2008 International Conference on Intelligent Computation Technology and Automation

Stochastic Dynamic Location-Routing-Inventory Problem in Closed-loop Logistics System for Reusing End-of-use Products

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Abstract

The reuse of end-of-use products is always an important issue. To optimize the closed-loop logistics system for reusing end-of-use products, an integrated optimization model for stochastic dynamic location-routing-inventory problem is presented in this paper. A two-phase heuristic algorithm is also proposed to solve the model. And the validity of the model and agorithm is demonstrated by an example.

1. Introduction

Facility location, vehicle routing and inventory control are three key problems in logistics system optimization. And these three problems are highly correlative[1,2]. So, it is necessary to optimize the combined location-routing-inventory problem (LRIP). For example, LRIP in logistics distribution systems is to allocate depots from several potential locations, to schedule vehicles' routes to meet customers' demands, and to determine the inventory policy (such as order quantity during each production run, reorder level for replenishment, etc.) based on the information of customers' demands, in order to minimize the total system cost (including location, transportation, and inventory costs).

Up to now, there are few literatures about LRIP in logistics distribution systems. Liu and Lee developed a mathematical model for LRP with multiple depots considering inventory strategy, and applied two-phase heuristic method to solve the problem[3]. Liu and Lin proposed a hybrid heuristic method combining Tabu Search with Simulated Annealing to solve the Combined LRIP[4]. Ambrosino and Scutellà studied some complex distribution network design problems which involve facility location, warehousing, transportation and inventory decisions. Two kinds of mathematical programming formulations are proposed together with a proof of their correctness[5]. Shen and Qi developed a nonlinear integer programming model and propose a Lagrangian relaxation based solution algorithm for a supply chain design problem that includes location, inventory, and routing decisions[6].

Owing to closed-loop logistics is beneficial to environmental protection and resource saving, the management of closed-loop logistics system (CLS) has attracted the attention of many scholars in recent years. Though the integrated optimization of LRIP in logistics distribution systems has been studied by few scholars, the similar problem has not been done for CLS. In this paper, an integrated optimization model for stochastic dynamic LRIP in CLS for reusing end-of-use products is developed and a two-phase heuristic algorithm is also proposed to solve the model.

2. Description of LRIP in CLS for Reusing End-of-use Products

The integrated optimization of LRIP in CLS for reusing end-of-use products is shown in Figure 1.

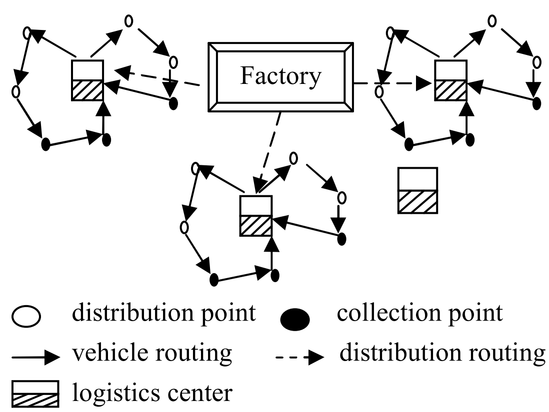

Figure 1. **CLS for reusing end-of-use products**

We define several policy of the CLS in accordance with its general situation as follows: (1) Location: where to build the logistics center among the candidates and how many? (2)Allocation: which

978-0-7695-3357-5/08 $25.00 © 2008 IEEE
DOI 10.1109/ICICTA.2008.181

customer should be allocated to which logistics center? (3)Routing: distribution-collection vehicles choose which route to gain the least transportation cost? (4) Inventory control: how long and how many is the new product delivered from the factory to the logistics center?

The system not only relates to strategic decision-making, for example, where to establish the logistics center, but also involves operational decisions, such as the choice of distribution-collection routing and the inventory control of each logistics center.

3. Mathematic Model

3.1. Notations and assumptions

We are dealing with the single-product, multi-period, multi-depot LRIP. The distribution and collection volume in every point are discrete random variables, which obey Poisson distribution in the planning horizon. In each period, every customer point can be either distribution point or collection point. The distribution need has a priority and the proportion of collection need which is not satisfied in a period can be collected in the next period. The interval of each period is longer than lead time. And we adopt the following notations.

N : number of customer points, which are distribution points or collection points

M : number of logistics centers

g: index of customer point or logistics center ($1 \le g \le N+M$)

h: index of customer point or logistics center ($1 \le h \le N+M$)

i: index of customer point ($1 \le i \le N$)

j: index of logistics center ($N+1 \le j \le N+M$)

F_j : establishing cost of logistics center j during the planning horizon

dis_{gh} : distance between point g and point h

T : length of the planning horizon, which have l periods.

