2.1 If V is a vector clock, prove that a $\longrightarrow$b if and only if V(a) $\leqslant$ V(b)

If a→b and a,b in the same channel, for example: channel i,

$V(a)=(a_1,a_2,a_3...a_i...a_n)$

$V(b)=(a_1,a_2,a_3...a_i+1...a_n)\geqslant V(a)$

If a→b and a,b in different channels, for example: a in channel i, b in channel j,

Assuming that there is another event b' and b'→b

V(b)[k]=max{V(a)[k],V(b')[k]} (k$\neq$j)

V(b)[k]=max{V(a)[k],V(b')[k]}+1 (k=j)

So V(b)$\geqslant$V(a)

If V(b)$\geqslant$V(a) and a,b in the same channel, obviously, a→b

If V(b)$\geqslant$V(a) and a,b in different channels, for example: a in channel I, b in channel j,

Assuming that b→a or a || b and there is another event a'(a'→a)

if b→a:

V(a)=max[V(a'), V(b)], V(a)[a]++, so V(a)$\geqslant$V(b) which contradict with V(b)$\geqslant$V(a)

If a || b:

According to the property of vector clock, neither V(a)$\leqslant$V(b) nor V(b)$\leqslant$V(a) when a and b are concurrent, in this case it also contradicts with V(b)$\geqslant$V(a)

So when V(b)$\geqslant$V(a), it should be a→b

To sum up, a→b if and only if V(a)$\leqslant$V(b)

2.2 Show that Lamport's mutual exclusion algorithm satisfies the Liveness property. Assuming that all channels are FIFO and there are no failures in channels or processes.

Liveness property: Every live request for CS is eventually granted.

Assuming that there are two processes p_i, p_j and two queues q_i, q_j

When process I send CS request:

q_i = (tsi,i)

q_j = (tsi,i)

process I enter CS.

Then before process I completes, process j send CS request (sorted by timestamp):

q_i = (tsi,i) (tsj,j)

q_j = (tsi,i) (tsj,j)

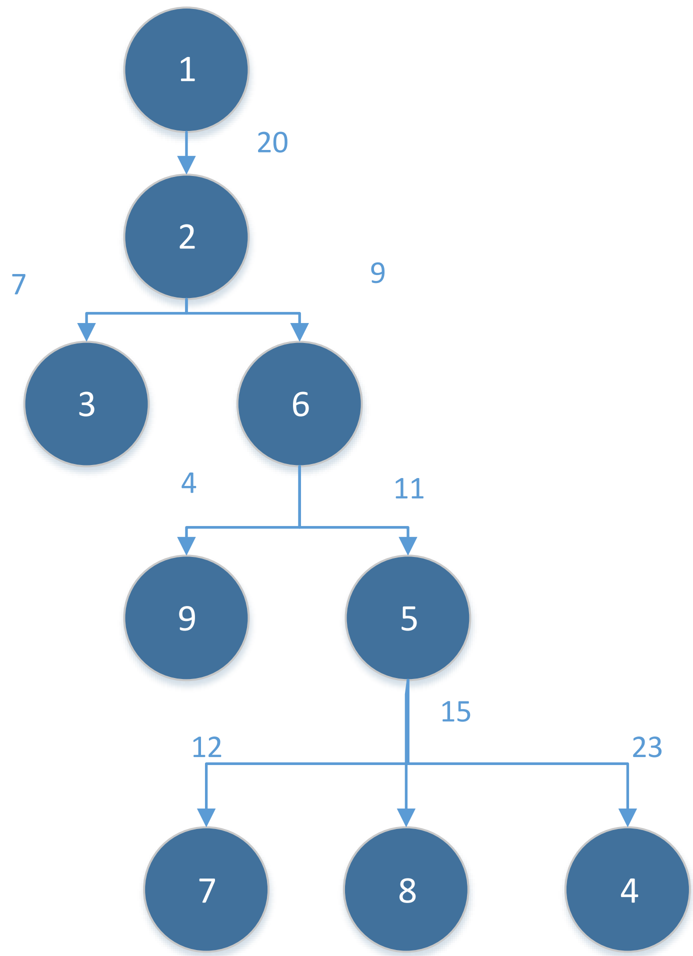
Because (tsj,j) is not at head of q_j , so process j cannot enter CS until process I have been released.

So every request for CS will finally be granted due to there are no failures in channel and processes and then the next request/process will enter CS which satisfies the Liveness property.

2.3 diameter of weighted network is 7, the paths are:

A-C-E-G-H, A-C-E-F-H, A-D-C-E-G-H, A-D-C-E-F-H, A-C-E-G-I-H, A-D-C-E-G-I-H
Diameter of unweighted network is 4, the paths are:
A-C-E-G-I, A-C-F-H-I

2.4b Using Prim's algorithm, find the minimum spanning tree of the following network



Start with node 1.

1. Link node 1 and 2, weight is 20
2. Link node 2 and 3, weight is 7
3. Link node 2 and 6, weight is 9
4. Link node 6 and 9, weight is 4
5. Link node 6 and 5, weight is 11
6. Link node 5 and 7, weight is 12
7. Link node 5 and 8, weight is 15
8. Link node 5 and 4, weight is 23