Multi-products location–routing problem integrated with inventory under stochastic demand

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Abstract: The location–routing problem (LRP) deals with simultaneously selecting (locating) one or more facilities from a set of potential facilities (locations), and assigning customers to the selected facilities. Since LRP and inventory decisions are inter-related, integrating them can lead to a more efficient supply chain. Therefore, to improve the operation of the supply chain process, this paper presents a model that integrates inventory control, as a tactical decision, with the LRP. The presented model considers the multi-product network under the fixed order interval inventory policy with stochastic demands for products. Moreover, third-party logistics offers excess space for selected warehouses if needed. This paper also presents a solution approach based on simulated annealing that solves problems with the size of 40 products and 350 customers in reasonable run time.

Keywords: location-routing problem; inventory; integrated supply chain.

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1 Introduction

As defined by Chopra and Meindl (2007), "a supply chain consists of all parties involved, directly or indirectly, in fulfilling a customer request. The supply chain includes not only the manufacturers and suppliers, but also transporters, warehouses, retailers, and even customers themselves". Blanchard (2007) reports that only in the USA, companies spend more than $1 trillion every year on their supply chain networks. Therefore, making improvements to the supply network can result in a substantial decrease in the network cost. Improvements to a supply chain can be made through its two main activities: production and logistics. While the former is associated with in-plant activities, such as production planning, material handling and shop floor tasks, the later considers the procurement of raw materials from suppliers to manufacturing plants and then transportation of finished goods from manufacturing plants to retailers. This study considers the logistics part of the supply chain. Craven and Islam (2007) reviewed logistics and supply chain management operational research models and presented some non-integrated models to formulate optimised logistics network. Although the improvement of the supply chain process is possible by optimising its individual drivers such as facility, inventory and transportation (Chopra and Meindl, 2007), the works of Burns et al. (1985), Daskin et al. (2002), Shu et al. (2005) and Miranda and Garrido (2004) indicate that integrating the supply chain drivers will lead to a lower network cost. Shen and Qi (2007) developed an integrated three-layer supply chain model, which simultaneously determines the strategic facility location decisions on the tactical inventory and shipment decisions. Nooral Haq and Kannan (2006) presented a five-tier supply chain including supplier, plant, distributor, whole seller and retailer. They developed an integrated supplier selection and multi-echelon distribution inventory model. Authors showed that the integrated decision system will results in a better solution than that of individual. Boudia and Prins (2009) proposed an integrated supply model where production and distribution problem were simultaneously determined. Results indicated that the integration model significantly obtains saving compares to non-integrated system. Shen (2007) presented a survey on the integrated supply chain models. He proposed that the physical structure of supply chain plays an important role in the network performance.

Locating plants and distribution facilities, and determining the routings as two logistical decisions in the design and operation of the supply chain can be integrated to improve the supply network. Location and routing when combined form the location–routing problem (LRP). LRP models allow decision makers to make three decisions simultaneously: determine the locations of facilities, allocate customers to those facilities and determine routings from facilities to customers. LRP has applications in many environments including postal delivery, news paper delivery, blood distribution, food distribution, school bus pick-up and delivery, waste disposal transportation, mobile healthcare and wherever the distribution centres (DCs) or retailers are close to customers or in situations where the customer demands are less than truck load (LTT) (Nagi and Salhi, 2007).

Although many LRP models exist in the literature, the integration of other components of logistics chain like inventory has not been fully studied. Therefore, it is desirable to enhance the performance of the supply chain process by integrating the inventory decisions with the LRP. Moreover, most of the research on LRP considers only a single product as oppose to most practical cases where multi-products are involved. Also, another practical issue which has not been integrated with LRP is the decision that companies make on the third-party logistics (3PLs) to overcome their space shortage constraint. This paper presents a three-layer LRP model integrated with inventory which considers stochastic demand at demand points, multiple capacitated depots and multiple capacitated homogenous vehicle fleets. Moreover, it does not consider any limitations on the delivery time and distance, and allows for 3PLs to supplement/replace the depots.

This paper is organised as follows. In Section 2, the state of the art review of the literature related to LRP is presented. In Section 3, the integrated LRP-inventory model is presented. Section 4 presents a solution methodology based on simulated annealing (SA) to solve the developed mathematical model. In Section 5, results are discussed and finally, in Section 6, conclusions and future research directions are presented.

2 Background and review of literature

Classical LRP models were proposed in the late 70s and early 80s. Some of the pioneer studies include Gillett and Johnson (1976), Jakobsen and Madsen (1980), Or and Pireskalla (1979), Golden et al. (1981), Golden and Assad (1986) and Laporte and Nobert (1981). Integration of LRP with warehousing and inventory decisions is a branch of LRP research that has not seen much attention. Perl and Daskin (1985) presented modified warehouse LRP (MWLRP), in which they introduced routing in the multi-depot location problem. They assumed a variable cost for potential depots per unit of throughput in addition to fixed depot cost. The model was decomposed into three sub-problems, and then a sequential method was applied to solve each sub-problem where the inter-dependency between sub-problems was considered. Berger et al. (2007) presented a set partitioning formulation of an un-capacitated LRP with distance constraint. They applied the alternate column generation technique which dramatically decreases the number of constraints, then, used a branch and bound algorithm to solve the LRP. Alumur and Kara (2007) formulated a hazardous waste management problem as an LRP. Considering total cost and transportation risk as a multi-objective problem, their three-level mixed integers LRP is subjected to disposal facilities capacity. Moreover, a minimum amount of requirement constraint is considered in the model. This constraint ensures that a facility (treatment centre) will be open if the amount of waste is greater than a pre-defined value.

Ambrosino and Scutella (2005) studied a complex distribution network design considering capacitated facility location, warehousing, capacitated transportation load and inventory levels. Proposing a four-tier logistic system consisting of plants, central depots, regional facilities and customers, they assumed two types of regional facilities; regional depots (hold inventory) and transit points (do not hold inventory). Moreover, they considered two types of customers; regular customers and big customers (big demand). They allowed big customers to be served by central depots. In contrast to most of the studies on LRP, their work assumed non-homogenous vehicle fleet. Mathematical formulations were presented for both static (fixed demand over time) and dynamic (known but variable demand overtime) models. Although they arrived at exact solutions for some small instances using CPLEX, exact solution could not be achieved for large instances. Burns et al. (1985) studied two distribution models. In one model, products from a supplier were transported directly to customers, in the other model, peddling (multiple visits) was allowed. Both models require spatial density[1] of customers rather than the precise locations of every customer. They showed that the optimal order quantity is attainable by calculating EOQ for direct delivery and full truck for the multiple visits. Liu and Lee (2003) proposed a two-layer, multi-depot LRP (MDLRP) model which considers the inventory decisions. The single product continuous review system was selected as the inventory policy and the inventory cost was added to the objective function. It was assumed that demands follow stochastic distributions overtime. They proposed a two-stage heuristics solution methodology for the proposed model. In the first stage, a *route first, location–allocation second* algorithm generated the initial solution and then in the second phase, the initial solution was improved. Computational results indicated that the algorithm achieves better solution than other algorithms that do not take into account the inventory cost. However, the proposed model is limited only to a single-product case. Moreover, adopting the fixed order system as the inventory policy may increase the transportation cost especially, when customers demand multiple products. Ago et al. (2007) allocated storage to raw materials and assigned them belt-conveyor as the transportation system. They assumed that requirements for raw materials are not fixed overtime but are determined. Although their work is not referred to as LRP, it is in fact formulated as a dynamic in-factory-LRP. In other words, the logistics system inside the factory is being improved while it considers the location of raw material in the storage and transportation of them to the production line simultaneously. They solved the problem by Lagrangian decomposition.

