An integrated model for distribution network design is proposed which explicitly represents the required level of customer service.

Distribution Networks: Facility Location, Transportation and Inventory

by Jossef Perl and Sompong Sirisoponsilp

Introduction

Effective logistics management must deal with a wide range of decisions. Logistics decisions may be categorised into three levels, depending on the scope, the investment requirement, and the time horizon and frequency of the decisions involved:

(1) at the highest level, *strategic decisions* are usually concerned with major capital commitments and with the efficient allocation of resources to the various components of logistics operations, over a relatively long period of time;

(2) *tactical decisions* typically deal with moderate capital investments and involve plans for annual, semi-annual, or seasonal time horizon. Decisions at this level can usually be reversed at lower cost than that which is required to reverse strategic decisions;

(3) at the lowest level, *operational decisions* deal with day-to-day operations; they are characterised by low capital investments and can be reversed at a relatively low cost.

The purpose of this hierarchical categorisation is twofold. First, it is consistent with the existence of an organisational structure within a firm, where each level of the organisation deals primarily with one of the levels of decisions outlined above. Second, the separation of the overall decision-making process into various stages (levels) makes the planning, design and operation of a logistics system tractable. It should be noted however that an interdependence exists not only among decisions within any given level, but also between decisions at different levels. Higher-level decisions often determine the boundaries for decisions in subsequent levels. On the other hand, evaluating higher-level decisions requires managers to consider the expected outcome of lower-level decisions.

Decisions related to the design of a distribution network can be classified into three basic components:

Facility location

Transportation

Inventory decisions.

A parallel classification can be used for the costs associated with a distribution network, which can be defined as facility costs, transportation costs and inventory costs. Table I presents the classification of facility location, transportation and inventory decisions at the three hierarchical levels of logistics decisions. In the context of distribution network design there is an interdependence among these three sets of decisions, which results in trade-offs between the associated cost components.

This article analyses the interdependence between facility location, transportation and inventory decisions and proposes an integrated model for distribution network design which represents this interdependence.

The proposed model can be expected to provide a more complete and accurate representation of the trade-offs that exist among the three cost components above, thereby leading to solutions that are closer to "true" optimality than those provided by existing mathematical models.

We consider the design of a distribution network as the process which consists of determining the following elements:

(1) number and locations of distribution centres (DCs);

(2) allocation of customers (markets) to DCs;

(3) flow pattern from supply sources (plants) to DCs;

(4) selected transportation services between plants and DCs;

(5) levels of inventories at the DCs.

The objective of the distribution network design process can be viewed as that of finding the optimal balance between facility, transportation and inventory costs.

Table I. *Classification of Facility Location, Transportation and Inventory Decisions into Three Hierarchical Levels*

Logistics Decisions	Strategic	Tactical	Operational
Facility Location	Number of DCs	Material handling equipment	
	Location of DCs		
	Assignment of DCs to supply sources		
	Allocation of demand to DCs		
Transportation	Mode	Carrier	Assignment of loads to vehicles
	Type of carriage	Shipment size	Routing/scheduling
			Crew assignment
Inventory	Total system inventory	Size of inventories at various locations	Control discipline at various locations
	Location of inventories	Levels of safety stock at various locations	

Decisions related to the design of a distribution network are among the most critical of logistics management decisions. Both the cost of a distribution system and the quality of customer service that can be provided by the system are significantly affected by the design elements stated above. Consequently, a great deal of effort has been devoted to the development of mathematical models to support decisions related to the design of distribution networks. However, the existing mathematical models have focused on individual components of the design problem, particularly on DC location, while ignoring or making some restrictive assumptions regarding the other components.

In recent years, the US freight transportation industry has gone through major regulatory reforms. These regulatory changes have significantly increased the spectrum of transportation choices available to shippers, thereby increasing the importance of considering multiple transportation options in the design of a distribution network. As a result, it has become increasingly more important explicitly to consider the interdependence between facility location, transportation and inventory decisions in distribution network design. It is no longer valid to assume a single unit transport cost when analysing DC location, or to assume that inventory decisions are related only to the number and locations of DCs and are independent of transportation decisions.

