Cosmic Ray Muon Laboratory

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Introduction

We have built an experiment to carry out some very fundamental measurements involving muons. The source of muons comes from cosmic rays. This brief write-up is meant to be a simple guide, not a cookbook or comprehensive document. It is based on experience with several classes. The physics that can be addressed with the setup is threefold and commonly depends on first collecting a sample of "stopped" muons; that is, cosmic ray muons that slow down and come to rest inside the experiment's fiducial (active) volume.

Cosmic ray muons evolve from a cascade of interactions that begins in the upper atmosphere, usually initiated by a very high energy stable particle, such as a proton, hitting the nuclei of the atmosphere. A picture of such a sequence is shown in Fig. 1a and a relative energy spectrum of the muons that arrive at roughly sea level is shown in Fig. 1b. It is these muons that we aim to catch. How many can we catch? A rule of thumb for muons passing through our setup is roughly $1\,\mathrm{cm}^{-2}\,\mathrm{min}^{-1}$. But that's not the number we can catch and stop. To stop them, we need massive energy absorbers to slow down the muons; we will only stop those that are already toward the lower part of the energy spectrum in Fig 1b. As the muons pass through the atmosphere, the building's roof and floors above, they lose energy. Some small fraction will be slow enough so that when they enter our apparatus, they are almost at rest. The absorbers (inert mass) in our experiment does the final slowing. Expect not more than a few per minute depending on how the setup is configured.

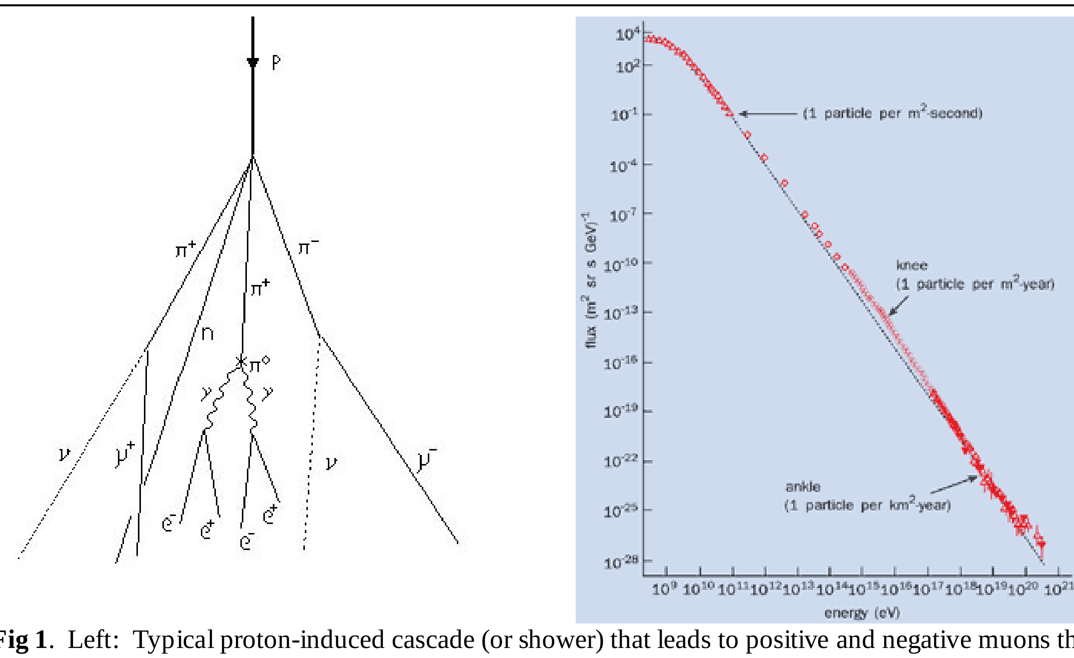

Fig 1. Left: Typical proton-induced cascade (or shower) that leads to positive and negative muons that can be observed at sea level. Right: Spectrum of primary energy of cosmic rays. The multi-GeV (i.e., 10^9 eV) range on the left side of the plot typically initiates the showers that result in multi-hundred MeV muons that can be stopped in an apparatus in an undergraduate laboratory.

Notice that both positive and negative muons are emitted in the cascade. The normal (positive) muon decay is

$$\mu^+ \to e^+ \nu_e \overline{\nu}_\mu,$$

where the only observable decay particle is the positron—the neutrinos interact with matter so feebly that they will "never" make a signal in our experiment.[1] The negative muon has the same sequence with the charges reversed and the "bar" for the antineutrino reversed. But the negative muon has other options besides decay that are discussed below.

The rest mass of the muon is 105.7 MeV (207 times greater than the electron mass) and it must be converted into the kinetic energy of the three emitted particles. A spectrum of the possible energy of the positron ranges from zero to half the muon mass; see Fig. 2.

Challenge: Calculate the decay kinematics and answer the question "why half" for the maximum energy of the positron. Here we can also assume that the neutrino masses are so tiny to be considered to be effectively massless.