Another branch of study which is very common in practice but has not been extensively studied in the literature is multi-commodity LRP model. Burks (2006) generalised the classical LRP mathematical model proposed by Perl and Daskin (1985) to show static mathematical formulation for multi-products LRP. Bowerman et al. (1995) presented an urban school bus-routing problem (USBRP) as an LRP where they considered the school at the first level, stations (depots) at the second and students at the third levels, respectively. Using the cluster-first, route-second decomposition method, they minimised their proposed multi-objective function using a heuristic solution. The model considered multi-products in its formulation. Gunnarsson et al. (2006) studied an integrated terminal location and ship-routing problem in Europe. A mathematical model was proposed to simultaneously decide on the optimal terminal locations and ship routing while customers demand multiple commodities. Yi and Ozdamar (2007) proposed an application of the dynamic facility capacitated LRP (CLRP) in a disaster situation. Their goal was to find the best location of temporary centres and shelters in affected areas to speed up medical care for less heavily wounded survivors considering the maximum coverage of the area. Their mixed integer, multi-commodity, network flow model considered capacitated vehicle and multi-commodity products pickup delivery assumptions. Moreover, heterogeneous fleet vehicles may pay multi-visits to the same customer in their route. However, their work relaxed some LRP classic assumptions; in contrast to most LRPs, it is not necessary for a vehicle to come back to the origin once it visits the last customer. A two-stage algorithm was proposed to solve the problem and a real-world earthquake situation verified the applicability of the algorithm.

Since the sub-problems of LRP, location–allocation and routing, are classified as NP-hard, LRP also belongs to this category. Therefore, most of the studies concentrate on developing heuristics as the solution approach. Bouhafs et al. (2006) proposed a two-phase algorithm for CLRP. In the first stage, a SA is used to find a good configuration of DCs while in the second stage, an ant colony system is applied to find good routing corresponding to this configuration. They tested their approach on some instances taken from the literature and reported that the computational results are reasonable. Yang and Li-Jun (2006) decomposed a capacitated facility and vehicle LRP to two sub-problems; location–allocation problem (LAP) and vehicle-routing problem (VRP). They modified the classic LRP objective function by adding a penalty term associated with infeasible solutions. Each sub-problem is iteratively and sequentially solved by the particle swarm optimisation method (PSO).

Nagi and Salhi (1996) presented a nested heuristic for the capacitated vehicle subject to distance limitation LRP. In their hierarchical approach, the location is the master problem while the routing is the sub-problem. In other words, the routing stage is embedded within the location phase. The routing length estimation (RLE) is used to estimate the routing costs. The fact behind using RLE is that computing the sum of radial distances is much easier to perform than determining the total routing cost. Computational results indicated that the proposed method produced better results than the sequential algorithm. Tuzun and Burk (1999) proposed a two-phase taboo search (TS) algorithm for the CLRP that iterates between location and routing phase in order to search for better solutions. They showed that the proposed algorithm is able to solve the LRP with 20 facilities and 300 customers efficiently. To prove the effectiveness of their algorithm, they compared it to ASVI algorithm proposed by Sirvastava (1993). A variant case of LRP allowing multiple use of vehicle subject to vehicle's time was presented by Lin and Kwok (2006). In their multi-objective function, they considered both vehicle capacity and travelling time limits among the homogenous vehicles. Two sequential approaches; SA and TS were proposed to solve the defined LRP before and after multi-route assignment. Computational results on real data showed that area characteristics played an important role in the two proposed heuristics. Since real data did not include the travelling time, the travelling times were estimated through a geographic information system (GIS). Moreover, they used the regression estimation for unavailable delivery time data. Caballero et al. (2007) presented a TS solution for a multi-objective, capacitated vehicle LRP subjected to time windows. The model consisted of weekly transportation of specific risk material (SRM) from some slaughterhouses to a set of incineration plants passing through some towns located at the road network. Applying the multi-objective meta-heuristic using an adaptive memory procedure (MOAMP), it is shown that the results are quite promising. Defining a saving matrix, Wu et al. (2002) applied SA to solve MDLRP by decomposing the problem to vehicle routing and location problems. Liu and Lin (2005) proposed a modified solution for the same model proposed by Liu and Lee (2003). They proposed an iterative algorithm which applies SA to solve LRP integrated with the inventory. Other heuristics were proposed by Albareda-Sambola et al. (2005), Melechovsky (2005), Wang et al. (2005), Prins et al. (2006) and Barreto et al. (2007).

In this paper, a mathematical model is presented that integrates the multiple products CLRP with the inventory decision under fixed order interval cost strategy. Such an inventory policy is more common in practice (1998) and usually easier to administer than the continuous strategy (2004). The presented model is an extension to and a modified version of the model by Liu and Lin (2005). To capture the real-world situation, a 3PL is added to the model, handling the depot space limitation if open depots are subjected to space limitations. A heuristic procedure based on SA is presented to solve the model. The SA-based algorithm is adopted from the work of Golden and Skiscim (1986) and uses two VRP procedures known as saving algorithms proposed by Clark and Wright (2007) and Wu et al. (2002). The Section 3 presents the development of the model.

3 Integrated LRP-inventory model

The focus of this research is on a three-layer LRP system consisting of plants (first layer), depots (second layer) and customers (third layer) integrated with inventory. It uses the fixed interval policy as the inventory strategy. The developed model possesses certain features that make it a useful decision-making tool for real-world applications. It considers multiple products and allows the use of 3PL in case of depot space limitation. It also assumes that customers' demands follow a statistical distribution. This capability allows utilising any particular distribution such as Normal, Poisson, etc. in practice. The following assumptions are made in the development of the model:

1. The locations of customers are pre-defined. However, demands are assumed to be stochastic.
2. Plants at the first level of the logistic network are considered capacitated.
3. The capacities of depots are limited. Therefore, depots hold a contract with 3PLs firms which can be used in case of space limitations.
4. The LTT property is assumed which indicates that the demand for each customer is less than a truck load. Therefore, each vehicle may visit more than one customer in each trip.
5. Each route starts and ends at the same depot.
6. Demand split is not allowed (i.e. a customer demand is supplied by only one depot).
7. Capacity of the homogenous vehicle(s) is limited.
8. A vehicle's route passes through only one depot. This means that there is no connection among depots.
9. The fixed interval order is used as the inventory policy. In this policy, the safety stock protection is needed for the order delivery lead time plus the order interval time. It is assumed that lead time is less than order interval time.
10. Demand during order interval follows a stochastic distribution with mean μ and standard deviation σ.

The following notations are used in this paper.

