Scattering Transform “Hunting invariants”

A naïve description of the Mallat’s team work:

https://www.di.ens.fr/data/scattering/

How to statistically characterize 2D structures ?

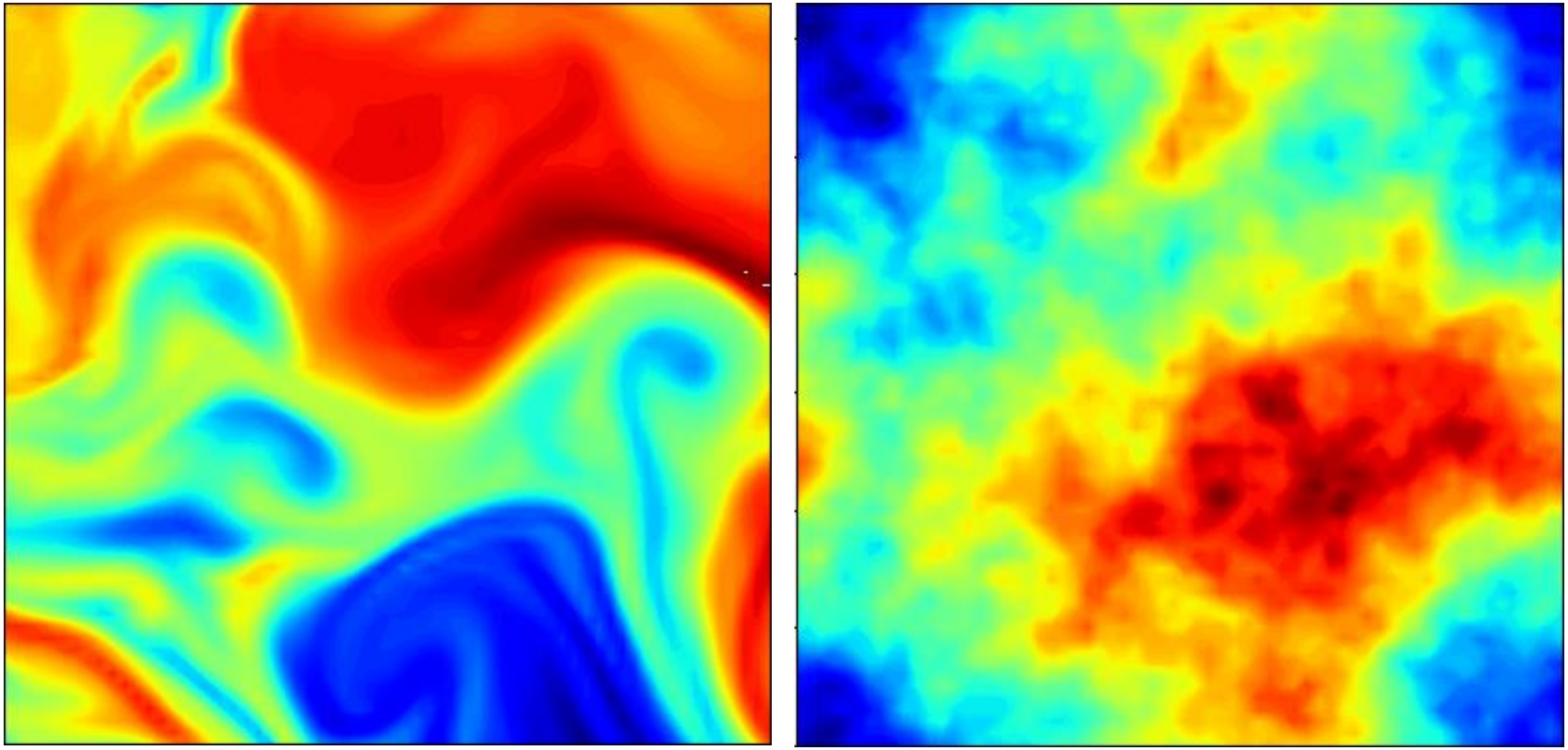