The next section reviews the literature on the three decision components of the distribution network design problem. In the third section we discuss the interdependence between facility location, transportation and inventory decisions. The proposed integrated model for distribution network design problem is presented in the fourth section. We close with a summary and presentation of our major conclusions.

Literature Review

This review is in three parts — facility location models, transportation choice models and inventory models — which reflect our proposed integrated mathematical model for the distribution network design problem, which represents the interdependence between facility location, transportation and inventory decisions.

The significant implications of facility location decisions on the cost and level of customer service that can be provided by a distribution system, have led to a great quantity of research on facility location models. A comprehensive review of this work is beyond our scope here and can be found elsewhere[1, 2]. Existing facility location models can be classified according to three approaches: optimisation, heuristic and simulation. In the last two decades, the focus has been on the development of optimisation models, which with today's computational capabilities can be used to solve large facility location problems. Consequently, while our review includes work representative of all three approaches, we concentrate on optimisation models.

Much of the existing work on transportation choice models has been conducted in the context of passenger transportation. Our review includes only freight transportation models which represent the trade-offs between transportation and inventory costs.

Very extensive research has been devoted to the development of inventory models[3]. However, our focus is on the relationship of inventory decisions to location and transportation decisions and therefore only studies dealing with the spatial aspects of inventory decisions are reviewed.

Facility Location Models

Early work on facility location modelling concentrated on heuristic methods for the Single-product Uncapacitated Facility Location Problem (SUFLP). Kuehn and Hamburger[4] developed a heuristic method which consists of two phases:

(1) a "main program" which adds DCs to the system one at a time until the addition of a DC does not achieve cost reduction;

(2) a "bump and shift" routine which attempts to improve the solution of the "main program" by dropping individual DCs or shifting them to other locations. The heuristic method of Feldman, Lehrer and Ray[5] is a modification of the Kuehn and Hamburger method which starts with all potential sites being used and eliminates DCs one at a time until no further cost reduction can be realised. Feldman *et al.* used a piecewise-linear warehousing cost function, to represent scale economy in warehousing.

Efroymson and Ray[6] proposed a Branch and Bound (B&B) method for solving a mixed-integer programming formulation of the SUFLP. Erlenkotter[7] presented a solution method for the mixed-integer programming formulation of the SUFLP, consisting of a simple ascent and adjustment procedure for solving the dual of the Linear Programming (LP) relaxation.

The Single-product Capacitated Facility Location Problem (SCFLP) is obtained from the SUFLP by adding capacity limitations at the facilities. Akine and Khumawala[8] developed a B&B algorithm for the SCFLP in which the lower bound was obtained by solving a transportation problem. Geoffrion and McBride[9] proposed a different B&B algorithm to the same model, in which the lower bound was obtained by Lagrangian relaxation. Soland[10] formulated the SCFLP as a non-linear programming model, which represented concave transportation and warehousing cost functions. He presented a B&B algorithm which used "local" linear approximations to the concave cost functions. Kelly and Khumawala[11] proposed a model for the SCFLP with non-linear warehousing cost, and a solution method which consisted of a sequence of expanded transportation problems.

The Multi-product Uncapacitated Facility Location Problem (MUFLP) and Multi-product Capacitated Facility Location Problem (MCFLP) are generalisations of the SUFLP and SCFLP, respectively. Klincewicz *et al.*[12] proposed a B&B algorithm for the MUFLP which distributed the fixed facility cost of potential facilities equally among products. The lower bound for the MUFLP solution was obtained from the lower bounds on the separate SUFLP sub-problems. Markland[13] developed a simulation model for the MCFLP, which explicitly considered customer service by incorporating a "penalty function" for the delay in satisfying customer demand. The model attempted to determine simultaneously the number and locations of plants and DCs and the level of inventory at each facility. Elson[14] proposed a mixed-integer programming model for the MCFLP, which in addition to the standard cost components, included facility expansion costs and savings from facility closure.

