Location-Routing-Inventory Problem with Stochastic Demand in Logistics Distribution Systems

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***Abstract*—Location-routing-inventory problem (LRIP) involves the decisions of three key problems in optimizing a logistics system, i.e. facility location-allocation, vehicle routing, and inventory control. In this paper, a model for the multi-period, multi-depot LRIP with stochastic customer demand in logistics distribution systems is developed. And a mixed genetic algorithm is proposed to solve the model. Finally, the validity of the model and algorithm are demonstrated by a numerical example.**

Keywords-*location-routing-inventory problem; logistics distribution systems; stochastic demand; mixed genetic algorithm*

I. INTRODUCTION

Facility location-allocation problem (LAP), vehicle routing problem (VRP), and inventory control problem are three key problems in optimizing a logistics system. Many scholars have studied the location-routing problem (LRP) which is the combination of LAP and VRP, but inventory control decisions are ignored in LRP. In fact, these three problems are highly correlative[1,2]. So, it is necessary to optimize the combined location-routing-inventory problem (LRIP).

There are few literatures about LRIP. Liu and Lee developed a mathematical model for LRP with multiple depots considering inventory strategy, and applied two-phase heuristic method to solve the problem[3]. Liu and Lin proposed a hybrid heuristic method combining Tabu Search with Simulated Annealing to solve the Combined LRIP[4]. Ambrosino and Scutellà studied some complex distribution network design problems which involve facility location, warehousing, transportation and inventory decisions. Two kinds of mathematical programming formulations are proposed together with a proof of their correctness[5]. Shen and Qi developed a nonlinear integer programming model and propose a Lagrangian relaxation based solution algorithm for a supply chain design problem that includes location, inventory, and routing decisions[6].

In this paper, a model for the multi-period, multi-depot LRIP with stochastic customer demand in logistics distribution systems is developed. And a mixed genetic algorithm is proposed to solve the model.

II. DESCRIPTION OF LRIP IN LOGISTICS DISTIRBUTION SYSTEMS EASE OF USE

The integrated optimization of LRIP in logistics distribution systems is shown in Figure 1. In a logistics distribution system, LRIP is to allocate logistics centers from several potential locations, to schedule vehicles' routes to meet customers' demands, and to determine the inventory policy based on the information of customers' demands, in order to minimize the total system cost. LRIP includes four kinds of decisions, that is: (1) Location decision: how to choose the depots from the potential depots and how many to choose? (2) Allocation decision: which customers should be allocated to open depot selected? (3) Routing decision: vehicles select which distribution routes to delivery? (4) Inventory control decision: how much should be the goods delivered from depots to the customers?

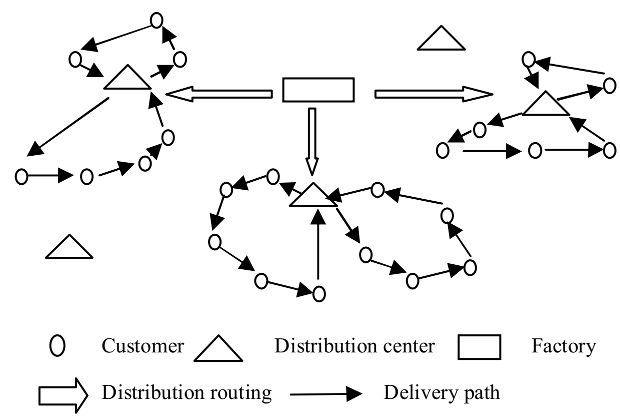

Figure 1. LRIP in logistics distribution systems

III. MATHEMATIC MODEL

A Assumptions

We are dealing with the single-product, multi-period, multi-depot LRIP. Each customer is served by exactly one vehicle. Each route is served by one vehicle. The total demand of each route is less than or equal to the vehicle service capacity. Each route begins and ends at the same depot. Fleet type is homogeneous (vehicle capacities are the same). The demand of each customer obeys Poisson distribution. Customers need to make an order for depots every period.