CK : capacity of vehicles

k : index of vehicles or routes ($1 \le k \le K$)

t : index of time periods of the planning horizon ($1 \le t \le l$)

c : unit cost of vehicles

u_j : cost of transport the new product from factory to logistics center j

μ_j : probability of being reused after a circulation

I_j : holding cost of unit useable product in one day in logistics center j

A_j : ordering cost of every time in logistics center j

L_j : lead time in logistics center j, $L_j < T/l$

$s_j(t)$: inventory level of logistics center j on the beginning of period t, where $s_j(0)=0$ means that the inventory is zero in beginning of the planning horizon

X_{ghkt} : 1, if point g immediately proceeds point h on route k in period t; 0 otherwise

O_j : 1, if logistics center j is opened; 0, otherwise

Y_{ij} : 1, if customer i is served by logistics center j; 0, otherwise

Z_{jkt} : 1, if route k is served by logistics center j in period t; 0 otherwise

$H_j(t)$: 1, if there is a ordering for new product in logistics center j; 0, otherwise

3.2. Model formulation

There is a relationship between distribution demand and collection demand in CLS, viz. collection demand is the result of distribution in former periods. Several new parameters are introduced as follows:

$E(d_i(t))$: expected distribution demand of customer i in period t, which obey the Poisson distribution

$R_i(xy)$: expected collection demand of customer i in period y from the distribution demand in period x, which obeys Poisson distribution

$E(p_i(y))$: expected collection demand of customer i in period y, which is the accumulation of the collection from the former distribution, and obeys the process of Poisson distribution δ_i , and $R_i(xy) = \sum_{t=x}^{y} E(d_i(t)) \cdot \frac{\delta_i^{(t-x)} e^{-\delta_i}}{(t-x)!}$

$\gamma_i(t)$: actual collection volume from customer i in period t

We define: when $E(d_i(y))=0$, customer i is a collection point in period y, and the expected volume of collection is $E(p_i(y))$, $E(p_i(y)) = E(p_i(x)) - \gamma_i(x) + \sum_{t=x}^{y} R_i(xt)$; otherwise, it's a distribution point.

Before the mathematic model is developed, three relevant costs are introduced, viz. the cost of establishing logistics center, transportation cost and inventory cost. The first item can be expressed as followed: $F_j \cdot o_j$. The transportation cost during the distribution-collection process in the whole planning horizon is $c \cdot \sum_{t=1}^{l}\sum_{k=1}^{K}\sum_{h=1}^{N+M}\sum_{g=1}^{N+M} dis_{gh} X_{ghkt}$. At the beginning of each period t, the volume that vehicle k prepare to start in the logistics center is $V_k^t(0) = \sum_{i\in I} E(d_i(t)\cdot Y_{ij}\cdot Z_{jkt}$, and $V_k^t(0) \le CK$. When customer i is a distribution point, $V_k^t(i) = V_k^t(i-1) - E(d_i(t))$, or a collection point, $V_k^t(i) = V_k^t(i-1) + \gamma_i(t)$, where $V_k^t(i) \le CK$, $\gamma_i(t) \le E(p_i(t))$. In addition, there is a delivery cost u_j from the factory to the opened logistics center j. As for inventory, the model for continuous review systems is adopted as inventory policy. When distribution-collection is accomplished in every period, we check the volume of useable product. And, $D_j(t+1) = \sum_{i=1}^{N} E(d_i(t+1))\cdot Y_{ij}$ is the total demand of distribution in period t, $P_j(t) = \sum_{i=1}^{N} \gamma_i(t)\cdot Y_{ij}$ is the volume of collection in period t. When $s_j(t) + \mu_j \cdot P_j(t) < D_j(t+1)$, it is need to ordering new, and the ordering volume is $D_j(t+1) - s_j(t) - \mu_j \cdot P_j(t)$. Otherwise, it is enough for the next period. Then the expression of holding cost is as follows: $IV_j = \sum_{t=1}^{l}(u_j + A_j)H_j(t) + I_j \cdot \frac{T}{l}\cdot \sum_{t=1}^{l}(s_j(t) + \mu_j P_j(t))$.