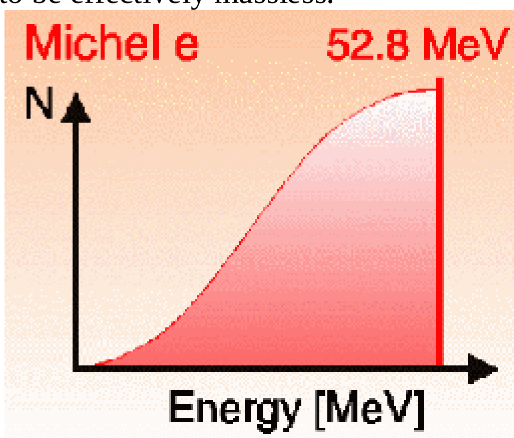

Fig. 2 Relative number of positrons emitted versus their kinetic energy in normal muon decay.

The experiment must be designed to observe not only the stopped muons, but also the decay positrons (or electrons) having energies as shown in Fig. 2. You will learn how the apparatus can do these things, but let's start with the physics.

Physics Experiments

Muon lifetime. The simplest experiment is measuring the lifetime of the positive muon to determine the Fermi constant, G_F (And, you won't find this experiment to be very simple.) The Fermi constant is a measure of the weak interaction strength, just as the fine-structure constant, α, is a measure of the electromagnetic strength. The muon lifetime τ_μ is ~2.197 μs and the relation to the Fermi constant is given by

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}(1+\delta),$$

[1] There are of course important neutrino experiments and they all involve huge detectors, usually placed deep underground to avoid backgrounds from cosmic rays. We can safely ignore the neutrinos here.

where δ represents a small quantum electrodynamics correction that is not important at the level of precision one can achieve in this laboratory (but it matters in a "real" experiment[2]).

The muon lifetime experiment involves establishing a "muon stop" signature, then opening up an electronic "gate" and waiting to observe a decay positron. The time between the stop and the decay is plotted and fit with an exponential form plus a background term. The "clock" used to determine the time difference between a muon stop and a decay positron has a full-scale range of 8 μs, about 3.6 lifetimes. The count rate of decays will have fallen by about a factor of 40 over the 8 μs. The "clock" is actually a time-to-digital converter (TDC), which must be calibrated to associate histogram bin numbers to real time units. (Note, we call the device to measure this the "slow TDC" in some of the notes and diagrams. It has one common "start" and 10 individual "stops" which are wired to the 10 different detectors used to detect the decay particle.)

Muon capture

When a positive muon stops in a material, it can only decay; it has no other disappearance channel. However, when a negative muon stops in a material, there is a competition between decay (at the same rate as the positive muon) and a processes known as "muon capture." Muon capture can be best described by the simple process on hydrogen,

$$\mu^- + p \rightarrow n + \nu_\mu ,$$

where we notice that the muon interacts with the proton, converting it to a neutron and emitting a muon-type neutrino. With a pure proton target (pure hydrogen gas for example), the capture process is only about 0.015% compared to the decay process. Therefore, in hydrogen, you wouldn't really notice any difference in the muon lifetime in our laboratory.[3] However, higher Z materials—such as the copper, aluminum or lead plates that can be inserted into the detector—have a much more rapid capture process, often far greater than the decay process. In fact, the capture rate increases roughly as Z^4 so the muon capture process is more likely to occur in a material like aluminum, $Z = 13$, compared to ordinary decay. What might you observe for negative muons stopping in aluminum if the experiment is designed to observe decays electrons? (See Fig. 3)

Because the cosmic ray muons include both positive and negative muons, at a ratio of about 1.2 : 1, we must consider what the lifetime histogram will look like based on a mixed-charge spectrum (and for one that we cannot tell the difference between positive and negative muon stops). Consider the observed lifetime τ_μ to be the reciprocal of the sum of rates that cause it to disappear. For positive muons, which can only decay, it's simply

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[2] Our UIUC MuLan experiment aims at a 1 ppm precision on the lifetime to get the Fermi constant to 0.5 ppm. See http://www.npl.uiuc.edu/exp/mulan/

[3] Our UIUC MuCap experiment has a goal to measure the capture rate in hydrogen to a precision of 1 percent. The physics is related to the induced pseudoscalar form factor of the nucleon. See http://www.npl.uiuc.edu/exp/mucapture/

$$\frac{1}{\tau_\mu} = \Gamma_{\text{decay}},$$

with $\Gamma_{\text{decay}} = 455\,\text{kHz}$. However, for negative muons

$$\frac{1}{\tau_\mu} = \Gamma_{\text{decay}} + \Gamma_{\text{capture}},$$

which must result in a shorter lifetime compared to the free lifetime. In fact, if the rate for capture in aluminum would be exactly 4 times greater than the decay, the observed lifetime would be

$$\frac{1}{\tau_\mu} = 455\,\text{kHz} + 4 \times 455\,\text{kHz} = 2275\,\text{kHz} = \frac{1}{0.44\,\mu\text{s}}.$$

You can use this type of analysis to determine the ratio of positive to negative muons in the sample of muons stopping in the apparatus and you can change the stopping medium to test the sensitivity to the capture rate law.

