**	**0.192**	**0.144**	**0.080**

Steps of TOPSIS

Step 3 (a): Determine Ideal Solution A*.

A* = {0.059, 0.244, 0.162, 0.080}

	Style	Rel.	Fuel	Cost
Civic	**0.046**	0.244	0.162	**0.106**
Saturn	**0.053**	**0.192**	**0.144**	**0.092**
Ford	0.059	**0.164**	**0.144**	**0.118**
Mazda	**0.040**	**0.192**	**0.144**	0.080

Steps of TOPSIS

Step 3 (b): Find Negative Ideal Solution A^-.

$A^- = \{0.040, 0.164, 0.144, 0.118\}$

	Style	Rel.	Fuel	Cost
Civic	0.046	0.244	0.162	0.106
Saturn	0.053	0.192	0.144	0.092
Ford	0.059	0.164	0.144	0.118
Mazda	0.040	0.192	0.144	0.080

Steps of TOPSIS

Step 4 (a): Determine Separation From Ideal Solution

$A^* = \{0.059, 0.244, 0.162, 0.080\}$

$S_i^* = [\Sigma (v_j^* - v_{ij})^2]^{½}$ for each row j

	Style	Rel.	Fuel	Cost
Civic	$(.046-.059)^2$	$(.244-.244)^2$	$(0)^2$	$(.026)^2$
Saturn	$(.053-.059)^2$	$(.192-.244)^2$	$(-.018)^2$	$(.012)^2$
Ford	$(.053-.059)^2$	$(.164-.244)^2$	$(-.018)^2$	$(.038)^2$
Mazda	$(.053-.059)^2$	$(.192-.244)^2$	$(-.018)^2$	$(.0)^2$

Steps of TOPSIS

Step 4 (a): Determine Separation From Ideal Solution S_i^*

	$\Sigma(v_j^* - v_{ij})^2$	$S_i^* = [\Sigma (v_j^* - v_{ij})^2]^{1/2}$
Civic	**0.000845**	0.029
Saturn	**0.003208**	0.057
Ford	**0.008186**	0.090
Mazda	**0.003389**	0.058

Steps of TOPSIS

Step 4: Determine Separation From Negative Ideal Solution s_i^-

	$\Sigma(v_j^- - v_{ij})^2$	$Si^- = [\, \Sigma\, (v_j^- - v_{ij})2\,]^{½}$
Civic	**0.006904**	0.083
Saturn	**0.001629**	0.040
Ford	**0.000361**	0.019
Mazda	**0.002228**	0.047

Steps of TOPSIS

Step 5: Calculate the relative closeness to the ideal solution $C_i^* = S^-_i / (S_i^* + S^-_i)$

	$S^-_i / (S_i^* + S^-_i)$	C_i^*		
Civic	**0.083/0.112**	0.74	←	BEST
Saturn	**0.040/0.097**	0.41		
Ford	**0.019/0.109**	0.17	←	WORSE
Mazda	**0.047/0.105**	0.45		