Creation and Measurement of a Uniform Magnetic Field

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ABSTRACT

To create a uniform magnetic field we assembled a magnet box with solenoids on top and bottom and 2 iron walls on the sides. The uniformity of magnetic field is under the rule of Maxwell's Equations. By using stepper motor system, we examined the uniformity of magnetic field at various heights, in accordance with the position where material plates are placed for muon magnetic moment measurement experiment within certain amount of deviation. Some suggested uniform region is given for the convenience of other groups.

INTRODUCTION

A uniform magnetic field is useful in many experiments. To create a uniform magnetic field, we adopt the design that 2 flat solenoids and 2 iron panels form a box shape. The magnetic field inside the box can be calculated using Maxwell's equations.

1 Magnetic Field inside the Box

The magnetic field inside the box is a result of the current in the wires on the top and bottom. These wires create very narrow solenoids. Normally, the magnetic field outside a solenoid is approximately zero away from the edges. The presence of two iron walls makes the magnetic field more complicated. Two of Maxwell's equations can be combined to determine the magnetic field inside the box. Ampere's law, written as

$$\oint \vec{H} \cdot d\vec{l} = I_{enc} \tag{1}$$

states that the path integral of the magnetic field divided by the permittivity for a closed loop equals the current enclosed by that loop. Using the loop shown in FIGURE 1 on next page, where the red walls contain the coils, Equation 1 can be written as

$$\frac{2\mu_0}{\mu} B_{iron} + B_{gap} = \frac{\mu_o NI}{L} \tag{2}$$

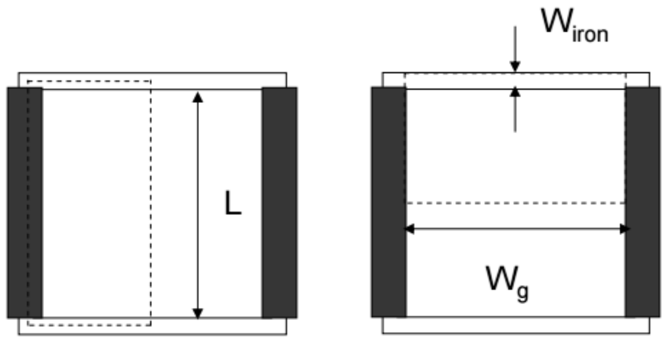


FIGURE 1. (Left) The loop used in Ampere's Law, dark part are solenoids. (Right) The 2D projection of closed Gauss surface used in Gauss's Law. W_{iron} is the width of iron walls and W_g is the total height of the magnet. (The graph is rotated 90˚ compared with actual setup.)

This equation cannot be solved on its own. Gauss's law, which states that the magnetic flux through a surface is zero, can be used to simplify Equation 2.

$$\oint \vec{B} \cdot d\vec{s} = 0 \tag{3}$$

The surface used is outlined in FIGURE 1. Equation 3 then simplifies to

$$2B_{iron}W_{iron} = B_{gap}W_{gap} \tag{4}$$

$$B_{iron} = \frac{W_{gap}}{2W_{iron}} B_{gap} \tag{5}$$

Equation 5 can be used to solve Equation 2 in terms of the magnetic field inside the box.

$$\left(1 + \frac{\mu_o W_{gap}}{\mu W_{iron}}\right) B_{gap} = \frac{\mu_o NI}{L} \tag{6}$$

The ratio of the permittivity is so small that equation 6 can be approximates as

$$B_{gap} = \frac{\mu_o NI}{L} \tag{7}$$

This equation proves that the magnetic field inside the box can be approximated as constant. It also has known values that can be plugged in to guess the magnetic field beforehand. The theoretical values for the magnetic field do not vary with height or width, although the actual values may.

2 Use in Muon Experiment [1]

The purpose of the magnet box is to be used in muon capture experiments. The muons in question are called cosmic ray muons. When certain high energy particles contact the Earth's atmosphere, they produce pions. Pions have a short lifetime, and decay into muons, which can be positive or negative. These muons will be caught by a dense material, such as copper, then decay. The result of these decays are measured at either end of the box. Having a magnetic field present causes the positive muons to precess before they decay. Negative muons do not precess, so the difference in the decays between the two can be used to determine the precession frequency. That can then be used to calculate the magnetic moment of the muons. The interaction between a known magnetic field and the magnetic moment of the muons is finally used to determine the lifetime of muons.

The diagrams showing our magnet's position in muon experiment setup are included in Appendix.

PROCEDURES

1 Assembling of the Magnet

The magnet box is shown in FIGURE 7. The top and bottom panels are wrapped many times with copper wire through which a current is passed. The two sides are iron. The size of the box is displayed in the diagram in Appendix. The resistance of the top coil is 3.6 **Ω**, and the bottom 3.3 **Ω**. Measurements were done with 33V and 18.5A being supplied to the coils. The temperature of the coils was kept at 104 ± 2 ˚F.

2 Setup of Stepper Motor System

Measurements of the magnetic field inside the box were conducted using a gauss meter connected to a stepper motor. The stepper motor has a range of 9 inches (22.86 cm) in two horizontal axis. The motor was kept in place, and the gauss meter adjusted in height and depth on the rail of the stepper motor system. A brass tube was used to hold the gauss meter, which allowed for close to 40cm of range from the stepper motor into the box. We also applied a second gauss meter to monitor the magnetic field at the center height of the iron wall. This gauss meter gives around 58 G throughout the whole measurement.

3 Detailed Procedure

The symmetry of the field inside the box allowed us to only take measurements for a quarter of a plane and extrapolate. The range of the stepper motor required each plane to be scanned in three regions. The first region started 1.6cm outside the box and 1.4cm from the inner north wall to avoid collisions. It used 30 steps of 0.3 inches into the box, and 20 steps of 0.4 inches across the width of the box. Regions 2 and 3 used 20 steps of 0.4 inches for both dimensions. The gauss meter was moved between scans so that the first inch of scans for regions 2 and 3 overlapped with the data from the previous scan.

For 145 mm, 155 mm, 191 mm, 196 mm and 240 mm scans, we move the starting point one step further to avoid collision with the tube where a fixed gauss meter is placed. This affects the graphs mentioned above by narrowing the range in width direction compared to other normal graphs.

DATA ANALYSIS

After collecting all magnetic field data in an area of 8 inches in width, 23 inches in depth at 8 different heights, we are able to visualize the magnetic field inside the magnet and extrapolate the region where magnetic field can be considered uniform within certain amount of error.

1 Visualization of Magnetic Field inside the Magnet

The area we measured for magnetic field is only 1/4 of the whole area at a certain height in the magnet due to the assumption that magnetic field is symmetric in width and depth direction. To get the full picture of the magnetic field, we mirror our collected data and plot it in Origin. We calibrate the width and depth axis to show the actual position of magnetic field inside our magnet. An example of 3D magnetic field graph at 103 mm is displayed as follows.

Magnetic field at other heights can also be visualized in this way. The graphs of magnetic field at 93 mm, 145 mm, 155 mm, 191 mm, 196 mm, 240 mm and 250 mm are included in the Appendix.

