Predictive Analysis with Transfer Operators and Analog Methods

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24 April 2019

LOPS

Outline:

- ***Transfer Operator and Markov Chain***

Application to Argo Float Displacements and the Evolution of Intermediate Water Masses

:::Physical Space:::

- ***Transfer Operators and Analog Methods***

Application to the Probabilistic Prediction of Interannual Climate Variations: ***PROCAST***

:::Phase Space:::

Transfer Operator and Markov Chain

Transfer Operator:

The goal is to describe a density (or a probability) at a later time (τ) by its knowledge at the current time (t).

$$\rho(t+\tau) = \mathbf{M}_\tau \rho(t).$$

Hence the solution is given by the knowledge of $\mathbf{M}$, the ***Transfer Operator***.

Markov Chain:

This relation can be reproduced sequentially, assuming the stationarity of $\mathbf{M}$, as a ***Markov Chain***.

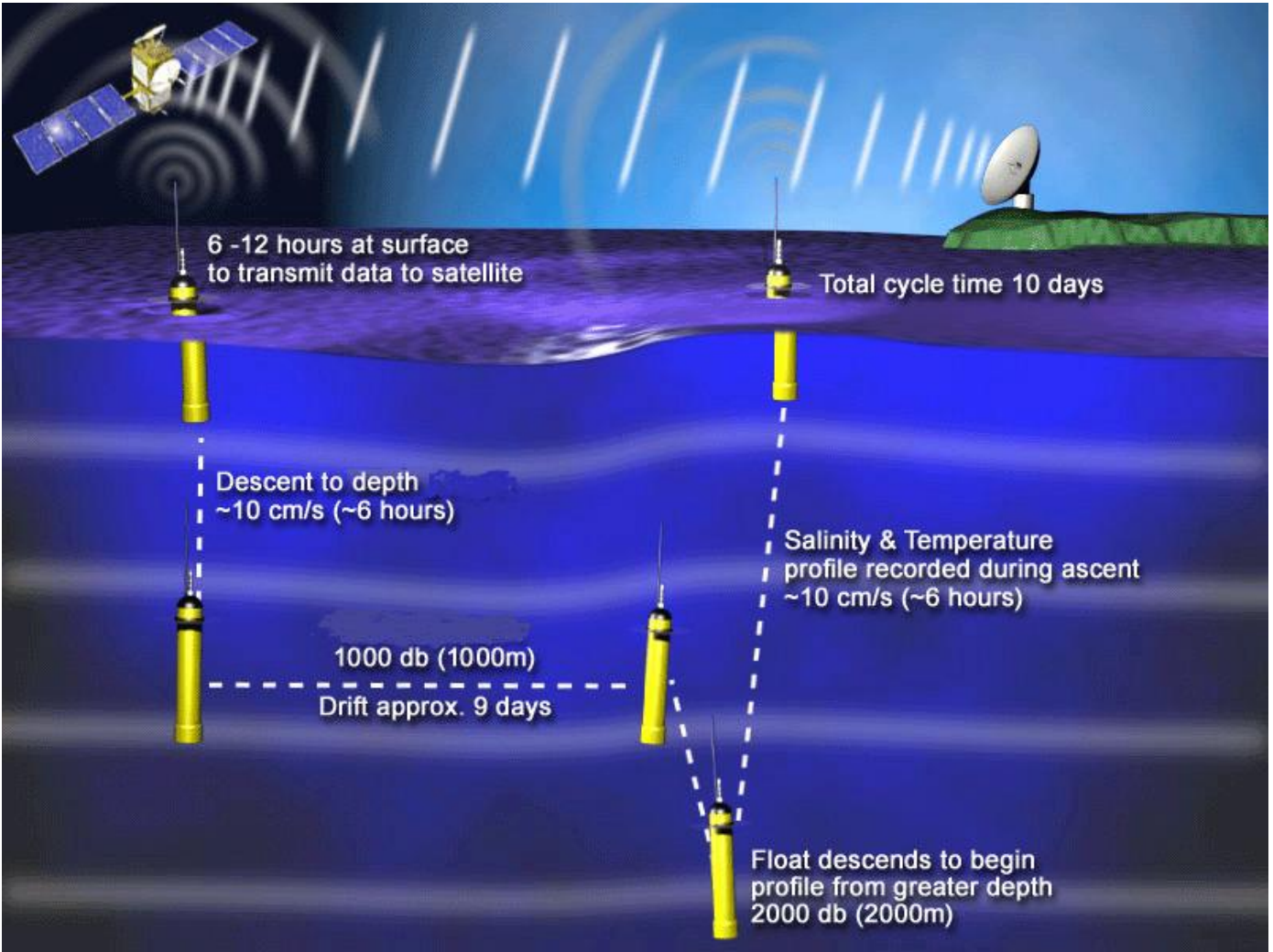
$$\rho(t+2\tau) = \mathbf{M}_\tau \mathbf{M}_\tau \rho(t),$$