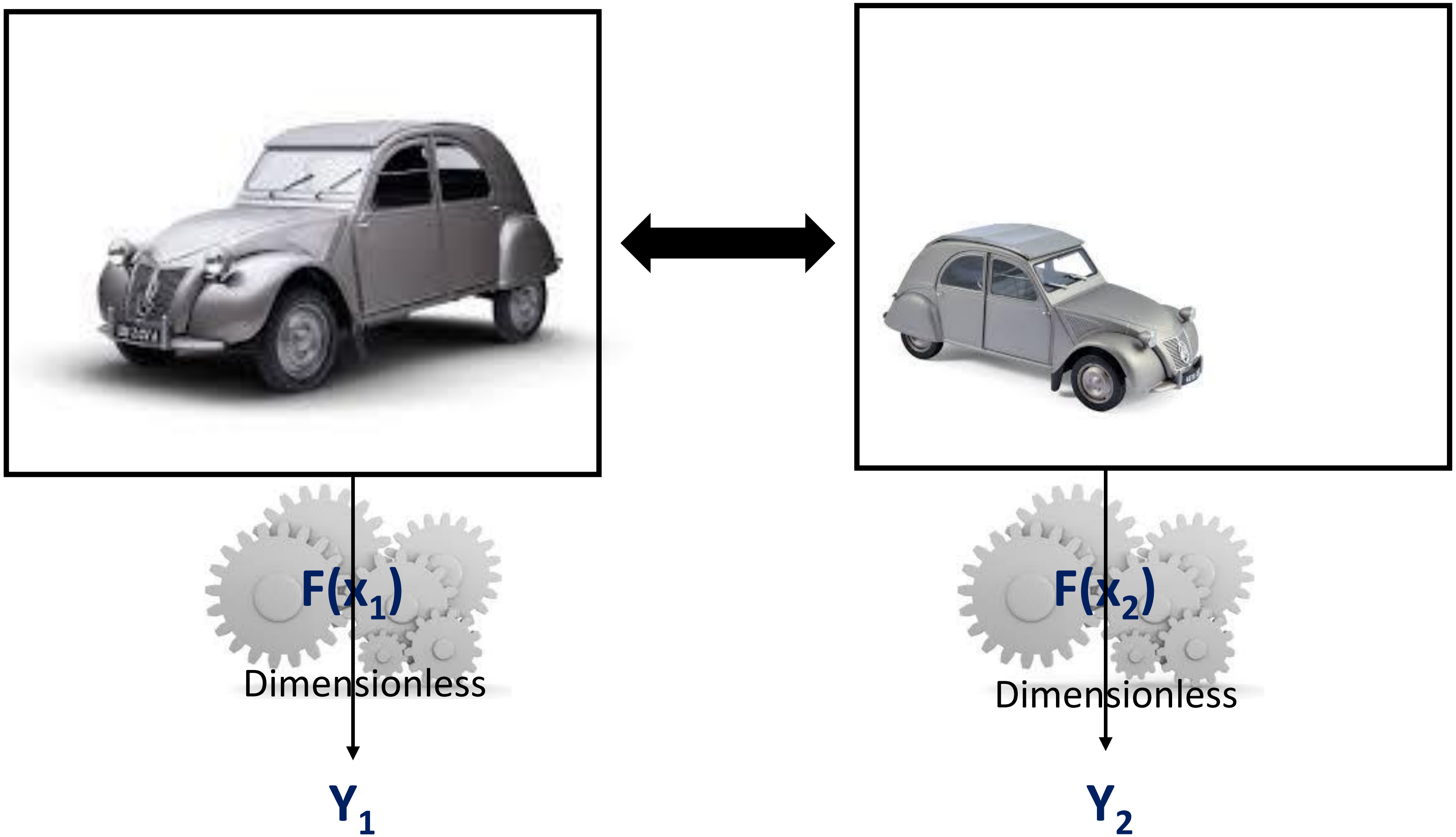
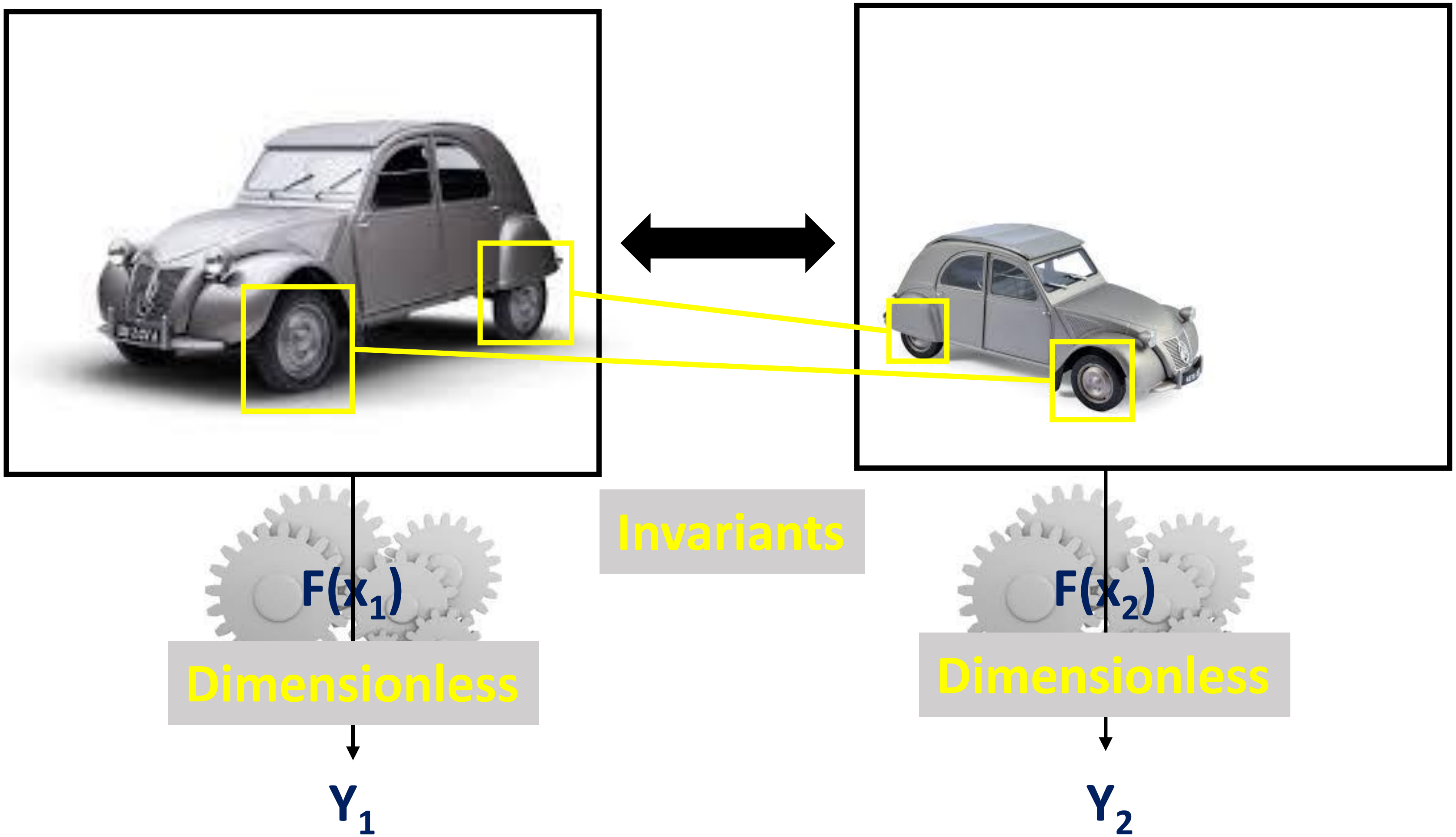