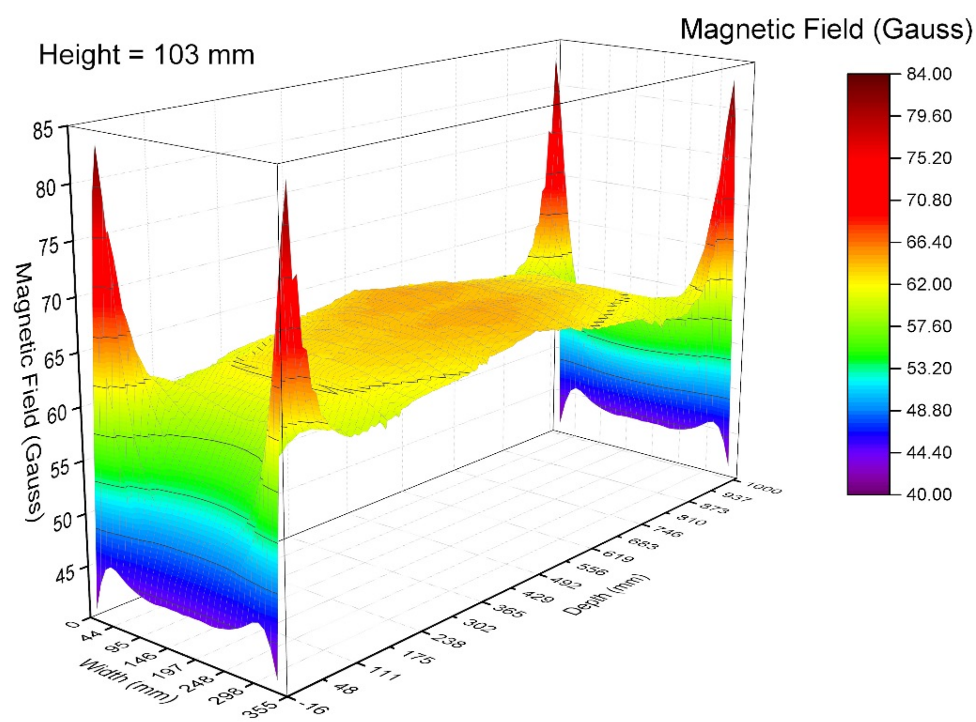


FIGURE 2. 3D Magnetic field for full range of the magnet at 103 mm, layer 4. Evidently, our magnet created a region where magnetic field is relatively uniform at around 65 G.

2 Magnetic Field in Width Direction

By roughly selecting a uniform magnetic field region, we calculated the average magnetic field in width direction at 103 mm height. The selected region is between 197.36 mm and 796.8 mm in depth direction indicated in the graph below.

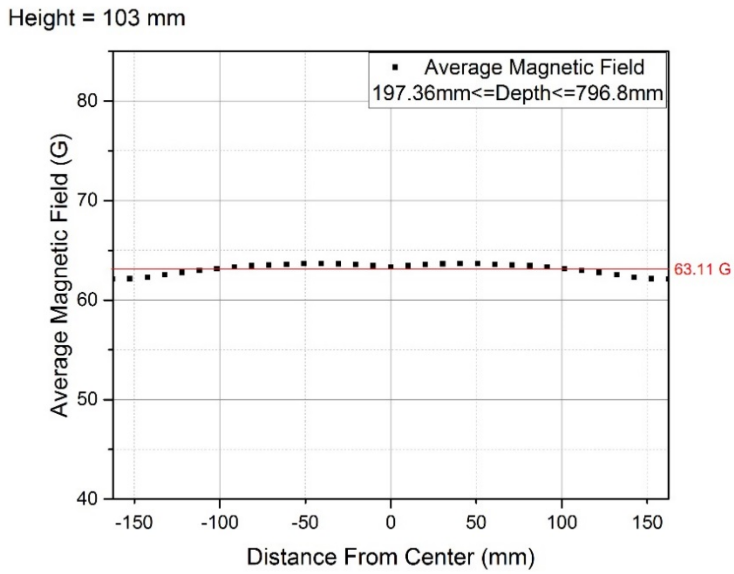


FIGURE 3. Average magnetic field in width direction at 103 mm between 197.36 mm and 796.8 mm in depth direction. The magnetic field is approximately uniform in width direction.

The average magnetic field in width direction at 103 mm is 63.11±0.55 G, with 0.87% deviation. The plot is basically the same for different heights we measured. Therefore, in a uniform region at each height, the average magnetic field is approximately uniform in width direction.

3 Magnetic Field in Depth Direction (Determine the Range of Uniform Magnetic Field)

Since the magnetic field can be considered as uniform field, we average the field in width direction and plot the magnetic field in depth direction. As an example, the magnetic field in depth direction at 103 mm is displayed as follows.

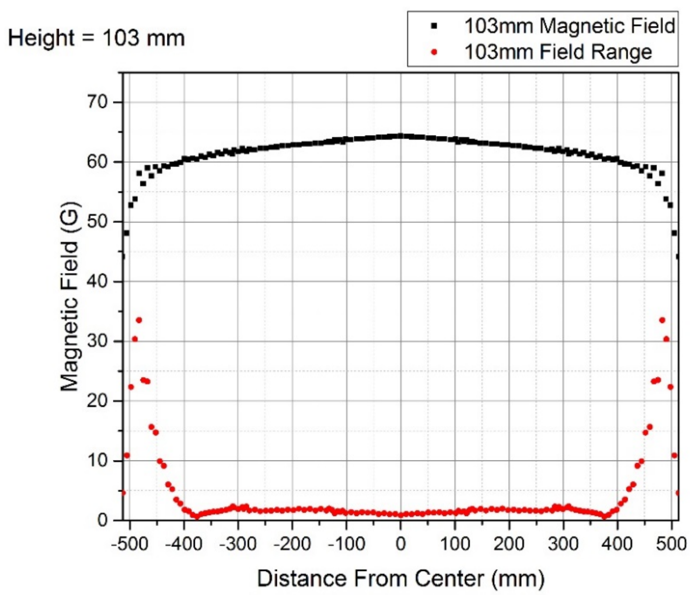


FIGURE 4. Magnetic field in depth direction at 103 mm is in black. Red scatter plot shows the range of magnetic field in width direction to indicate the variance.

The magnetic field plot in depth direction is the most important tool to determine the range of uniform field. The black scatter plot shows the magnetic field with respect to the depth position of our magnet. At the center, the magnetic field is relatively uniform. But at both ends, which are the openings of our magnet, the magnetic field is evidently affected by edge effect. At places near the iron wall at both ends, the field is considerably high while at center of the openings it is much lower. Thus we utilize a statistical quantity, range, to determine the area where edge effect dominates and the result is satisfying. In the red scatter plot, we can find the region where the magnetic field has a small range and thus a relatively uniform distribution in width direction.

To determine the region in a more accurate way, we calculate the standard deviation in every measured region centered at the middle in the depth direction of the magnet. Here we translate the depth coordinates and place the origin at the middle of magnet depth. We also average the magnetic field for each height since we only want to find one range for all of the layers. Then we calculate the average field over regions centered at the center in depth direction. The percentage deviation is given by standard deviation dividing the average magnetic field in such region at different depths.

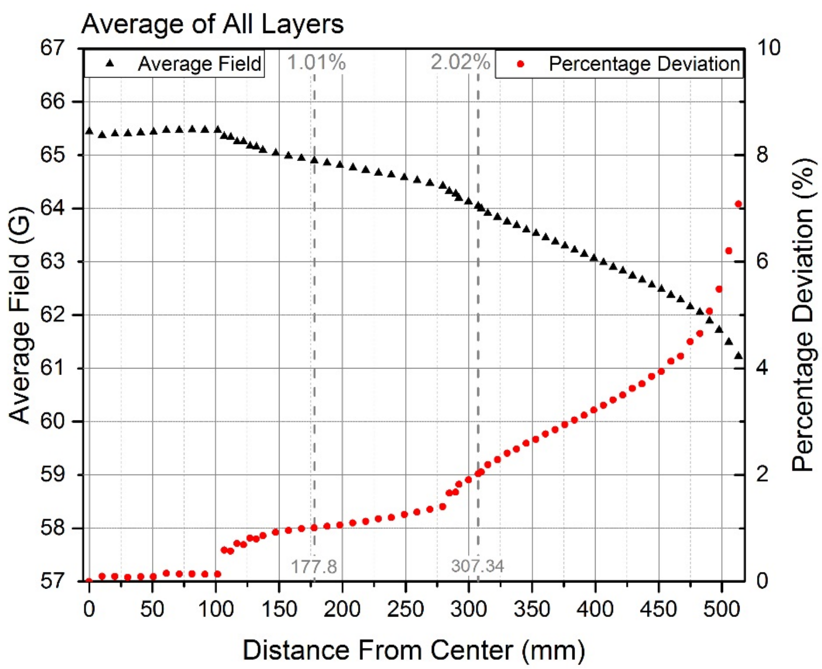


FIGURE 5. Average Field and percentage deviation of magnetic field in depth direction. For example, in a 355.6 mm region (177.8 mm from center) located at the center of depth direction of the magnet, the measured magnetic fields at all data points have a percentage deviation of 1.01% and the average field in this region is 64.90 G.