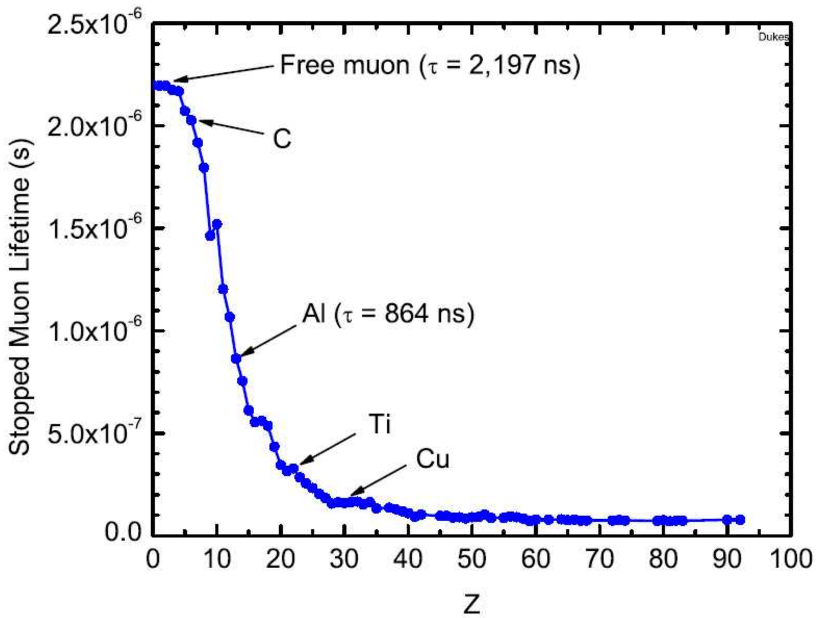


Fig 3. Plot of muon lifetime in different materials.

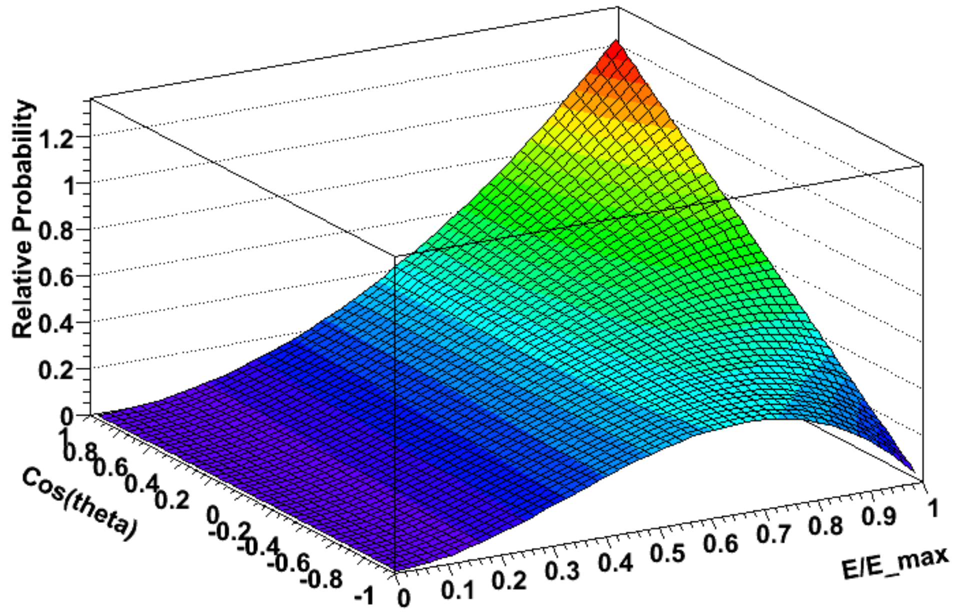


Fig. 4. Decay pattern for emitted positrons from muon decay with respect to the muon spin direction. The E/E_{max} axis is the energy of the outgoing positron, from 0 to 53 MeV. The $\cos(\theta)$ axis is the angle with respect to the spin. Thus, the "most probable" emission is in the direction of the spin and at the highest energies.

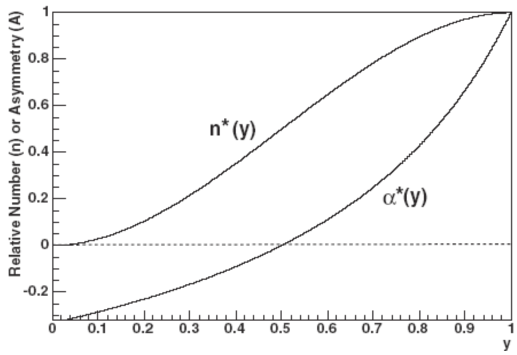
Muon magnetic moment

The muon is a "point-like" Dirac particle, just like its lighter cousin, the electron. Accordingly, it should have a gyromagnetic ratio of 2 (with corrections from quantum electrodynamics and other quantum loops making a small "anomalous" magnetic moment correction of about 1 part in one thousand to this number[4]. We will embark on a very challenging laboratory experiment to measure g. It requires (spin) polarized muons, a precession magnet, and an apparatus that can distinguish "which way" the decay positrons go. While this type of experiment can be carried out rather easily using a standard muon beam at many accelerators around the world, it will be a bit of a chore for us to do it with an ill-defined cosmic ray source. But, in this Phys403 class we have done it and measured the muon g factor to several percent precision.

