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Characterization of facility assignment costs for a location-inventory model under truckload distribution

Sıla Çetinkaya[1], Burcu B Keskin[2]* and Halit Üster[1]

[1]Texas A&M University, College Station, TX, USA; and [2]University of Alabama, Tuscaloosa, AL, USA

We consider a two-stage distribution system, where the first stage consists of potential distribution centres (DCs) and the second stage consists of geographically dispersed existing retailers. Our goal is to determine the set of open DCs and assignment of open DCs to retailers simultaneously with inventory decisions of retailers. In addition to the DC-specific fixed facility location costs, we explicitly model the inventory replenishment and holding costs at the retailers and truckload transportation costs between the DCs and the retailers. The transportation costs are subject to truck/cargo capacity, leading to an integrated location-inventory problem with explicit cargo costs. We develop a mixed-integer nonlinear model and analyse its structural properties leading to exact expressions for the so-called implied facility assignment costs and imputed per-unit per-mile transportation costs. These expressions analytically demonstrate the interplay between strategic location and tactical inventory/transportation decisions in terms of resulting operational costs. Although both the theory and practice of integrated logistics have recognized the fact that strategic and tactical decisions are interrelated, to the best of our knowledge, our paper is the first to offer closed-form results demonstrating the relationship explicitly. We propose an efficient solution approach utilizing the implied facility assignment costs and we demonstrate that significant savings are realizable when the inventory decisions and cargo costs are modelled explicitly for facility location purposes.

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1. Introduction

Both the theory and applications of supply chain management concentrate heavily on three important logistical decisions to include facility location, inventory, and transportation decisions (Ballou, 1998). In this context, the integration of inventory and transportation decisions has received increasing attention both in academia and practice (Çetinkaya, 2004). Likewise, integrated decision-making in the context of simultaneous optimization of facility location and inventory decisions has also attracted significant interest (Keskin and Üster, 2012). The former line of research (i) investigates the impact of inbound and/or outbound transportation costs and decisions on inventory optimization and (ii) demonstrates that significant cost savings are realizable through simultaneous consideration of inventory and transportation costs and decisions. The latter line of research (i) highlights the role of transportation costs as one of the critical links contributing to the interplay between facility location and inventory problems, and (ii) explores and quantifies the value of integrated location-inventory models.

Transportation-related costs do not only impact facility location decisions, they also have an immediate impact on the frequency of inventory replenishment decisions. However, the existing work in location-inventory models ignores the fact that *transportation-related costs are often subject to truck/cargo capacity, and hence cargo costs must be modelled explicitly*. Explicit consideration of cargo costs is particularly important in the context of truckload transportation, which is the main focus of the current paper.

We consider a firm managing a two-stage distribution system operating a fleet of trucks each with a finite cargo capacity. The first stage includes a set of potential distribution centres (DCs) representing cross-docking points subject to facility-specific fixed location costs. Since the DCs are not inventory keeping locations, they are replenished on an as-needed basis by an external supplier incurring a sunk cost, so that there is no need to explicitly model the external supplier. The second stage consists of geographically dispersed existing retailers facing store-specific constant demand rates and operating under the assumptions of the classical Economic Order Quantity (EOQ) model (Zipkin, 2000). That is, each retailer is replenished by a single DC and holds inventory to meet the constant demand.

**Correspondence: Burcu B Keskin, Department of Information Systems, Statistics, and Management Science, University of Alabama, 355 Alston Hall, Tuscaloosa, AL 35487-0226, USA.*

E-mail: bkeskin@cba.ua.edu

Authors are listed in alphabetical order.

Hence, we model the inventory replenishment and holding costs at the retailers along with the dispatch and distance-based transportation costs associated with (direct) shipments of replenishments to the retailers from their respective DCs. We explicitly account for the general transportation costs associated with direct shipments between the retailers and their respective DCs, delivered using the fleet of trucks. Each truck has a finite cargo capacity leading to more general transportation costs than those that are typical in location theoretic models.

The resulting transportation costs are known as cargo (or truckload) costs and depend on both the distance travelled and cargoes used, that is, quantity shipped. Hence, this type of transportation cost function can be characterized as distance-based transportation costs subject to cargo capacity, and it is a practical generalization of the unit transportation costs that are commonly used in the classical fixed charge facility location problem (FCFLP) (Daskin, 1995).

We propose a mixed-integer nonlinear programming model for the purpose of determining (i) the number and selection of open DCs, (ii) the assignment of each retailer to an open DC, and (iii) the inventory policy (ie, when and how much to order at) of each retailer. The model is aimed at minimizing the sum of (i) fixed facility location costs of open DCs and (ii) transportation costs (subject to cargo capacities) between the DCs and the retailers, along with (iii) purchasing, (iv) inventory replenishment, and (v) holding costs at the retailers. Due to the simultaneous consideration of DC-specific fixed facility location costs along with inventory- and transportation-related costs, the proposed model is representative of potential trade-offs between strategic and tactical decision-making. In fact, the model is a practical extension of the FCFLP to consider integrated location-inventory decisions under cargo costs.

Examining the structural properties of the model, we present exact expressions for the so-called *implied facility assignment costs* (see expressions (5) and (11)) and *imputed per-unit per-mile transportation costs* (see expression (9)). The corresponding analytical expressions demonstrate the interplay between *strategic* facility location and *tactical* inventory replenishment decisions along with the impact of transportation-related parameters, that is, costs and capacities, on this interplay. Based on these results, we suggest an efficient solution approach for the model and demonstrate its use relative to a practical benchmark approach through numerical results. The benchmark relies on sequential decision-making, where the facility location decisions precede inventory and transportation decisions.