Percentage deviation increases as the selected region is longer. This is expected by the theory since the closer to the middle of the magnetic the more uniform the field will be. A smaller percentage deviation suggests a more uniform magnetic field in a certain region, and also means a better estimation of magnetic field.

With certain amount of tolerance of error, one can easily find the range of the region from the graph above, where magnetic field can be regarded as uniform. The Distance From Center coordinate can be easily converted into the real position in the depth direction using the conversion table in the Appendix.

We also display the average magnetic field and percentage deviation in a table in the Appendix for the convenience of following groups.

4 Magnetic Field in Height Direction

In total we measure the magnetic field for four layers, with measurements at two heights in each layer. To obtain the average magnetic field for each layer, we average the two heights and conduct similar manipulation as what we have done for depth direction in section 3. The value for magnetic field can be read in the table in Appendix. The result is as follows.

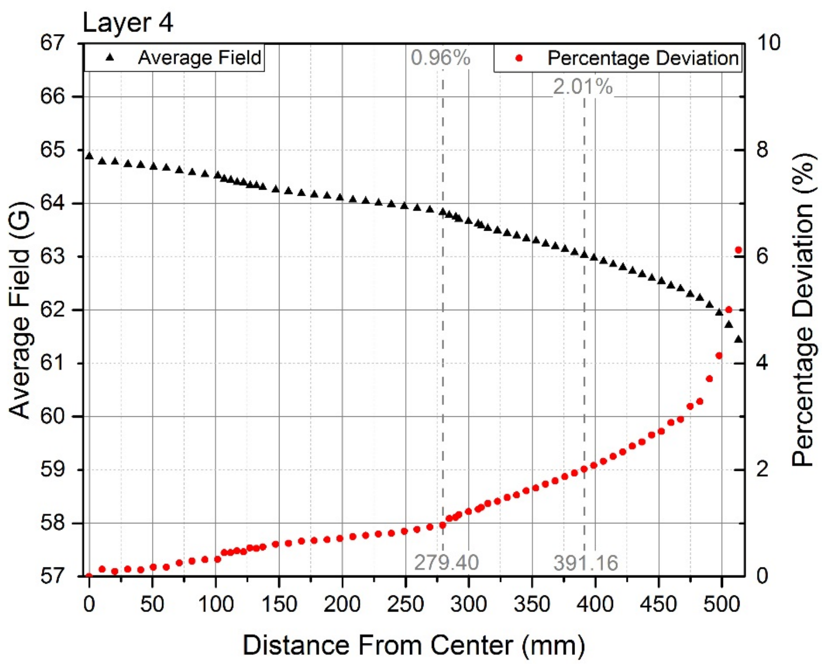


FIGURE 6. Average Field and percentage deviation of magnetic field in depth direction for Layer 4. For example, in a 558.80 mm region (279.40 mm from center) located at the center of depth direction of the magnet, the measured magnetic fields at all data points have a percentage deviation of 0.96% and the average field in this region is 63.84 G.

We apply similar analyzing technique to other three layers and attach other graphs and tables in the Appendix.

5 Some Suggested Regions (Indicated in Graphs by Grey Vertical Lines)

(1) Averaging all layers

Percentage Deviation	Region	Magnetic Field Strength
1.01%	319.28 mm ~ 674.88 mm	64.90 ± 0.65 G
2.02%	189.74 mm ~ 804.42 mm	64.05 ± 1.29 G

(2) Layer 1

Percentage Deviation	Region	Magnetic Field Strength
1.00%	268.48 mm ~ 725.68 mm	65.14 ± 0.65 G
2.06%	204.98 mm ~ 789.18 mm	64.59 ± 1.33 G

(3) Layer 2

Percentage Deviation	Region	Magnetic Field Strength
1.01%	339.60 mm ~ 654.56 mm	65.51 ± 0.66 G
2.01%	204.98 mm ~ 789.18 mm	64.63 ± 1.30 G

(4) Layer 3

Percentage Deviation	Region	Magnetic Field Strength
1.05%	375.16 mm ~ 619.00 mm	65.23 ± 0.69 G
1.92%	217.68 mm ~ 776.48 mm	64.89 ± 0.99 G

(5) Layer 4

Percentage Deviation	Region	Magnetic Field Strength
0.96%	217.68 mm ~ 776.48 mm	63.84 ± 0.61 G
2.01%	105.92 mm ~ 888.24 mm	63.03 ± 1.27 G

TABLE 1. Some suggest uniform regions. Two percentage deviations and the magnetic field strength for each region is given.

DISCUSSION

1 Edge Effect

Due to the limited size of our magnet in the depth direction, the magnetic field inside the magnet is substantially influenced by edge effect. The edge effect consists of two parts: the openings on both ends and the iron wall on the sides.

The openings greatly increase the magnetic field at rims of two ends of the magnet and lower it at center of two ends. In the 3D magnetic field graph, 4 peaks at corners and 2 valleys on both ends exhibit the influence of edge effect by the openings.

The iron walls on the other way lower the field in inner region of the magnet. In the magnetic field in width direction graph, the field strength near the sides is slightly lower than that at the center. But this effect is minor compared with the previous one.

2 Discontinuity in Overlapping Regions

In several graphs, there are discontinuity in overlapping regions. These step-like discontinuities exactly correspond to the overlapping regions in our measurement.

We can provide several possibilities about why these discontinuities occur. Due to limitation in measurement apparatus, we cannot investigate these systematic errors in a deeper way.

First, the gauss meter, along with the brass pipe which holds it, was obviously shaking while the stepper motor system was taking the data. It could only be solved by using a stiffer pipe.

Second, as we extended the pipe into the magnet, it began to bend slightly, this also influence the measurement.

Third, when installing the gauss meter onto the pipe before each measurement, it was hard to tell if the gauss meter was pointing forward horizontally.

These possibilities in total lead to the discontinuities in magnetic field measurement.

CONCLUSION

By measuring the magnetic field at various height in four layers, we obtain the region where magnetic field can be considered as uniform field within certain amount of deviation. The analysis gives a relatively uniform distribution of magnetic field with a few percent of deviation. The regions within 1% deviation are typically around 300 mm in length, while those within 2% deviation 600 mm in length. Meanwhile, the following groups can pick an arbitrary possible percentage deviation by referring to the table for certain layer to find a specific range of region.

REFERENCE

[1] Amsler, C. The Determination of the Muon Magnetic Moment from Cosmic Rays. Swiss Federal Institute of Technology. Zurich, 1974.

