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Theory and Methodology

Combined location–routing problems: A synthesis and future research directions

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Abstract

Over the past few decades, the concept of integrated logistics systems has emerged as a new management philosophy which aims to increase distribution efficiency. Such a concept recognizes the interdependence among the location of facilities, the allocation of suppliers and customers to the facilities, and the vehicle route structure around depots. As such, it coordinates a broader spectrum of location and routing options available to logistics managers and consequently avoids the suboptimization of distribution solutions. Reflecting the increasing importance of integrated logistics systems, an extensive body of combined location–routing literature has developed in less than 30 years. In this paper, we synthesize the past evolution of location–routing literature and then explore promising research opportunities in incorporation of more realistic aspects, algorithmic design, and model complexity. © 1998 Elsevier Science B.V.

1. Introduction

Today's logistics environments are often characterized by deregulation and globalization. In the wake of deregulation and globalization, many logistics managers are faced with tough location, allocation, and transportation problems. As the managers wrestled with these problems, management scientists have developed more efficient problem solving techniques using the concept of integrated logistics systems. The crux of such techniques is the combined location–routing model. In general terms, the combined location–routing model solves the joint problem of determining the optimal number, capacity and location of facilities (domiciles) serving more than one customer/supplier, and finding the optimal set of vehicle schedules and routes. Its major aim is to capitalize on distribution efficiency resulting from a series of coordinated, non-fragmented movements and transfer of goods.

The main difference between the location–routing problem (LRP) and the classical location–allocation problem is that, once the facility is located, the former requires a visitation of customers/suppliers through tours, whereas the latter assumes the straight-line or radial trip from the facility to the customer/supplier. Therefore, the classical loca-

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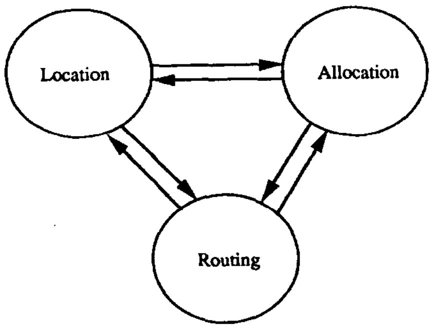

Fig. 1. Interdependence among location, allocation and routing (source: Perl, 1983).

tion–allocation problem ignores tours when locating facilities and subsequently may lead to increased distribution cost (Salhi and Rand, 1989). In contrast with the classical location–allocation problem, LRP considers tours and then seeks the optimal facility location and route design simultaneously so that it can interrelate those two decisions (see Fig. 1). The conceptual foundation of LRP studies dates back to Von Boventer (1961), Maranzana (1965), Webb (1968), Lawrence and Pengilly (1969), Christofides and Eilon (1969), and Higgins (1972). Although these earlier studies are far from capturing the total complexity of LRP, they first recognized the close interface between location and transportation decisions. Later, Cooper (1972, 1976) generalized the transportation–location problem which aimed to find the optimal location of supply sources and to minimize the transportation cost from such sources to destinations. His work was further refined by Tapiero (1971) who incorporated time-related complications into the generic transportation–location model. All of these earlier studies, however, were not designed to establish tours on the transportation network and therefore might not be perceived as true forms of LRP studies.

Watson-Gandy and Dohrn (1973) may be one of the first authors credited to consider the multiple-drop ('tour') nature of the vehicle routes within the location–transportation framework. With the added task of establishing tours within the location–transportation network, LRP is much harder to solve than the generic transportation–location model. Nevertheless, we have witnessed the development of true LRP studies in late 1970s and early 1980s. These studies include Or and Pierskalla (1979), Jacobsen and Madsen (1978), Harrison (1979), Jacobsen and Madsen (1980), Nambiar et al. (1981), Laporte and Nobert (1981), and Madsen (1983). Such popularity of LRP studies almost parallels the advent of an integrated logistics concept and the growth of international trade which necessitated distribution efficiency. Recognizing the increasing importance of LRP, we focus our attention on the comprehensive review of LRP studies throughout the paper. Therefore, in this paper we may not survey the progress of numerous articles which stressed the interrelationship between the location and transportation decisions, but did not take into account the tour nature of vehicle routes. Examples of such articles include Kaufman and Broeckx (1981), Current et al. (1984), Drezner et al. (1985), Orgyczack et al. (1989), and Koksalan et al. (1995).

In light of the above, the main purposes of this paper are to:

1. Comparatively review the existing LRP literature with respect to its algorithmic development and added realities;
2. Develop the hierarchical taxonomy and classification scheme which may help identify which aspects of LRP have been insufficiently explored and how far the existing LRP modelling efforts have progressed toward real-world needs;
3. Synthesize the past trend of LRP studies;
4. Offer some suggestions and directions for future LRP studies.