Perhaps the most significant optimisation model for the MCFLP is that developed by Geoffrion and Graves[15]. The model includes quadruply subscripted flow variables to represent the entire movement from plants to customers through DCs. This offers the additional flexibility of representing special plant-to-customer transportation rates and service-related measures such as maximum allowable total delivery time. Another notable feature of this model is the ability to accommodate a set of "logical constraints" such as: specifying required characteristics on the set of DCs to be established; representing surrogate measures of customer service; and accommodating non-linear transportation and warehousing costs through piecewise-linear approximations. Geoffrion and Graves proposed a solution algorithm based on Bender's Decomposition method, which reduced the computational effort involved in making a sequence of related runs for sensitivity analysis.

Transportation Choice Models

Freight transportation choice models attempt to represent the trade-off between direct transportation cost, and inventory costs affected by transportation decisions. The existing models can be classified as single origin-destination models, and network-level models.

Baumol and Vinod[16] proposed the "Inventory-Theoretic" model for determining a shipper choice of transportation option, in a single market. The Inventory-Theoretic model can be viewed as a cost model which provides the total cost (transportation + inventory) associated with each transportation option. In the basic Inventory-Theoretic model the total cost associated with each transportation option is specified as the sum of direct transportation cost, in-transit carrying cost, ordering cost, cycle stock cost and safety stock cost. Two central assumptions of the Inventory-Theoretic model are:

(1) constant unit direct transportation cost which is independent of shipment size;

(2) stochastic elements of demand and lead time which follow the Poisson distribution.

The determination of optimal transportation decisions involves analysing, for each transportation option, the order quantity which minimises the total cost. Das[17] suggested that the Poisson assumption of the Inventory-Theoretic model is inaccurate and results in overestimation of the required safety stock. He showed that the choice of transportation option is affected by the mathematical formula of safety stock cost.

Constable and Whybark[18] proposed an alternative version of the Inventory-Theoretic model which explicitly represented the two components of safety stock cost, i.e., carrying cost and back-order cost. Langley[19] reformulated the Inventory-Theoretic model to represent the dependence of unit transportation cost on order quantity (shipment size). Langley observed that:

(1) the relative importance of the direct transportation component of total cost increased with demand;
(2) higher product value resulted in smaller optimal shipment size due to the effect on inventory carrying cost;
(3) the validity of the claim that minimising direct transportation cost would result in lowest total cost was dependent on the relationship between transportation rate and shipment size and on the value of the product.

A series of studies conducted at the General Motors Research Laboratories analysed the trade-offs between transportation and inventory costs in a network with multiple origins and destinations[20, 21, 22]. These studies considered two alternative transportation strategies: direct shipping, and indirect shipping via a consolidation terminal. Given the flows between all origin-destination pairs, the objective of these studies was to determine simultaneously the routing strategies for all origin-destination pairs and the shipment sizes that minimised the sum of transportation cost and cycle stock inventory cost.

Spatial Inventory Models

As stated earlier, we have included in this review only studies which attempted to represent the spatial aspects of inventory decisions. Existing spatial inventory models can be classified as aggregate versus disaggregate models. An aggregate model attempts to determine total system inventory without explicitly representing inventory decisions for each stocking point. In a disaggregate model, the inventory cost at each individual stocking point is modelled in detail and the objective is to minimise the system inventory cost as the sum of inventory costs at individual stocking locations.