Accordingly, the model can be formulated as follows:

$$\min \ SC = \sum_{j=N+1}^{N+M}(F_j + IV_j + u_j)\cdot O_j + c\cdot\sum_{t=1}^{l}\sum_{k=1}^{K}\sum_{h=1}^{N+M}\sum_{g=1}^{N+M} dis_{gh}\cdot X_{ghkt} \quad (1)$$

$$\text{s.t.} \quad \sum_{k=1}^{K}\sum_{g=1}^{N+M} X_{gikt} = 1, \quad \forall i,t \quad (2)$$

$$\sum_{g=1}^{N+M} X_{ghkt} - \sum_{g=1}^{N+M} X_{hgkt} = 0, \quad \forall h,k,t \quad (3)$$

$$\sum_{h=1}^{N+M}\sum_{g=1}^{N+M} X_{ghkt} \le 1, \quad \forall k,t \quad (4)$$

$$V_k^t(i) \le CK, \quad i \in \{0,1,2,\cdots,N\} \quad (5)$$

$$\gamma_i(t) \le E(p_i(t)), \quad \forall i \quad (6)$$

$$\sum_{h=1}^{N+M} X_{ihkt} + \sum_{h=1}^{N+M} X_{jhkt} - Y_{ij} \le 1, \quad \forall i,j,k,t \quad (7)$$

$$X_{ghkt} \in \{0,1\}, \quad \forall g,h,k,t \quad (8)$$

$$O_j \in \{0,1\}, \quad \forall j \quad (9)$$

$$Y_{ij} \in \{0,1\}, \quad \forall i,j \quad (10)$$

$$Z_{ikt} = \{0,1\}, \quad \forall i,k,t \quad (11)$$

$$Y_j(t) \in \{0,1\}, \quad \forall j \quad (12)$$

In the above formulation, the objective function (1) is to minimize the total system cost, which is the sum of establishing cost of logistics centers, transportation cost and inventory cost. Constraint (2) states that every customer point appears in only one route. Constraint (3) states that every point entered by the vehicle should be the same point the vehicle leaves. Constraint (4) states each route cannot be served by more than one logistics center. Constraint (5) and (6) insures that in any period, in any point the total load is less than the capacity of the vehicle and the actual collection volume is equal to or less than the expected volume. Constraint (7) states a customer can be allocated to a depot only if there is a route passing by that customer. Constraints (8)-(12) insure the integrality of decision variables.

4. A Two-phase Heuristic Algorithm

Since finding the optimal solution for this problem is a NP-hard problem, we propose a two-phase heuristic method. Phrase 1, all the collection points are grouped in accordance with the collection volume, then open the transfer stations with the minimum collection cost, compute the total cost at this time, and get the initial solution. Phrase 2, close an open transfer station with the minimum total cost. If it is less than the total cost, update the current solution set, otherwise keep the original solution set. The solution is altered continuously to yield another feasible solution with a reduced total cost until no additional total cost reductions are possible. The detailed procedure is as follows:

Phase 1: finding the initial solution

Step1. Set k=1, t=1, allocate each customer point to the nearest logistics center. Calculate the *NOD* which is the number of opened logistics centers. Establish void sets $E_j(t)$ and $F_j(t)$ for $j \in \{1,2,\cdots,NOD\}$, put each logistics center j into $E_j(t)$, and put the corresponding customer points for logistics center j into $F_j(t)$. Set j=1.

Step2. Choose the nearest distribution point to the logistics center j, put it into $E_j^k(t)$, and delete it from $F_j(t)$.

Step3. Choose the nearest customer to the former as the next candidate point.

Step4. Is the candidate customer a distribution point? If yes, go to step5; Otherwise, put it into $E_j^k(t)$, and delete it from $F_j(t)$, go to step6.

step5. Is the volume of all the distribution points which are chose or about to be chose into $E_j^k(t)$ less than the capacity of vehicle? If yes, put it into $E_j^k(t)$, and delete it from $F_j(t)$, go to step 6. Otherwise, set k=k+1, put it into set $E_j^k(t)$, and delete it from $F_j(t)$, go to step 6.

Step6. Is $F_j(t)$ empty? If yes, go to step7; otherwise, go to step 3.

Step7. Using the C-W method to search the shortcut in $E_j^k(t)$ ensuring that the first customer form logistics center j is a distribution point, and gain the route $\{V_j^k(t)\}$. Calculate the transportation cost, and gain the actual collection volume in every collection point.

Step8. Calculate the total collection volume $P_j(t)$. Is $P_j(t)+s_j(t)<Q_j(t+1)$? If yes, there is a order for new, the quantity is $D_j(t+1)-s_j(t)-P_j(t)$; Otherwise, there is no order.

Step9. Calculate the operation cost of the logistics center j in period t, that is $SC_j(t)=\sum_{k=1}^{K}\sum_{h\in E_j}\sum_{g\in E_j} dis_{gh}\cdot X_{ghk}+IV_j(t)$.

Step10. Set t=t+1. Is $t\le l$? If yes, return to step2; otherwise, calculate $SC_j=\sum_{t=1}^{l}SC_j(t)+F_j$.