2. Survey methodology

The LRP literature surveyed in this paper was selected from two major sources: (1) an on-line computerized search (i.e., ABIC, BPI, and dialogue information service) of various business, mathematics, and operations research journals using keywords such as location, location–allocation, transportation, vehicle routing, vehicle scheduling, location–routing, and integrated logistics; (2) the bibliographic list of past survey articles dealing with transportation network, location, routing, and combined location and routing problems. In addition to the initial list of the literature, we included a few more articles related to LRP by scanning the reference sections of the initially selected LRP articles. However, it is noted that we included here only those articles which have appeared in English language publications and excluded proceeding papers, working papers, internal technical reports, and doctoral and master's theses. Although our selection of articles for this review was intended to be as extensive and exhaustive as possible, we apologize for any inadvertent oversight of some relevant articles.

3. Taxonomy and classification

Various classification schemes were available in the literature to categorize either pure vehicle routing/scheduling problems (e.g., Bodin, 1975; Bodin and Golden, 1981; Ronen, 1988) or location–allocation problems (e.g., Krarup and Pruzan, 1979; Aikens, 1985; Current et al., 1990). We used these as guidelines and have developed our own taxonomy and classification schemes for LRP. Our taxonomy and classification schemes are far more comprehensive than are the earlier attempts (Madsen, 1981, 1983; Balakrishnan et al., 1987; Laporte, 1988; List et al., 1991) of categorizing LRP. As shown in Table 1(a) and (b), we use a two-way classification to categorize LRP studies. In one dimension, LRP studies were classified in terms of their problem perspectives. In another dimension, LRP studies were grouped into their algorithmic developments. Each dimension is further broken down into appropriate subcategories. Using this taxonomy, each LRP study reviewed in this paper was placed within at least one subcategory along each of the two dimensions. In addition, each reviewed article was categorized according to its application area, if it focused on the specific application rather than theoretical advances.

4. Problem perspectives

In a broad sense, there are two distinct classes of LRP studies with regard to their problem perspectives. One of these classes is a single-stage LRP which is primarily concerned with both the location of facilities serving customers and the establishment

Table 1

a. Classification of LRP with regard to its problem perspective

I.	Hierarchical level A. Single stage B. Two stages
II.	Nature of demand/supply A. Deterministic B. Stochastics
III.	Number of facilities A. Single facility B. Multiple facilities
IV.	Size of vehicle fleets A. Single vehicle B. Multiple vehicles
V.	Vehicle capacity A. Uncapacitated B. Capacitated
VI.	Facility capacity A. Uncapacitated B. Capacitated
VII.	Facility layer A. Primary B. Secondary/intermediate
VIII.	Planning horizon A. Single period (static) B. Multiple periods (dynamic)
IX.	Time windows A. Unspecified time with no deadline B. Soft time windows with loose deadlines C. Hard time windows with strict deadlines
X.	Objective function A. Single objective B. Multiple objectives
XI.	Types of model data A. Hypothetical B. Real-world

b. Classification of LRP with regard to its solution method

I.	Exact algorithm A. Direct tree search/Branch and bound B. Dynamic programming C. Integer programming D. Nonlinear programming
II.	Heuristic A. Location–allocation-first, route-second B. Route-first, location–allocation-second C. Savings/insertion D. Improvement/exchange

of outbound delivery routes around those facilities (see Fig. 2). Another class is a two-stage LRP which expands its problem scope to consider two-layers of the production–distribution network involving both outbound (delivery) and inbound (pickup) distribution processes. These classes can be further subdivided into two types of models: deterministic and stochastic. The deterministic LRP model assumes that the nature of location/routing parameters such as demand/supply size is known and fixed with certainty, whereas the stochastic counterpart consider an uncertain and random nature of those parameters. Within each of these broader categories, there are nine subcategories that reflect many practical aspects of LRP (see Table 1(a)). These subcategories include two layers of distribution facilities: primary and secondary. A primary facility represents either the origin or destination of a vehicle journey. Some examples of primary facilities are manufacturing plants, hospitals, waste collection centers, airports, and landfills. On the other hand, a secondary facility represents an intermediate or a transshipment point such as a military depot, a warehouse, a distribution center, a waste transfer station, a consolidation terminal, and a break-bulk terminal. Many LRP studies which were concerned with the location of secondary facilities typically assume that primary facilities are located at known and fixed sites as shown in Fig. 3. The main intent of these classification schemes is to help academicians and practitioners alike to correctly identify the specific scope of the LRPs they encounter and then encourage them to capture more realistic aspects of LRPs.