The remainder of the paper is organized as follows. Next, we proceed with a summary of the related literature. In Section 3, we introduce the notation and model. In Section 4, we prove useful structural properties of the model and develop an efficient solution approach relying on these properties. We present our numerical results in Section 5. Finally, in Section 6, we summarize our findings and conclude by discussing the potential generalizations of this work.

2. Related literature

Since our proposed model takes into account for optimization of inventory decisions under cargo costs along with facility location decisions, it is a natural extension of the existing work on integrated inventory-transportation models (eg, see Çetinkaya (2004) for a review paper, and see Çetinkaya *et al* (2009) and Rieksts and Ventura(2008) for examples of recent papers on integrated transportation-inventory models). This class of models can also be classified further as (i) quantitative models that investigate the impact of transportation costs on inventory decisions without explicitly optimizing transportation decisions, that is, shipment/dispatch policy or shipment/dispatch policy parameters, and (ii) quantitative models that are aimed at simultaneous optimization of inventory and transportation decisions. The current paper draws from the literature on the former class of models with general inbound/outbound transportation costs arising in the context of truckload transportation. For example, see the results in Toptal *et al* (2003) regarding the buyer–vendor coordination under cargo costs.

Although the interaction between facility location and inventory decisions has been recognized as early as 1960s (Heskett, 1966), quantitative research exploring this interaction in the context of integrated location-inventory models is relatively new. Existing work can be broadly categorized as (i) quantitative models that investigate the impact of inventory costs on facility location decisions without explicitly optimizing inventory decisions, that is, inventory policy or inventory policy parameters (eg, Erlebacher and Meller (2000); Nozick and Turnquist (1998, 2001); Miranda and Garrido (2004); Croxton and Zinn (2005); Shen and Daskin (2005), and (ii) quantitative models that are aimed at simultaneous optimization of location and inventory decisions (eg, McCann (1993); Daskin *et al* (2002), Drezner *et al* (2003); Shen *et al* (2003); Üster *et al* (2008); Keskin *et al* (2010), Keskin and Üster (2012); Teo and Shu (2004); Shu *et al* (2005). Our proposed model falls into this second category of models, and it fills a gap in the literature by investigating the impact of cargo costs.

Potential applications of the two-stage problem setting of interest include vendor-managed inventory (VMI) and contract manufacturing (Üster *et al*, 2008; Keskin *et al*, 2010). In the former example, the firm is a manufacturer operating multiple facilities (DCs) that supply inventory for multiple retailers, and it has the central authority regarding the source, timing, and quantity of resupply. In the latter example, the firm can be considered as a brand owner operating multiple retailers and it is searching for contract manufacturers (DCs) to satisfy the demand at the retailers. The key characteristic of the current paper is the explicit modelling of cargo costs as they apply in truckload transportation for both of these examples. Also, in both of the examples, explicit modelling of cargo costs has strong motivations, especially in the context of distribution of (i) fast-moving items in high volume packaging, such as potato chips, and (ii) bulky items, such as office furniture, where both

VMI and contract manufacturing practices are widely applicable (Çetinkaya *et al,* 2009; Mutlu and Çetinkaya, 2010).

Another possible application of the problem can be found in the context of the so-called vendor selection problem of a firm, which is in the process of specifying (i) the selection of vendors to work with and (ii) annual supply patterns for each selected vendor (Weber *et al*, 1991). While the vendor selection process is complex and has both qualitative and quantitative aspects, it can be supported by mathematical constructs of discrete facility location models (Current and Weber, 1994; Keskin *et al*, 2010). In particular, our proposed location-inventory model captures important practical characteristics of the vendor selection problem and provides a foundation for further analytical work generalizing the existing literature in the area. The solution offers not only a selection of vendors (DCs) and the annual supply quantities for each selected vendor, but also the detailed supply plan including the replenishment frequency/quantity for each retailer under explicit cargo costs. Such information is useful both for supply and transportation contracts.

3. General model and notation

We introduce the notation in Table 1 and provide an illustration of the two-stage distribution system of interest in Figure 1. We have three sets of decision variables.

1. The first set is associated with selecting the DCs. For each DC j, $j \in \mathcal{J}$, the binary decision variable X_j is set to 1 if DC j is open, and set to 0, otherwise.
2. The second set pertains to the assignment of retailers to DCs. For retailer i, $i \in \mathcal{I}$, and DC j, $j \in \mathcal{J}$,

$$Y_{ij} = \begin{cases} 1, & \text{if retailer i is assigned to DC } j, \\ 0, & \text{otherwise.} \end{cases}$$

   Note that by letting assignment variables assume only binary values, we ensure that each retailer will receive shipments of replenishment quantities from only one DC. This property is known as *single-sourcing* in the location literature. For the uncapacitated FCFLP, even if the assignment variables are allowed to be fractional, due to the lack of capacity restrictions, the assignment variables naturally assume binary values.
3. Finally, the third set relates to inventory decisions of the retailers. For each retailer i, $i \in \mathcal{I}$, Q_i and T_i denote the replenishment quantity and reorder interval of retailer i, respectively. We also define **Q** as the vector of the replenishment quantities and **T** as the vector of the reorder intervals. Clearly, $T_i = Q_i/D_i$, for all $i \in \mathcal{I}$. Hence, for the rest of the analysis, we use **Q** to represent inventory decisions, keeping in mind that the corresponding reorder intervals can be easily obtained from the replenishment quantities.