B Notations

N : number of customers
M : number of candidate distribution centers or depots
T : length of the planning horizon
i: index of customers ($1 \le i \le N$)
j: index of depots ($N+1 \le j \le N+M$)
t: index of periods ($1 \le t \le T$)
V_j : number of delivery vehicles (or routes) of depot *j*
v_{jt} : index of routes in depot *j* during period *t* ($1 \le v_{jt} \le V_j$)
F_j : establishing cost of depot *j*
L_j : lead time of depot *j*
CK : vehicle capacity
B_j : cost of dispatching vehicle from depot *j* every time
A_j : order cost of depot *j* from factory every time
Q_j : order volume of depot *j* every time
α : traveling cost/unit distance
h^+ : holding cost/unit product
hs : shortage cost/unit product
R_j : reorder level for replenishment of depot *j*
$B(R_j)$: expected shortage number of depot *j* during the lead time L_j
$C_i(t)$: demand of customer *i* during period *t*
$Q_i(t)$: delivery quantity for customer *i* during period *t*, $Q_i(t) \le C_i(t)$
D_{ji} : total demand of customer *i* served by depot *j* during the planning horizon
$D_j(t)$: total demand of customers served by depot *j* during period *t*
DL_j : total demand of customers served by depot *j* during lead time L_j
D_j : total demand of customers served by depot *j* during the planning horizon
d_{pq} : distance from node *p* to node *q* ($1 \le p, q \le N+M$)
$P_j(k)$: probability density function in depot *j* demand in lead time L_j, that is $P_j(k) = \frac{(L_j \times \lambda)^k e^{-(L_j \times \lambda)}}{k!}$
Z_j : 1, if depot *j* is established; 0 otherwise
Y_{ij} : 1, if customer *i* is allocated to depot *j*; 0 otherwise
$X_{pqv_{jt}}$: 1, if point *p* immediately proceeds point *q* on route v_{jt} ; 0, otherwise

C Model analysis

1) Inventory cost analysis

The total demand of customer *i* served by depot *j* during the planning horizon is $D_{ji} = \sum_{t=1}^{T} E[C_i(t)] = \sum_{t=1}^{T} \lambda_i$. The total demand of customers served by depot *j* during period *t* is $D_j(t) = \sum_{i=1}^{N} C_i(t).Y_{ij}$. During the lead time L_j, the total demand of customers served by depot *j* is $DL_j = \sum_{t=1}^{L_j} D_j(t)$. During the planning horizon, the total demand of customers served by depot *j* is $D_j = \sum_{t=1}^{T} D_j(t)$. The holding number during lead time L_j is $\sum_{k=1}^{R_j} (R_j - k) P_j(k)$. The shortage number during lead time L_j is $B(R_j) = \sum_{k=R_j+1}^{\infty} (k - R_j) P_j(k)$. The reorder level of depot *j* is $R_j = DL_j \times \sum_{i=1}^{N} Y_{ij}$. The optimal ordering volume for depot *j* is

$$Q_j^* = \sqrt{\frac{2D_j[A_j + B_j + hs \cdot \sum_{k=R_j+1}^{\infty} (k - R_j) P_j(k)]}{h^+}}$$

So the total inventory cost for all open depots is

$$\sum_{j=N+1}^{N+M} \left\{ (A_j + B_j) \cdot \frac{D_j}{Q_j^*} + h^+ \cdot [\frac{Q_j^*}{2} + \sum_{k=1}^{R_j} (R_j - k) P_j(k)] + hs \cdot \sum_{k=R_j+1}^{\infty} (k - R_j) P_j(k) \cdot \frac{D_j}{Q_j^*} \right\} .Z_j$$

2) Transportation cost analysis

The total delivery cost for depot *j* during the planning horizon is $\sum_{t=1}^{T} \sum_{p=1}^{M+N} \sum_{q=1}^{M+N} d_{pq} X_{pqv_{jt}}$. So, total delivery cost for all open depots is $\sum_{j=N+1}^{N+M} \sum_{t=1}^{T} \sum_{p=1}^{V} \sum_{q=1}^{V} d_{pq} X_{pqv_{jt}}$.

3) Model formulation

Accordingly, the model can de formulated as follows:

$$Min\ \ TotalCost = \sum_{j=N+1}^{M+N} F_j Z_j + \sum_{j=N+1}^{M+N} \left\{ (A_j + B_j). \frac{D_j}{Q_j} + h^+ \cdot [\frac{Q_j}{2} + \sum_{k=1}^{R_j} (R_j - k).P_j(k)] + hs \cdot \sum_{k=R_j+1}^{\infty} (k - R_j).P_j(k) \cdot \frac{D_j}{Q_j} \right\} .Z_j + \sum_{j=N+1}^{M+N} \sum_{t=1}^{T} \sum_{p=1}^{M+N} \sum_{q=1}^{M+N} d_{pq}.X_{pqv_{jt}} \quad (1)$$

$$\text{s.t.} \quad \sum_{j=N+1}^{N+M} \sum_{v_{jt}=1}^{V_j} \sum_{q=1}^{N+M} X_{iqv_{jt}} = 1, \quad \forall i, t \quad (2)$$

$$\sum_{q=1}^{N+M} X_{pqv_{jt}} = \sum_{q=1}^{N+M} X_{qpv_{jt}}, \quad \forall p, v_{jt} \quad (3)$$