APPENDIX

1 The Magnet Box and Stepper Motor System

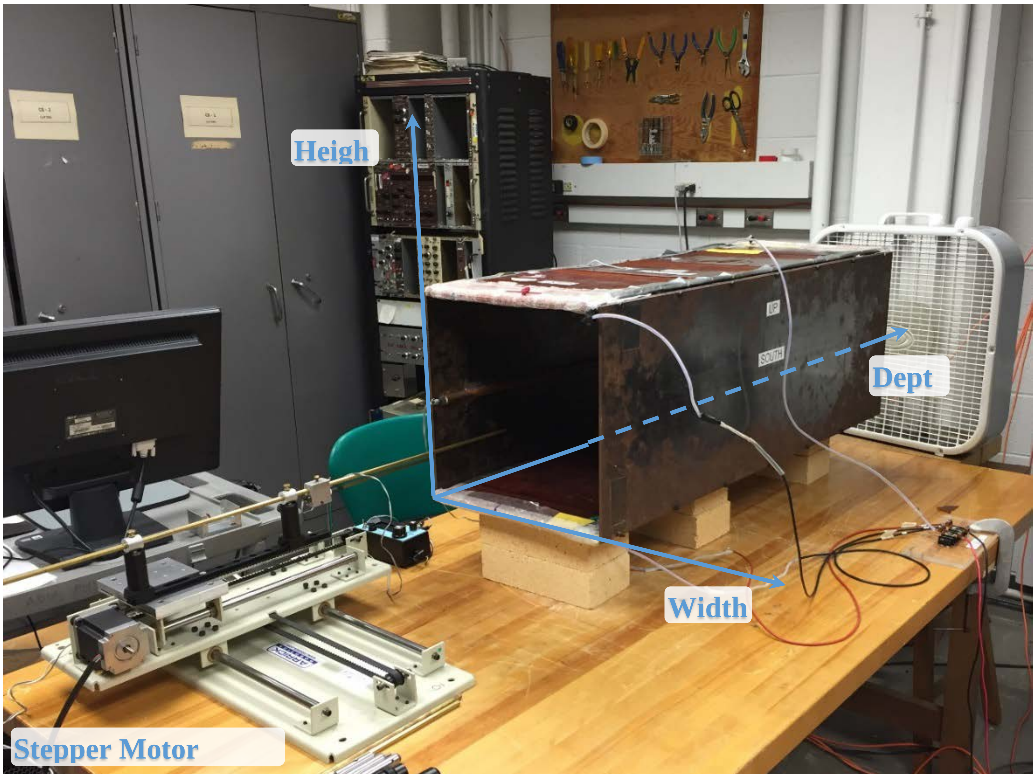


FIGURE 7. The structure of the magnet box and the stepper motor system for measuring the magnetic field. The coordinate is defined in such way for our experiment and analysis.

2 Diagrams of the Experiment Apparatus

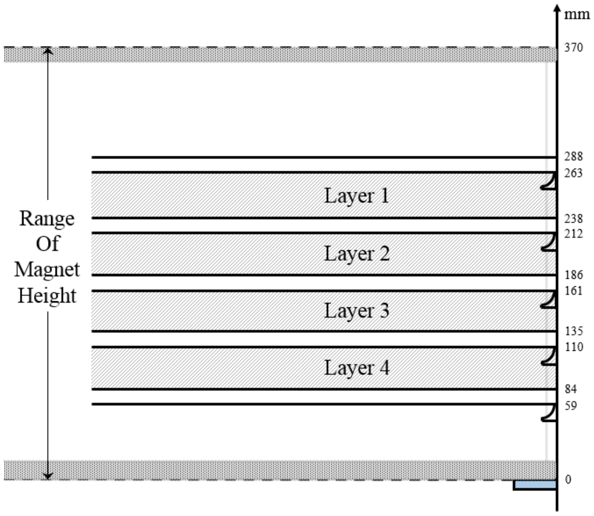


FIGURE 8. The diagram shows a part of the muon experiment apparatus where our magnet is placed in. Grey translucent graph indicates our magnet. Four filled layers are the spacing where heavy materials will be located.

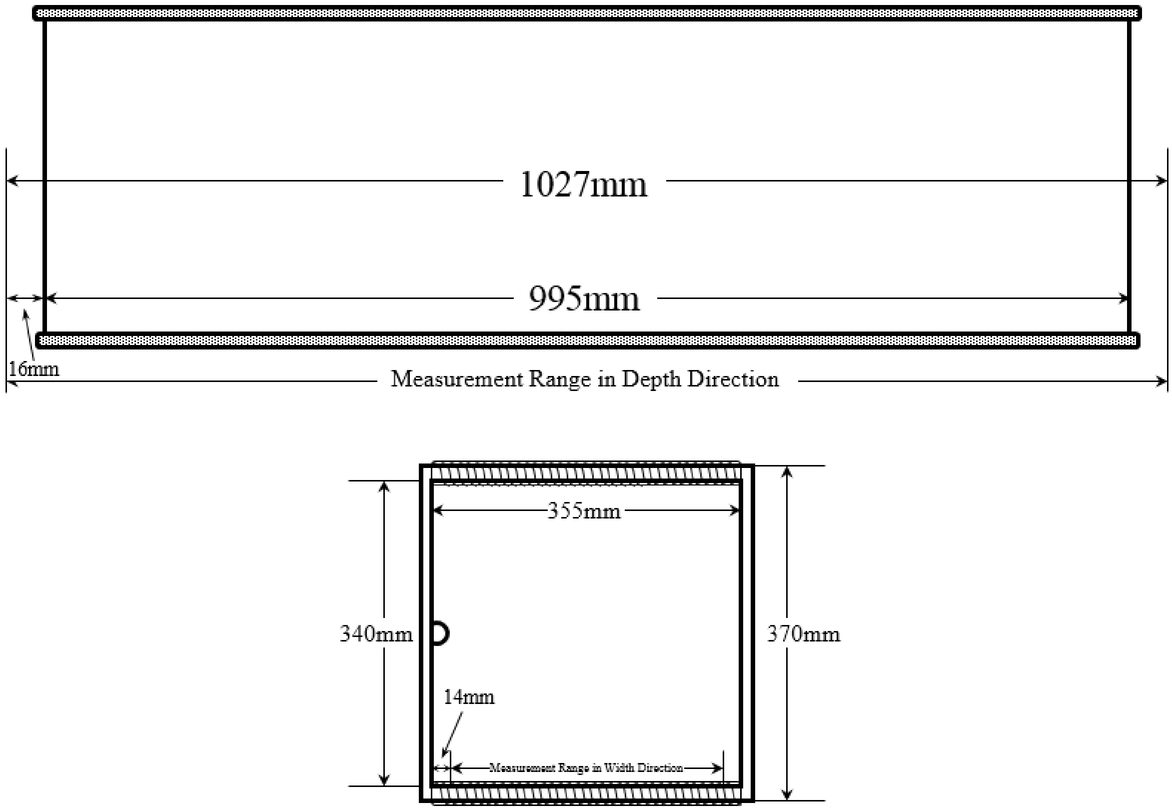
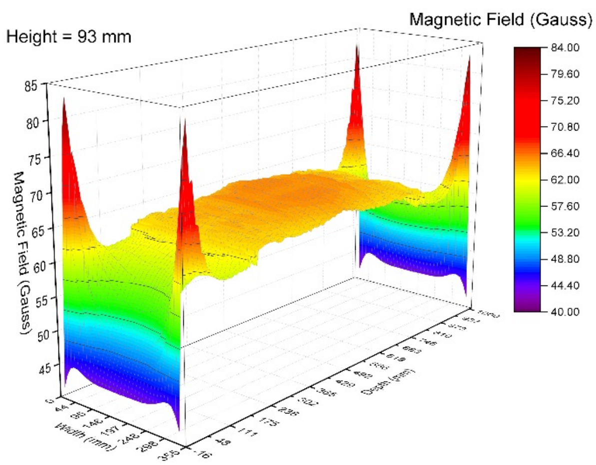
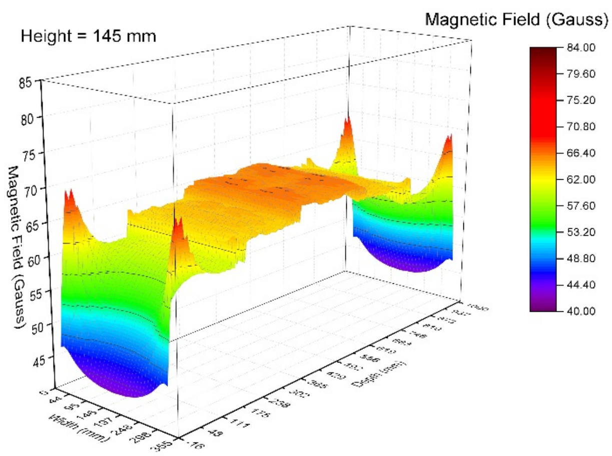


FIGURE 9. The diagram shows the dimension of our magnet. The upper is a side-perspective projection while the lower one views from the end. The magnet is placed with coils on the top and bottom. Ranges of our measurement is also labeled.

