

## A LOCATION-INVENTORY-ROUTING-PROBLEM WITH PERISHABLE PRODUCTS

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### *Abstract:*

*We introduce a formulation for the inventory-location problem with routing costs. A single supplier distributes a single product to multiple retailers with deterministic demand through a set of distribution centers. The product is a perishable product with a certain shelf-life. The problem is to determine how many warehouses to open, where to locate them, and which customers to allocate to them. We develop a mathematical model for the problem, and solve small size instances of the problem using GAMS. Our numerical studies demonstrate the benefits of integrating different supply chain decision levels.*

### *Keywords:*

*Integrated Supply Chain models; Location models; Vehicle routing; Inventory; Perishable products*

## 1 Introduction

Researchers and practitioners often classify supply chain decisions into strategic, tactical, and operational, based on time horizon of impact. Strategic decisions deal with the facilities location and they affect the firm over significantly longer time periods than others. Tactical decisions are related to inventory management. Finally, distribution decisions are considered operational and have the lowest time horizon of impact. Historically, these decisions were treated separately. This led to sub-optimality and excessive costs. Recently, supply chain managers and researchers have realized the importance of integration of supply chain decisions. Many researchers showed significant savings when considering a combination of the aforementioned decisions into a single model. Many models presented in the literature combine two of the supply chain decisions into one single model. However, few models integrate all three decisions and solve them simultaneously.

Perishable products provide extra challenges to the supply chain due to their limited shelf-life. As such, storing these goods for more than their designated shelf-time leads to expiry and they must be discarded. Hence, quantities delivered to retailers are limited by the shelf-life of goods as well as the retailer's holding capacity.

The objective of our model is to, at minimum cost, meet the following requirements: decide how many warehouses are needed, where to locate the opened warehouses, and how to allocate customers to them; find out how much to keep in inventory at any time period; and understand how to build the vehicles' routes starting from opened warehouse to customers and back to that warehouse. The model is formulated as a Mixed Integer Program (MIP). The model is solved simultaneously to avoid sub-optimal solutions.

## 2 Literature Review

The supply chain management involves three decisions: facilities location, inventory management, and distribution and routing. Initially, these decisions were studied separately. Location decisions have been studied extensively. For example, Hindi and Pienkosz [1999] studied the capacitated single-source plant location. Drezner and Scott [2010] studied the location of a single plant for a given set of customers. Moreover, Jayaraman [1998] studied the capacitated multi-product warehouse location problem. Pirkul and Jayaraman [1998] extended the previous model by also locating a predetermined number of plants. Furthermore, Amiri [2006] formulated a model to find the numbers, locations, and capacities of both warehouses and plants for a single product. On inventory management, Chen et al. [2001] studied a two-echelon distribution system. Yang and Wee [2002] analyzed an integrated single-vendor multi-buyers inventory system of a deteriorating item. Wee and Yang [2004] presented a single-producer, multi-distributors and multi-retailers inventory

system. Furthermore, Axsater et al. [2007] considered two-level inventory with one warehouse and multiple retailers. Routing decisions can be formulated in different ways. A summary of the development of vehicle routing research can be found in Bertsimas and Simchi-Levi [1996]. Moreover, Laporte et al. [2000] and Toth and Vigo [2002] surveyed the heuristics for vehicle routing problems. They included extensive computational results. Bektas [2006] concluded that the characteristics of the multiple salesman problem seem appropriate for real life applications and it is possible to extend the problem to a wide variety of vehicle routing problems by incorporating some additional side constraints. He provided an overview of these multiple salesman problems.

Later on, researchers integrated two of the above problems. Integrated models consist of a set of problems, rather than one problem. Simultaneous solutions of the model would, in general, present significant cost savings compared with previous approaches. Nagy and Salhi [2007] present a survey of location-routing problems. They propose a classification scheme and look at a number of problem variants. Both exact and heuristic algorithms were considered. Furthermore, Inventory-routing problems are surveyed in Baita et al. [1998] and Moin and Salhi [2006]. Finally, several papers studied location-inventory problems. Daskin et al. [2002] and Shen et al. [2003] study a joint location-inventory model with the risk pooling proposed by Eppen [1979]. Both models include location, shipment, and nonlinear safety stock inventory costs. While Daskin et al. [2002] apply Lagrangian relaxation, Shen et al. [2003] use Column generation to solve their model. Recently, Diabat et al. studied multi-echelon joint location-inventory model deals with the distribution of a single product. Their model is formulated as a nonlinear mixed-integer program, and solved problems of sizes ranging from 49 to 150 nodes using Lagrangian relaxation.

Based on this previous work, researchers tended to integrate all the three components of the supply chain management into one model. Max Shen and Qi [2007] modified Daskin et al. [2002] model and proposed a stochastic model that considers the location, inventory and routing costs. They approximate the shipment from a warehouse to its customers using a vehicle routing model. They used Lagrangian relaxation to solve sub-problems. Ahmadi Javid and Azad [2010] present a model which simultaneously optimizes location, inventory and routing decisions without approximation. They show that the approximation in [Max Shen and Qi, 2007] is only applicable under some restrictive assumptions. The model is formulated as a mixed integer convex program. They proposed a hybrid algorithm of Tabu Search and Simulated Annealing.