$$\sum_{j=N+1}^{N+M}\sum_{i=1}^{N} X_{ijv_{jt}} \le 1, \qquad \forall v_{jt} \tag{4}$$

$$\sum_{q=1}^{N+M} X_{iqv_{jt}} + \sum_{q=1}^{N+M} X_{jqv_{jt}} - Y_{ij} \le 1, \qquad \forall i, j, v_{jt} \tag{5}$$

$$\sum_{i=1}^{N}\sum_{q=1}^{N+M} Q_i(t) Y_{ij} X_{iqv_{jt}} \le CK, \qquad \forall j, v_{jt} \tag{6}$$

$$Z_j \in \{0,1\}, \qquad \forall j \tag{7}$$

$$y_{ij} \in \{0,1\}, \qquad \forall i, j \tag{8}$$

$$X_{pqv_{jt}} \in \{0,1\}, \qquad \forall p, q, v_{jt} \tag{9}$$

In the above formulation, the objective function (1) is to minimize the total system cost, including establishing cost of depots, transportation cost and inventory cost. Constraint (2) states each customer appears in only one route during period *t*. Constraint (3) insures that every point entered by the vehicle should be the same point the vehicle leaves during period *t*. Constraint (4) insures a route can not be served by multiple depots. Constraint (5) a customer can be allocated to a depot only if there is a route passed by that customer. Constraint (6) insures the total delivery quantity for route v_{jt} is less than or equal to vehicle service capacity. Constraints (7)-(9) insure the integrality of decision variables.

IV. A MIXED GENETIC ALGORITHM

LRIP belongs to the class of NP-hard problem, so it is usually solved by heuristic algorithm. In this paper, a mixed genetic algorithm is proposed as shown in Figure 2. The detailed procedure for this method is as follows:

Step1. Initialize parameters

Load the candidate (*X*, *Y*) of *N* customers, the location (*X*, *Y*) of *M* depots. Randomly generate the demand $C_i(t)$ (obeys Poisson distribution) of each customer during the planning horizon. Compute the total demand D_{ji} of each customer during the planning horizon.

Step2. Initialize population

Randomly generate a population with *S* chromosomes based on single dimensional array which consists of *M* binary values, representing decision variables related to depots. For example, {0,1,01,1,0,0,0,1,0} represent depot 2,4,5,9 got open.

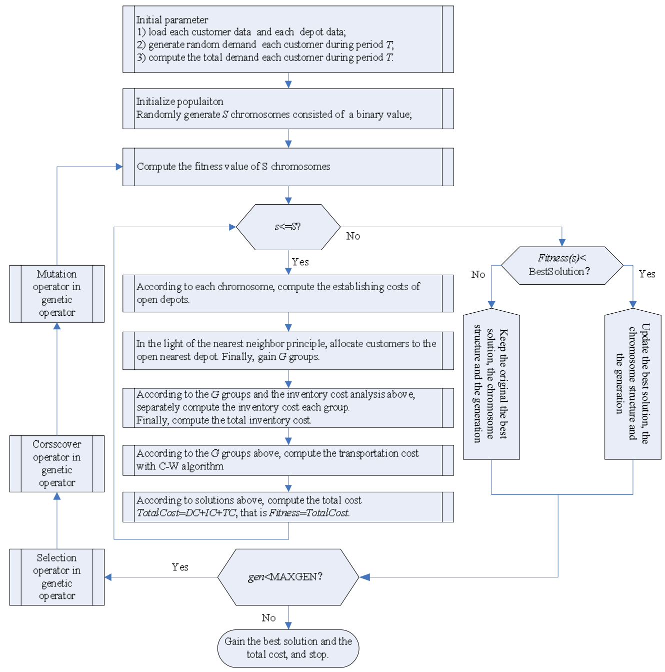

Figure 2. The flowchart of the mixed genetic algorithm

Step3. Compute the fitness value *Fitness*(*s*) , if *s*≤*S*, continue; Otherwise, go to *Step4*.

According to each chromosome, compute the depot establishing cost: $DC = \sum_{j=N+1}^{N+M} F_j \times Z_j$.

In the light of the nearest neighbor principle, allocate customers to the open nearest depot and gain $G = \sum_{j=N+1}^{M+N} Z_j$ groups. According to the inventory cost analysis above, compute the inventory cost of each group as follows:

$$IC_g = (A_j + B_j)\cdot\frac{D_j}{Q_j} + h^+ \cdot [\frac{Q_j}{2} + \sum_{k=1}^{R_j}(R_j - k)P_j(k)] +$$
$$hs \cdot \sum_{k=R_j+1}^{\infty}(k - R_j)P_j(k)\cdot\frac{D_j}{Q_j}$$

In the end, the total inventory cost $IC = \sum_{g=1}^{G} IC_g$.