3 3D Magnetic Field Graph





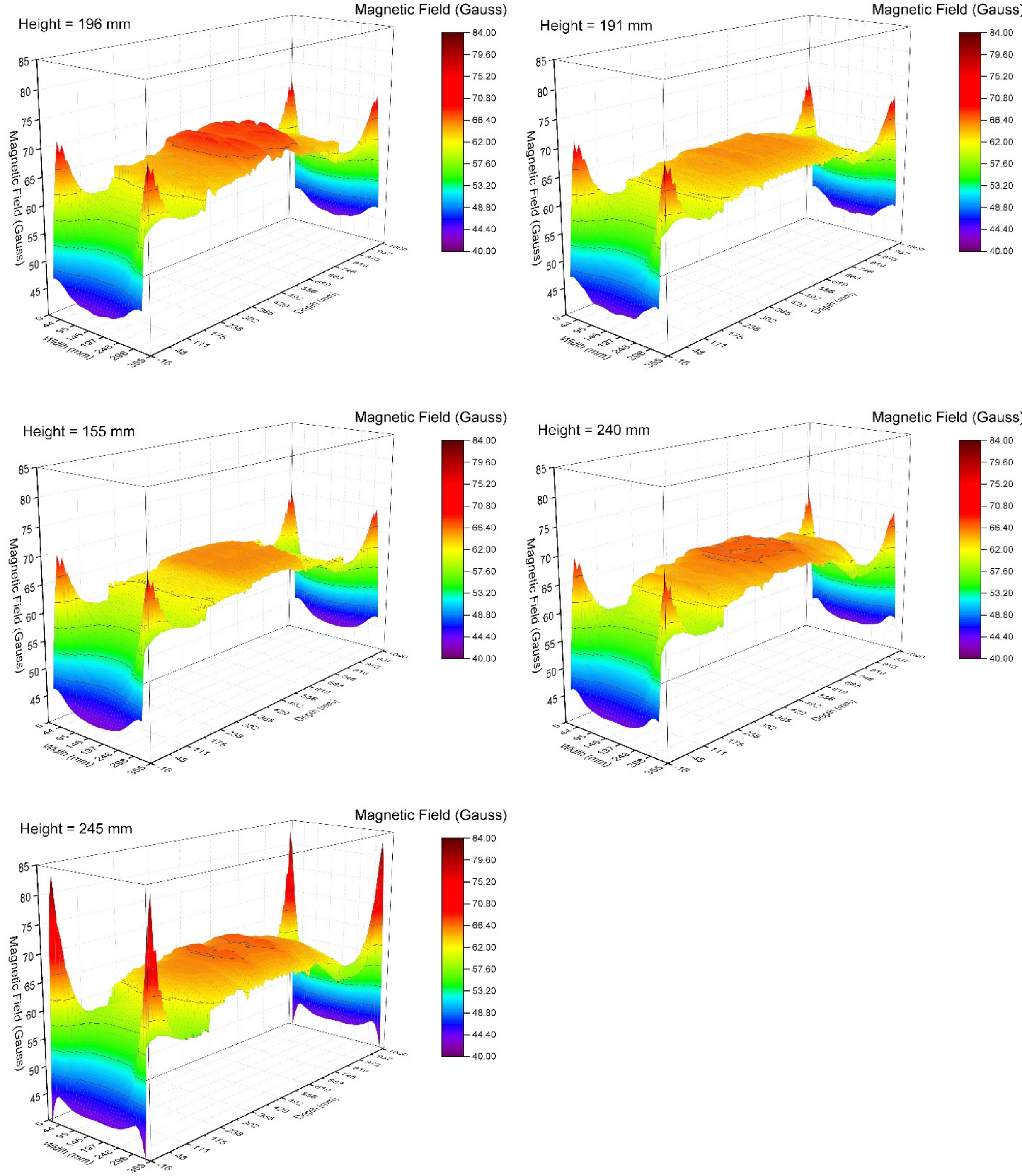
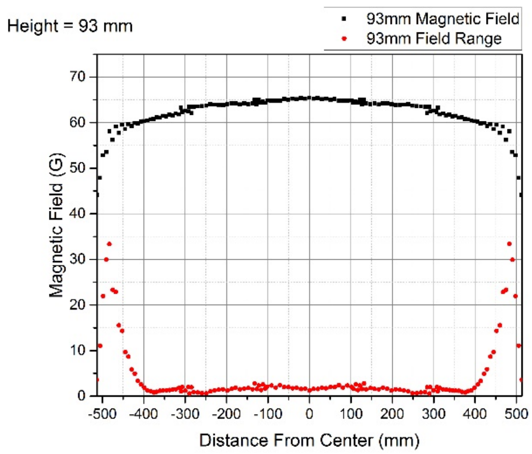
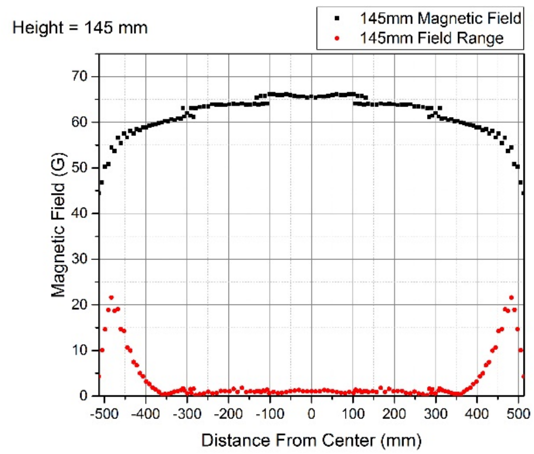
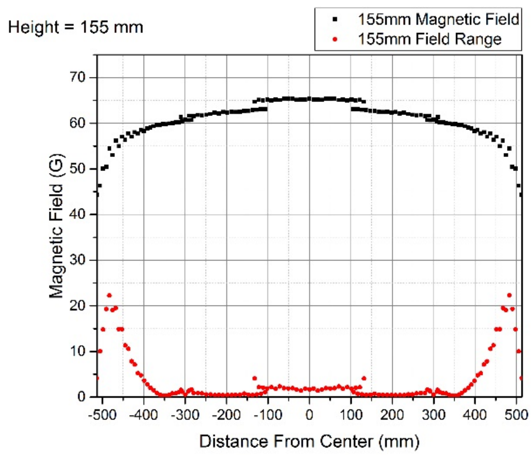


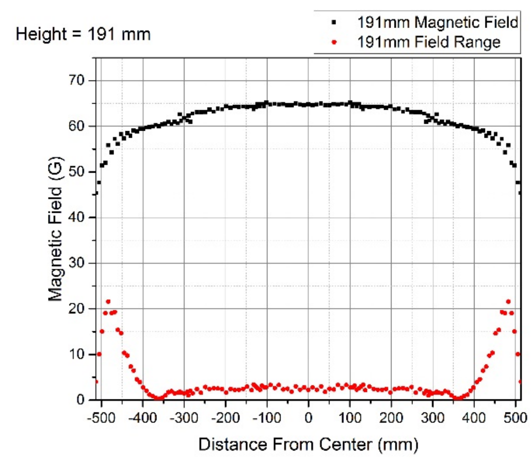
FIGURE 10. Magnetic field vs. width and depth. Red indicates stronger magnetic field and violet indicates weaker field. Four spikes in the corner and obvious drops on both ends represent the edge effect.