$I = \{i; i = 1, 2, \ldots, n\}$: set of customers (third layer)

$J = \{j; j = n + 1, n + 2, \ldots, n + m\}$: set of depots (second layer)

$S = \{s; s = n + m + 1, \ldots, n + m + k\}$: set of plants (first layer)

$V = \{v; v = 1, \ldots, V\}$: set of vehicles

$P = \{p; p = 1, \ldots, P\}$: set of products

$H = I \cup J = \{h;\ h = 1, 2, \ldots, n + \mathrm{m}\}$: set of nodes on the network including depots and customers

A_p: order cost of product p

H_p: holding cost of product p per unit per unit time

H': holding cost at the third-party per unit of product (fixed for all products) per unit time

L_{hgv}: total distance travelled from point (node) h to point (node) g on route v

m: transportation cost per unit distance

C_{sj}: cost of direct shipment of one unit of product from plant s to depot j

C_{pj}: processing (purchasing) cost of product p at depot j

FV: fixed dispatching cost per order

K_p: stock out cost per unit of product p

d_{hvp}: demand of point h on route v for product p during order interval time (random variable)

μ_{hvp}: expected demand of point h on route v for product p during order interval time

D_{ip}: annual demand of product p for customer i

D_{hvp}: annual demand of point h on rout v for product p

I_{hvp}: maximum inventory level of point h on route v for product p

$E\ (d_{hvp} > I_{hvp})$: expected shortage of point h on rout v for product p during order interval time

$P\ (d_{hvp} > I_{hvp})$: probability of shortage of point h on route v for product p during order interval time

F_j: fixed depot opening cost of depot j

K_{sp}: capacity space for product p at plant s (number of units)

K_{jp}: capacity space of product p at depot j (number of units)

K_v: capacity of vehicle v for all products. The limitation is on the total number of products serving route v not on individual product.

T_{ghv}: order interval for point g to h on route v in year. Connection gh is served on route v every T_{ghv} year.

N_v: number of customers assigned to route v

The following decision variables are used in the model.

Z_{ghv}: routing decision – 1 if there is an immediate connection from point g to point h on route v, and 0 otherwise

X_j: location decision – 1 if depot j is open and 0 otherwise

Y_{ij}: allocation decision – 1 if customer i is assigned to depot j and 0 otherwise

W_{sjp}: quantity of product p shipped from plant s to depot j

B_{jp}: quantity of product p that should use 3PL due to space limitation of depot j.

3.1 Model formulation

The objective of the LRP model is to minimise the system's total annual cost. In the classical LRP model, the system's cost is the sum of fixed depot cost, variable warehousing cost and delivery cost. Adopting the classical LRP objective function, the system cost is modified based on the presented network characteristics. Equation (1) indicates the objective function which contains six terms. The first term $\left(\sum_{j=n+1}^{n+m} F_j X_j\right)$ shows the fixed depot establishing cost in case a new depot(s) is to be opened. The second term, the direct transportation cost $\left(\sum_{s=n+m+1}^{n+m+k}\sum_{j=n+1}^{n+m} C_{sj}\sum_{p=1}^{p} W_{sjp}\right)$, is the shipping cost of products from plants to depots. This cost depends on the distance from the plant to the depot. The third term represents the total annual inventory and routing costs consisting of four sub-terms: purchasing (processing) cost $\left(\sum_{j=n+1}^{n+m}\sum_{p=1}^{p}\sum_{i=1}^{n} D_{ip}C_{pj}Y_{ij}\right)$, 3PLs cost $\left(\sum_{j=n+1}^{n+m}\sum_{p=1}^{p} H'B_{jp}\right)$ and inventory and logistics cost (the last two terms in Equation (1)). The 3PLs cost is a linear function of the quantity of product p which should use 3PL due to space limitation of depot j. The third sub-term indicates the combined routing and inventory cost. The inventory policy assumed is the fixed order interval policy adopted from Silver et al. (1998) and Tersine (1994) modified for a multi-product application. The last sub-term stipulates the routing cost from the last customer to the depot. Since there is no inventory cost from last customer to the depot, only routing cost should be considered in the model. There are also 14 sets of constraints in the model. The constraint sets (2)–(5) force only one depot to be assigned to a route. Constraint (2) assigns one and only one route to any customer. Constraint (3), known as flow conservation constraints (Golden and Assad, 1986), shows that any node belonging to the set of depots and customers should be entered and departed by the same vehicle. Constraint (4) indicates that any vehicle on the network can depart a depot only once. This prevents the vehicle to travel more than once for the customers located on the same routings. Moreover, it does not allow the vehicle to pass other depots. Constraint (5) illustrates that there is at least one connection from sets of depots and any customer(s) to the rest of the customers. These constraints are known as connectivity constraints. Constraint (6) connects the routing decision to the allocation decision. If there is a route from a customer to a depot, the customer should be assigned to that depot. Constraint (7) represents the 3PL constraint. If there is a limitation on the capacity of a depot for a product, the corresponding cost should be included in the total cost. For per unit limitation of product p at depot j, a cost of H' is charged on the network cost. Constraint (8) implies the capacity limitation of a plant. It ensures that the total units of product p delivered to all open depots are less than the space limitation of the plants. The constraint set (9) indicates that for any selected depot, the sum of the products shipped from all plants should be equal to the sum of the product demanded by customers. The constraint set (10) implies that for any route, the total quantity of all products should be less than the capacity of the vehicle. Appendix shows how the equation is developed. Equations (11)–(13) indicate the binary requirement of decision variables. Equations (14) and (15) show non-negative restriction on quantity and 3PL decision variables.

$$\begin{aligned}
\text{Min} & \sum_{j=n+1}^{n+m} F_j X_j + \sum_{s=n+m+1}^{n+m+k} \sum_{j=n+1}^{n+m} C_{sj} \sum_{p=1}^{p} W_{sjp} + \sum_{j=n+1}^{n+m} \sum_{p=1}^{p} \sum_{i=1}^{n} D_{ip} C_{pj} Y_{ij} \\
& + \sum_{j=n+1}^{n+m} \sum_{p=1}^{p} H' B_{jp} \\
& + \sum_{\substack{h\in H \\ h\neq g}} \sum_{\substack{g\in H \\ h\neq g}} \sum_{v\in V} \left[\sqrt{2\sum_{p=1}^{P} H_p D_{hvp} \sum \left(A_p + FV + mL_{hgv} + K_p E\left(d_{hvp} > I_{hvp}\right)\right)} \right. \\
& \left. + \sum_{p=1}^{P} H_p \left(I_{hvp} - \mu_{hvp}\right) + \sum_{p=1}^{P} H_p E\left(d_{hvp} > I_{hvp}\right) \right] Z_{ghv} \\
& + \sum_{\substack{h\in I \\ h\neq g}} \sum_{\substack{g\in H \\ h\neq g}} \sum_{v\in V} \frac{1}{T_{ghv}} Z_{ghv} \sum_{g\in J} mL_{hgv} Z_{hgv}
\end{aligned} \tag{1}$$

s.t.

$$\sum_{v\in V} \sum_{h\in H} Z_{ihv} = 1 \quad i \in I \tag{2}$$

$$\sum_{g\in H, g\neq h} Z_{hgv} - \sum_{g\in H, g\neq h} Z_{ghv} = 0 \quad v \in V, h \in H \tag{3}$$

$$\sum_{i\in I} \sum_{j\in J} Z_{jiv} \leq 1 \quad v \in V \tag{4}$$

$$\sum_{g\in R} \sum_{h\in \bar{R}} \sum_{v\in V} Z_{ghv} \geq 1 \quad \forall (R, \bar{R}) \quad R \subset G, J \subseteq R \tag{5}$$

$$-Y_{ij} + \sum_{h\in H} \left(Z_{ihv} + Z_{jhv}\right) \leq 1 \quad v \in V, i \in I, j \in J \tag{6}$$

$$\sum_{s\in S} W_{sjp} - K_{jp} X_{jp} \leq B_{jp} \quad p \in P, j \in J \tag{7}$$

$$\sum_{j=n+1}^{m} W_{sjp} - K_{sp} X_{sp} \leq 0 \quad p \in P, s \in S \tag{8}$$

$$\sum_{s \in S} W_{sjp} - \sum_{i \in I} D_{ip} Y_{ij} = 0 \quad p \in P, j \in J \tag{9}$$

$$\sum_{\substack{h \in H \\ g \neq h}} \sum_{\substack{g \in H \\ g \neq h}} \left(\sum_{p \in P} \mu_{hvp} - \frac{\sum_{p \in P} D_{hvp} T_{ghv}}{2} \right) Z_{ghv} \leq K_v \quad v \in V \tag{10}$$

$$Z_{ghv} = 0{,}1 \quad g \in H, h \in H, v \in V \tag{11}$$

$$X_j = 0{,}1 \quad j \in J \tag{12}$$

$$Y_{ij} = 0{,}1 \quad i \in I, j \in J \tag{13}$$

$$W_{sjp} >= 0 \quad s \in S, j \in J, p \in P \tag{14}$$

$$B_{jp} >= 0 \quad j \in J, p \in P \tag{15}$$

4 Solution methodology

An iterative heuristic algorithm is presented to solve the developed model. The algorithm consists of two steps. In the first step, an initial solution is generated, and in the second step, the initial solution is improved through an iterative process in which two SA modules are embedded.