$$\vdots$$

$$\rho(t+k\tau) = \mathbf{M}_\tau^k \rho(t).$$

Transfer Operator and Markov Chain

Application to Argo Float Displacements:



Scientific Question:

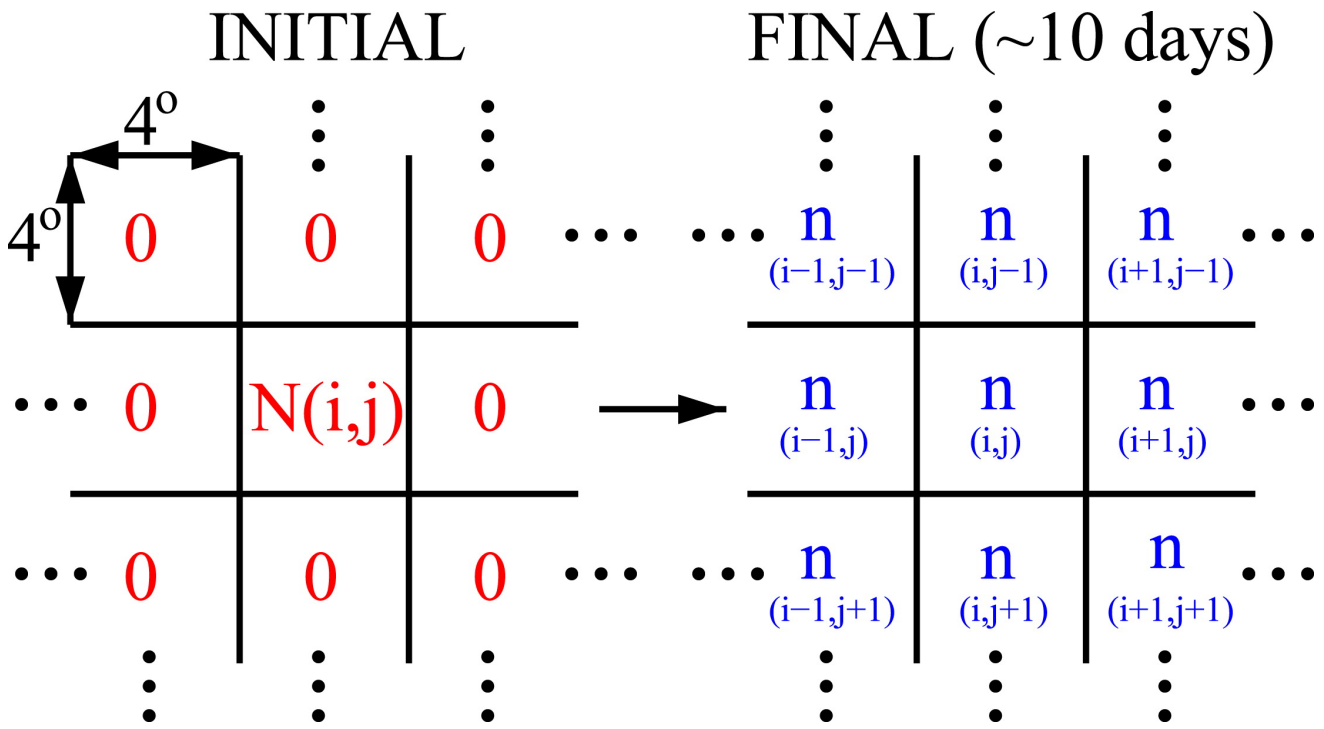
What is the fate of Intermediate Water Masses?