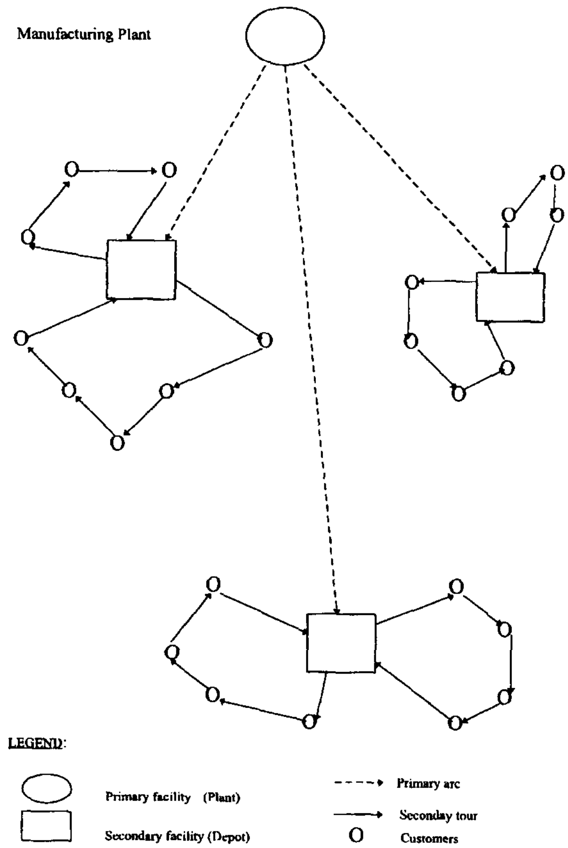

Fig. 3. A typical secondary facility location–routing problem.

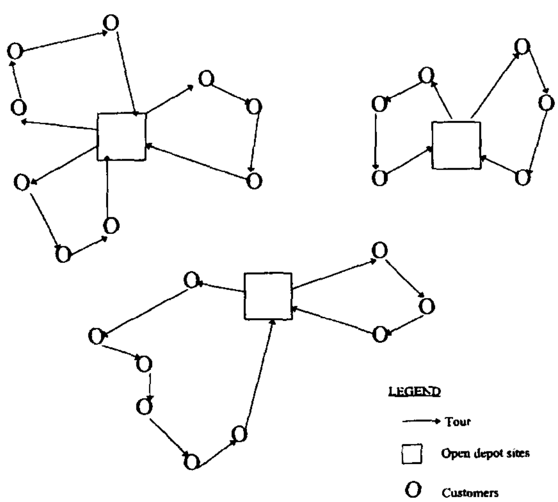

Fig. 2. A single-stage location–routing problem.

5. Algorithmic developments

Two distinct types of solution methods have been used for solving LRPs. These are exact algorithms and heuristics. Analogous to the vehicle routing problem (VRP) classification scheme suggested by Laporte (1992), exact algorithms for the LRP can be classified into four categories: (1) direct tree search/branch-and-bound, (2) dynamic programming, (3) integer programming, and (4) non-linear programming. Since location–allocation problems and vehicle routing problems (VRPs) are often NP-hard, LRP which combines those two difficult problems will often prohibit the use of pure exact algorithms. As such, the multi-phase decomposition solution procedures which can ease problem complexity is commonly used for solving LRPs. These procedures may be comprised of some combinations of four algorithms: (1) location–allocation-first, route-second, (2) route-first, location–allocation-second, (3) savings/insertion, and (4) tour improvement/exchange. Among these four, both savings/insertion and tour improvement/exchange heuristics were often used to solve the subproblem of LRP (i.e., vehicle route design within the LRP framework).

To elaborate, savings/insertion heuristics used for LRP are based on the well-known Clarke and Wright (1964) or the Rosenkrantz et al. (1977). In general, to obtain solutions with larger distance/cost savings, these heuristics are repeated using a different combination of customer nodes. The advantages of these heuristics are their efficiency for forming reasonably good clusters for the customer nodes and their flexibility for incorporating practical scope such as hard time windows into the LRP (Bodin, 1975; Ballou and Agarwal, 1988). Thus, savings/insertion heuristics can be particularly useful for constructing the routes of vehicles with variable driving speed. The savings/insertion heuristics, however, implicitly ignore vehicle fleet size and may create severe computational difficulty with the increasing number of customer nodes (Ballou and Agarwal, 1988; Laporte, 1992).

Tour improvement/exchange heuristics originally developed by Lin (1965) build a vehicle route in such a way that a feasible route is altered continuously to yield another feasible route with a reduced overall cost until no additional cost reductions are possible. As such, unlike savings/insertion methods, these heuristics maintain feasibility throughout the entire solution procedure (Bodin et al., 1983).