Image classification/recognition



How to a set of invariant rules if x_1 is the same than x_2 : $Y_1 \approx Y_2$

Image classification/recognition



How to a set of invariant rules if x_1 is the same than x_2 : $Y_1 \approx Y_2$

Define Invariants

- Rotation
- Position
- Size
- Deformation
- Colours
- Etc.

Use power spectrum ?

- Power spectrum is position invariant thanks to the phase killing, but:
  - No more localisation.

- Wavelet provides frequency/position information but are not at all invariants (e.g. position).

$\Rightarrow$A mathematical transform is needed to make this usable.

Neural Network approach

- Teach the Neural Network with a large training data set to provide the same answer for any elements of the same classe.

⇒Learn “blindly” invariants

- Internal processing is almost unknown.

⇒Not usable for the scientific post analysis.

What is scattering transform ?

The first three layers of the 2D scattering of an image x are defined by

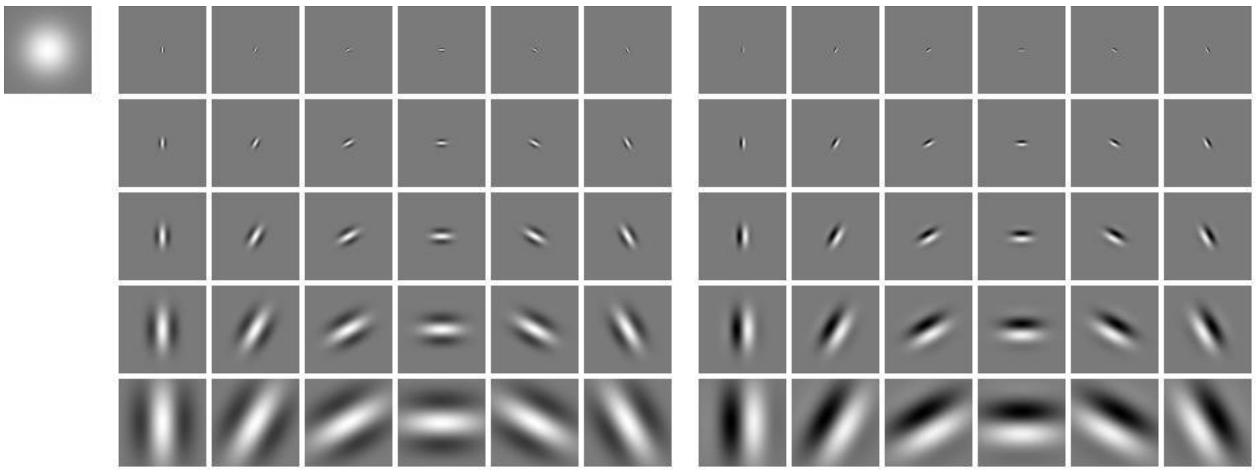
$$S_0x=\{x\star\varphi\}$$
$$S_1x=\{|x\star\psi_{j1,\theta 1}|\star\varphi\}$$
$$S_2x=\{||x\star\psi_{j1,\theta 1}|\star\psi_{j2,\theta 2}|\star\varphi\}$$

where the symbol $\star$ denotes the spatial convolution, φ is an averaging window and $\psi_{j,\theta}$ is a wavelet dilated by 2^j and rotated by $\pi(\theta-1)/L$.

The output variable S is a cell array, whose elements are the different layers of the scattering transform : S{m+1} corresponds to the m^{th} layer S_mx.

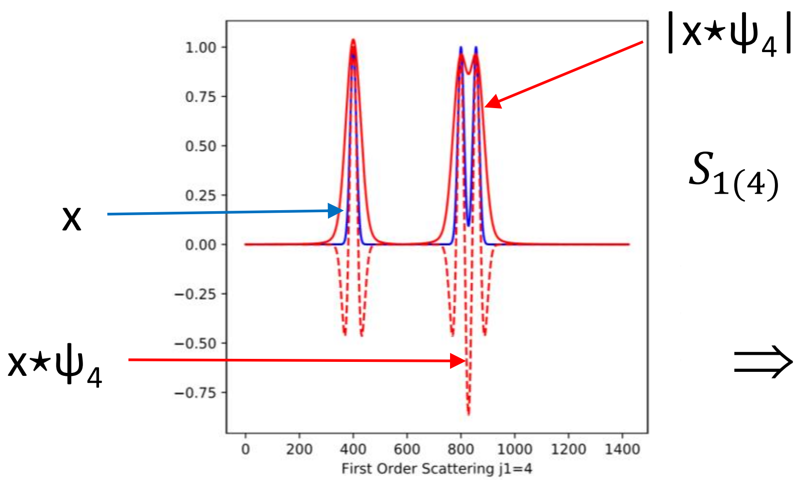
[1] Deep Scattering Spectrum, Andén J. and Mallat. S., Submitted to IEEE Transactions on Signal Processing, 2011.

What is scattering transform ?



The top left image corresponds to φ. The first left half corresponds to the real parts of $\psi_{j,\theta}$, arranged according scales (rows) and orientations (columns). The right half image corresponds to the imaginary part.