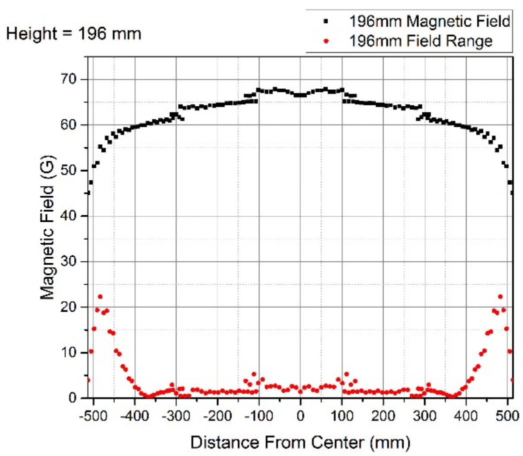
4 Magnetic Field in Depth Direction

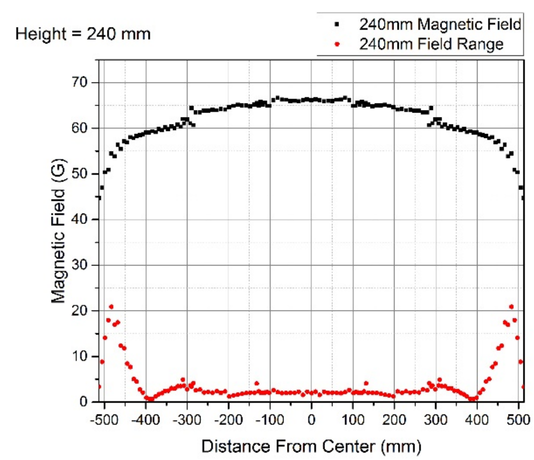












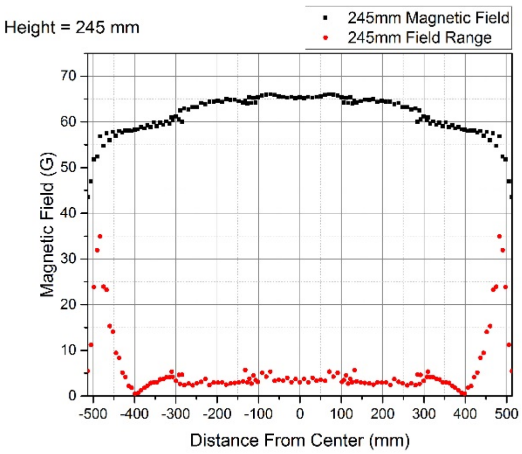


FIGURE 11. Magnetic field in depth direction at different heights is in black. Red scatter plot shows the range of magnetic field in width direction to indicate the variance.

5 Average Field and Percentage Deviation for Each Layers

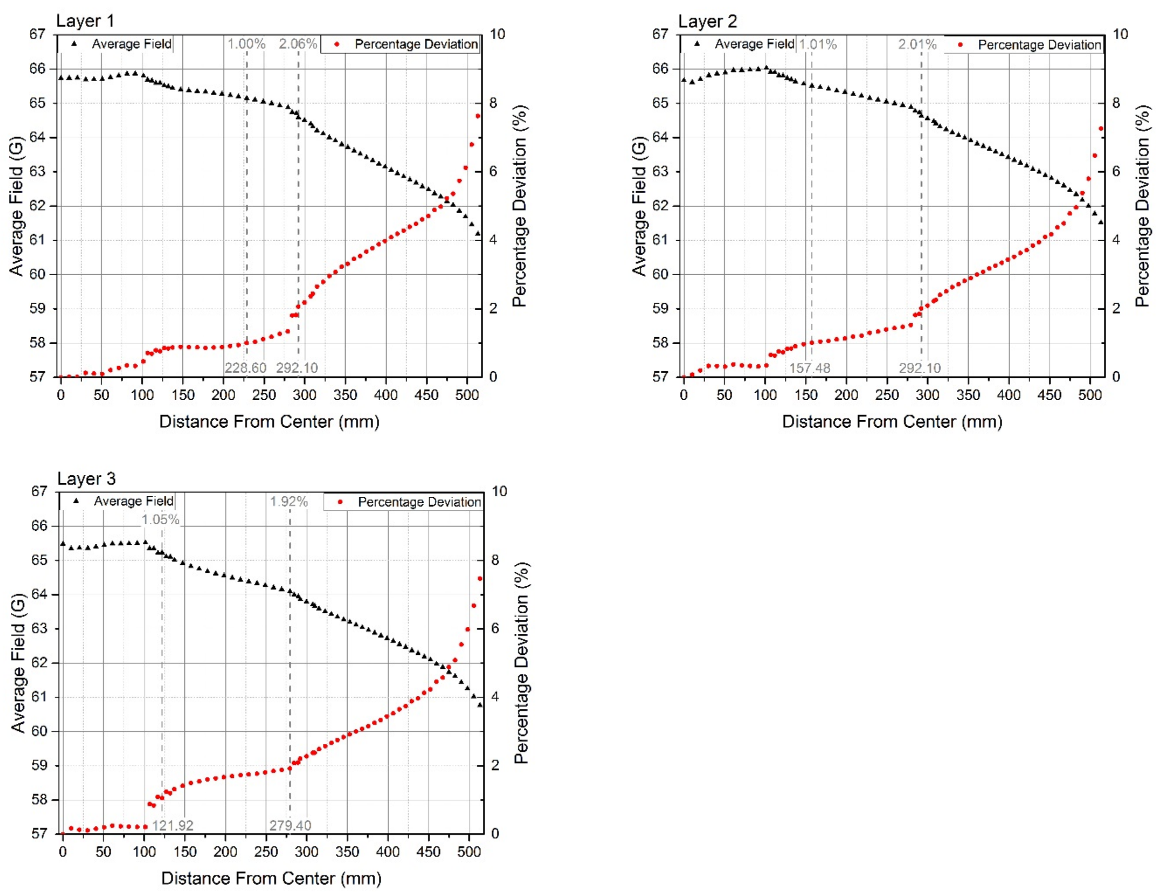


FIGURE 12. Average Field in depth direction over regions that have lengths two times the x coordinate and percentage deviation of magnetic field in depth direction for Layer 1, 2, and 3.

6 Coordinate Conversion Table

Distance From Ct. mm	Depth mm	Distance From Ct. mm	Depth mm	Distance From Ct. mm	Depth mm	Distance From Ct. mm	Depth mm	Distance From Ct. mm	Depth mm
513.08	-16.00	314.96	182.12	111.76	385.32	-121.92	619.00	-330.20	827.28
505.46	-8.38	309.88	187.20	106.68	390.40	-127.00	624.08	-337.82	834.90
497.84	-0.76	307.34	189.74	101.60	395.48	-132.08	629.16	-345.44	842.52
490.22	6.86	299.72	197.36	91.44	405.64	-137.16	634.24	-353.06	850.14
482.60	14.48	292.10	204.98	81.28	415.80	-147.32	644.40	-360.68	857.76
474.98	22.10	289.56	207.52	71.12	425.96	-157.48	654.56	-368.30	865.38
467.36	29.72	284.48	212.60	60.96	436.12	-167.64	664.72	-375.92	873.00
459.74	37.34	279.40	217.68	50.80	446.28	-177.80	674.88	-383.54	880.62
452.12	44.96	269.24	227.84	40.64	456.44	-187.96	685.04	-391.16	888.24
444.50	52.58	259.08	238.00	30.48	466.60	-198.12	695.20	-398.78	895.86
436.88	60.20	248.92	248.16	20.32	476.76	-208.28	705.36	-406.40	903.48
429.26	67.82	238.76	258.32	10.16	486.92	-218.44	715.52	-414.02	911.10
421.64	75.44	228.60	268.48	0.00	497.08	-228.60	725.68	-421.64	918.72
414.02	83.06	218.44	278.64	-10.16	507.24	-238.76	735.84	-429.26	926.34
406.40	90.68	208.28	288.80	-20.32	517.40	-248.92	746.00	-436.88	933.96
398.78	98.30	198.12	298.96	-30.48	527.56	-259.08	756.16	-444.50	941.58
391.16	105.92	187.96	309.12	-40.64	537.72	-269.24	766.32	-452.12	949.20
383.54	113.54	177.80	319.28	-50.80	547.88	-279.40	776.48	-459.74	956.82
375.92	121.16	167.64	329.44	-60.96	558.04	-284.48	781.56	-467.36	964.44
368.30	128.78	157.48	339.60	-71.12	568.20	-289.56	786.64	-474.98	972.06
360.68	136.40	147.32	349.76	-81.28	578.36	-292.10	789.18	-482.60	979.68
353.06	144.02	137.16	359.92	-91.44	588.52	-299.72	796.80	-490.22	987.30
345.44	151.64	132.08	365.00	-101.60	598.68	-307.34	804.42	-497.84	994.92
337.82	159.26	127.00	370.08	-106.68	603.76	-309.88	806.96	-505.46	1002.54
330.20	166.88	121.92	375.16	-111.76	608.84	-314.96	812.04	-513.08	1010.16
322.58	174.50	116.84	380.24	-116.84	613.92	-322.58	819.66		