Let's start by considering a very pure source of positive muons. Imagine a case where all muon spins pointed up. This is not uncommon as muons are born from pion decay to a muon and a neutrino with different back-to-back spin states because the pion has spin = 0

[4] The UIUC group has been involved in a high-precision measurement of the muon's anomalous magnetic moment. The g factor is now known to approximately 0.5 ppb (that's parts per billion). The physics of determining the anomalous part so precisely—to 0.54 ppm so far—is related to an equally precise prediction from the standard model of particle physics. The present comparison has experiment different from theory by 3.4 standard deviations, a possible hint of new physics beyond the standard model.

and the spin-1/2 neutrino is essentially massless and has an intrinsic left-handed helicity (like a spin direction). The muon is forced to be emitted with a complementary orientation to the spin. For positive muons, the spin is backwards with respect to its momentum. Of course it's a little more involved, but let's consider this for now. The distribution of positrons from the positive muon decay is shown schematically in Fig. 4 for all outgoing positron energies and for all angles. You can make your own plots (Fig 5) , using the following expressions:



$$\frac{dP(y,\theta^*)}{dy d\Omega} = (1/2\pi) n^*(y)[1 - \alpha^*(y)\cos\theta^*] \quad \text{with}$$

$$n^*(y) = y^2(3 - 2y) \quad \text{and}$$

$$\alpha^*(y) = \frac{q}{e}\frac{2y-1}{3-2y}.$$

Fig. 5. Probability in COM of decay (n) and asymmetry (α) versus energy; on the right are simple expressions to make these plots.

For certain materials—good insulators such as plastic, and good conductors, such as aluminum or copper—the polarization (spin direction) of the muon will remain unchanged as the muon slows down and stops in the material. In our ideal experiment, the stopped muons will have their spins pointing up. A transverse magnetic field, applied to the region where the muons are stopped, will then cause the spin to rotate end over end about the axis of the magnetic field. The rotation frequency is given by[5]

$$\omega = \frac{geB}{2m_\mu c}$$

which leads to a practical and useful number that a 1 μs rotation period is caused by a 75 G field. You can plug in your own numbers and check this. For the magnet in the experiment, the field is about 55 G and we have to prepare and measure this carefully. What is the expected period? The idea in the experiment is to observe the muon rotation period and use the period to determine g – the rest of the factors above are well-known constants. Figure 6 gives an example from the Fall 2008 class for a plot of the difference between Up-going and Down-going decay positrons versus time. The period is evident and from that a good measure of g was obtained (with a concurrent measure of B).

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[5] Hints to do this Use Bohr magneton for the muon. An hbar is in the denominator. Use conversion (hbar-c) = 200 MeVfm. Use c = 3 x 10^8 m/s. Bohr magneton (electron) is 5.788 x 10^{-11} MeV T^{-1}.

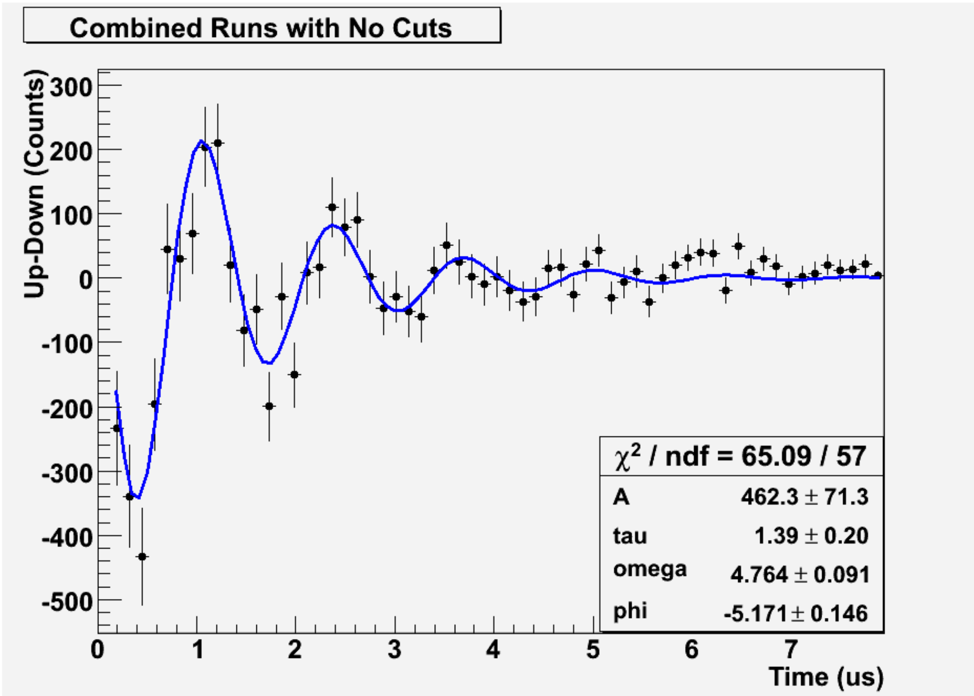

Fig. 6. Up – Down decays versus time, after normalization, giving a period that is related to the muon g factor. Data from Fall 2008 class.