Rosenfield *et al.*[23] proposed an analytic aggregate model which represented an approximate relationship between system safety stock and the number of DCs in the distribution systems. The model assumes an exponential relationship between the average demand allocated to a DC and the standard deviation of that demand. The total safety stock, which is assumed to be proportional to the standard deviation of demand allocated to a DC, is shown to increase with the number of DCs. This analytic aggregate model considers only the effect of demand uncertainties on inventory per facility and does not account for the effect of the locations of DCs on inventory requirements.

The disaggregate spatial inventory models have focused on inventory decisions in multi-echelon systems, where a lower-level DC replenishes its supply from the inventory of a higher-level DC. Van Beek[24] investigated strategies for locating inventories in a two-level distribution system, consisting of a plant, a central DC, and four local DCs. The objective was to determine the distribution strategy that minimises the sum of ordering costs and total inventory carrying cost over all stocking points in the system. Assuming a fixed-order system and inventory level prior to replenishment following a logistic density function, Van Beek's model provides the average inventory at a DC as a function of order quantity, average demand per period, variance of demand per period, average replenishment lead time, variance of replenishment lead time and allowable probability of stock-out. For each alternative distribution strategy, the optimal set of order quantities at all the DCs was found by minimising the cost defined above, with respect to all the order quantities simultaneously.

Hanssman[25] proposed a model for analysing inventory decisions under elastic demand, i.e. demand was affected by delivery time. Given the locations of inventories and the relationship between demand and delivery time, the model attempts to determine simultaneously the level of inventories at various locations in order to maximise the expected profit (revenue minus inventory cost). Assuming a fixed period inventory system and demand following a normal distribution, the average inventory at each DC was expressed as a function of average lead time, average stockout time at the supply source, delivery time, average demand per period and standard deviation of demand per period. The model was used to determine the profit maximising delivery time by first maximising revenue with respect to average delivery time for all stocking points simultaneously, and then substituting the profit maximising delivery time in the inventory model to obtain the average inventory at each DC.

Interdependence between Location, Transportation and Inventory Decisions

Existing mathematical models have focused on individual components of the network design problem, particularly on DC location, while ignoring or making some restrictive assumptions regarding the other components. Specifically, existing DC location models:

(1) assume pre-specified transportation choices;
(2) most often assume that unit transportation cost is independent of shipment quantity;
(3) do not explicitly represent inventory cost as a component of the location objective.

Due to the interdependence between facility location, transportation and inventory decisions (to be discussed below) a model for distribution network design model *should consider transportation decisions as output not input* and should explicitly represent the associated inventory cost.

As shown in Table I, strategic facility (DC) location decisions include the determination of number of facilities, locations of facilities, assignment of facilities to supply sources, and allocation of demand (customers) to facilities, while decisions regarding material handling equipment can be viewed as tactical. Facility-related decisions at the operational level include those related to the quantities of labour and equipment to be used at each facility and the scheduling of these resources.

Similarly, transportation decisions can be classified into the three hierarchical levels of managerial logistics decisions. Strategic transportation decisions include choice of transportation mode and choice of type of carriage, i.e. common, contract or private carriage. Different modes and different types of carriage within a given mode differ in their cost characteristics and therefore differ in the relationship of unit cost to volume (or weight) as well as in their service characteristics. Typical tactical transportation decisions include selection of a specific carrier within the chosen mode, and the determination of shipment frequency (or equivalently shipment size). In most cases, unit transportation cost can be expected to decrease with shipment size. Finally, typical transportation decisions at the operational level include assignment of loads to vehicles, and routing and scheduling of vehicles and crews.

Inventory decisions at the strategic level are concerned with total level of inventories in the system, and location of inventories. The significance of the decisions regarding location of inventories results from the fact that total inventory cost usually increases with the number of stocking points. In a multi-echelon distribution system, related to the decisions regarding the location of inventories is the allocation of safety stock among stocking points at different levels. For example, the issue of inventory location in a two-level system with central DCs (or plants) and field DCs would involve the choice between two inventory strategies, i.e. "independent" versus "coupled" system. An "independent" system places much of the safety stock at central DCs (or plants) in order to protect the entire system against demand variations during procurement lead times. The safety stock at field DCs in an independent system covers demand variations only over the replenishment lead times. Alternatively, a coupled system stores much of the safety stock at the field DCs. In a coupled system safety stocks at field DCs protect the system against demand variations during both procurement and replenishment lead times.