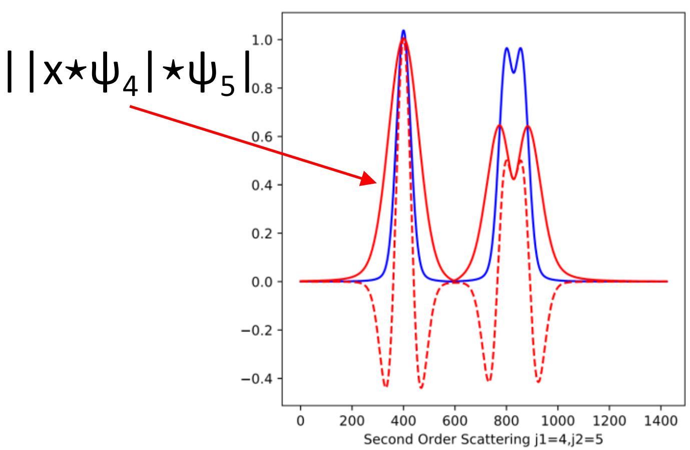
Some plot to understand

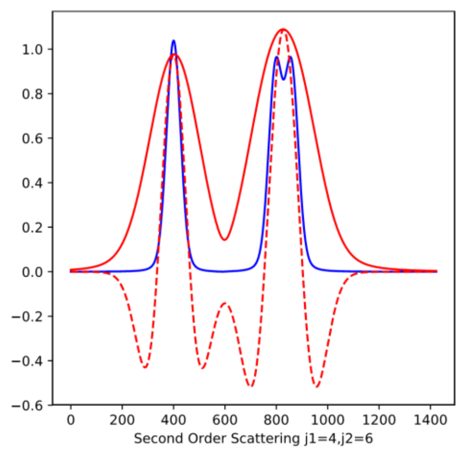


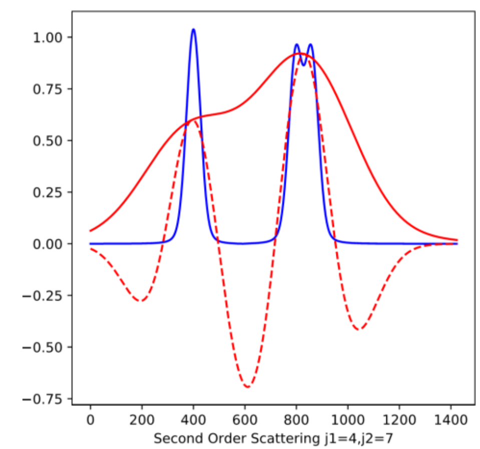
$$S_{1(4)} = \sum_{pixels} |x \star \psi_4|$$