4.1 Generating initial solution

The procedure to generate the initial solution is an extended version of an algorithm presented by Liu and Lee (2003). The algorithm is a route first, location–allocation second algorithm. A customer is randomly selected and assigned to a route (vehicle). A customer closest to the selected customer is then chosen and assigned to the same route. The process is repeated until all customers and all routes are assigned. The routes are assigned to the depots that are physically closest to them. For this, using Equation (16), an (x, y) coordinate is assigned to each route. The equation identifies the weighted-average coordinate of a route. Using this coordinate values, the average distances from a given route to all depots are calculated and then the shortest path is chosen.

$$X_v = \frac{\sum_{i=1}^{Nv} \sum_{p=1}^{p} D_{ip} x_i}{\sum_{i=1}^{Nv} \sum_{p=1}^{p} D_{ip}} \quad \text{and} \quad Y_v = \frac{\sum_{i=1}^{Nv} \sum_{p=1}^{p} D_{ip} y_i}{\sum_{i=1}^{Nv} \sum_{p=1}^{p} D_{ip}} \tag{16}$$

where X_v and Y_v indicate the coordinate of route v, (x_i, y_i) shows the coordinate of customer i and N_v is the number of customers assigned to route v. Since there is limitation on the capacity of depots, a penalty cost which is the 3PL cost is charged to the network if the total demand assigned to a depot exceeds the depot capacity.

4.2 Initial solution improvement procedure

To improve the initial solution, the problem is decomposed into two sub-problems: location/allocation problem and VRP. The initial solution is used as an input to the location–problem (also known as depot improvement). The solution to the location problem is improved by applying a SA procedure until a stopping criterion is satisfied. The improved location problem is then input to the vehicle-routing module (also known as route improvement) which consists of two sequential sub-modules: a SA and a saving procedure. The improved vehicle routing is then fed into the location problem and such an improvement procedure iterates until a stopping criterion is satisfied. In Sections 4.2.1 and 4.2.2, depot and route improvement algorithms are discussed in details.

4.2.1 Depot improvement algorithm

Figure 1 shows the depot improvement algorithm. A SA approach, adopted from Golden and Skiscim (1986) improves the depot configuration. The algorithm begins by setting the SA parameters; starting temperature, final temperature, cooling rate, epoch and equilibrium measure. At each temperature of the SA, a set of selective changes are made to the depot configuration. The change is made by randomly applying one of the exchange, switch or drop procedures as defined later in this section.

Two criteria are used to accept the proposed network change. As the primary criterion, if the cost of the network after the change has been made (CNC) is less than the initial network cost (INC), or if the value of $e^{(-\mathrm{CNC})/(\mathrm{INC})}$ is less than a randomly generated number from the interval (0, 1), the candidate change is accepted; otherwise, another candidate change is generated. This process continues until a pre-determined number of accepted candidate changes have been generated. The average cost of the generated solution set is then compared to the INC. If the values are sufficiently close, the generated solution is considered as a good solution at temperature T (SA is in equilibrium). Under such a condition, the temperature of SA is dropped as much as the pre-determined cooling rate value and the algorithm is repeated until the final temperature is achieved. However, if the average cost of generated solution is not sufficiently close to the initial cost, a new set of candidates is generated.

Procedure exchange

a Randomly select an open depot.

b Find a closed depot nearest to the selected open depot.

c Randomly select a route from the selected open depot.

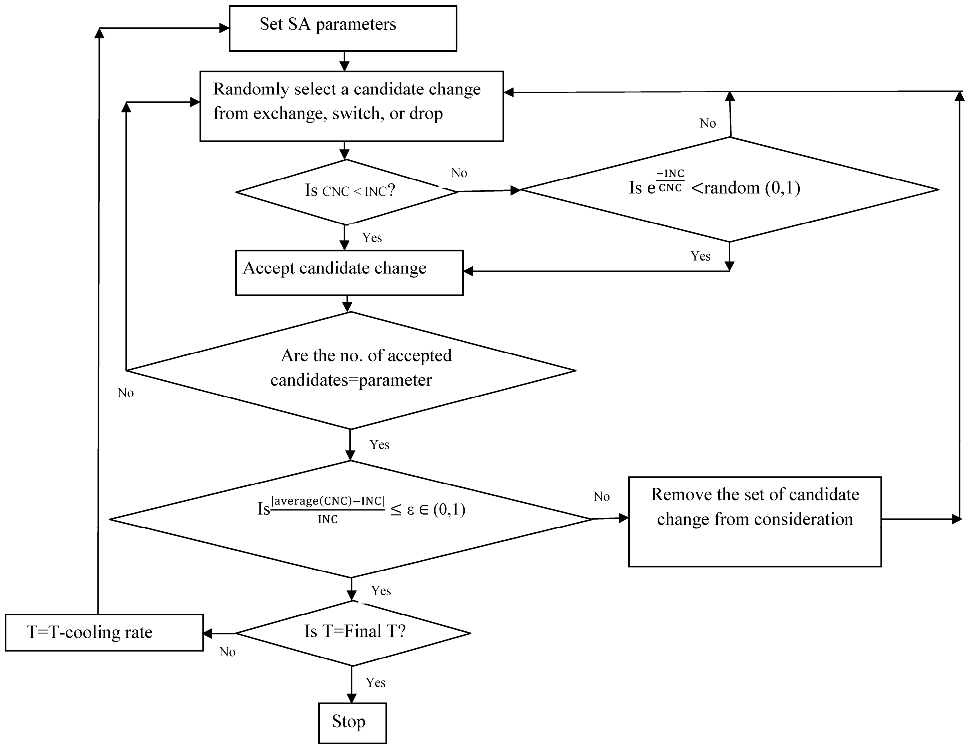
d Remove the selected route in step c and assign it to the selected closed depot forcing it to open.

e If there is no more routes in the selected open depot, close the depot. Otherwise; leave the depot open.

f Apply the saving algorithm proposed by Wu et al. (2002) to modify the customer sequence. Note that the saving algorithm is applied at each iteration of the exchange procedure.

g A candidate change is generated by exchange procedure.

Figure 1 Depot improvement algorithm



Procedure switch

a Randomly select two open depots.

b Randomly select two routes, one from each selected open depots.

c Exchange the routes.

d Apply saving algorithm proposed by Wu et al. (2002) to modify the customer sequence.

e A candidate change is generated by switch procedure.

Procedure drop

a Select an open depot randomly.

b Select a route on the selected open depot randomly.

c Select another open depot randomly.

d Assign the selected route at step b to the depot chosen at step c forcing the depot to open.