6. Annotated literature review

Adopting a typology somewhat similar to that of Current and Min (1986) and Current and Marsh (1993), we develop the following format of an annotation entry. First, the main features of each reviewed article are identified with its first six categories of the problem perspective: hierarchical level, nature of demand/supply, number of facilities, size of vehicle fleets, vehicle and facility capacity. Second, the articles grouped by their respective subcategory in the taxonomy are listed in chronological order. If there are multiple entries in the same year, the articles are listed in the alphabetical order of their main author's surname. These articles are further annotated with regard to the last five subcategories of the problem perspective. Third, the articles are categorized with respect to their solution method(s). Finally, the articles are identified with their specific application areas if appropriate.

To summarize, the format is comprised of:

single or multiple stage(s), deterministic or stochastic, single or multiple facility(ies), single or multiple vehicle(s), uncapacitated or capacitated vehicle(s)/facility(ies);

author(s), (year of publication), title:
- primary or secondary facility layer, static or dynamic, unspecified or soft/hard time windows, single or multiple objectives, hypothetical or real-world data used;
- *specific types of exact or heuristic algorithms used;
- a specific focus of the study (algorithmic design or potential/actual application areas if appropriate).

Annotation:

Single stage, deterministic, single facility, single vehicle, uncapacitated vehicle, uncapacitated facility

Ghosh et al. (1981), "A generalized reduced gradient based approach to round-trip location problem"
- primary facility, static, unspecified time windows, single objective, hypothetical data used;
- exact algorithm (non-linear programming);
- non-rectilinear round-trip distance location.

Stowers and Palekar (1993), "Location models with routing considerations for a single obnoxious facility"
- primary facility, static, unspecified time windows, multiple objectives (minimization of both location and transportation risks), hypothetical data used;
- exact algorithm (non-linear programming);
- continuous obnoxious location–routing.

Single stage, deterministic, single facility, multiple vehicles, uncapacitated vehicles, uncapacitated facility

Laporte and Nobert (1981), "An exact algorithm for minimizing routing and operating costs in depot location"
- secondary facility, static, unspecified time windows, single objective, hypothetical data used;
- exact algorithm (two branching strategies);
- symmetric traveling salesman tour-location.

Averbakh and Berman (1994), "Routing and location–routing p-delivery men problems on a path"
- primary facility, static, unspecified time windows, single objective, no computational result (i.e., no usage of data);
- exact algorithm (dynamic programming);
- delivery men problem.

Single stage, deterministic, single facility, multiple vehicles, capacitated vehicles, uncapacitated facility

Chien (1993), "Heuristic procedures for practical-sized uncapacitated location-capacitated routing problems"
- secondary facility, static, unspecified time windows, single objective, hypothetical data used;
- heuristic (insertion method);
- warehouse location–routing (private sector).

Single stage, deterministic, multiple facilities, single vehicle, uncapacitated vehicle, uncapacitated facilities

Srisvastava (1993), "Alternate solution procedures for the location–routing problem"
- secondary facilities, static, unspecified time windows, single objective, hypothetical and randomly generated data used;
- heuristics (savings method and clustering);
- examination of effects of cost structure, customer spatial distribution, and number of depots on the performances of savings- and clustering-based heuristics.

Single stage, deterministic, multiple facilities, single vehicle, uncapacitated vehicle, capacitated facilities

Zografos and Samara (1989), "Combined location–routing model for hazardous waste transportation and disposal"
- primary facilities, static, unspecified time windows, multiple objectives (minimization of disposal/routing risks and travel time), hypothetical data used;
- exact algorithm (mixed-integer goal programming);
- hazardous waste location–routing.

Single stage, deterministic, multiple facilities, multiple vehicles, uncapacitated vehicles, uncapacitated facilities

Laporte et al. (1983), "Hamiltonian location problems"
- secondary facilities, static, unspecified time windows, single objective, hypothetical data used;
- exact algorithms (integer programming);
- presentation of exact algorithms and formulations for both Hamiltonian and non-Hamiltonian depot location problem.

Laporte and Dejax (1989), "Dynamic location–routing problems"
- secondary facilities, dynamic, unspecified time windows, single objective, hypothetical data used;
- Both exact algorithm (integer programming) and heuristic (partitioning algorithm);
- vehicle fleet/route configuration and depot location.

List and Mirchandani (1991), "An integrated network/planar multiobjective model for routing and siting for hazardous materials and wastes"
- primary facilities, static, unspecified time windows, multiple objectives (minimization of risk, cost, and equity), real-world data used;
- exact algorithm (a multiobjective route generation method);
- hazardous material location–routing (public sector).

ReVelle et al. (1991), "Simultaneous siting and routing in the disposal of hazardous wastes"
- primary facilities, static, unspecified time windows, multiple objectives (minimization of transportation cost and perceived risk), real-world data used;
- exact algorithms (integer programming, shortest paths, and a weighting method);

- hazardous waste disposal site location–routing (public sector).