Experimental Setup

At the end of this document, we show a Table of the approximate evolution of the experiment over the course of a semester. In each 2-week period students will advance the setup and obtain different sets of data.

The experiment is shown in Fig. 7. It involves 14 thin plastic scintillators, each viewed on both ends by photomultiplier tubes (i.e., 28 PMTs total). When a charged particle passes through a scintillator (muon or positron), it deposits about 2 MeV per cm of travel. This energy deposition is—by processes that will be explained later—converted into light, then into an electronic "pulse." A typical pulse has a width of 10 ns or so and an amplitude, once the PMT voltage is set, of about 300 – 400 mV. Your first step will be to calibrate the PMTs using "straight-through" muons (that is, muons that don't stop in the apparatus). Each PMT should respond similarly. The energy deposition is statistical and it follows a so-called "Landau distribution," shown in Fig. 8. Here's a clip from extracted from Wikipedia containing the definition,

> The **Landau distribution** is a probability distribution … For numerical purposes it is more convenient to use the following equivalent form of the integral,
>
> $$p(x) = \frac{1}{\pi}\int_0^\infty e^{-t\log t - xt}\sin(\pi t)\,dt.$$
>
> where log refers to the logarithm base e. The Landau distribution is used in physics to describe the fluctuations in the energy loss of a charged particle passing through a thin layer of matter. (Wikipedia)

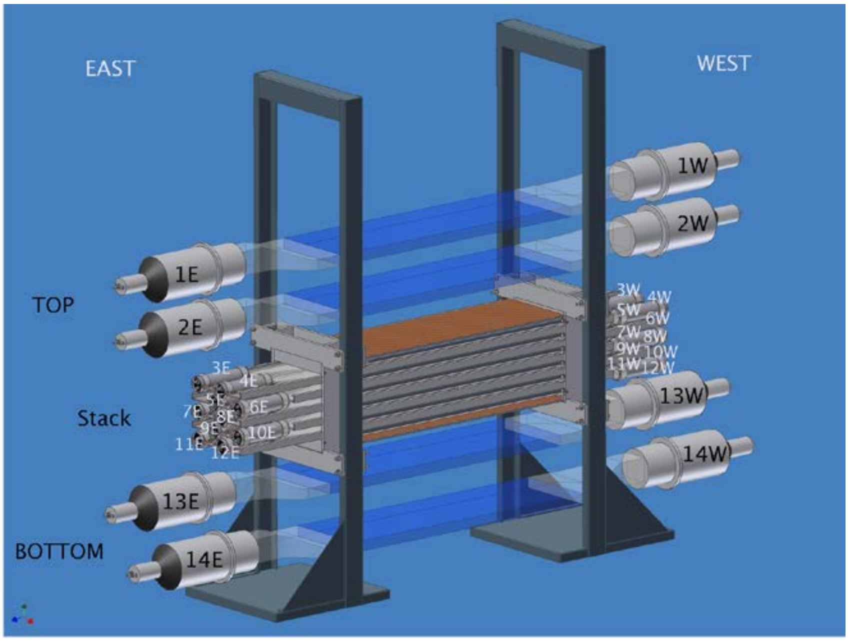

Fig. 7 This figure depicts the layout of the scintillators and the PMTs. The Top and Bottom scintillators were used as a trigger to activate the stack. The stack is housed inside a box magnet.

Each PMT signal is split into two equal parts. One is integrated to measure the energy deposition (i.e., was it a good pulse or a runt pulse from some noise; there is little to no real information in the amount of energy deposited here. The thin scintillator does not measure the energy of the muon or of the positron, only that a charged particle passed through the detector. It gives (largely) the same response for a muon passing through and a decay positron passing through.) The second output is discriminated, meaning it is turned into a standard electronic logic signal. This logic signal is used to establish coincidences, trigger, and time of stops or decays. A logic diagram is shown in Fig. 9. **Study it**. We will discuss the employed units and how they work in class. A few of you will have to rebuild the logic based on this diagram. You can modify the diagram and make your own if you like. I'm always up for an improved setup.

Basic Tasks to Build the Experiment

The experiment has been largely taken down so that each class can start fresh and rebuild the electronics and test the system. Improvements have been made. With all the physical handling, the detectors should be checked for light-tight wrapping (use an analog scope for this and a flashlight). There may be small tears in the light-tight wrapping of the scintillators that must be repaired. The electronics has also been used hard and some channels will be faulty. A lot of our time will be spent debugging electronics and cables – this is a real, and important learning experience so embrace it.