Tactical inventory decisions may be viewed as those dealing with the determination of size of inventories at various DCs, and relative proportion of cycle and safety stocks at various DCs. Finally, inventory decisions at the operational level are those dealing with the selection of a "control discipline". A control discipline specifies the frequency of inventory checks and the inventory level at which a replenishment order should be made. Clearly, there is a strong interdependence between inventory decisions at the three levels. Of primary importance in the context of distribution network design, is the interdependence between decisions regarding the total level of inventories in the system, the location of inventories, and the sizes of inventories at different locations.

Figure 1. *Interdependence between Facility Location, Transportation and Inventory Decisions.*

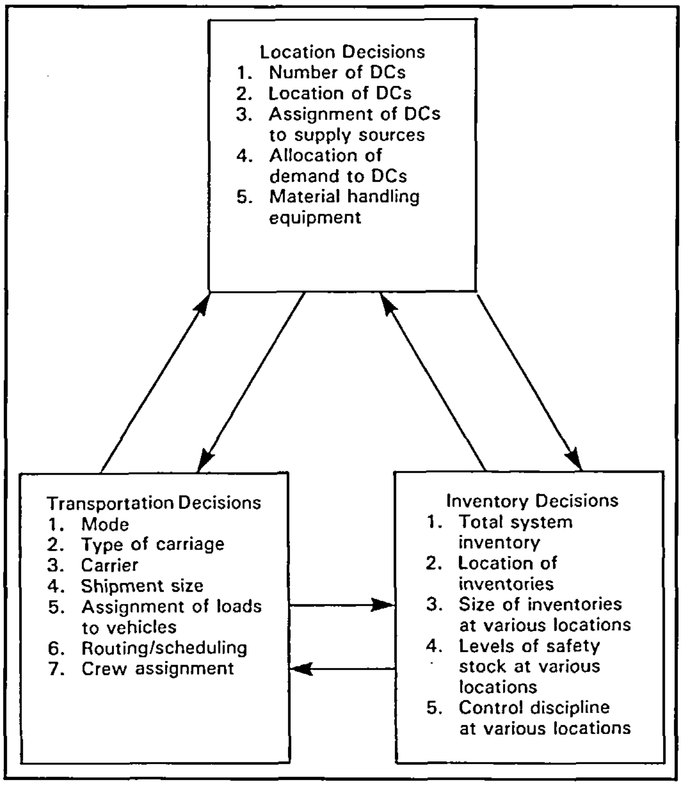

Figure 1 provides a schematic representation of the interdependence between facility location, transportation and inventory decisions. The following analysis uses this schematic representation to demonstrate the interdependence between the three components of distribution network design, showing the specific relationships between the various elements of these sets of decisions, both in the clockwise and counterclockwise directions. These relationships are discussed with reference to the three hierarchical levels of decisions shown in Table I.

Consider the relationships represented by Figure 1 in the clockwise direction, facility location decisions affect inventory decisions both at the strategic and tactical levels. At the strategic level, an increase in the number of DCs increases total system inventory. As the number of DCs increases, the optimal level of safety stock at individual DCs, required to maintain a pre-specified level of customer service, may decrease but the total safety stock in the system increases since the safety stock is divided among more locations. The allocation of demand points to DCs and the distances between plants and DCs, as determined by facility location decisions, affect inventory decisions at the tactical level. First, the allocation of demand to DCs determines the sizes of inventories at various locations. Second, the distances between plants and DCs affect transit lead times. Longer transit lead times increases the variability between scheduled arrival times of replenishments and actual arrival times, thereby increasing the optimal levels of safety at the DCs.