Single stage, deterministic, multiple facilities, multiple vehicles, capacitated vehicles, uncapacitated facilities

Gillett and Johnson (1976), "Multi-terminal vehicle-dispatch algorithm"
- secondary facilities, static, unspecified time windows, single objective, hypothetical data used (from previous literature);
- heuristic (multi-terminal sweep algorithm);
- terminal location–routing (private sector).

Jacobsen and Madsen (1978), "On the location of transfer points in a two-level newspaper delivery system – A case study"
- secondary facilities, static, hard time windows, single objective, real-world data used;
- heuristics (location–allocation-first, route-second using the savings and tree-tour methods);
- newspaper transfer point location-routing (private sector).

Jacobsen and Madsen (1980), "A comparative study of heuristics for a two-level routing-location problem"
- secondary facilities, static, hard time windows, single objective, real-world data used;
- heuristics (location–allocation-first, route-second using the savings method);
- newspaper distribution (private sector).

Srivastava and Benton (1990), "The location–routing problem: Considerations in physical distribution system design"
- secondary facilities, static, hard time windows, single objective, hypothetical data used;
- heuristics (savings method and clustering);
- implication of environmental factors for the performance of location-routing heuristics.

Single stage, deterministic, multiple facilities, multiple vehicles, capacitated vehicles, capacitated facilities

Nambiar et al. (1981), "A large scale location–allocation problem in the natural rubber industry"
- primary facilities, static, hard time windows, single objective, real-world data used;
- heuristics (location–allocation-first, route-second);
- central rubber processing factory location–allocation–routing (public sector).

Perl and Daskin (1984), "A unified warehouse location–routing methodology"
- secondary facilities, static, unspecified time windows, single objective, real-world data used;
- heuristic (route-first, location–allocation-second);
- warehouse location–routing (private sector).

Perl and Daskin (1985), "A warehouse location–routing problem"
- secondary facilities, static, unspecified time windows, single objective, real world data used;
- heuristic (route-first, location–allocation-second);
- warehouse location–routing (private sector).

Laporte et al. (1986), "An exact algorithm for solving a capacitated location–routing problem"
- secondary facilities, static, unspecified time windows, single objective, hypothetical data used;
- exact algorithm (integer programming);
- development of a formulation and an exact algorithm for the generic capacitated location–routing problem.

Bookbinder and Reece (1988), "Vehicle routing considerations in distribution system design"
- secondary facilities, static, unspecified time windows, single objective, hypothetical data used;
- heuristics (location–allocation-first, route-second with a generalized assignment method);
- multicommodity warehouse location–routing (private sector).

Laporte et al. (1988), "Solving a family of multi-depot vehicle routing and location–routing problem"
- secondary facilities, static, hard time windows, single objective, hypothetical data used;
- exact algorithm (a modified branch and bound method);
- asymmetric location–routing.

Hansen et al. (1994), "A heuristic solution to the warehouse location–routing problem"
- secondary facilities, static, unspecified time windows, single objective, hypothetical data used;

- heuristic (savings method);
- warehouse location–routing (private sector).

Single stage, stochastic, single facility, single vehicle, uncapacitated vehicle, uncapacitated facility

Burness and White (1976), "The traveling salesman location problem"
- primary facility, static, unspecified time windows, single objective, hypothetical data used;
- exact algorithm (an iterative procedure);
- traveling salesman location.

Simchi-Levi and Berman (1988), "A heuristic algorithm for the traveling salesman location problem on networks"
- primary facility, static, unspecified time windows, single objective, hypothetical data used;
- heuristic (tour approximation and 1-median);
- a repair vehicle service location–routing.

Jamil et al. (1994), "The traveling repairperson home base location problem"
- primary facility, static, unspecified time windows, single objective, hypothetical data used;
- heuristic (Fibonacci search);
- traveling repairperson home base location.

Berman et al. (1995), "Location–routing problems with uncertainty"
- primary facility, static, unspecified time windows, single objective, hypothetical data used;
- both exact algorithm (non-linear programming) and heuristics (spacefilling curve and iterative improvement algorithms);
- probabilistic traveling salesman location.

Single stage, stochastic, single facility, multiple vehicles, capacitated vehicles, uncapacitated facility

Simchi-Levi (1991), "The capacitated traveling salesmen location problem"
- secondary facility, static, unspecified time windows, single objective, no computational result;
- heuristics (an iterative improvement algorithm and 1-median);
- probabilistic traveling salesman location.

Single stage, stochastic, single facility, multiple vehicles, uncapacitated vehicles, uncapacitated facilities

Daskin (1987), "Location, dispatching, and routing models for emergency services with stochastic travel times"
- primary facility, static, unspecified time windows, multiple objectives (maximization of expected coverage, minimization of expected response time of the first arriving vehicle, and minimization of the expected difference between the first- and second-arriving vehicle response time), hypothetical data used (from previous literature);
- exact algorithm (branch-and-bound techniques on the LP relaxation);
- emergency vehicle location–routing.