Table 1 Notation

$\mathcal{I}$	set of retailers, $\mathcal{I} = \{1, \ldots, m\}$.
$\mathcal{J}$	set of potential DCs, $\mathcal{J} = \{1, \ \ldots \ , n\}$.
D_i	annual demand rate faced by retailer i, $\forall i \in \mathcal{I}$.
h_i	inventory holding cost rate for each unit of inventory at retailer i, $\forall i \in \mathcal{I}$.
K_i	fixed inventory replenishment cost of retailer per replenishment order i, $\forall i \in \mathcal{I}$.
d_{ij}	distance between retailer i, abcd and DC j, $\forall j \in \mathcal{J}$.
p_{ij}	fixed surcharge cost to retailer i, abcd from DC j, $\forall j \in \mathcal{J}$.
r_{ij}	per-mile cost to retailer i, $\forall i \in \mathcal{I}$ from DC j, $\forall j \in \mathcal{J}$.
f_j	(annual) fixed facility location cost of DC j, $\forall j \in \mathcal{J}$.
C_T	cargo capacity of a truck.

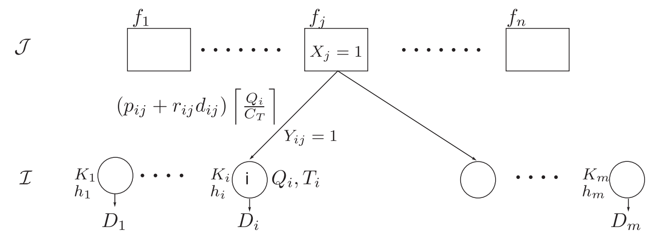

Figure 1 Problem setting.

As we noted earlier, in addition to the DC-specific fixed facility location costs, we explicitly model the inventory replenishment and holding costs at the retailers and truckload transportation costs between the DCs and the retailers. Transportation operations between the DCs and retailers incur both *fixed* (eg, administrative, maintenance, dispatching, etc) and *variable costs* (eg, fuel, labour, depreciation, etc). Hence, we express the transportation cost associated with one trip from DC j, $j \in \mathcal{J}$ to retailer i, $i \in \mathcal{I}$ as $p_{ij} + r_{ij}d_{ij}$, where p_{ij} represents the fixed surcharge cost and r_{ij} denotes the per-mile cost for a truck. The transportation cost is subject to cargo capacity. That is, the number of trips/trucks required by retailer i, $i \in \mathcal{I}$, for a replenishment quantity of Q_i is given by $\lceil Q_i/C_T \rceil$, where C_T is the cargo capacity. Then, the total transportation cost from DC j, $j \in \mathcal{J}$, to retailer i, $i \in \mathcal{I}$, for a replenishment quantity of Q_i is $(p_{ij} + r_{ij}\, d_{ij}) \lceil Q_i/C_T \rceil$.

Consequently, the integrated location-inventory problem with cargo costs can be formulated as the following mixed-integer nonlinear program denoted by *IFLP*:

$$\text{Minimize} \sum_{j \in \mathcal{J}} f_j X_j + \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \left(\frac{(p_{ij} + r_{ij}\, d_{ij}) \left\lceil \frac{Q_i}{C_T} \right\rceil}{Q_i} \right) D_i Y_{ij}$$

$$+ \sum_{i \in \mathcal{I}} \left(\frac{K_i D_i}{Q_i} + \frac{1}{2} h_i Q_i \right) \qquad \text{(IFLP)}$$

subject to

$$\sum_{j \in \mathcal{J}} Y_{ij} = 1, \qquad \forall i \in \mathcal{I} \qquad (1)$$

$$Y_{ij} \leqslant X_j, \qquad \forall\, i \in \mathcal{I} \text{ and } \forall j \in \mathcal{J} \qquad (2)$$

$$X_j \in \{0, 1\}, \qquad \forall j \in \mathcal{J} \tag{3}$$

$$Y_{ij} \in \{0, 1\}, \qquad \forall i \in \mathcal{I} \text{ and } \forall j \in \mathcal{J} \tag{4}$$

$$Q_i \geqslant 0, \qquad \forall i \in \mathcal{I} \tag{5}$$

The objective function of *IFLP* minimizes the total annual costs to include the fixed facility location costs associated with the open DCs, the transportation cost from DCs to retailers, and the inventory replenishment and holding costs at the retailers. Constraints (1) ensure that the demand of each retailer is satisfied. Constraints (2) establish that each retailer will be assigned to an open DC. Finally, constraints (3) and (4) ensure integrality, and constraints (5) ensure nonnegativity. Observe that the cargo capacity is modelled in the objective function rather than in the constraints. This is because the replenishment quantities of the retailers imply the number of trucks needed along with the resulting transportation costs. Hence, one can argue that the overall transportation capacity is *installable*, which is true in many practical settings due to the abundance of contract and for-hire carriers in the truckload industry.

4. Structural properties and proposed solution approach

Let us proceed with analysing the structural properties of *IFLP* that are useful to develop an efficient solution approach. To this end, let us first suppose that cargo capacity is very large, that is, infinity, so that each shipment can be sent using one truck.

4.1. Special case: $C_T \to \infty$

In this case, the transportation cost is given by $p_{ij} + r_{ij}\, d_{ij}$, $\forall i \in \mathcal{I}$ and $\forall j \in \mathcal{J}$ and the *IFLP* reduces to

$$\text{Minimize} \sum_{j\in\mathcal{J}} f_j X_j + \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \frac{(p_{ij} + r_{ij} d_{ij}) D_i Y_{ij}}{Q_i} + \sum_{i\in\mathcal{I}} \left(\frac{K_i D_i}{Q_i} + \frac{h_i Q_i}{2} \right) \tag{6}$$

subject to (1), (2), (3), (4), and (5).