Strategic and tactical inventory decisions affect transportation decisions at all three levels. Clearly, the location of inventories determine the set of destinations for replenishment operations, which affect strategic and tactical transportation decisions regarding mode choices, type of carriage and choice of carrier. The tactical inventory decisions with regard to the levels of safety and cycle stocks at a DC, have a significant effect on the choice of transportation mode, type of carriage and shipment frequency (shipment size). With a pre-specified level of customer service, a decision to reduce safety stock at the facility would require the use of faster and more reliable transportation mode, or a more "exclusive" type of carriage. A decision to change the average level of cycle stock held at the facility would lead to a change in shipment size. Such a change in shipment size may or may not be accompanied by a change in transportation mode and/or type of carriage being used.

The effect of unit transportation cost (as determined by transportation decisions) on strategic facility location decisions has been the typical focus of existing mathematical models for DC location. The set of transportation decisions shown in Figure 1 determines the unit transportation cost between any origin and any destination in the distribution network. These transportation decisions also determine how unit transportation cost between any given origin and any given destination changes with the volume of flow and travel distance. A set of transportation decisions that results in a relatively slow decrease of unit transportation cost with volume and a relatively rapid increase of unit cost with distance, would lead to the optimal number of DCs being relatively large, thereby reducing travel distances in the costly delivery operations. Conversely, a set of transportation decisions that would lead to a relatively rapid decrease of unit transportation cost with volume and relatively slow increase with distance, would lead to the optimal number of DCs being relatively small, thereby increasing the volumes of shipments between individual origin-destination pairs.

To establish the existence of a complete interdependence between facility location, transportation and inventory decisions, we analyse the relationship presented in Figure 1 in the counter-clockwise direction. First, the set of strategic facility location decisions shown in Figure 1 determines the flow pattern in the distribution network, i.e. the origins, destinations and volumes of flow on the various links of the network. We should note that an origin of flow may be at a plant (inbound flow) or at a DC (outbound flow), while a destination of flow may be at a DC (inbound flow), or at a market (outbound flow). Given the flow pattern as determined by facility location decisions, the optimal transportation decisions at all three levels are those which minimise the cost of "serving" that flow pattern, while satisfying the requirements with regard to the level of customer service.

The effect of strategic and tactical transportation decisions on inventory decisions follows directly from the earlier discussion. At the strategic level, the choice of a transportation mode or type of carriage affects the variability between the scheduled arrival times of shipments and actual arrival times thereby affecting the optimal level of total systems inventory by affecting the optimal levels of safety stock at the DCs. A faster transportation mode or more "exclusive" type of carriage would reduce the optimal levels of safety stock. Furthermore, a faster transportation mode or more "exclusive" type of carriage would affect the inventory control discipline by reducing the replenishment lead times. At the tactical level, changes in shipment frequency (shipment size) would change the average level of cycle stock at the DCs.

Finally, strategic inventory decisions with regard to total system inventory affect location decisions. A decision to reduce the overall levels of inventories in the system, particularly the levels of safety stocks, would lead to a lower optimal number of stocking locations, and therefore to the establishment of fewer DCs. Second, the optimal number of DCs would also be affected by the choice between an independent and coupled inventory systems. Since the level of safety stock at each field DC in an independent system is usually lower than in a coupled system, the penalty in terms of higher inventory levels which would result from increasing the number of DCs is generally lower under an independent system. Consequently, the optimal number of DCs would tend to be higher under an independent inventory system than under a coupled system.

Integrated Model for Distribution Network Design

This section presents a mathematical model for the distribution network design problem, which explicitly represents the trade-offs between facility, transportation and inventory costs. The proposed model has not yet been tested numerically. We are currently developing an efficient computerised solution method. Upon completion of this development, the model will be tested and the results will be reported in future publications. An integrated model which represents the interdependence between facility location, transportation and inventory decisions, differs from existing DC location models primarily in the representation of the design objective. There are no basic differences in the set of constraints which defines either physical limitations of the network or the range of acceptable designs. Therefore, our discussion of the integrated model focuses on the mathematical representation of the design objective. The proposed integrated model minimises the Total Distribution Cost.