Supply chains with perishable products have been studied in different lines of research. Some researchers extended the economic order quantity (EOQ) policy for inventory models which include perishable products. For example, Giri and Chaudhuri [1998] proposed an inventory model for a perishable product where the demand rate is a function of the on-hand inventory, and the holding cost is nonlinear. Moreover, Padmanabhan and Vrat [1995] proposed stock-dependent selling rate model where the backlogging function was assumed to be dependent on the amount of demand backlogged. Dye and Ouyang [2005] extended their model by introducing a time-proportional backlogging rate. In a different line of research, Hsu et al. [2007] extended the vehicle routing problem with time-windows discussed in a number of papers (e.g. Koskosidis et al. [1992] and Sexton and Choi [1986]), by considering the randomness of the perishable food delivery process. However, both EOQ and VRP extensions lack the integration of inventory and transportation decisions. Hence, the problem of sub-optimality might arise.

Recently, Le et al. [2011] combined inventory and routing components into one model. They proposed a column generation-based heuristic to solve the model. They showed significant savings when using their model. On the other hand, Shen et al. [2003] presented a joint location-inventory model for blood distribution system. Consequently, we try to extend these models and integrate location, inventory, and routing components into one model.

### 3 Model Formulation

#### 3.1 Model Description

This model deals with the distribution of a single commodity from a single manufacturer to a set of retailers,  $I$ , through a set of warehouses that can be located at various pre-determined sites,  $W$ . The retailers deal with deterministic demand. The goods are distributed by a homogeneous fleet of vehicles of the same capacity. In this model, it is assumed that out-of-stock situations never occur. Moreover, the goods have a fixed shelf-life, which is measured by the number of time periods. Retailers' demand is deterministic but it may vary across the time periods. Inventory holding costs vary slightly across time. The inventory holding cost for warehouses was assumed the same for all candidate warehouses and, therefore, was neglected. Inventory level at retailers is limited by two constraints, namely: physical capacity at retailers site and shelf-life of products. It was assumed in Le et al. [2011] that perishability dominates the physical capacity of retailers storage site in defining the upper bound inventory. We use the same assumption. Consequently, upper bound inventory level at customer sites is only defined by the perishability restrictions.

We define a feasible route as a route in which a vehicle starts from a candidate warehouse, visits a number of retailers, and comes back to the same warehouse. As such, a feasible route does not pass by more than one warehouse. As a result, the number of possible feasible routes will be  $W \times 2^I$ , where  $W$  and  $I$  are the number of warehouses and retailers, respectively. Feasible routes and the associated parameters ( $\alpha_{ir}$  and  $\beta_{rw}$ , as explained later) are required to define the model and, therefore, generated before solving the model. We assume vehicle capacity to be larger than the maximum customer demand at any time period. Moreover, in any time period, each vehicle travels at most on one route, and customers are visited at most once.

Three major cost components are considered in the objective function of the model. They are as follows:

- (i) warehouse fixed-location cost: the cost to establish and operate a warehouse;
- (ii) retailer unit-inventory holding cost: the cost to store products at retailer; and
- (iii) routing cost: the cost associated with delivering the goods from warehouse to retailers.

### 3.2 Notation

To formulate the problem, we use a similar notation as that used in Le et al. [2011], as follows.

-Sets

$W$	$\triangleq$	Set of candidate warehouses $W = 0, \dots,  W $
$I$	$\triangleq$	Set of retailers, $I = 0, \dots,  I $
$V$	$\triangleq$	Set of nodes, $V = W \cup I$
$T$	$\triangleq$	Set of time periods $T = 1, \dots,  T $
$R$	$\triangleq$	Set of all feasible routes
$K$	$\triangleq$	Set of homogeneous vehicles $K = 1, \dots,  K $

-Parameters

$f_w$	$\triangleq$	Fixed cost of opening and operating warehouse $w \in W$
$C$	$\triangleq$	Vehicle capacity
$\tau_{max}$	$\triangleq$	Maximum shelf-life
$d_{it}$	$\triangleq$	Demand of customer $i \in I$ in time period $t = 1, \dots, T, \dots, T + \tau_{max} - 1$
$u_{it}$	$\triangleq$	Upper bound inventory level at customer $i \in I$ in time period $t \in T$ , $u_{it} = \left( \sum_{\tau > t}^{\tau < t + \tau_{max}} d_{i\tau} \right)$
$h_{it}$	$\triangleq$	Inventory holding cost of customer $i \in I$ in time period $t \in T$
$I_{i0}$	$\triangleq$	Inventory level at customer $i \in I$ at the beginning of time period $t = 1$
$\alpha_{ir}$	$=$	$\begin{cases} 1 & \text{if route } r \in R \text{ visits customer } i \in I; \\ 0 & \text{otherwise.} \end{cases}$
$\beta_{rw}$	$=$	$\begin{cases} 1 & \text{if route } r \in R \text{ visits warehouse } w \in W; \\ 0 & \text{otherwise.} \end{cases}$
$c_{vv'}$	$\triangleq$	Transportation cost from node $v \in V$ to node $v' \in V$
$c_r$	$\triangleq$	Transportation cost of route $r \in R$