S1x={|x⋆ψ$_{j1}$|}
S$_2$x={||x⋆ψ$_{j1}$|⋆ψ$_{j2}$|}

⇒Sum makes the position invariants

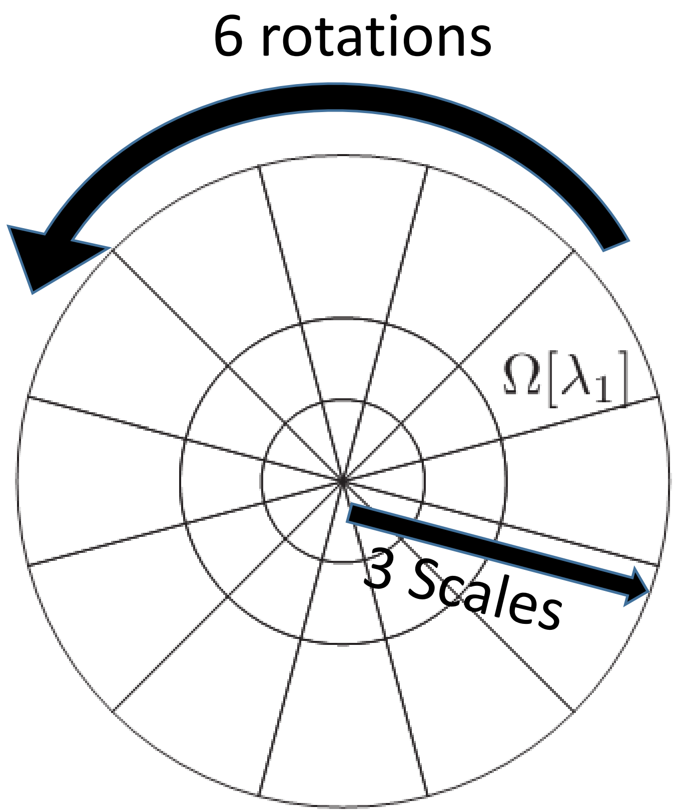




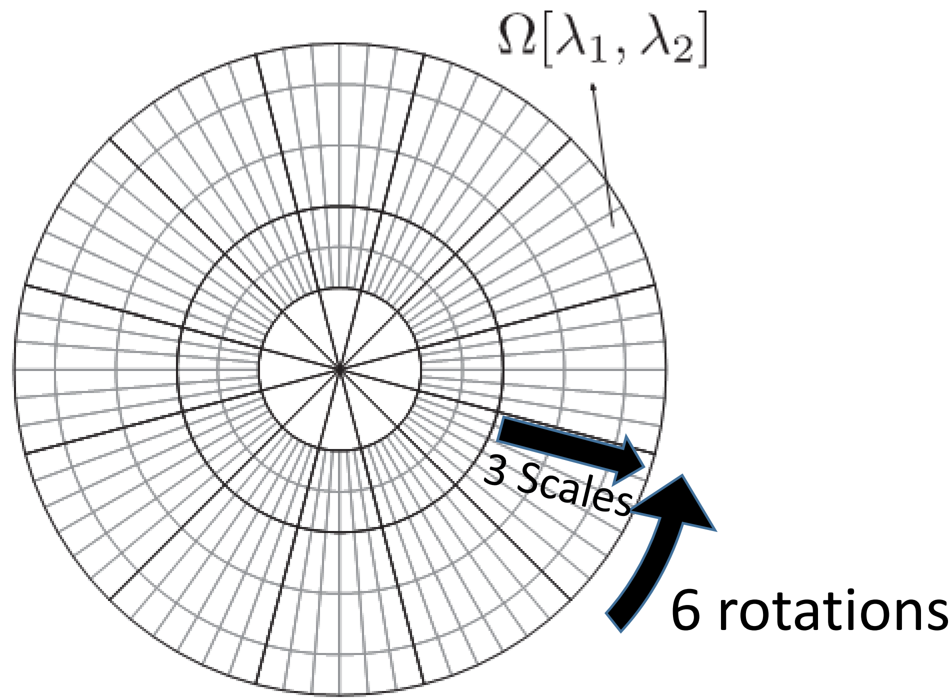


$$S_{2(4,5)} = \sum_{pixels} ||x \star \psi_4| \star \psi_5| \qquad S_{2(4,5)} = \sum_{pixels} ||x \star \psi_4| \star \psi_6| \qquad S_{2(4,5)} = \sum_{pixels} ||x \star \psi_4| \star \psi_7|$$