Averbakh and Berman (1995), "Probabilistic sales-delivery man and sales-delivery facility location problems on a tree"
- primary facility, static, unspecified time windows, single objective, no computational result (i.e., no usage of data);
- exact algorithm (non-linear programming);
- sales-delivery location–routing.

Single stage, stochastic, multiple facilities, multiple vehicles, uncapacitated vehicles, uncapacitated facilities

Harrison (1979), "A planning system for facilities and resources in distribution networks"
- secondary facilities, static, unspecified time windows, single objective, real-world data used;
- heuristics (location–allocation-first, route-second using the savings method);
- warehouse location–routing (private sector).

Single stage, stochastic, multiple facilities, multiple vehicles, capacitated vehicles, uncapacitated facilities

Laporte et al. (1989), "Models and exact solutions for a class of stochastic location–routing problems"
- secondary facilities, static, unspecified time windows, single objective, hypothetical data used;

Table 2
Frequency listing of LRP articles by publication outlets

Journal	Total number of LRP articles published
Transportation Science	9
European Journal of Operational Research	7
Journal of Business Logistics	2
Omega	2
AIIE Transactions	1
Annals of Operations Research	1
Computers and Operations Research	1
Decision Sciences	1
Facility Location: A Survey of Applications and Methods [a]	1
Interfaces	1
Journal of the Operational Research Society	1
Operations Research	1
Spatial Analysis and Location–Allocation Models [a]	1
Scientific Management of Transport Systems [a]	1
The International Symposium on Locational Decisions [a]	1
Transportation Research	1
Transportation Research Board	1

[a] Note: Some books are included, because they contained a collection of articles relevant to the location–routing literature.

- exact algorithm (branch-and-bound);
- determination of the vehicle fleet size and collection routes with random customer supplies.

Multiple stages, deterministic, multiple facilities, multiple vehicles, capacitated vehicles, uncapacitated facilities

Or and Pierskalla (1979), "A transportation location–allocation model for regional blood banking"
- secondary facilities, static, unspecified time windows, single objective, real-world data used;
- heuristics (location–allocation-first, route-second with insertion algorithms);
- blood bank location–routing (private sector).

Multiple stages, deterministic, multiple facilities, multiple vehicles, capacitated vehicles, capacitated facilities

Min (1996), "Consolidation terminal location–allocation and consolidated routing problems"
- secondary facilities, static, unspecified time windows, single objective, real-world data used;
- both exact algorithm (integer programming) and heuristic (location–allocation-first, route-second with clustering);
- terminal location–routing (private sector).

7. Summary and future research directions

Since the concept of logistics cuts across almost every function of the firm's activities involving facility location, warehousing, storage, traffic control, inventory planning, materials handling, and purchasing, an integration of the traditionally fragmented logistics function has become a central part of forward looking firms' strategic plan over the last several decades (LaLonde et al., 1985). Thus the continued success of operations research (OR) in logistics might rest with the OR community's effort and ability to model practical logistics problems in an integrated way (Mercer and Rand, 1977). In this paper we annotated 33 articles reflecting such an effort and ability. The articles annotated in this paper have been published in 17 different journals and books that were devoted to the refereed articles. The diversity of the publication outlets represents the interdisciplinary nature of location–routing problems (see Table 2 for the number of articles per journal). *European Journal of Operational Research* and *Transportation Science* are two dominant journal outlets for combined location–routing research. A majority of these appeared in the 1980s and 1990s during which firms were shifting their focus from expansion to 'rightsizing' or improved productivity. Significant productivity gains can be achieved through the design of combined location–routing models that may determine true least-cost solutions to a logistic problem in light of both strategic (facility location) and operational (vehicle routing) policy. Increasing popularity of combined location–routing models reflects such a trend. Despite considerable past research efforts geared toward the combined location–routing problem, some key dimensions of an integrated logistics model have not been fully incorporated. The specific areas of future research that we believe may offer the highest potential reward include the following.

7.1. Stochasticity

A majority (73%) of the annotated LRP literature have tilted heavily toward the development of deterministic models. In practice, however, the number, demand, and location of customers as well as travel times of vehicles may not be known a priori and consequently should be treated as random variables. In light of this, future studies should consider probabilistic dimensions of customer demand patterns and variations in vehicle travel times. For this line of research, previous articles by Laporte et al. (1989) and Berman et al. (1995) can be good theoretical foundations. To build more conceptual foundations, interested readers should also refer to an excellent survey of stochastic vehicle routing problems by Stewart et al. (1982).