The electronics employs standard NIM units, including leading-edge discriminators, coincidence units, and counting units. The pulses from the detectors are integrated by CAMAC analog-to-digital converters (ADCs) and the time that a detector fires is recorded by two different time-to-digital (TDC) recorders. NIM units are only supplied power by the crate. CAMAC units are supplied power and a USB interface to our linux-based data acquisition system. CAMAC units can be "read out" by the computer. The principle of the TDC is that it give the time with respect to a "trigger" and you will establish what the "trigger" is using the NIM logic. A typical trigger might be a hit in the top several layers (above the stack that is housed in the magnet) and no hit by any counters below the stack. That could indicate (inefficiently) that a muon has arrived and **might** have stopped in our setup. That's already interesting enough for our system to record what happened. Later we can make cuts to enhance the probability of a muon stop. Here is what is recorded for all events.

1) The pulse integration in all of the scintillators (except the bottom two).
2) Some "fast TDC" signals we will initially ignore, but later define based on your interests. They record the time from the trigger to something happening in the next 100 ns (which is fast).
3) The trigger starts a "slow" TDC, which looks for hits in the stack counters over the next 8 microseconds (about 3.6 muon lifetimes). When a counter fires during that interval, the TDC records its time. If a counter does not fire, the TDC records a "time out" (a maximum time). You will have to calibrate channels to time with a special procedure that will be explained in the lab. But, basically, 4000 channels is very close to 8 microseconds.

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The data acquisition (DAQ) system is custom built by a former Yakov Kulinich, a former Phys 403 TA. It is run on a linux system and records the data listed above as histograms in a .root file. Each semester, students will start out with a program that makes the data readily available and establishes a basic analysis loop. You must understand the physics and the logic of the experiment to write C++ analysis code that allow us to get the lifetime, magnetic moment and make other checks. The programming is relatively straight forward, and there are patterns in the setup that you can exploit to make the code simple.

How we start up the semester: (these tasks will take some weeks … many weeks)

1. Test each PMT and scintillator for light-tightness
2. Setup a cosmic ray "straight-through" trigger and collect energy spectrum for each PMT.
    a. Adjust high voltages to get the most-probable value (MPV) of the fit at about channel 300 of the ADC. ROOT histograms will be made but can currently only be viewed offline. Using ROOT, a fit to a Landau function can be performed over a limited range of the ADC histograms. (See Figure 5). The "MPV" (most probable value; think "peak") can be obtained. We would like to set all of these to a relatively common value (e.g, 400 on the ADC scale). That allows us to set the discriminators to a common threshold for all counters.
    b. Iterate
3. Setup the triggers for step #2, and the software for the analysis (clearly to be done at about the same time)

4. Make a stopped muon trigger and start a lifetime run.
    a. We can change the stopping material here.
    b. What materials do you want in the stack? Let's start with lead (Z = 82). Why???
    c. Try the same with only aluminum. Can you see the difference?
5. Assemble the box magnet and map it. Instructions will be given
6. Put in the box magnet and prepare a long run for the magnetic moment.
7. Write analysis code to find the stopping layer of the captured muon.
8. Write analysis code to sort the up and down decays of the muon.
9. Work on data fitting to extract the oscillation frequency.

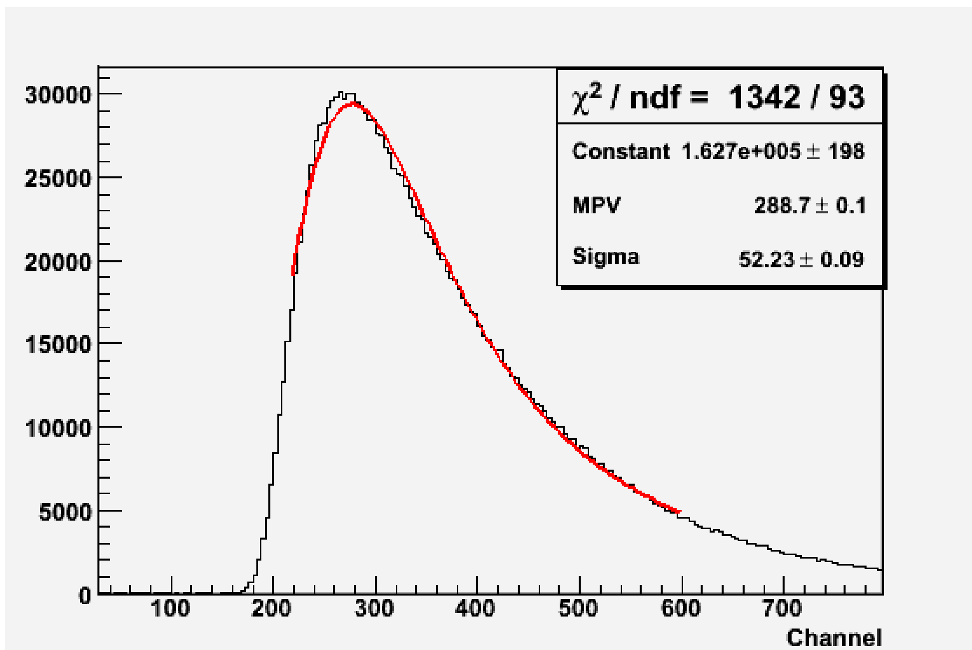

Figure 8. Example of an ADC distribution for one counter and a fit using a Landau function. It's not a great fit, but give an idea of what we are trying to calibrate.