Scattering transform synthetic view

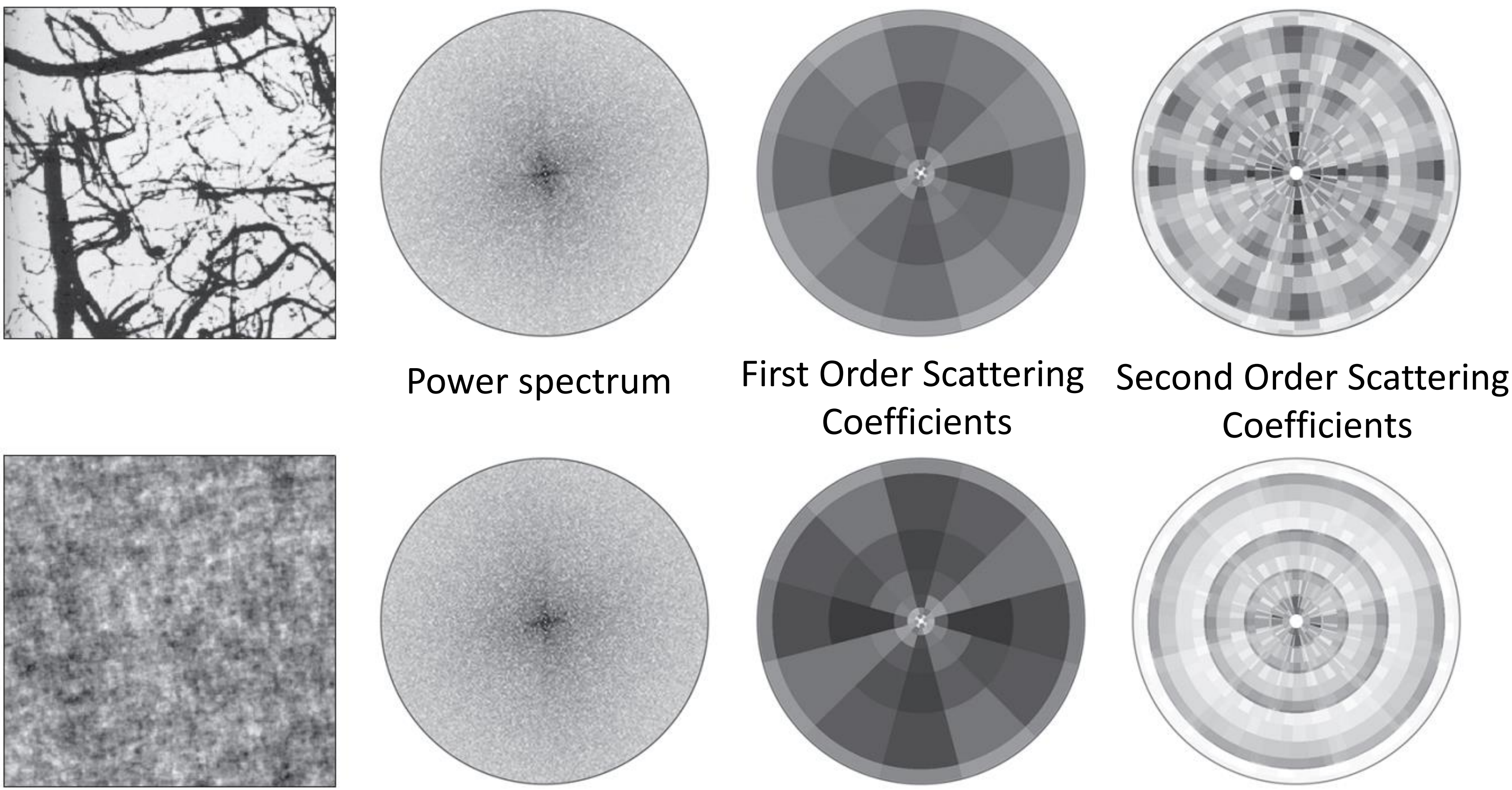


First Order Scattering Coefficients



Second Order Scattering Coefficients

A 2D example


Power spectrum
First Order Scattering Coefficients
Second Order Scattering Coefficients

How to manage rotation invariant

$$S_0x=\{x\star\varphi\}$$
$$S_1x=\sum_{\theta 1}\{|x\star\psi_{j1,\theta 1}|\star\varphi\}$$
$$S_2x=\sum_{\theta 1,\theta 2}\{||x\star\psi_{j1,\theta 1}|\star\Psi_{\theta_2,j_2,k_2}|\star\varphi\}$$

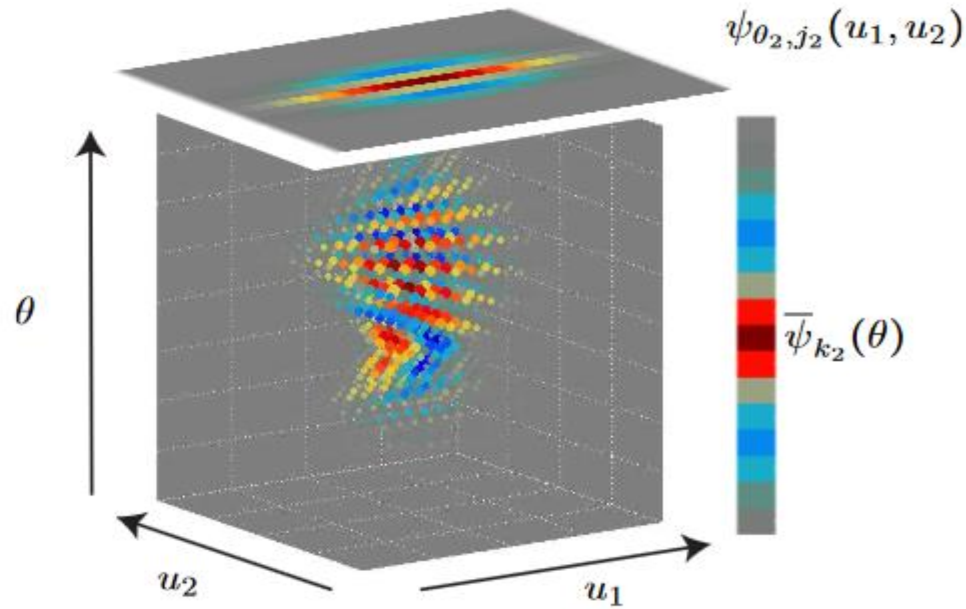
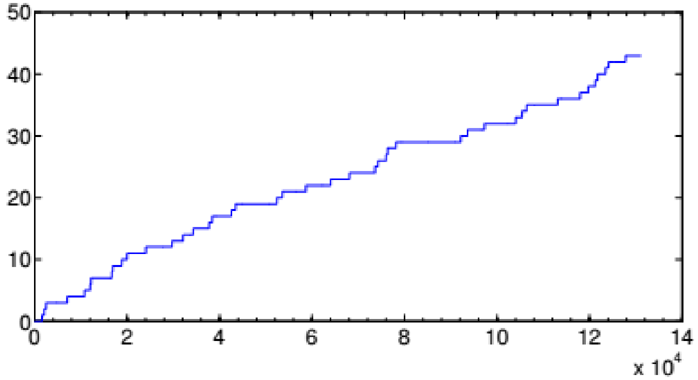