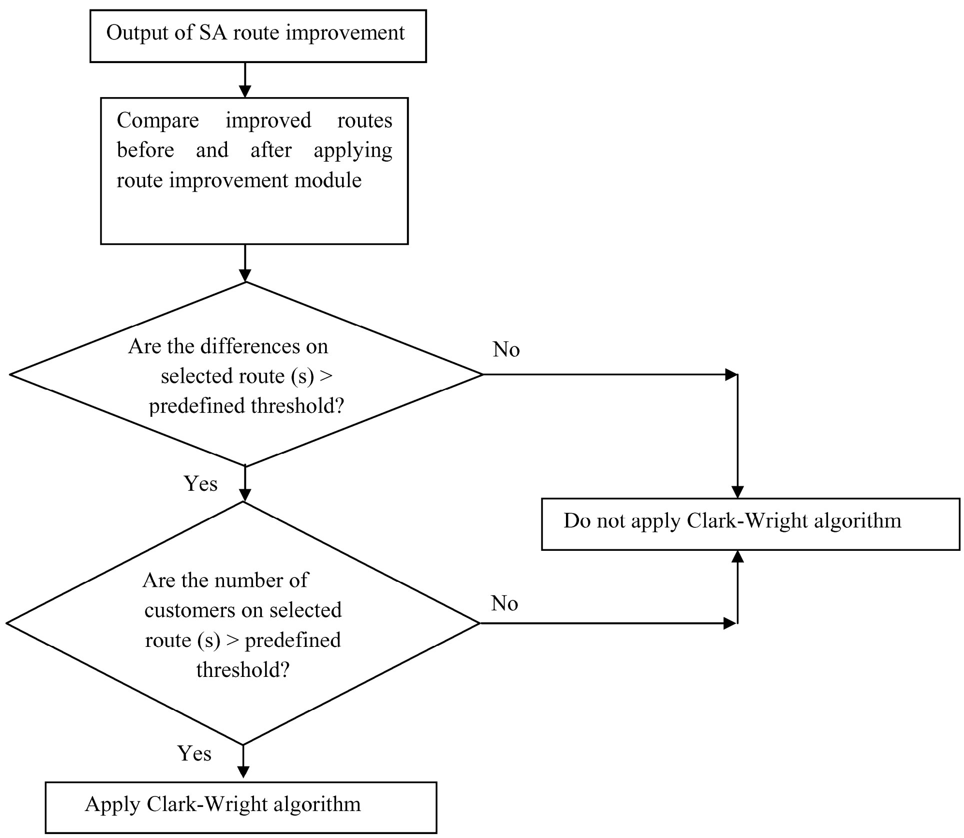
e Apply saving algorithm proposed by Wu et al. (2002) to modify the customer sequence.

A candidate change is generated by the application of drop procedure.

4.2.2 Route improvement algorithm

The route improvement algorithm uses two sub-modules: a SA procedure and a saving procedure. The SA is similar to that used for depot improvement except that it utilises insert and swap procedures These procedures are defined at the end of this section. The saving procedure is called Clark–Wright algorithm (Larsen and Odoni, 2007) and it is a conditional procedure that follows the SA. Figure 2 shows how the Clark–Wright algorithm is applied.

Figure 2 Conditions on applying Clark–Wright algorithm



The two decision conditions shown in the procedure eliminate many unnecessary routing changes from consideration. The first condition indicates that the routes holding modifications less than the pre-defined value are not considered by Clark–Wright algorithm. The reason is that by applying few numbers of modifications on a route sequence, the network cost probably will not substantially be affected. Similarly, the latter condition implies that when the number of customers on a route is not large enough, the route is not an appropriate candidate for Clark–Wright algorithm. The reason is that a route sequence holding only few numbers of customers does not play an important role in the network cost. The insert and swap procedures are described as follows:

Procedure insert

a Randomly choose two routes 1 and 2.

b Randomly choose one customer from route 1.

c If there is enough capacity on route 2, insert the selected customer to route 2 using the saving algorithm proposed by Wu et al. (2002); otherwise, go back to step a.

d A candidate change is generated by insert procedure.

Procedure swap

a Randomly choose two routes 1 and 2.

b Randomly choose a customer on route 1.

c Find the nearest customer on route 2 to the selected customer to route 1.

d Exchange the position of selected customers on routes 1 and 2.

e Apply saving algorithm proposed by Wu et al. (2002).

f A candidate change is generated by swap procedure.

5 Results and analysis

There are two conditions under which the model can be used in practice: designing a new network or redesigning/modifying an existing one. With the design of a new network, the locations of customers and potential plants may be pre-determined while the locations of potential depots are to be identified. In the case where a network already exists, one or more parameters of the network can be changed to analyse their effects on the system.

The developed solution algorithm was coded with Matlab 7.5.0 (R2007b). Matlab allows the use of pre-programmed functions which simplify the programming of the procedures. There is no benchmark problem in the literature with which the solution algorithm can be compared. Hence to validate the model and the developed solution algorithm, a small size problem was generated, modelled and solved. The problem size is small enough to allow for a complete enumeration to find the optimal solution. A network consisting of one plant, two potential depots and three customers with two products was designed and solved. Results indicated that the developed algorithm generated an optimal solution.

To further evaluate the capability of the algorithm in solving real-world problems and to analyse the effect of different parameters on the performance of the network, test cases were generated based on different network configurations (defined in Table 1) and were solved. The evaluations were done based on two criteria: network cost and runtime. As shown in this table, the smallest test case is a network consisting of one plant, ten potential depots, ten products and 50 customers. A total of 378 problems was generated and solved. Each problem was solved five times and then the average network cost and runtime were calculated.

While some of the parameters of the model, like dispatching cost, are set at fixed values, others are selected at random from the intervals shown in Table 2. The annual demand for any customer and any product is randomly selected from the Uniform distribution $U(350, 450)$. Demand during lead time is assumed to follow a Poisson distribution with the mean randomly taken from $U(10, 20)$. The location of customers, depots and plants are randomly generated from a square with the side of 200 which is (–100, 100) on the x and y axes.

Table 1 Major network parameters and their levels

Parameter	*No. of levels*	*Level*
No. of plants	3	1, 3, 5
No. of depots	3	10, 20, 30
No. of products	3	10, 20, 40
No. of customers	7	50, 100, 150, 200, 250, 300, 350
Vehicle capacity	2	Low = 1,900, high = 4,500
Total test problems	378	

Table 2 Parameters of the mathematical model and their values/range for test problems

Range	*Unit*	*Parameter*
$U(10, 20)$	Per customer per product	Demand during order interval time
$U(350, 450)$	Per customer per product	Annual demand
$U(14{,}000, 15{,}000)$	Number of product	Depot capacity/product
160,001	Number of product	Plant capacity for any product any plant
$U(400, 410)$	\$/order	Order cost/product
$U(1, 1.5)$	\$/unit of product p/year	Holding cost
$U(40, 45)$	\$/unit of product p	Shortage cost
300	\$/order	Dispatching cost
2	\$/mile	Indirect transportation cost
$U(0.005, 0.01)$	\$/mile	Direct transportation cost
$U(125{,}000, 135{,}000)$	\$	Depot fixed cost
$U(2, 3)$	\$/unit of product p/year	3PL holding cost

As stated in a previous section, the developed solution algorithm adopts a SA procedure from Golden and Skiscim (1986). For analysis purposes, the parameters of this algorithm are selected as follows:

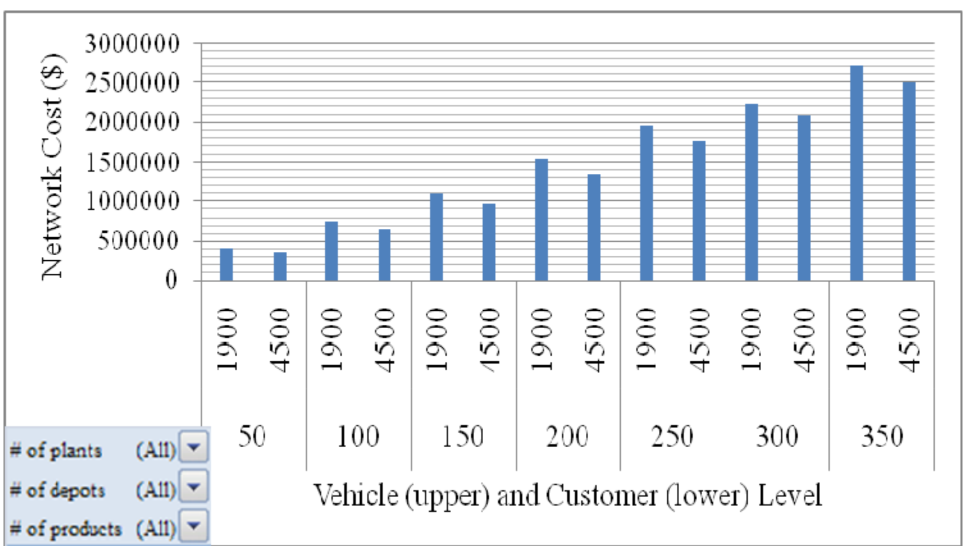
starting temperature = 200; final temperature = 60; cooling rate = 15; epoch = 5; $\varepsilon = 0.5$.

To ensure timely termination of the algorithm with good results, the following termination criterion is selected. When the percentage of improvement in total cost of the network from one iteration to the next is very small, repeating the algorithm will not improve the objective function significantly and hence, the algorithm will terminate. In this study, the termination condition is set to 1% difference between costs of the network in two consecutive iterations.

5.1 Network cost

Average network cost can be used as a measure to determine optimal network parameters for any network. It is apparent that when the number of customers and/or products increases, the cost of the network is expected to increase as well. However, when all other parameters in the model remain the same, it is possible to make certain decisions on parameters of interest. One such decision is the type of vehicle fleet, low capacity or high capacity. Figure 3 shows that for the network under consideration, on average, the network cost is lower when the high-capacity vehicle is used. This result is reasonable because the number of required vehicles is less when the high-capacity vehicles serve the customers compared to low-capacity vehicles. The statistical analysis indicates that the cost difference between low- and high-capacity vehicles is significant.

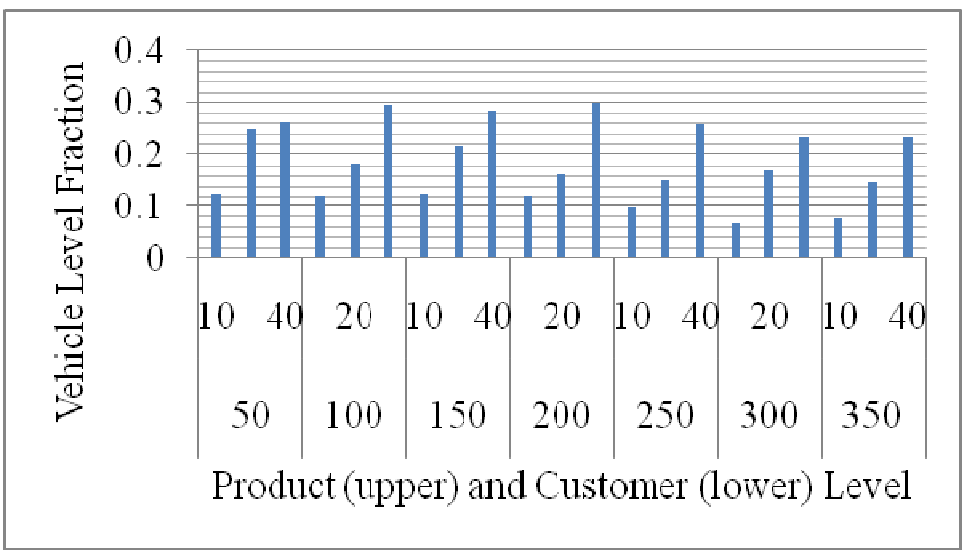
Figure 3 Effect of changes in vehicle capacity and number of customers on total network cost (see online version for colours)



It is also desirable to consider the effect of other parameters such as number of depots, plants and products on the cost difference between low- and high-capacity vehicle scenarios. Further analyses indicate that number of plants and depots do not have a significant effect on the cost difference. Since this conclusion is quite obvious, the corresponding figures are not shown here. However, the cost difference is significantly dependent on the number of products. Figure 4 compares the network cost under two different conditions; when network uses low-vehicle capacity vs. high-vehicle capacity. The cost difference is represented as a percentage of the low-capacity vehicle cost on the *y*-axis. The ratio is computed at different levels of customers and products (*x*-axis). For example, the 25% ratio for a network including 50 customers and 20 products indicates that the cost of choosing a high-capacity vehicle is 25% lower than that of a low-capacity vehicle. It is seen that as the number of products increases, the cost ratio at any customer level also increases. For example, with 100 customers, the ratio is about 12.29% ($741,322 against $650,250). This fraction increases to 18.42% and 29.64% for product levels at 20 and 40, respectively.

In summary, the analysis only indicates that the developed model can be used to perform different types of analysis and can be used as a decision tool. Our limited analysis for the stated network indicates that using the high-capacity vehicle fleet for the network leads to saving more money than that of low-capacity vehicle. Besides, the saving will increase when the number of products increases. This conclusion may or may not hold for other networks due to large number of variables and parameters involved. However, the model can be used to perform such analysis for any network and appropriate conclusions can be drawn based on the observed pattern.

Figure 4 Ratio of high- to low-vehicle network cost against different levels of products and customers (see online version for colours)



5.2 Run time

Using run time to evaluate the performance of a procedure is much dependent on the computer, that is, used for the execution of the procedure. However, it can be used to study the effect of network parameters and to perform comparative evaluation of the procedure. The test problems were run on Pentium (R) CPU 3.00 GHz and average run time was calculated.

Figures 5 and 6 show average run time in seconds (*y*-axis) against different levels of product, customers and vehicle capacity (*x*-axis). It is shown that in general, run time for the model increases with an increase in the number of products, customers and vehicle capacity. This result is quite expected. As seen, the only unexpected run time belongs to a network with 100 customers, 40 products and 4,500 vehicle capacity. No explanation could be given for this observation.