7.2. Time windows

House and Karrenbauer (1982) observed that improved transportation performance in terms of greater speed and reliability have decreased the significance of spatial parameters such as distance in logistics model design, while increasing the importance of time value and rate of flow over time as the critical dimension of logistics modelling. Nevertheless, our review of the past LRP literature reveals an oversight of time restrictions or windows on deliveries and pickups. Only five annotated LRP articles (less than 16%) account for hard time windows and none considers soft time windows. Considering that some customers often impose service deadlines and desirable service time restrictions, the existing LRP studies should be extended to consider the presence of time windows. Such extension will be especially important with the advent of Just-In-Time (JIT) logistics principle.

7.3. Multiple periods

To date, all the LRP articles but Laporte and Dejax (1989) have been restricted to developing static LRP models, with little attempt to capture the changing nature of LRP parameters over time. For instance, warehouse location cost may vary over time with the fluctuations of warehousing employee wages and interest rates. Similarly, transportation cost may vary constantly with changes in negotiated contract rates and the extent of empty miles. Also, in a given time planning horizon, a company may have to deal with decisions of relocating existing facilities and re-routing/re-scheduling vehicles. As such, LRP parameters are inherently time-sensitive. Hence, the incorporation of dynamic nature into the LRP model would greatly enhance the realism associated with real-time or on-line logistics operations.

7.4. Multiple objectives

Golden and Baker (1985) observed that the multi-objective context is a rule rather than an exception in both private and public sector logistics operations. For example, the minimum cost route, primarily based on the spatial dispersion of customer nodes, may fail to meet customers' needs for on-time delivery services. Accordingly, most real-world location–routing problems are characterized by more than one conflicting objectives. Although five annotated LRP articles have recognized the multiobjective nature, they mainly focused on finding the origin–destination path rather than solving node-covering problems that required complete Hamiltonian tours. Though methodologically challenging, future

LRP research should address multiobjective decision problems involving combined location and node-covering route problems.

7.5. The vertical integration of a value-added supply chain

Traditionally, logistics activities have been separated into two distinctive areas: an inbound flow focusing on pickup operations and an outbound flow facilitating deliveries of finished goods. Thus much of the annotated LRP literature (94%) attempted to create only the optimal outbound or inbound routes around location configurations, but not both outbound and inbound routes. However, firms are increasingly compelled to take a total supply chain approach that views inbound and outbound flows as interrelated steps (Chapman, 1994). Such an approach implies that logistics decisions involving location and routing should be more closely coordinated and centrally monitored across the entire supply chain. Therefore, a great research opportunity exists for developing multi-stage LRP models that determine simultaneously the sequences of pickups and deliveries by exploring the relationship between inbound and outbound flows.

7.6. The horizontal integration of multi-faceted logistics operations

The earlier logistics survey conducted by Lambert et al. (1978) reported that a majority (74%) of the surveyed logistics managers already have adopted the integrated logistics concept. The existing LRP modelling efforts, however, lag behind such a practical demand. In particular, LRP models should examine the interactions among location, routing, and inventory control decisions. For instance, the level of inventory has a significant effect on the capacity and number of warehouses as well as the choice and route of transportation modes. Thus inventory control is intimately related to facility location and vehicle routing (Daskin, 1985). Due in part to added complexity, no LRP literature to date has explored the intricacies of location, routing, and inventory control. One starting point may be the recent article by Perl and Sirisoponsilp (1988) which designed a network model recognizing the interdependence among location, transportation, and inventory.

7.7. Benchmarks for solution efficiency

There are at least 19 LRP articles which developed some forms of exact algorithms. Although direct tree search methods (i.e., branch and bound) and integer programming techniques were commonly used for solving LRPs, nearly half of the exact algorithms developed so far depended upon streamlined methods designed for the specific case of LRPs. Therefore, exact algorithms capable of solving the more generic case of LRPs need to be developed in the future. The exact algorithm introduced by Laporte et al. (1988) may serve as a useful guideline for such an endeavor. The development of such an exact algorithm is important in that it can be a meaningful benchmark for measuring the solution efficiency of LRP heuristics.

Table 3 presents the most frequently used LRP heuristics. Considering the complexity of LRP, it is not surprising that the most popular approach is decomposition, followed by a savings method. Typical decomposition heuristics include both location–allocation-first, route-second and route-first, location–allocation-second. Both algorithms partition an original LRP into a sequence of tractable smaller

Table 3
Solution methods used for LRP

Solution methods	Number of articles
I. Exact algorithm	
(a) direct tree search/branch and bound	4
(b) dynamic programming	1
(c) integer programming	6
(d) nonlinear programming	4
(e) others	4
Total	19 [a]
II. Heuristic	
(a) location–allocation-first, route-second	7
(b) route-first, location–allocation-second	2
(c) savings/insertion	8
(d) tour improvement/exchange	2
(e) others	7
Total	26 [a]

[a] Note: There are some multiple entries for each category because some LRP articles used more than one type of solution methods.

subproblems and inductively link those problems by using the solution output of one problem as the input to the next sequence of subproblems. Despite its popularity, the decomposition method has its limitations. Since the decomposition method solves LRP sequentially, it may not suffice for analyzing trade-offs between location and routing factors at the same level of decision hierarchy. That is to say, it would be desirable for us to solve the whole LRP problem concurrently.