The above formulation is referred as the uncapacitated *IFLP* (*UIFLP*) in the remainder of the paper. Now, suppose that Y_{ij} values are known for all $i \in \mathcal{I}$ and $j \in \mathcal{J}$. Then, it is easy to verify that the second part of the objective function (6) is convex in $\mathbf{Q}$ for each retailer $i \in \mathcal{I}$ and

$$Q_i = \sqrt{\frac{2\left(K_i + \sum_{j\in\mathcal{J}} (p_{ij} + r_{ij}\, d_{ij}) Y_{ij} \right) D_i}{h_i}}. \tag{7}$$

Consequently, substituting expression (7) in the objective function (6), we have

$$\sum_{j\in\mathcal{J}} f_j X_j + \sum_{i\in\mathcal{I}} \sqrt{2\left(K_i + \sum_{j\in\mathcal{J}} (p_{ij} + r_{ij} d_{ij}) Y_{ij} \right) D_i h_i}$$
$$= \sum_{j\in\mathcal{J}} f_j X_j + \sum_{i\in\mathcal{I}} \sqrt{A_i + \sum_{j\in\mathcal{J}} B_{ij} Y_{ij}}, \tag{8}$$

where $A_i = 2K_i D_i h_i$ and $B_{ij} = 2(p_{ij} + r_{ij} d_{ij}) D_i h_i$ for $i \in \mathcal{I}$ and $j \in \mathcal{J}$.

With these simplifications, the *UIFLP* is now reduced to determining the selection of DCs and retailer-DC assignments so that the revised objective function (8) is minimized while satisfying the constraints (1), (2), (3), and (4). The following optimality property and theorem lead to an important corollary associated with the *UIFLP* and are eventually useful for a simplified solution approach for the *IFLP*.

Property 1: *Consider the* UIFLP *and let* $\mathcal{J}' = \{j \in \mathcal{J} X_j = 1\}$ *denote the set of open DC locations. Then, the optimal assignments of open DCs to the retailers are given by*

$$Y_{ij} = \begin{cases} 1, & \text{for} \quad j = \arg\min_{j\in\mathcal{J}'} \{p_{ij} + r_{ij} d_{ij}\}, \\ 0, & \text{otherwise}, \end{cases}$$

for all $i \in \mathcal{I}$ *and* $j \in \mathcal{J}$ *That is, for the UIFLP, given the set of open DCs, optimal retailer-DC assignments are based on the transportation-related costs only.*

Proof: All proofs are presented in the Appendix.

Theorem 1 *The objective function (8) of UIFLP is equivalent to*

$$\sum_{j\in\mathcal{J}} f_j X_j + \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} C_{ij} Y_{ij}, \tag{9}$$

where

$$C_{ij} = \sqrt{2(K_i + p_{ij} + r_{ij}\, d_{ij}) D_i h_i}, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}, \tag{10}$$

represents the resulting facility assignment cost *of retailer* $i \in \mathcal{I}$ to DC $j \in \mathcal{J}$, and hence it is referred as the 'implied facility assignment cost'.

The implied facility assignment cost C_{ij} is, in fact, the optimal sum of annual inventory- and transportation-related expenses at retailer $i \in \mathcal{I}$ if served by DC $j \in \mathcal{J}$, and hence it is also known as the minimum *annual operational cost* associated with the retailer-DC pair *i-j*. It is important to emphasize that expression (5) explains the impact of operational costs on location-related cost parameters. Although both the theory and practice of recent supply chain management initiatives, such as VMI and integrated logistics, have recognized the fact that *strategic* (eg, location) and *tactical* (eg, inventory/transportation) decisions are interrelated, to the best of our knowledge, no previous work has offered closed-form and/or analytical results demonstrating the relationship explicitly in terms of resulting *operational* costs.

Corollary 1 *The UIFLP reduces to the classical uncapacitated FCFLP, and hence it can be solved effectively using the existing techniques in the literature* (Daskin, 1995).

In the traditional literature on the FCFLP, due to the lack of an alternative approach, the assignment costs are approximated/estimated by $\alpha d_{ij} D_i$, where α is the per-unit per-mile transportation cost. As we demonstrated above, however, the implied assignment cost C_{ij}, $i \in \mathcal{I}$ and $j \in \mathcal{J}$, depends on the optimal sum of annual inventory- and transportation-related expenses at retailer i when served by DC j.

It is easy to observe that, in general, there is no unique constant $\alpha > 0$, such that

$$\alpha d_{ij} D_i = \sqrt{2(K_i + p_{ij} + r_{ij} d_{ij}) D_i h_i}, \quad \forall\, i \in \mathcal{I}, j \in \mathcal{J},$$

that is, the traditional, approximate assignment costs computed using a fixed α value are perhaps very different than the actual assignment costs. However, letting

$$\alpha_{ij} \equiv \frac{\sqrt{2(K_i + p_{ij} + r_{ij}\ d_{ij}) D_i h_i}}{d_{ij} D_i}, \quad \forall\, i \in \mathcal{I}, j \in \mathcal{J}, \qquad (11)$$

we can easily compute a so-called *imputed per-unit per-mile transportation cost* for a given network configuration with a set of open DCs $J' = \{j \in J{:}X_j = 1\}$:

- First, recalling Property 1 and expression (11), we define

$$\alpha^*_{ij} = \begin{cases} \alpha_{ij} = \frac{\sqrt{2(K_i + p_{ij} + r_{ij}\ d_{ij}) D_i h_i}}{d_{ij} D_i} & \text{for } j = \arg\min_{j \in \mathcal{J}'} \{p_{ij} + r_{ij} d_{ij}\}, \\ 0, & \text{otherwise}, \end{cases} \qquad (12)$$

for all $i \in \mathcal{I}$ and $j \in \mathcal{J}$.