TABLE 2. This table is used for converting Distance From Center coordinates to actual Depth coordinates. For example, after reading 121.92 mm from the third graph in FIGURE 12, one can use this table to find the real depth coordinates, 375.16 mm ~ 619.00 mm.

7 Average Magnetic Field Strength and Percentage Deviation Table

Distance From Center (mm)	Avg. Mag. Field Strength (G)	Standard Deviation (G)	Percentage Deviation (%)
513.08	61.23	4.34	7.08
505.46	61.49	3.82	6.21
497.84	61.72	3.39	5.49
490.22	61.89	3.14	5.07
482.60	62.05	2.89	4.65
474.98	62.16	2.80	4.50
467.36	62.29	2.63	4.23
459.74	62.37	2.58	4.13
452.12	62.48	2.46	3.94
444.50	62.56	2.41	3.85
436.88	62.66	2.32	3.71
429.26	62.73	2.27	3.62
421.64	62.82	2.20	3.50
414.02	62.90	2.14	3.41
406.40	62.99	2.08	3.30
398.78	63.06	2.03	3.21
391.16	63.14	1.97	3.12
383.54	63.22	1.91	3.03
375.92	63.30	1.86	2.94
368.30	63.37	1.80	2.85
360.68	63.45	1.76	2.77
353.06	63.53	1.69	2.67
345.44	63.60	1.65	2.59
337.82	63.68	1.58	2.49
330.20	63.75	1.53	2.40
322.58	63.83	1.46	2.28
314.96	63.91	1.40	2.19
309.88	63.99	1.31	2.05
307.34	64.05	1.29	2.02
299.72	64.12	1.22	1.90
292.10	64.19	1.17	1.82
289.56	64.28	1.08	1.68
284.48	64.32	1.07	1.66
279.40	64.42	0.90	1.40
269.24	64.47	0.87	1.35
259.08	64.53	0.84	1.30
248.92	64.57	0.81	1.26
238.76	64.63	0.78	1.20
228.60	64.67	0.76	1.17
218.44	64.72	0.73	1.13
208.28	64.76	0.71	1.10
198.12	64.81	0.68	1.06
187.96	64.85	0.67	1.03
177.80	64.90	0.65	1.01
167.64	64.94	0.64	0.99
157.48	64.98	0.62	0.96
147.32	65.03	0.60	0.92
137.16	65.09	0.56	0.86
132.08	65.15	0.52	0.80
127.00	65.17	0.53	0.81
121.92	65.25	0.45	0.69
116.84	65.25	0.47	0.71
111.76	65.33	0.37	0.57
106.68	65.35	0.38	0.59
101.60	65.46	0.09	0.13
91.44	65.47	0.09	0.13
81.28	65.48	0.09	0.14
71.12	65.47	0.09	0.14
60.96	65.47	0.10	0.15
50.80	65.43	0.06	0.09
40.64	65.42	0.06	0.09
30.48	65.40	0.05	0.08
20.32	65.40	0.06	0.09
10.16	65.37	0.06	0.10
0.00	65.44	0.00	0.00

TABLE 3. This table is for extracting data from FIGURE 5.

8 Magnetic Field Strength and Percentage Deviation Table for Layer 1

Distance From Center (mm)	Avg. Mag. Field Strength (G)	Standard Deviation (G)	Percentage Deviation (%)
513.08	61.19	4.67	7.63
505.46	61.46	4.18	6.79
497.84	61.69	3.78	6.12
490.22	61.86	3.55	5.74
482.60	62.03	3.32	5.36
474.98	62.14	3.25	5.22
467.36	62.27	3.10	4.98
459.74	62.37	3.05	4.88
452.12	62.48	2.94	4.71
444.50	62.57	2.89	4.61
436.88	62.68	2.81	4.48
429.26	62.77	2.76	4.39
421.64	62.86	2.69	4.28
414.02	62.95	2.64	4.19
406.40	63.05	2.58	4.09
398.78	63.14	2.51	3.98
391.16	63.23	2.45	3.88
383.54	63.33	2.39	3.77
375.92	63.42	2.33	3.67
368.30	63.52	2.25	3.54
360.68	63.61	2.19	3.45
353.06	63.71	2.11	3.31
345.44	63.80	2.06	3.23
337.82	63.91	1.96	3.07
330.20	64.00	1.89	2.96
322.58	64.11	1.78	2.78
314.96	64.20	1.70	2.65
309.88	64.32	1.57	2.45
307.34	64.40	1.52	2.36
299.72	64.50	1.41	2.18
292.10	64.59	1.33	2.06
289.56	64.70	1.17	1.81
284.48	64.74	1.17	1.81
279.40	64.88	0.87	1.34
269.24	64.93	0.83	1.27
259.08	65.00	0.77	1.18
248.92	65.05	0.73	1.12
238.76	65.10	0.67	1.03
228.60	65.14	0.65	1.00
218.44	65.19	0.61	0.94
208.28	65.23	0.60	0.92
198.12	65.27	0.58	0.88
187.96	65.30	0.57	0.87
177.80	65.33	0.56	0.86
167.64	65.35	0.57	0.87
157.48	65.38	0.57	0.88
147.32	65.40	0.58	0.89
137.16	65.44	0.57	0.87
132.08	65.49	0.55	0.84
127.00	65.51	0.56	0.85
121.92	65.59	0.50	0.76
116.84	65.59	0.52	0.79
111.76	65.67	0.45	0.69
106.68	65.69	0.47	0.71
101.60	65.80	0.31	0.47
91.44	65.87	0.22	0.33
81.28	65.86	0.23	0.35
71.12	65.80	0.18	0.28
60.96	65.75	0.14	0.21
50.80	65.70	0.07	0.10
40.64	65.70	0.07	0.11
30.48	65.69	0.08	0.13
20.32	65.74	0.01	0.01
10.16	65.74	0.01	0.01
0.00	65.73	0.00	0.00

TABLE 4. This table is for extracting data from FIGURE 12, Layer 1.