7.8. New application to real-world decision problems

Successful application of LRP models and solution algorithms to real-world decision problems can not only bring a broader spectrum of location–routing options, but also provide compelling evidence of their efficacy and practicality. Aside from well-researched problem settings such as depot/warehouse location–routing problems, diverse applications of LRP models and solution algorithms to untouched actual problems appear to be fruitful areas of future research. A specific example of such applications includes the design of a beer distribution system involving three layers of facilities: a brewery, warehouses, and pubs (see Fig. 4). Another untapped application potential includes the simultaneous configuration of transfer station location and solid waste transportation routes (see Fig. 5).

In closing, regardless of various research challenges ahead of us, the trend in the number of LRP

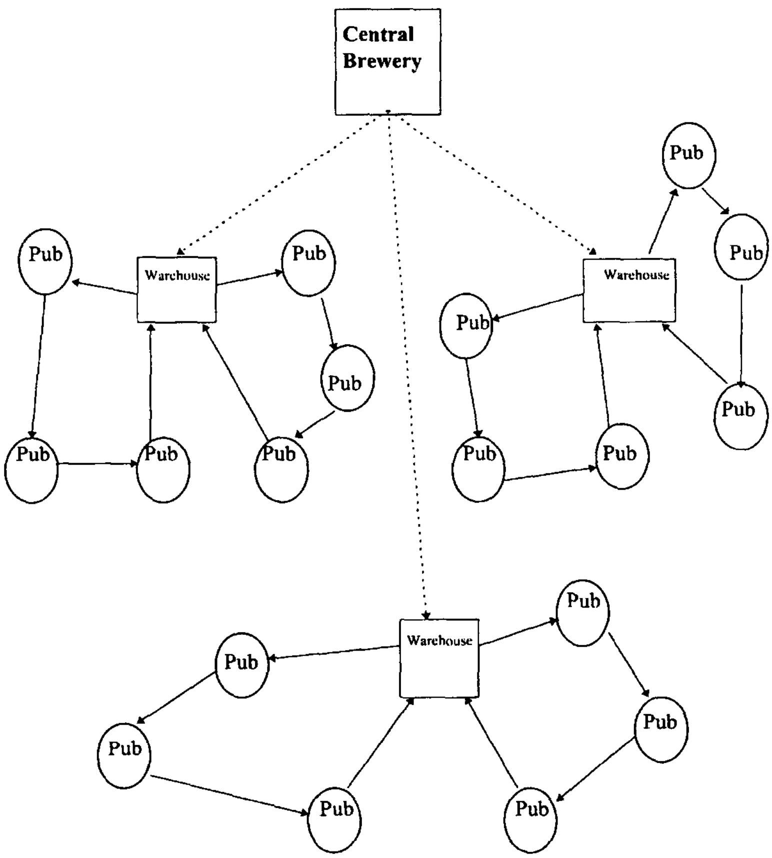

Fig. 4. A beer distribution system.

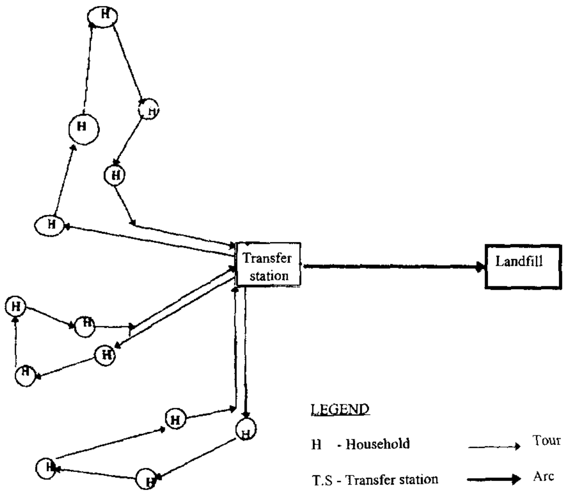

Fig. 5. A solid waste location–routing problem.

articles in the past two decades reveals sustained growth. Future LRP research may explode, however, if there is an algorithmic 'breakthrough' which allows logistics professionals to solve realistic sized problems of significant practical value. Such a breakthrough does not seem to be impossible, given that the trend in the number of algorithmic developments also shows a moderate growth in the last two decades. We hope this review will serve to stimulate such advances and remain optimistic that LRP will be an exciting and evolving field for years to come.

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