Table of approximate flow of experiments in a given Semester

Cycle	Tasks and Goals	Comment
C1-1	Wire up the basic trigger electronics Wire up the CAMAC readout of the TDCs and ADCs Start the DAQ and take a simple lifetime spectrum Use Pb sheets as stopping media	You will need to set the HV values to try to balance the gains of the PMTs using a straight-thru muon trigger. You should obtain a free muon lifetime in a couple days of running (Remove the Pb sheets)
C1-2	"Perfect" setup and fine tune electronics Document the performance of each counter Write software to see the stopping distribution by layer Get a high-quality, high-statistics muon lifetime with Pb. Calculate GF (Fit the lifetime plot)	This will depend on previous group's stopping point but usually there will be lots to debug and fine tune once the basic setup is built. Use Vcal.C to get MPV values for all PMTs simultaneously. Develop a voltage to gain curve and select the appropriate voltage for each.
C2-1	Change to Al, Graphite, and possibly Cu plates and make several runs, concentrating on the "capture" of negative muons. Learn to do a more complicated fit with ROOT	Goal here is capture rate vs Z. Take and re-analyze the Pb data from previous group. Least important is the graphite data.
C2-2	Time to build and map the magnet. On the one hand, this is historically tricky to get right. Do careful planning here. We need the variance of the B field and an "absolute" calibration we can use later. You will need temp. stability, so be patient. In parallel, calibrate the TDC using a special circuit.	Here, we are preparing for the muon magnetic moment run. The goal here depends on where we are to date. To make the muon magnetic moment work, we need all layers and counters to work consistently. The job here is to finalize that study before we "wrap up" the counters in the box magnet.
C3-1	Install magnet, install Hall probe, use Copper plates for stopping media. Write code to define "up" and "down" decays. Begin a long run; Check it!!!	You should obtain preliminary magnetic moment data here but mostly things should be left running well. You can also get a good muon lifetime data set in Copper
C3-2	Write analysis software to get the magnetic moment. Use ADCs to cut on the entrance	This is the "money" data set so work it as hard as possible to extract the best information.

	location in the longitudinal coordinate to better choose a uniform magnetic field. Work with instructor to obtain a good fit function	

A Note on the Electronics

If you follow the logic diagram below precisely, the experiment still may not work. Triggers and gates need to be tuned to work properly. Trigger thresholds must be set to a reasonable level. The widths of the output pulses must be set wide enough that small differences in timing don't prevent corresponding pulses from overlapping (ex. As inputs to an AND gate) but must be narrow enough that random noise above trigger threshold is very unlikely to overlap with another channel's random noise.

Gates must also be understood and tuned based on the equipment. In general, a gate is a logic high that occurs for the duration of interest. It can be thought of like a fence gate where, on logic high the gate is opened, and things can pass through. As soon as the gate input returns to logic low the gate is closed, and nothing can pass again. In our setup, the veto of scintillator layers 13 and 14 behave like an inverted gate. Figure 9 shows an oscilloscope trace where the green pulse on the bottom is the veto "gate". The veto means that on logic high the gate is closed and nothing (our other three triggers) is allowed to pass, but during logic low the gate is open. For this reason, we pass our pulses through a variable delay module to ensure they fall within the veto.

For some equipment the "gate" may behave more like a start trigger, where the module begins a function as soon as it sees the leading edge of a logic high and ends after a set period.

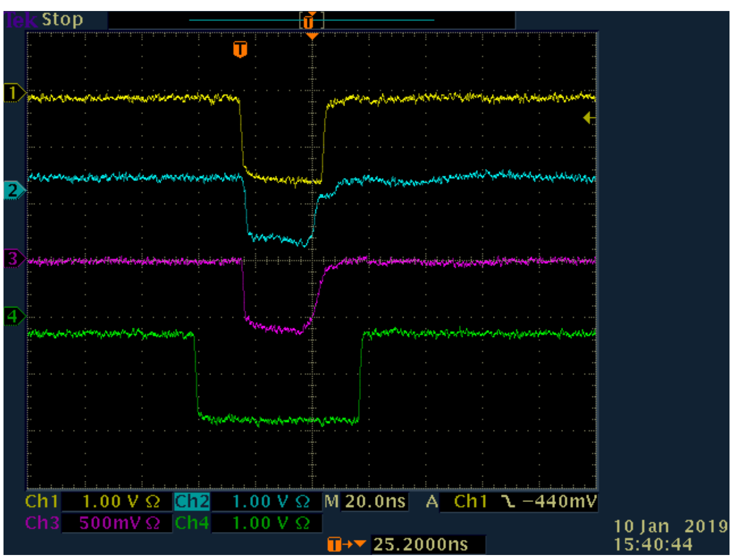

Figure 9. An oscilloscope trace from scintillators 1,2 and 3 above the veto pulse from scintillators 13 and 14