Step11. Set j=j+1. Is $j\le NOD$? If yes, return to step2; otherwise, go to step12.

Step12. Calculate the total system cost $SC=\sum_{j=1}^{NOD}SC_j$, and set $SC'=SC$, gain the initial routes $\{V_j^k(t)\}$ for $j\in\{1,2,\cdots,NOD\}$.

Phase 2: improving the initial solution

Step13. Close an opened logistics center with the minimum total system cost SC, and set NOD=NOD-1.

Step14. Is $SC<SC'$? If yes, update the total cost $SC'=SC$ and the routes $\{V_j^k(t)\}$, and go to step15; otherwise, go to step15.

Step15. Is NOD=1? If yes, go to step16; otherwise, return to step13.

step16. Set *Max_swap*=0, default value=M/2(M is the number of candidate logistics centers).

step17. Randomly choose a closed logistics center to replace an opened logistics center, and calculate the new total system cost SC.

Step18. Is $SC<SC'$? If yes, update the best solution $SC'=SC$ and the routes $\{V_j^k(t)\}$; otherwise, *Max_swap*=*Max_swap*+1.

Step19. Is *Max_swap* $\le$ default value? If yes, go to step17; otherwise, the best solution SC' and $\{V_j^k(t)\}$ is obtained and stop.

5. An Illustrative Example

We assume that there are 20 customer points and 5 candidate logistics centers. The expected distribution-collection demand is created randomly which obey Poisson distribution with $\lambda_i=2$, $\delta_i=1$. CK is 125 units, c is 2/unit distance, l is 50 weeks, L_j is 2 days, A_j is 5 for each time, I_j is 10/unit/day, and μ_j is 0.9. The other parameters values of the candidate logistics centers and customers are shown in Table 1 and Table 2, respectively.

Table 1. **Parameter values of logistics centers**

Logistics centers	Location (coordinate)	Managing cost	μ_j
D1	(51,30)	12000	25
D2	(61,59)	10000	20
D3	(13,77)	12000	30
D4	(16,20)	13000	10
D5	(34,45)	15000	15

Table 2. **Parameter values of customer points**

C1	C2	C3	C4	C5
(74,20)	(60,89)	(95,75)	(76,69)	(52,18)
C6	C7	C8	C9	C10
(18,54)	(22,98)	(2,58)	(69,23)	(87,14)
C11	C12	C13	C14	C15
(49,8)	(55,83)	(45,53)	(55,21)	(45,12)
C16	C17	C18	C19	C20
(61,32)	(3,55)	(31,96)	(33,91)	(45,65)

The above heuristic algorithm is coded using Visual C++ 6.0 programming language to solve this problem. First, grouping all the customer points, we get 4 distribution groups. According to the minimum collection cost, we open four logistics centers: L1, L2, L3, and L5. Then according to the distribution demand, we optimize the internal routing in every period. For brevity, we give the distribution road in the first period: L1 serves V1 and V2, L2 serves V3, L3 serves V4, and L5 serves V5. Where V_1^1 ={L1,C16,C1,C10,L1}, V_1^2 ={L1,C5,C20,C11,L1}, V_2^1 ={L2,C4,C2,C12,C3, L2}, V_3^1={L3,C12,C18,C7,L3}, V_5^1={L5,C6,L5}. And SC^1=81394.

Then close L5 with the minimum cost, C6 be served by L2, and SC^2 =75623. Because SC2<SC1, update the solution set.

Unceasingly close an open logistics center with the minimum cost until there is only one opened. By computing, the optimization result is $SC = SC^2 = 75623$. Where the opened logistics centers is L1, L2, and L3, the routes in the first period is V_1^1 ={D1,C16,C9,C1, C10,D1}, V_1^2 ={D1,C14,C5,C20,C11,D1}, V_2^1 ={D2,C3, C4,C2,C12,D2}, V_2^2 ={D2,C13,C15,D2}, V_3^1 ={D3, C19,C18,C7,D3}, V_3^2 ={D3,C8,C17,C6,D3}. For brevity, the routes in other periods are omitted.

6. Conclusions

In this paper, the integrated optimization of LRIP in CLS for reusing end-of-use products is studied and a stochastic dynamic model is presented. Since the issue of solving the model is a NP-hard problem, a two-phase heuristic algorithm is proposed and tested by an illustrative example. Next, we will further consider the LRIP with time window, expecting to solve the practical problem better.

Acknowledgements

This work was supported by National Natural Science Foundation of China (No.70502025), National Social Science Foundation of China (07CJY019), and Key Research Institute of Humanities and Social Science in University of Sichuan Province (Sichuan Research Center of Circular Economy), China (No. XHJJ-05).

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