Total Distribution Cost = sum of the costs of:

Warehousing, trunking, delivery, in-transit inventory, plant and DC cycle stocks, safety stock (1)

The representation of warehousing cost (WC) is similar to that of existing DC location models, consisting of a fixed cost and a linear variable cost:

$$WC_j = W_j + v_j\overline{D}_j \tag{2}$$

where WC_j = average total warehousing cost at DC location "j"; W_j = fixed cost at DC location "j"; v_j = unit variable cost at DC location "j"; D_j = average total demand allocated to DC "j".

As discussed earlier, one of the shortcomings of existing DC location models is the implicit assumption that the unit trunking cost is constant and does not depend on shipment quantity. The proposed integrated model avoids this assumption. Unit trunking cost is related to shipment size as follows:

$$t = a + b/Q = a + bF/X \tag{3}$$

where t = unit trunking cost; Q = shipment size; X = total quantity shipped; F = shipping frequency; a, b = non-negative constants.

Based on equation (3), the total trunking cost of shipments between plant "i" and DC "j" on transportation option "m" is given by:

$$CT_{ijm} = t_{ijm}X_{ijm} = [a_{ijm} + b_{ijm}/Q_{ijm}] \times X_{ijm}$$

$$= [a_{ijm} + b_{ijm}F_{ijm}/X_{ijm}] \times X_{ijm} \tag{4}$$

$$= a_{ijm}X_{ijm} + b_{ijm}F_{ijm}\ V_{ijm}$$

where, CT_{ijm} = total trunking cost for shipments between plant "i" and DC "j" by transportation option "m";

t_{ijm} = unit trunking cost for shipping between plant "i" and DC "j" by transportation option "m";

X_{ijm} = total quantity shipped from plant "i" to DC "j" by transportation option "m";

F_{ijm} = shipment frequency of transportation option "m" from plant "i" to DC "j";

a_{ijm}, b_{ijm} = non-negative constants which characterise transportation option "m" from plant "i" to DC "j".

The representation of delivery cost is the same as that used in most existing warehouse location models:

$$CD_{jk} = d_{jk} \times Y_{jk} \tag{5}$$

where, CD_{jk} = delivery cost from DC "j" to demand point "k"; d_{jk} = unit delivery cost from DC "j" to demand point "k"; Y_{jk} = quantity shipped from DC "j" to demand point "k".

Inventory cost in the integrated model consists of three components: in-transit inventory cost, cycle stock cost and safety stock. In-transit stock can be represented as the product of quantity shipped and average lead time:

$$I_{ijm} = \overline{L}_{ijm} \times X_{ijm} \tag{6}$$

where, I_{ijm} = in-transit stock for shipments from plant "i" to DC "j" by transportation option "m"; and $\overline{L}_{ijm}$ = average lead-time for shipments from plant "i" to DC "j" by transportation option "m".

Using equation (6), the average carrying cost of in-transit inventory can be represented as follows:

$$CI_{ijm} = c_mI_{ijm} = c_m\overline{L}_{ijm} \times X_{ijm} \tag{7}$$

where, CI_{ijm} = carrying cost of in-transit inventory for shipments from plant "i" to DC "j" by transportation option "m"; and c_m = unit carrying cost for in-transit inventory per unit transit time, on transportation option "m".

Assuming a constant production rate, the average cycle stock held at a plant is equal to one-half the average quantity shipped. Thus, cycle stock cost at the plant is given by:

$$CCP_{ijm} = 0.5cp_i\ Q_{ijm} = 0.5cp_i\ X_{ijm}/F_{ijm} \tag{8}$$

where, CCP_{ijm} = cycle stock cost at plant "i" associated with shipments to DC "j" by transportation option "m"; and cp_i = unit carrying cost at plant "i".