Figure 5 Average run time (second) against product and customer levels (see online version for colours)

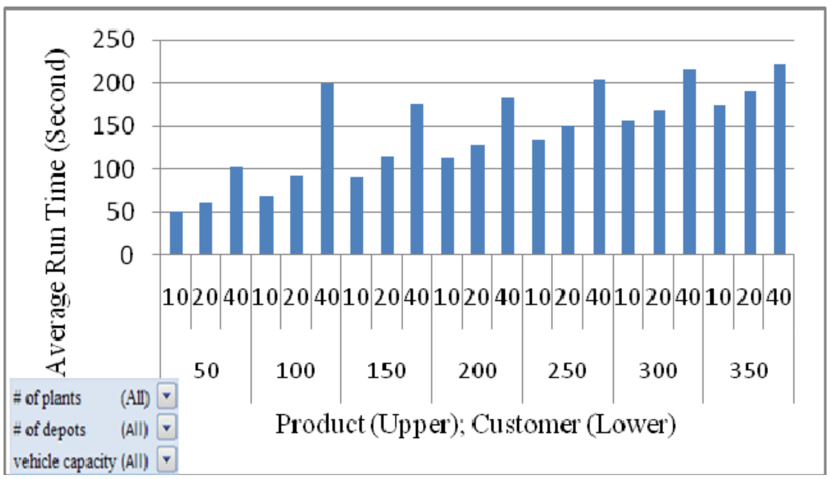
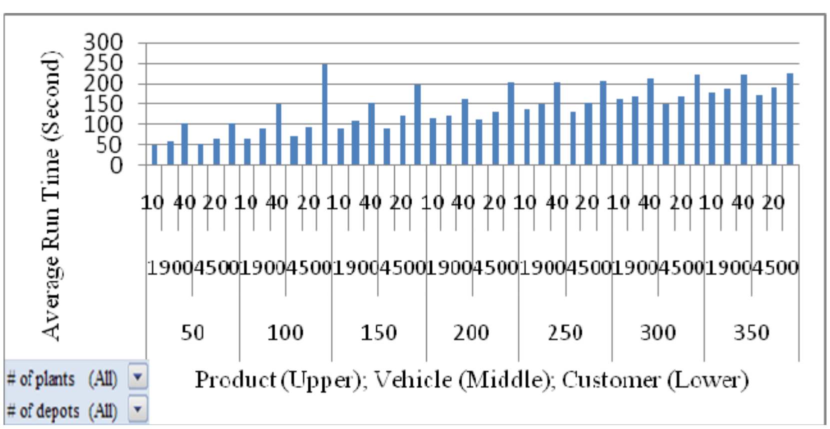
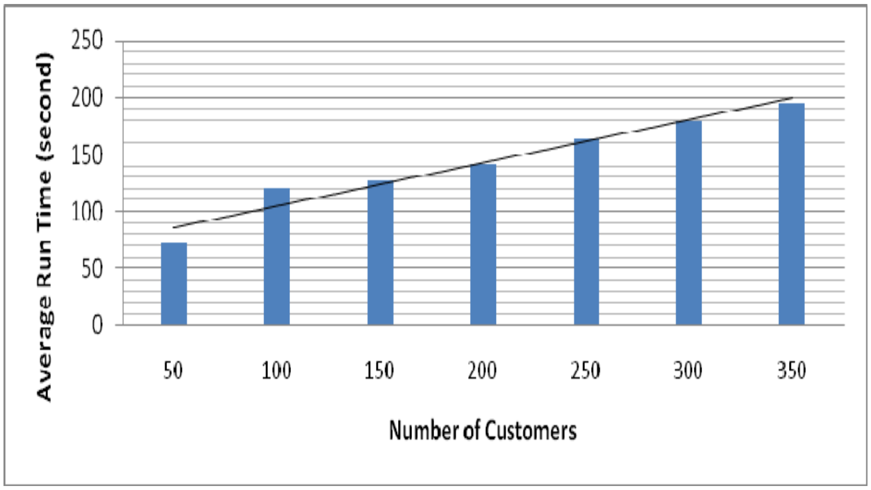


Figure 6 Average run time (second) against product, vehicle level and customer (see online version for colours)



Another item of interest was to observe how big a problem the presented algorithm can solve and how the change in run time is related to problem size (Figure 7). In this analysis, the largest problem size which is considered contains 350 customers. As shown, the average run time follows a linear pattern as the number of customers increases. This linear relationship between problem size and time indicates that much larger size problems can also be solved with the proposed algorithm.

Figure 7 Linear relationship between run time and problem size (see online version for colours)



6 Conclusions

A model for a three-layer LRP integrated with the inventory decisions is presented in this paper. The model assumes multiple products network with a fixed interval strategy as the inventory policy. The model takes the capacitated depot LRP into consideration; therefore, a third-logistics party handles the depots space limitation if needed. Since the model belongs to Np-hard class, a heuristic SA-based algorithm is presented as the solution methodology. The presented algorithm is tested and the results that while selecting high-vehicle capacity lead to saving more cost on the network, the run time for such size problems are quite reasonable.

It is suggested to extend this study in terms of both modelling characteristics and solution methodology to capture real situation better while generating more promising results. For example, it is desirable to integrate other inventory policies with LRP and then compare them to observe the best inventory strategy. In terms of solution, other heuristic methods can be examined. Genetic algorithm may generate good results for integrated LRP with inventory if a good chromosome representation is proposed to define the solution. More saving on the network cost is achievable if other supply chain activities such as plant location problem and packing problem are added to the LRP integrated with the inventory.

References

Ago, M., Nishi, T. and Konishi, M. (2007) 'Simultaneous optimization of storage allocation and routing problems for Belt-conveyer transportation', *Journal of Advanced mechanical Design, Systems, and Manufacturing*, Vol. 1, No. 2, pp.250–261.

Albareda-Sambola, M., Diaz, J.A. and Fernandez, E. (2005) 'A compact model and tight bounds for a combined location–routing problem', *Computer and Operation Research*, Vol. 32, No. 3, pp.407–428.

Alumur, S. and Kara, B.Y. (2007) 'A new model for hazardous location routing problem', *Computer and operation Research*, Vol. 34, No. 5, pp.1406–1423.

Ambrosino, D. and Scutella, M.G. (2005) 'Distribution network design: new problems and related models', *European Journal of Operational Research*, Vol. 165, No. 3, pp.610–624.

Barreto, S., Ferreiera, C., Paixia, J. and Santos, B.S. (2007) 'Using clustering analysis in a capacitated location routing problem', *European Journal of Operational Research*, Vol. 179, No. 3, pp.968–977.

Berger, R.T., Caullard, C.R. and Daskin, M.S. (2007) 'Location routing problem with distance constraint', *Transportation Science*, Vol. 41, No. 1, pp.29–43.

Blanchard, D. (2007) *Supply Chain Management Best Practices*. Hoboken, NJ: John Wiley & Sons, Inc.

Boudia, M. and Prins, C. (2009) 'A memetic algorithm with dynamic population management for an integrated production–distribution problem', *European Journal of Operational Research*, Vol. 195, No. 3, pp.703–715.

Bouhafs, L., Hajam, A. and Koukam, A. (2006) 'A combination of simulated annealing and ant colony system for the capacitated location routing problem', *Lecture Notes in Computer Science*, Vol. 4251, pp.409–416.

Bowerman, R., Hall, B. and Calamai, P. (1995) 'A multi objective optimization approach to urban school bus routing: formulation and solution method', *Transportation Research, Part A*, Vol. 22, No. 2, pp.107–123.

Burks, Jr., R.E. (2006) 'An adaptive taboo search heuristic for the location routing pickup and delivery problem with time windows with a theater distribution application', PhD Dissertation, Air force institute of technology.

Burns, L.D., Hall, R.W., Blumenfeld, D.E. and Daganzo, C.F. (1985) 'Distribution strategies that minimize transportation and inventory costs', *Operations Research*, Vol. 33, No. 3, pp.469–490.

Caballero, R., Gonzalez, M., Guerrero, F.M., Molina, J. and Paralera, C. (2007) 'Solving a multi-objective location routing problem with a meta-heuristic based on taboo search: application to a real case in Andalusia', *European Journal of Operational Research*, Vol. 177, No. 3, pp.1751–1763.