### -Decision Variables

- $I_{it}$  = Inventory level of customer  $i \in I$  at the end of time period  $t \in T$   
 $\theta_{rt}$  =  $\begin{cases} 1 & \text{if route } r \in R \text{ is selected in time period } t \in T; \\ 0 & \text{otherwise.} \end{cases}$   
 $m_w$  =  $\begin{cases} 1 & \text{if warehouse is opened at location } w \in W; \\ 0 & \text{otherwise.} \end{cases}$   
 $a_{irt}$  = Quantity delivered to customer  $i \in N$  by route  $r \in R$  in time period  $t \in T$

### 3.3 The Model

The MIP formulation of the this problem can be stated as follows:

$$\min \quad \sum_{w \in W} f_w m_w + \sum_{t \in T} \left( \sum_{r \in R} c_r \theta_{rt} + \sum_{i \in I} h_{it} I_{it} \right) \quad (1)$$

$$s.t. \quad \sum_{r \in R} \alpha_{ir} \theta_{rt} \leq 1 \quad \forall i \in I, t \in T \quad (2)$$

$$\sum_{i \in I} a_{irt} \leq C \theta_{rt} \quad \forall r \in R, t \in T \quad (3)$$

$$I_{it-1} + \sum_{r \in R} \alpha_{ir} a_{irt} = d_{it} + I_{it} \quad \forall i \in N, t \in T \quad (4)$$

$$I_{it} \leq u_{it} \quad \forall i \in N, t \in T \quad (5)$$

$$\theta_{rt} \leq \sum_{w \in W} \beta_{rw} m_w \quad \forall r \in R, t \in T \quad (6)$$

$$\sum_{r \in R} \theta_{rt} \leq |K| \quad \forall t \in T \quad (7)$$

$$\theta_{rt} \in \{0, 1\} \quad \forall r \in R, t \in T \quad (8)$$

$$m_w \in \{0, 1\} \quad \forall w \in W \quad (9)$$

$$a_{irt}, I_{it} \geq 0 \quad \forall i \in I, r \in R, t \in T \quad (10)$$

The objective function(1) shows the sum of costs included in this model. The first term represents the cost of opening and operating the selected warehouses, whereas the second term represents the transportation cost. The last term is the inventory holding cost. Constraints (2) guarantee that a customer is visited once at most in any time period. Constraints (3) account for the vehicle capacities. Inventory balance equations are represented in constraints (4). Constraints (5) ensure that the inventory level at a customer never exceeds the total demand in the next  $(\tau_{max} - 1)$  consecutive time periods. Constraints (6) guarantee that routes start and end with open warehouses only. Constraint (7) limit the maximum number of routes at any time period to be no greater than the number of vehicles. Finally, constraints (8) and (9) restrict  $\theta_{rt}$  and  $m_w$  to be binary, and constraints (10) ensure that quantities to be shipped to customers and inventory levels are non-negative.

## 4 Experimental Analysis

In this section, we provide computational examples of the model developed. The analysis was based on a sample data of four retailers and two warehouses. The location of retailers and warehouses was randomly chosen. The demand of each customer was a random integer in the interval  $[10, 100]$  for each time period. Transportation cost was set equal to the distance between nodes;  $c_{vv'} = \sqrt{(X_v - X_{v'})^2 + (Y_v - Y_{v'})^2}$ . Furthermore, Inventory cost for customer  $i$  is a random number in the interval  $[4.6, 5]$ . Moreover, capacity of the fleet of vehicles is 110% of total demand of customers,  $\sum_{i \in I} \sum_{t \in T} d_{it}$ . Hence, the number of vehicles was  $\lceil \frac{1.1}{C} \{ \sum_{i \in I} \sum_{t \in T} d_{it} \} \rceil$ . Beginning inventory,  $I_{i0}$ , was set equal to a random inte-

ger in the interval  $[0, d_{i1} + d_{i2}]$ . Moreover, shelf life,  $\tau_{max}$  is 2. vehicle capacity,  $C$ , is  $1.5d_{max}$ , where  $d_{max} = \max\{d_{it} : i \in I, t \in T\}$ . Finally, we simulated the model over 5 time periods.

To solve these instances, we used the CPLEX MIP solver in the GAMS<sup>©</sup> modeling language. It was shown that introducing the location/allocation decisions into one single model can change the retailer-warehouse assignment, operated warehouses, and the total cost.

## 5 Conclusion

In this paper, we introduced a joint location-inventory-routing model for supply chains with perishable products. The model was formulated as Mixed Integer Program (MIP). We have shown a significant savings when compared with multi-step optimization models.

Possible extensions of this model might be to use varying shelf-life time for products, depending on time of the year or temperature. Moreover, in this model it is assumed that goods maintain their values up to the expiry date, when they will be of no value. A more realistic scenario might be that products lose their value gradually throughout their lifetime.

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