- Next, utilizing expression (7), we solve

$$\min_{\alpha} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \left(\alpha - \alpha^*_{ij}\right)^2. \qquad (13)$$

- The corresponding unique minimizer is then given by

$$\alpha = \frac{\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \alpha^*_{ij}}{|\ \mathcal{I}\ |}, \qquad (14)$$

and it is called the imputed per-unit per-mile transportation cost because it is a proxy for the counterpart per-unit per-mile transportation cost commonly used in location theory.

Here the adjective 'imputed' is in the spirit of similar terminology for traditional inventory control models with implicit cost parameters. For example, see Nahmias (2009, pp 272–276), regarding the computation of the imputed shortage penalty associated with the approximate (Q, R)-model and Çetinkaya and Parlar (1998) for the deterministic EOQ model with backorders under service measure constraints. The imputed per-unit per-mile transportation cost is then useful for cost allocation, transit cost analysis, and benchmarking with unit costs.

4.2. General case

Given the above formal results, let us examine the general problem where C_T is considered explicitly. In this case, the objective function of *IFLP* is nonlinear. Fortunately, as we explain below, this nonlinearity can be eliminated by reducing the *IFLP* to the classical uncapacitated FCFLP. Consequently, the problem at hand can be solved via the techniques developed for the uncapacitated FCFLP.

Now, observe that if **X** and **Y** are known, then the remaining problem is a multi-retailer EOQ-model with a generalized replenishment cost structure. Further, given **X** and **Y**, the *IFLP* is decomposable for each retailer $i \in \mathcal{I}$. More specifically, let $\mathcal{I}_j = \{i \in \mathcal{I} Y_{ij} = 1\}$, for any $j \in \mathcal{J}$. Then, for each DC $j \in \mathcal{J}$ and each retailer $i \in \mathcal{I}_j$, we have the following EOQ problem with a generalized replenishment cost structure:

$$\min_{Q \geqslant 0} g_{ij}(Q) = \frac{\left(K_i + (p_{ij} + r_{ij}\ d_{ij}) \left\lceil \frac{Q}{C_T} \right\rceil \right) D_i}{Q} + \frac{1}{2} h_i Q. \qquad (15)$$

This problem can be solved using the Generalized EOQ Algorithm below developed by Toptal *et al* (2003; see Algorithm 1 on p 991).

Generalized EOQ Algorithm

For retailer $i \in I$ and DC $j \in J$:

1: Compute $\sqrt{2K_i D_i / h_i}$.

2: Let N denote the integer multiple of C_T such that
$NC_T < \sqrt{2K_i D_i / h_i} \leqslant (N+1) C_T$. Compute
$Q^{N+1}_{ij} = \sqrt{\frac{2D_i(K_i + (N+1)(p_{ij} + r_{ij} d_{ij}))}{h_i}}$.

If $Q^{N+1}_{ij} \geqslant (N+1) C_T$, then go to Step 3. Otherwise, go to Step 4.

3: $Q^*_{ij} = \arg\min\{g(NC_T), g((N+1)C_T)\}$. Stop.

4: $Q^*_{ij} = \arg\min\left\{g(NC_T),\ g(Q^{N+1}_{ij})\right\}$. Stop.

We note that the optimal Q^*_{ij} resulting from the above algorithm is the preferred order quantity of retailer $i \in \mathcal{I}$ and DC $j \in \mathcal{J}$ under sole sourcing, and it is given by

$$\arg\min\left\{g_{ij}(Q^{N+1}_{ij}), g_{ij}(NC_T), g_{ij}((N+1)C_T)\right\}.$$

Considering all the retailers $i \in \mathcal{I}$ and DCs $j \in \mathcal{J}$, we can obtain the optimal replenishment quantities of the retailers for alternative DCs via the Generalized EOQ Algorithm, and we can set the corresponding implied facility assignment costs using Theorem 2 below.

Theorem 2 *The objective function of IFLP can be expressed as in expression (4) by using*

$$C_{ij} = g_{ij}(Q^*_{ij}) = \frac{\left(K_i + (p_{ij} + r_{ij}d_{ij})\left\lceil Q^*_{ij}/C_T\right\rceil\right)D_i}{Q^*_{ij}} + \frac{1}{2}h_i Q^*_{ij}, \quad (16)$$

where

$$Q^*_{ij} = \arg\min\left\{g_{ij}(Q^{N+1}_{ij}),\ g_{ij}(NC_T),\ g_{ij}((N+1)C_T)\right\},$$

$$Q^{N+1}_{ij} = \sqrt{\frac{2D_i(K_i + (N+1)(p_{ij} + r_{ij}d_{ij}))}{h_i}}, \quad \text{and}$$

N is the integer multiple of C_T

$$\text{s.t. } NC_T < \sqrt{2K_iD_i/h_i} \leqslant (N+1)C_T.$$

Corollary 2: *The* IFLP *reduces to the classical uncapacitated FCFLP, and hence it can be solved effectively using the existing techniques in the literature* (Daskin, 1995).

Consequently, our general problem *IFLP* also reduces to the uncapacitated FCFLP. Hence, it can be solved using the well-known and effective techniques developed for the uncapacitated FCFLP.