9 Magnetic Field Strength and Percentage Deviation Table for Layer 2

Distance From Center (mm)	Avg. Mag. Field Strength (G)	Standard Deviation (G)	Percentage Deviation (%)
513.08	61.51	4.47	7.26
505.46	61.77	4.00	6.47
497.84	62.00	3.59	5.80
490.22	62.17	3.34	5.38
482.60	62.35	3.09	4.96
474.98	62.46	2.99	4.78
467.36	62.60	2.81	4.49
459.74	62.69	2.74	4.38
452.12	62.81	2.62	4.18
444.50	62.89	2.57	4.09
436.88	62.99	2.48	3.94
429.26	63.08	2.43	3.85
421.64	63.17	2.35	3.71
414.02	63.25	2.29	3.63
406.40	63.34	2.23	3.52
398.78	63.42	2.18	3.44
391.16	63.50	2.12	3.34
383.54	63.58	2.07	3.26
375.92	63.66	2.02	3.17
368.30	63.75	1.96	3.08
360.68	63.82	1.91	3.00
353.06	63.91	1.85	2.89
345.44	63.99	1.80	2.81
337.82	64.07	1.74	2.71
330.20	64.15	1.69	2.63
322.58	64.24	1.61	2.51
314.96	64.32	1.55	2.41
309.88	64.41	1.45	2.26
307.34	64.47	1.43	2.23
299.72	64.55	1.35	2.09
292.10	64.63	1.30	2.01
289.56	64.72	1.19	1.84
284.48	64.77	1.18	1.81
279.40	64.89	0.99	1.53
269.24	64.94	0.96	1.48
259.08	64.99	0.93	1.44
248.92	65.04	0.91	1.40
238.76	65.10	0.87	1.33
228.60	65.15	0.85	1.30
218.44	65.22	0.79	1.22
208.28	65.26	0.77	1.19
198.12	65.32	0.74	1.13
187.96	65.36	0.73	1.11
177.80	65.42	0.70	1.06
167.64	65.46	0.68	1.04
157.48	65.51	0.66	1.01
147.32	65.57	0.63	0.97
137.16	65.63	0.59	0.90
132.08	65.69	0.55	0.84
127.00	65.73	0.55	0.83
121.92	65.80	0.48	0.73
116.84	65.82	0.50	0.76
111.76	65.90	0.42	0.63
106.68	65.91	0.43	0.66
101.60	66.02	0.23	0.35
91.44	65.98	0.21	0.31
81.28	65.98	0.22	0.33
71.12	65.96	0.23	0.35
60.96	65.96	0.25	0.38
50.80	65.90	0.21	0.31
40.64	65.87	0.22	0.33
30.48	65.81	0.22	0.33
20.32	65.70	0.13	0.20
10.16	65.61	0.05	0.08
0.00	65.67	0.00	0.00

TABLE 5. This table is for extracting data from FIGURE 12, Layer 2.

10 Magnetic Field Strength and Percentage Deviation Table for Layer 3

Distance From Center (mm)	Avg. Mag. Field Strength (G)	Standard Deviation (G)	Percentage Deviation (%)
513.08	60.76	4.54	7.47
505.46	61.02	4.08	6.68
497.84	61.25	3.66	5.98
490.22	61.44	3.40	5.54
482.60	61.61	3.13	5.08
474.98	61.73	3.02	4.88
467.36	61.88	2.83	4.57
459.74	61.97	2.76	4.45
452.12	62.09	2.63	4.23
444.50	62.18	2.57	4.13
436.88	62.29	2.47	3.97
429.26	62.37	2.42	3.88
421.64	62.46	2.34	3.74
414.02	62.55	2.28	3.65
406.40	62.64	2.21	3.53
398.78	62.72	2.16	3.44
391.16	62.80	2.09	3.33
383.54	62.88	2.04	3.25
375.92	62.96	1.99	3.16
368.30	63.04	1.94	3.08
360.68	63.12	1.89	3.00
353.06	63.19	1.84	2.91
345.44	63.27	1.80	2.84
337.82	63.35	1.74	2.75
330.20	63.42	1.69	2.67
322.58	63.51	1.63	2.57
314.96	63.58	1.58	2.48
309.88	63.67	1.51	2.38
307.34	63.71	1.51	2.38
299.72	63.79	1.45	2.28
292.10	63.86	1.41	2.21
289.56	63.94	1.34	2.10
284.48	63.99	1.33	2.08
279.40	64.09	1.23	1.92
269.24	64.15	1.20	1.87
259.08	64.21	1.18	1.84
248.92	64.27	1.16	1.80
238.76	64.33	1.14	1.76
228.60	64.38	1.12	1.74
218.44	64.43	1.11	1.73
208.28	64.49	1.09	1.69
198.12	64.55	1.07	1.66
187.96	64.62	1.05	1.63
177.80	64.68	1.03	1.60
167.64	64.75	1.00	1.55
157.48	64.83	0.97	1.49
147.32	64.91	0.92	1.41
137.16	65.01	0.85	1.31
132.08	65.10	0.78	1.20
127.00	65.11	0.80	1.24
121.92	65.23	0.69	1.05
116.84	65.21	0.71	1.09
111.76	65.35	0.55	0.84
106.68	65.35	0.57	0.88
101.60	65.51	0.14	0.21
91.44	65.50	0.13	0.21
81.28	65.49	0.14	0.22
71.12	65.49	0.15	0.23
60.96	65.49	0.16	0.25
50.80	65.44	0.13	0.20
40.64	65.40	0.10	0.15
30.48	65.36	0.07	0.11
20.32	65.36	0.09	0.13
10.16	65.35	0.12	0.18
0.00	65.48	0.00	0.00

TABLE 6. This table is for extracting data from FIGURE 12, Layer 3.

11 Magnetic Field Strength and Percentage Deviation Table for Layer 4

Distance From Center (mm)	Avg. Mag. Field Strength (G)	Standard Deviation (G)	Percentage Deviation (%)
513.08	61.44	3.76	6.12
505.46	61.72	3.09	5.00
497.84	61.94	2.56	4.14
490.22	62.08	2.30	3.71
482.60	62.22	2.04	3.28
474.98	62.29	1.99	3.19
467.36	62.40	1.84	2.95
459.74	62.45	1.80	2.89
452.12	62.54	1.70	2.72
444.50	62.60	1.66	2.65
436.88	62.67	1.58	2.52
429.26	62.73	1.53	2.45
421.64	62.80	1.47	2.34
414.02	62.86	1.42	2.25
406.40	62.92	1.36	2.16
398.78	62.98	1.31	2.08
391.16	63.03	1.27	2.01
383.54	63.08	1.22	1.94
375.92	63.14	1.18	1.87
368.30	63.19	1.13	1.79
360.68	63.24	1.10	1.73
353.06	63.29	1.05	1.66
345.44	63.34	1.02	1.61
337.82	63.39	0.97	1.53
330.20	63.44	0.94	1.48
322.58	63.49	0.90	1.41
314.96	63.53	0.87	1.37
309.88	63.58	0.82	1.29
307.34	63.61	0.80	1.26
299.72	63.66	0.77	1.21
292.10	63.70	0.74	1.16
289.56	63.74	0.71	1.11
284.48	63.78	0.69	1.08
279.40	63.84	0.61	0.96
269.24	63.87	0.59	0.92
259.08	63.91	0.57	0.88
248.92	63.94	0.54	0.85
238.76	63.98	0.52	0.81
228.60	64.01	0.51	0.79
218.44	64.04	0.49	0.77
208.28	64.07	0.48	0.74
198.12	64.10	0.45	0.71
187.96	64.13	0.44	0.69
177.80	64.16	0.43	0.67
167.64	64.19	0.42	0.66
157.48	64.22	0.40	0.62
147.32	64.26	0.39	0.60
137.16	64.30	0.36	0.55
132.08	64.33	0.34	0.52
127.00	64.34	0.35	0.54
121.92	64.39	0.30	0.46
116.84	64.39	0.31	0.48
111.76	64.43	0.29	0.44
106.68	64.46	0.29	0.44
101.60	64.52	0.21	0.32
91.44	64.54	0.20	0.32
81.28	64.58	0.19	0.29
71.12	64.61	0.16	0.25
60.96	64.66	0.11	0.17
50.80	64.68	0.11	0.17
40.64	64.72	0.08	0.12
30.48	64.73	0.09	0.13
20.32	64.78	0.06	0.09
10.16	64.78	0.09	0.13
0.00	64.88	0.00	0.00

TABLE 7. This table is for extracting data from FIGURE 6.