According to the *G* groups above, compute the transportation cost with C-W algorithm as follows:

a) If *g*≤*R* (first *g*=1 then *g*=*g*+1), go to *step b)*; Otherwise, go to *step f)*;

b) Compute the distance d_{pq} between two nodes (include depot node) in group *g*;

c) If period *t*≤*T* （first *t*=1, *t*=*t*+1）, go to *step d)*; Otherwise, go to *step a)*;

d) Compute the demand $C_i(t)$ each node in group *g*;

e) According to the distance d_{pq} in *step b)* and the demand $C_i(t)$ in *step d)*, compute the transportation cost *TC(t)* during period *t* by using C-W algorithm, then go to *step c)*;

f) Compute the total transportation cost $TC = \sum_{t=1}^{T} TC(t)$;

Finally, compute the total cost *TotalCost=DC+IC+TC*, and $Fitness(s) = TotalCost$.

Step4. Is $Fitness(s) < Bestsolution, s = 1,2...S$? If yes, update best-solution, the chromosome structure and the generation, and go to *Step5*; Otherwise, keep original solution, the chromosome structure and the generation, and go to *Step5.*

Step5. Genetic Operators

- Selection operator: adopt roulette wheel selection operator;
- Crossover operator: adopt two-point crossover operator, and the crossover probability *Pc*=0.8;
- Mutation operator: randomly exchange two bit values between two chromosomes, the mutation probability *Pm*=0.4.

Step6. Stop condition

- *gen*=*gen*+1;
- If *gen*<*MAXGENS*, go to step 3; Otherwise, go to Step7.

Step7. Obtain the optimal solution and the total cost *SC*, and stop.

V. AN ILLUSTRATIVE EXAMPLE

We assume that there are 10 candidate depots and 30 customers. *CK* is 150 units, cm is 1/unit distance, α' and α is 1/unit distance, *hs* is 2/unit/year, h^+ is 0.05/unit, L_j is 2 for each time, $A_j + B_j$ is 25 each time, F_j is 3000 each depot, *T* has 300 periods. $C_i(t) = \frac{\lambda^{[k]} e^{-\lambda}}{[k]!}$ (generate $C_i(t)$ with acceptance rejection method). The coordinates of all nodes are shown in Table 1 and Table 2.

The above algorithm is coded by using Visual C++ 6.0 programming language to solve the model, where population size=10, crossover rate=0.8, mutation rate=0.4, and maximum number of generations=300.

TABLE I. COORDINATES OF DEPOTS

D1	D2	D3	D4	D5
(43,49)	(1,12)	(41,30)	(5,58)	(24,19)
D6	D7	D8	D9	D10
(16,20)	(30,45)	(40,0)	(7,33)	(54,57)

TABLE II. COORDINATES OF CUSTOMERS

C1	C2	C3	C4	C5
(15,3)	(18,24)	(2,59)	(9,6)	(49,54)
C6	C7	C8	C9	C10
(33,10)	(30,50)	(24,59)	(3,35)	(33,21)
C11	C12	C13	C14	C15
(45,27)	(46,6)	(24,32)	(28,33)	(2,0)
C16	C17	C18	C19	C20
(28,60)	(38,51)	(3,54)	(25,2)	(45,57)
C21	C22	C23	C24	C25
(48,8)	(36,14)	(42,27)	(24,35)	(25,28)
C26	C27	C28	C29	C30
(9,26)	(24,11)	(17,11)	(11,12)	(6,38)

The final result is: the best solution is gained at the 63 generation, and the minimum total cost is 109266. Here, depots D3, D4, D7, and D9 are opened. And the routes in the first period is {D3,C10,C22,C6,C19,C27,C25,D3}, {D3,C23,C12, C21,C11,D3}, {D4,C3,C18, D2}, {D7,C7,C20,C5,C17,D7}, {D7,C24,C13,C14,D7}, {D7,C8,C16,D7}, {D9,C26,C29,C4, C15,C1,C28,C2,D9}, {D9,C9,C30,D9}. For brevity, the delivery routes in other periods are omitted.

VI. CONCLUSIONS

In this paper, the integrated optimization of locating depots, vehicle routing and inventory control problem in logistics distribution systems is studied and a stochastic model for LRIP is presented. As the issue of solving the model is a NP-hard problem, a mixed genetic algorithm is proposed and tested by an illustrative example. Next, we will further consider the dynamic model with time window and vehicle limited, expecting to solve the practical problem better.

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