Observe that, as in the case of *UIFLP*, the new implied assignment cost in expression (11) represents the optimal annual sum of inventory- and transportation-related expenses at retailer i if served by DC j, that is, it is the minimum *annual operational cost* associated with the retailer-DC pair i-j under explicit cargo costs and capacities. Also, as before, our results can be used to compute the corresponding imputed per-unit per-mile transportation cost subject to cargo cost and capacity for a given network configuration with open DCs $\mathcal{J} = \{j \in \mathcal{J}\ X_j = 1\}$ and retailer-DC assignments $\mathcal{I}_j = \{i \in \mathcal{I} Y_{ij} = 1\}$ for $j \in \mathcal{J}'$. For this purpose, expression (9) is directly applicable if we simply we redefine

$$\alpha^*_{ij} = \begin{cases} \alpha_{ij} \equiv \frac{g_{ij}(Q^*_{ij})}{d_{ij}D_i} = \frac{\left(K_i + (p_{ij} + r_{ij}d_{ij})\left\lceil Q^*_{ij}/C_T\right\rceil\right)D_i}{Q^*_{ij}d_{ij}D_i} + \frac{h_iQ^*_{ij}}{2d_{ij}D_i},\ \forall\, i \in I, j \in J, & \text{for} \quad i \in \mathcal{I}_j \\ 0, & \text{otherwise,} \end{cases}$$

for all $i \in \mathcal{I}$ and $j \in \mathcal{J}$.

5. Computational study

We proceed with the results of a computational study for which the proposed solution approaches have been implemented using C++ and run on a Pentium IV 3.2Ghz machine with 1 GB memory. The purposes of the computational study include (i) quantifying the benefit of integrated decision-making via comparison of the results obtained under sequential decision-making, (ii) examining the impact of network size on the value of integrated decision-making, and (iii) demonstrating the effectiveness of our proposed solution approach.

The comparisons in (i) above build on the idea that without the use of integrated location-inventory model developed in this paper, one must rely on sequential decision-making, where the facility location decisions precede inventory decisions. That is, our comparative results are relative to a benchmark model, called *BM*, representative of the sequential decision-making approach common in the practice and literature (eg, see Daskin *et al* (2002); Sahyouni *et al* (2007); Keskin and Üster (2012)).

Under the *BM*, we first solve the following uncapacitated FCFLP problem, referred as *BM-FLP*:

$$\text{Minimize} \quad \sum_j f_jX_j + \sum_i \sum_j C_{ij}Y_{ij} \qquad (BM\text{-}FLP)$$

subject to

$$\sum_{j \in \mathcal{J}} Y_{ij} = 1, \qquad \forall i \in \mathcal{I}, \qquad (17)$$

$$Y_{ij} \leqslant X_j, \qquad \forall i \in \mathcal{I} \text{ and } \forall j \in \mathcal{J}, \qquad (18)$$

$$X_j \in \{0, 1\}, \qquad \forall j \in \mathcal{J}, \qquad (19)$$

$$0 \leqslant Y_{ij} \leqslant 1, \qquad \forall i \in \mathcal{I} \text{ and } \forall j \in \mathcal{J}, \qquad (20)$$

For the purpose of solving this problem, due to the lack of a better approach, we have to utilize crude estimates of the assignment costs C_{ij}, $i \in \mathcal{I}$ and $j \in \mathcal{J}$. In keeping with the traditional facility location literature, we set $C_{ij} = \hat{\alpha}d_{ij}D_i$, where $\hat{\alpha}$ is the per-unit per-mile transportation cost, which is set to 1 in our computational study. As we demonstrated in the previous section, however, the resulting assignment cost C_{ij}, $i \in \mathcal{I}$ and $j \in \mathcal{J}$, depends on the the optimal sum of annual inventory- and transportation-related expenses at retailer i when served by DC j. Hence, perhaps, the main weakness of the sequential approach is the imprecise nature of the way in which assignment costs are estimated.

Note that the *BM-FLP* is a typical formulation of the uncapacitated FCFLP. As we noted earlier, although the assignment variables are modelled as a fraction of the demand of the retailer that is served by a DC, since the facilities are uncapacitated, they will assume integer values. And in our computational tests, we solve this problem to optimality using CPLEX 9.0.

Let $\mathbf{X}^{BM}$ and $\mathbf{Y}^{BM}$ denote the optimal solution of the *BM-FLP*. Next, given the assignment variables Y^{BM}_{ij} for all $i \in \mathcal{I}$ and $j \in \mathcal{J}$, the replenishment quantities of the retailers are determined by solving

$$\min_{Q \geqslant O} \sum_{i \in \mathcal{I}} \left(\frac{K_iD_i}{Q_i} + \frac{1}{2}h_iQ_i + \sum_{j \in \mathcal{J}} \frac{(p_{ij} + r_{ij}\ d_{ij})\left\lceil \frac{Q_i}{C_T}\right\rceil D_iY^{BM}_{ij}}{Q_i} \right). \ (BM\text{-}Inv)$$

The optimal replenishment quantity of retailer $i \in \mathcal{I}$, denoted by Q^{BM}_i, can now be obtained using the Generalized EOQ Algorithm discussed in the previous section.