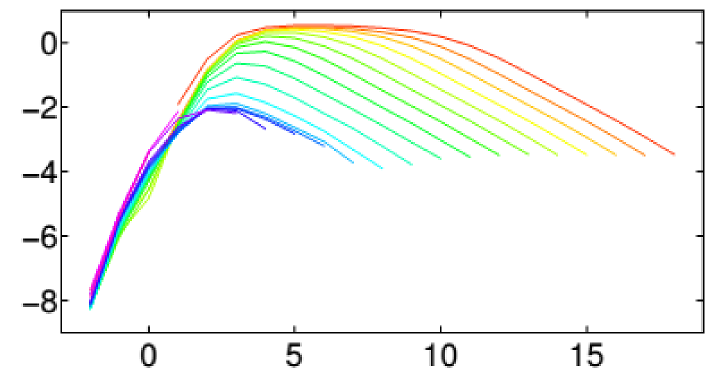
Figure 5: A three dimensional roto-translation convolution with a wavelet $\Psi_{\theta_2,j_2,k_2}(u_1,u_2,\theta)$ can be factorized into a two dimensional convolution with $\psi_{\theta_2,j_2}(u_1,u_2)$ rotated by θ and a one dimensional convolution with $\overline{\psi}_{k_2}(\theta)$.

[1] Rotation, Scaling and Deformation Invariant Scattering for Texture Discrimination, Sifre L. and Mallat S., Proceedings in IEEE CVPR 2013 conference, 2013.

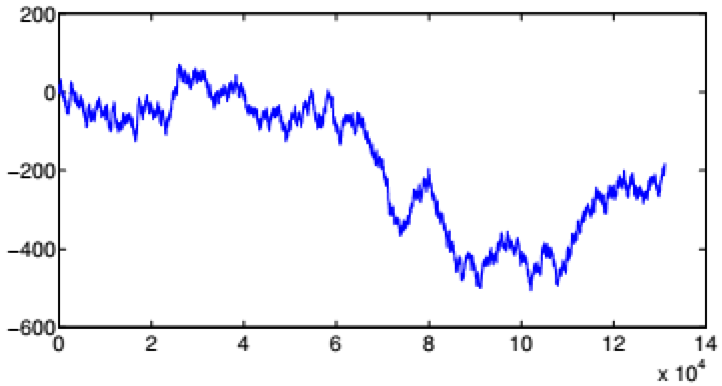
Some theoretical description



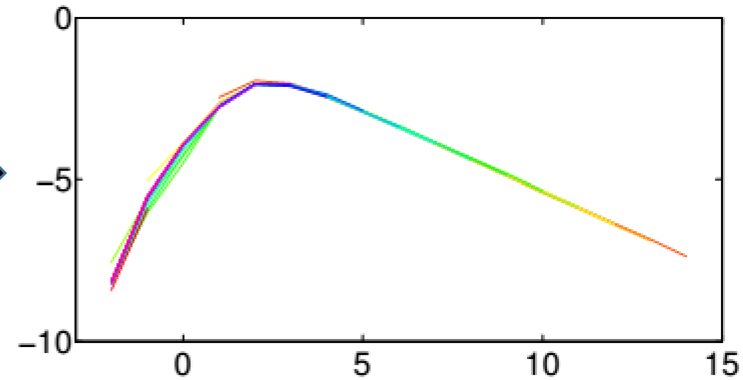
Realization of a Poisson process



J2-J1



Realization of a Brownian motion



J2-J1

And now the T.P.:

Test 1D computation:

- testscat_1D.py

Test 2D computation:

- testscat_2D.py

More see:

https://www.di.ens.fr/data/scattering/

Short Biblio:

- **1D stats:**

Intermittent process analysis with scattering moments; Bruna, J. and Mallat, S. and Bacry,E. and Muzy, J.-F., 2015, **arXiv:1311.4104**

- **Scattering Transform reader digest:**

Invariant Scattering Convolution Networks; Bruna, J. and Mallat, S. , 2013, **arXiv:1203.1513v2**