Similarly, under the assumption that the outbound flow from a DC is uniform, the cycle stock cost at the DC is given by:

$$CCW_{ijm} = 0.5cw_j\ Q_{ijm} = 0.5cw_j\ X_{ijm}/F_{ijm} \tag{9}$$

where, cw_j = unit inventory carrying cost at DC "j".

The representation of safety stock cost is the most complex element of the integrated model; a complete discussion of the development of the safety stock representation is beyond the scope of this article. The approach to this representation is based on the Inventory-Theoretic model discussed in the second section. Given the pattern of demand, the characteristics of lead-time and the desired level of customer service, the safety stock requirement at a single stocking location is computed under an assumed "(R, Q) inventory system" with continuous review. An (R, Q) inventory system is one in which a pre-specified quantity "Q" is ordered when the inventory level reaches a pre-specified order point "R". It is also assumed that the system is designed as a coupled system in which the plants carry minimal safety stock and the lead-time needed in the analysis of safety stock includes procurement lead-time as well as replenishment lead-time. The safety stock cost is represented as follows:

$$SSC_j = \left[\frac{q}{\beta - p}\right] \times \left[\frac{\sum_i \sum_m F_{ijm}(\overline{L}_{ijm}\, \sigma_j^2 + \overline{D}_j^2\, \nu_{ijm}^2)^{1/2}}{\sum_i \sum_m F_{ijm}}\right] \qquad (10)$$

where, SSC_j = safety stock cost at DC "j"; β = allowed probability of stockout during order cycle; σ_j = standard deviation of demand at DC "j"; ν_{ijm} = standard deviation of replenishment lead-time from plant "i" to DC "j" by transportation option "m"; and p, q = non-negative parameters.

There are three basic differences between the integrated model represented by equations (2)-(10) and existing DC location models:

(1) It represents the cost associated with all three decision components, i.e. facility cost transportation cost and inventory cost;

(2) it represents multiple transportation options, i.e. *transportation choices are considered as output, not input;*

(3) it explicitly represents the required level of customer service.

Since it is primarily the decisions with regard to the trunking component of transportation that are interrelated with inventory decisions at the DCs, the focus is on representing multiple transportation options in trunking operations. In the integrated model, a transportation option is defined by the following elements: fixed cost per shipment; variable cost per unit shipped; unit in-transit carrying cost per unit time; shipping frequency; average lead-time and standard deviation of lead-time. Customer service is represented by the allowed probability of stockout during an order cycle.

Summary and Conclusions

Decisions related to the design of a distribution network are among the most critical logistics management decisions, since they affect both distribution cost and the quality of customer service that can be provided. Existing mathematical models for distribution network design have focused on individual components of the design problem, while ignoring or making restrictive assumptions regarding the other components. Such a component-by-component approach is likely to result in "sub-optimal" solutions.

A full understanding of the relationship between the decisions associated with faculty, transportation and inventory costs is needed if one is adequately to represent the trade-offs between them in a mathematical model.

This article has discussed the specific elements of the interdependence beween facility location, transportation and inventory decisions, and proceeded to present an integrated mathematical model for the distribution network design problem, which explicitly represents the trade-offs between facility, transportation and inventory costs. There are three basic differences between the proposed integrated model and existing DC location models: it represents all three cost components of the design problem; it represents multiple transportation options; and it explicitly represents the required level of customer service. The proposed model has not yet been tested numerically. An efficient computerised solution method is currently under development. Upon completion of this development, the model will be tested and the results will be reported in future publications.

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Jossef Perl is Assistant Professor in the Department of Civil Engineering, College of Engineering, at the University of Maryland, College Park, MD, where Sompong Sirisoponsilp is a PhD candidate.

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