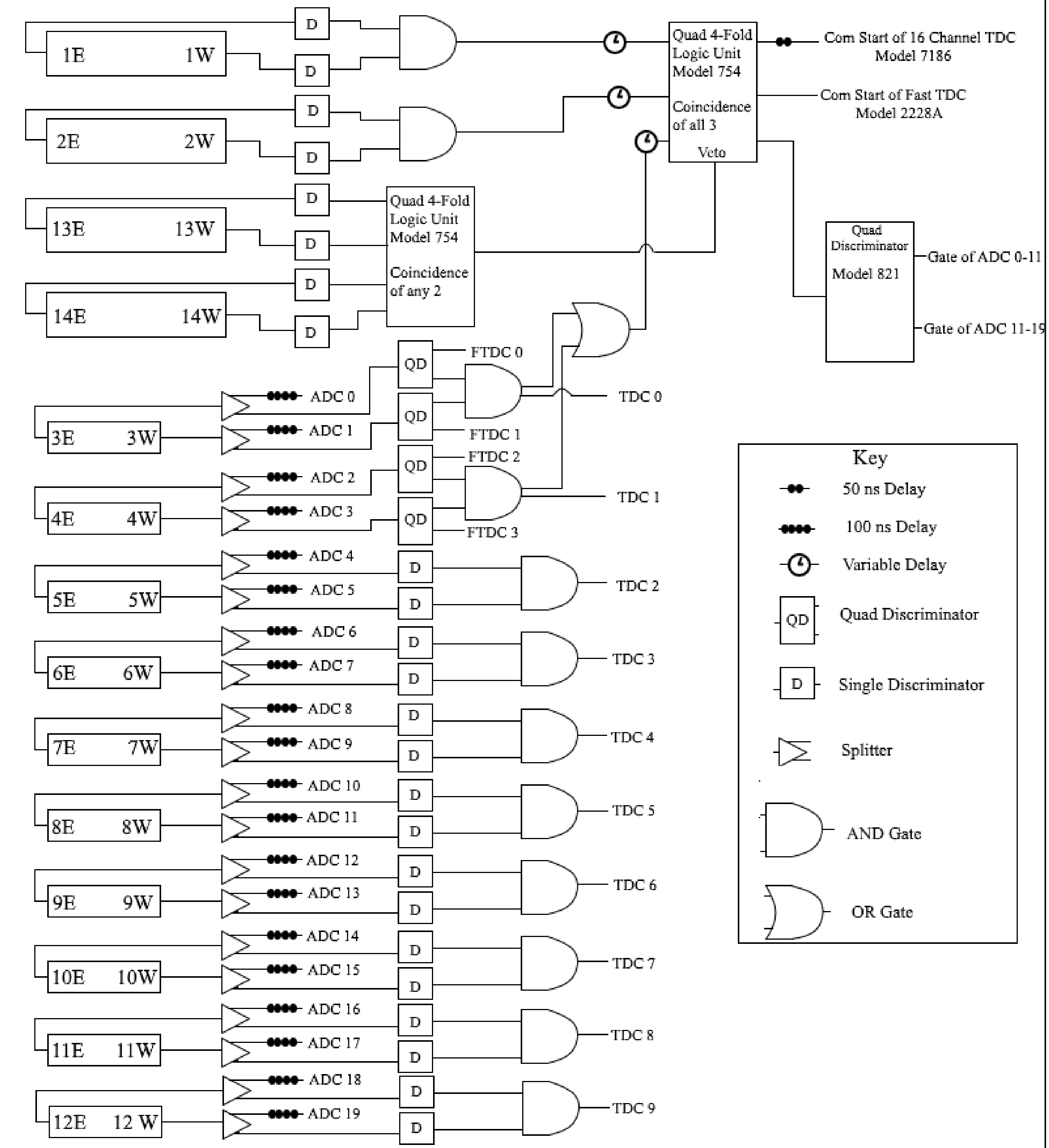

Fig. 9. Logic diagram of the cosmic ray muon experiment in Phys. 403

MuonDAQ installation and run instructions

1. Open a terminal in the linux machine (esb-5103-linux-02.ews.illinois.edu)
2. Execute the following commands in the terminal (copy-paste them and press enter)
    a. module load root
    b. git clone http://github.com/YakovKulinich/MuonLab
    c. cd MuonLab
    d. source setup.sh
3. Now MuonDAQ has been installed
    a. cd MyDaq/run
    b. ./muonDAQ
4. A graph should appear with a text box and buttons that say start and stop
    a. Enter a name for the data file in the text box
    b. Press start to begin taking data
    c. Go get some tea or coffee
    d. Press stop to end data taking
5. The run should finalize and output a "yourfilename".root in ~/MuonLab/MyDaq/run

If any of the steps listed here fail, consult the readme contained in ~/MuonLab/MyDaq for troubleshooting instructions and ask the instructor/TA for help. The instructor/TA may need to run some commands as the root user.