Table 2 Parameter ranges

Parameter	D_i	K_i	h_i	p_{ij}	r_{ij}	d_{ij}	f_j
Range	U[350, 1400]	U[75, 300]	U[5, 10]	U[425, 1700]	U[180, 120]	U[1, 150]	U[100 000, 150 000]

Table 3 Comparison with the *BM*

	DATA SET		*GAIN (%)*			*OPEN DCs*			*IMPUTED COST, α*		
	$\|I\|$	$\|J\|$	*Min*	*Ave*	*Max*	*%DC*	*Ave*	*Max*	*Min*	*Ave*	*Max*
Uncapacitated Case	25	10	10.74	20.39	30.19	89	1.56	4	0.083	0.174	0.390
	25	20	13.76	22.85	32.57	78	1.32	4	0.070	0.176	0.495
	25	30	11.93	24.39	31.45	81	1.47	4	0.071	0.178	0.431
	50	10	2.38	20.88	25.71	100	2.21	4	0.085	0.180	0.411
	50	20	2.86	22.44	28.58	92	2.01	5	0.083	0.181	0.361
	50	30	4.89	23.82	30.26	94	2.00	4	0.098	0.186	0.390
	100	10	17.45	21.23	24.70	99	2.73	5	0.106	0.173	0.282
	100	20	8.48	23.25	26.99	98	3.23	6	0.107	0.181	0.277
	100	30	15.31	24.45	28.07	94	2.85	6	0.106	0.182	0.255
Capacitated Case, $C_T = 50$	25	10	4.37	34.85	60.17	91	1.79	4	0.331	0.726	1.962
	25	20	3.36	37.40	58.21	90	1.74	4	0.357	0.738	1.723
	25	30	5.39	37.48	60.63	88	2.13	5	0.276	0.695	1.370
	50	10	6.60	39.52	61.42	100	2.77	6	0.386	0.723	1.318
	50	20	7.10	39.96	58.79	94	2.45	5	0.363	0.707	1.260
	50	30	5.09	40.86	61.07	97	2.53	5	0.395	0.723	1.519
	100	10	8.34	39.65	59.64	100	3.62	7	0.406	0.702	1.219
	100	20	2.89	40.38	60.31	98	3.23	6	0.322	0.710	1.142
	100	30	5.25	41.26	59.35	99	3.67	7	0.486	0.700	0.994

The cost of *BM*, denoted by Z^{BM}, is then calculated by substituting $\mathbf{X}^{BM}$, $\mathbf{Y}^{BM}$, and $\mathbf{Q}^{BM}$, obtained from the solutions of the *BM-FLP* and the *BM-Inv*, in the objective function (6). Hence, the percentage gain by using the IFLP over $BM(\%) = 100(Z^{BM} - Z^{IFLP})/Z^{BM}$, where Z^{IFLP} is the optimal cost of the *IFLP* obtained by CPLEX 9.0.

The counterpart benchmark model for *UIFLP* is the same except that in *BM-Inv* we let $C_T \to \infty$, so that $\lceil Q_i/C_T \rceil \to 1$, for all $i \in \mathcal{I}$. The corresponding optimal replenishment quantities in the uncapacitated case can then be obtained by using the traditional EOQ formula:

$$Q_i^{BM} = \sqrt{\frac{2\left(K_i + \sum_{j \in \mathcal{J}} (p_{ij} + r_{ij}\ d_{ij}) Y_{ij}^{BM}\right) D_i}{h_i}}, \quad \forall i \in \mathcal{I}.$$

The percentage gain by using the *UIFLP* over the counterpart *BM* is given by

$$100\left(Z^{BM} - Z^{UIFLP}\right)/Z^{BM},$$

where Z^{UIFLP} is the cost of the *UIFLP*, which is obtained by the Lagrangian relaxation heuristic (see Daskin, 1995, p 250) in our numerical results below.

As we noted earlier, one of the main goals of the computational study is to measure the impact of integrated decision-making for different configurations of distribution systems, that is, for different numbers of retailers and potential DCs. For this purpose, we report results from nine different data sets, where both the number of retailers and the number of potential DCs have three alternatives. Each data set consists of either 25, 50, or 100 retailers and 10, 20, or 30 potential DCs. In each group, we have 100 problem instances, generated randomly using the uniform distributions given in Table 2, resulting in a total of 900 problem instances with cargo capacities of $C_T = 50$ (units) and $C_T \to \infty$ (uncapacitated case).

We report the minimum, average, and maximum percentage gains by *IFLP* and *UIFLP* over the counterpart *BM*s in Table 3. We also report information regarding the number of open DCs. That is, we present the average and maximum differences in the number of open DCs under the *IFLP* (*UIFLP*) and its counterpart *BM*. Under the '%DC' column, we present the percentage of instances where the *IFLP* (*UIFLP*) leads to fewer open DCs than its counterpart *BM*. Finally, we report the minimum, average, and maximum imputed per-unit per-mile transportation costs, under column α, for each of the data sets.

Unless otherwise is stated, we discuss the numerical results for *IFLP* while noting that similar observations also apply for

UIFLP. For all of the data sets in Table 3, we observe that significant savings are obtained over the *BM* when using the *IFLP*, with average gains of more than 34%. One interesting observation is that, for a given number of retailers, the average gains increase as the number of potential DCs increase. For instance, from data set 1 to data set 2, the average gain increases to 34.85% from 37.40%. A similar pattern is observed between data sets 4, 5, and 6 and data sets 7, 8, and 9 as well. In other words, when the number of potential DCs increases for a fixed number of retailers, the *IFLP* is more efficient than the *BM* in determining open DCs and assignments of DCs to retailers, which translates to an increasing gap between the costs of the *IFLP* and *BM*.

Another interesting observation from Table 3 is regarding the difference in the number of open DCs in the solutions of the *BM* and *IFLP*. In most of the instances, the number of open DCs for *IFLP* is less than that of for *BM*. On average, this difference is at least one DC for data sets with smaller networks (data sets 1, 2, and 3) and can be as high as three DCs on average for data sets with larger networks (data sets 8 and 9). Due to the explicit consideration of the transportation-related expenses and the impact of inventory decisions on open DC locations, *IFLP* generally requires fewer number of DCs. For instance, for data set 4, in all of the instances, the number of open DCs with the *IFLP* is less than with the *BM*. This is because the *IFLP* evaluates the trade-off among the fixed facility location costs and the operational facility assignment costs including the inventory- and transportation-related expenses, more efficiently than the *BM*.