Chopra, S. and Meindl, P. (2007) *Supply Chain Management: Strategy, Planning, and Operation* (3rd ed.). Upper Saddle River, New Jersey: Prentice Hall.

Craven, B.D. and Islam, S.M.N. (2007) 'Logistics and supply chain management: a review of some operations research approaches', *Int. J. Industrial and Systems Engineering*, Vol. 2, No. 4, pp.444–453.

Daskin, M., Coullard, C. and Shen, Z.J. (2002) 'An inventory location model: formulation, solution algorithm, and computational results', *Annals of Operations Research*, Vol. 110, pp.83–106.

Gillett, B.E. and Johnson, J.G. (1976) 'Multi terminal vehicle dispatch algorithm', *Omega*, Vol. 4, No. 6, pp.711–718.

Golden, B.L. and Assad, A. (1986) 'Perspective on vehicle routing: exciting new developments', *Operations Research*, Vol. 34, No. 5, pp.803–810.

Golden, B.L., Ball, M. and Bodin, L.D. (1981) 'Current and future research directions in network optimization', *Computer and Operation Research*, Vol. 8, No. 2, pp.71–81.

Golden, B.L. and Skiscim, C.C. (1986) 'Using simulated annealing to solve routing and location problems', *Naval Research Logistics Quarterly*, Vol. 33, No. 2, pp.261–279.

Gunnarsson, H., Ronnqvist, M. and Carlsson, D. (2006) 'A combined terminal location and ship routing problem', *Journal of the Operational Research Society*, Vol. 57, No. 8, pp.928–938.

Jakobsen, S.K. and Madsen, O.B.G. (1980) 'A comparative study of heuristics for two level location routing problem', *European Journal of Operational Research*, Vol. 5, No. 6, pp.378–387.

Laporte, G. and Nobert, Y. (1981) 'An exact algorithm for minimizing routing and operating costs in depot location', *European journal of operational research*, Vol. 6, No. 2, pp.224–226.

Larsen, R.C. and Odoni, A.R. (2007) *Urban Operation Research: Logistical and Transportation Planning Methods* (2nd ed.). MA: Dynamic Ideas Belmont.

Lin, C.K.Y. and Kwok, R.C.W. (2006) 'Multi objective meta-heuristics for a location routing with multiple use of vehicle on real data and simulated data', *European Journal of Operational Research*, Vol. 175, No. 3, pp.1833–1849.

Liu, S.C. and Lee, S.B. (2003) 'A two phase heuristic method for the multi depot location routing problem taking inventory control decision into consideration', *Int. J. Advanced Manufacturing Technology*, Vol. 22, pp.941–950.

Liu, S.C. and Lin, S.B. (2005) 'A heuristic method for the combined location routing and inventory problem', *Int. J. Advanced Manufacturing Technology*, Vol. 26, pp.372–381.

Melechovsky, J. (2005) 'A meta-heuristics to solve location routing problem with non linear costs', *Journal of Heuristics*, Vol. 11, Nos. 5–6, pp.375–391.

Miranda, P.A. and Garrido, R.A. (2004) 'Incorporating inventory control decisions into a strategic distribution network design model with stochastic demand', *Transportation Research Part E: Logistics and Transportation Review*, Vol. 40, No. 3, pp.183–207.

Nagi, G. and Salhi, S. (1996) 'A nested location routing heuristic using route length estimation', *Studies in Locational Analysis*, Vol. 10, pp.109–127.

Nagi, G. and Salhi, S. (2007) 'Location–routing: issues, models and methods', *European Journal of Operational Research*, Vol. 177, No. 2, pp.649–672.

Nooral Haq, A. and Kannan, G. (2006) 'Design of an integrated supplier selection and multi-echelon distribution inventory model in a built-to-order supply chain environment', *Int. J. Production Research*, Vol. 44, No. 10, pp.1963–1985.

Or, I. and Pireskalla, W.P. (1979) 'A transportation location allocation model for regional blood banking', *AIIE Transactions*, Vol. 11, No. 2, pp.86–95.

Perl, J. and Daskin, M.S. (1985) 'A warehouse location routing problem', *Transportation Research B*, Vol. 19, No. 5, pp.381–396.

Prins, C., Prodhon, C. and Calvo, R.W. (2006) 'A memetic algorithm with population management (MA|PM) for the capacitated location routing problem', *Lecture Notes in Computer Science*, Vol. 3906, pp.183–194.

Shen, Z.J.M. (2007) 'Integrated supply chain design models: a survey and future research direction', *Journal of Industrial and Management Operation*, Vol. 3, No. 1, pp.1–27.

Shen, Z.J.M. and Qi, L. (2007) 'Incorporating inventory and routing costs in strategic location models', *European Journal of Operational Research*, Vol. 179, No. 2, pp.372–389.

Shu, J., Teo, C-P. and Shen, Z-J.M. (2005) 'Stochastic transportation inventory network design problem', *Operations Research*, Vol. 53, No. 1, pp.48–60.

Silver, E.A., Pyke, D. and Peterson, R. (1998) *Inventory Management and Production Planning and Scheduling* (3rd ed.). New York, NY: John Wiley & Sons.

Sirvastava, S. (1993) 'Alternate solution procedures for location routing problem', *OMEGA International Journal of Management Science*, Vol. 21, No. 4, pp.497–506.

Tersine, R.J. (1994) *Principles of Inventory and Materials Management* (4th ed.). New Jersey: PTR Prentice-Hall.

Tuzun, D. and Burke, L. (1999) 'A two phase taboo search approach to the location routing problem', *European Journal of Operational Research*, Vol. 116, No. 1, pp.87–99.

Wang, X., Sun, X. and Fang, Y. (2005) 'A two phase hybrid heuristic search approach to the location–routing problem', *IEEE International Conference on Systems, Man, and Cybernetics*, Vol. 4, pp.3338–3343.

Winston, W.L. (2004) *Operations Research: Applications and Algorithms* (4th ed.). Canada: Brooks/Cole-Thomson Learning.

Wu, T., Low, C. and Bai, J. (2002) 'Heuristic solutions to multi depot location–routing problems', *Computers and Operation Research*, Vol. 29, No. 10, pp.1393–1415.

Yang, P. and Li-Jun, B. (2006) 'An integrated optimization problem in logistics and PSO solution', *International Conference on Service System and Service Management*, Vol. 2, pp.965–970.

Yi, W. and Ozdamar, L. (2007) 'A dynamic logistic coordination model for evacuation and support in disaster response activities', *European Journal of Operational Research*, Vol. 179, No. 3, pp.1177–1193.

Note

[1] Special density indicates the weight of customers in a region but does not identify the exact location of demand points like LRP assumption.

Appendix

Winston (2004) formulates the expected on hand inventory for a single product system run under the fixed order interval as:

$$\text{Expected on hand inventory} = I - \mu + D\frac{T}{2}$$

Therefore, the expected order quantity can be calculated as:

$$\text{Expected order quantity} = \mu - D\frac{T}{2}$$

Extending the single product system to the multiple (*p* product) results in:

$$\text{Expected order quantity} = \sum_{p \in P} \mu_p + \sum_{p \in P} \left(\frac{D_p}{2} \right) T$$

The total number of order quantities on any route should satisfy the vehicle capacity constraint. In addition, the occupied space for all products are assumed equal and the total expected summation of demands on a route can be approximated by central limit theorem. Therefore, for any point *h* on route $v \in V$:

$$\left(\sum_{p \in P} \mu_{hvp} - \frac{\sum_{p \in P} D_{hvp} T_{hv}}{2} \right) \le K_v$$