Our final observation from Table 3 is that the imputed per-unit per-mile transportation costs, denoted by α, range between 0.070 and 0.495 with an average of 0.179 for the uncapacitated case while they range between 0.276 and 1.962 with an average of 0.714 for the case with cargo capacity. Hence, one can explicitly quantify the impact of the cargo capacities on transportation costs. Additionally, recalling that BM utilizes an α value of 1, which is higher than the average α, for each of the data sets, one can reason why *BM* opens more facilities compared to *IFLP*.

6. Conclusions

In this paper, we analyse a generalized location-inventory problem under cargo costs. Although a growing body of literature examines the interaction between strategic facility location and tactical inventory decisions, our model is the first to analyse the impact of cargo costs and capacities on this interaction. We propose an efficient solution approach that exploits the structural properties of the problem. We also offer exact analytical expressions for estimating the facility assignment costs. We prove that each of these expressions is, in fact, the optimal annual sum of inventory- and transportation-related expenses at a given retailer i if served by a given DC j, and hence it represents the minimum *annual operational cost* associated with the retailer-DC pair i-j. Our numerical results demonstrate substantial savings with our approach over a practical benchmark model.

The results of this paper can be extended in several directions. One noteworthy extension would consider optimization of inventory decisions at both the DC and retailer levels. Other areas for future research include (i) explicit modelling of dynamic and stochastic nature of demand, (ii) consideration of dispatch and storage capacities along with cargo capacities, and (iii) evaluating the impact of uncertainty of demand on the models, solution approaches, and results. Last but not least, VMI contract design with location, transportation, and inventory considerations remains a prosperous area for future research.

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Appendix

Proofs

Proof of Property 1: For retailer $i \in \mathcal{I}$, let j^* be the open DC with the lowest transportation-related costs, $p_{ij^*} + r_{ij^*}\, d_{ij^*}$, among all the open DCs. Then, using (8) it is easy to verify that the resulting cost of assigning retailer $i \in \mathcal{I}$ to DC j^* is (see the proof of Theorem 1):

$$\sqrt{2\left(K_i + p_{ij^*} + r_{ij^*}\; d_{ij^*}\right) D_i h_i}. \tag{21}$$

Now, assume that in the optimal assignment, retailer i is assigned to open DC j^o. The resulting cost of this assignment is

$$\sqrt{2\left(K_i + p_{ijo} + r_{ijo}\; d_{ijo}\right) D_i h_i}. \tag{22}$$

Furthermore, since j^o is the optimally selected DC,

$$\sqrt{2\left(K_i + p_{ij^o} + r_{ij^o} d_{ij^o}\right) D_i h_i} \leqslant \sqrt{2\left(K_i + p_{ij^*} + r_{ij^*} d_{ij^*}\right) D_i h_i}. \tag{23}$$

However, for $p_{ij^*} + r_{ij^*} d_{ij^*} \leqslant p_{ij^o} + r_{ij^o} d_{ij^o}$,

$$\sqrt{2\left(K_i + p_{ij^*} + r_{ij^*} d_{ij^*}\right) D_i h_i} \leqslant \sqrt{2\left(K_i + p_{ij^o} + r_{ij^o} d_{ij^o}\right) D_i h_i}. \tag{24}$$

Using (23) and (24), we conclude that $j^o = j$, and this completes the proof. □

Proof of Theorem 1: For any retailer $i^o \in \mathcal{I}$ and any potential DC $j^o \in \mathcal{J}$, if $Y_{i^o j^o} = 1$, the resulting assignment cost denoted by $TAC_{i^o j^o}$ is simply

$$TAC_{i^o j^o} = \frac{K_{i^o} D_{i^o}}{Q_{i^o}} + \frac{h_{i^o} Q_{i^o}}{2} + \frac{D_{i^o}(p_{i^o j^o} + r_{i^o j^o} d_{i^o j^o})}{Q_{i^o}},$$

becuase $Y_{i^o j} = 0$ for $j \in \mathcal{J} \setminus \{j^o\}$. Observe that the above expression of $TAC_{i^o j^o}$ is simply the sum of the annual inventory- and transportation-related expenses for retailer i^o if served by DC j^o. Then, the replenishment quantity for this retailer that optimizes $TAC_{i^o j^o}$ is

$$Q^*_{i^o j^o} = \sqrt{\frac{2(K_{i^o} + p_{i^o j^o} + r_{i^o j^o} d_{i^o j^o}) D_{i^o}}{h_{i^o}}}.$$

Hence, the optimal value of $TAC_{i^o j^o}$, denoted by $C_{i^o j^o}$, for retailer i^o and DC j^o is equal to

$$C_{i^o j^o} = \sqrt{2(K_{i^o} + p_{i^o j^o} + r_{i^o j^o} d_{i^o j^o}) D_{i^o} h_{i^o}}.$$

Calculating C_{ij} values for all $i \in \mathcal{I}$ and $j \in \mathcal{J}$ in the same manner completes the proof. □

Proof of Corollary 1: The corollary directly follows from Theorem 1. □

Proof of Theorem 2: Proof is similar to the proof of Theorem 1, and hence it is omitted. □

Proof of Corollary 2: The corollary directly follows from Theorem 2. □

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