Specific Challenge:

Can we identify the behavior of Intermediate Water Masses using the Argo float displacement during their 10-day journey at 1,000-m depth?

Transfer Operator and Markov Chain

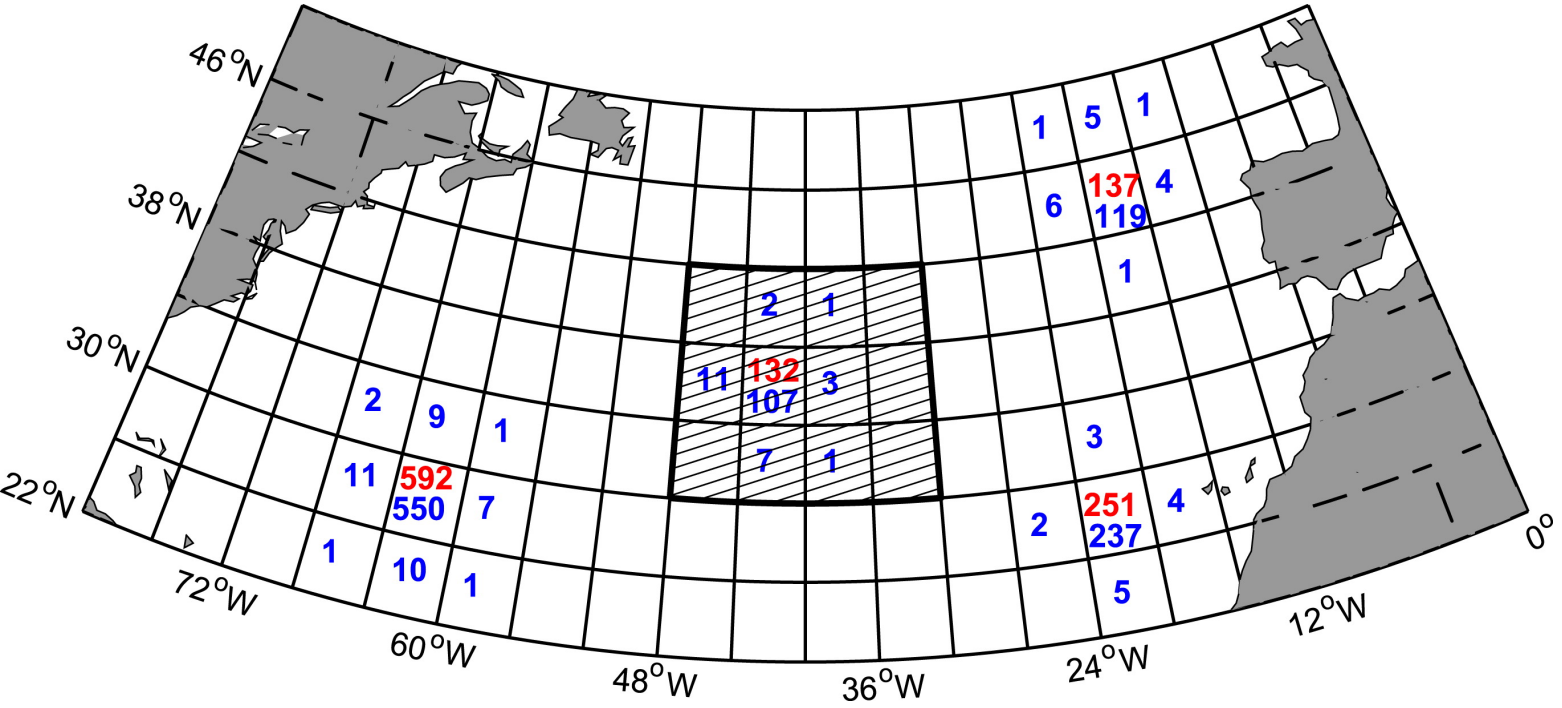
Building the Transfer Operator:



Hence, one could define a probability of transition as n/N

Transfer Operator and Markov Chain

- ***Example of probability computation:***



$$\mathbf{M}_{i_0 j_0} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2/132 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/132 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 11/132 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 107/132 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3/132 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7/132 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/132 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Reproducing this transfer probability for all possible locations $\{i_0, j_0\}$, the transfer operator can be defined as:

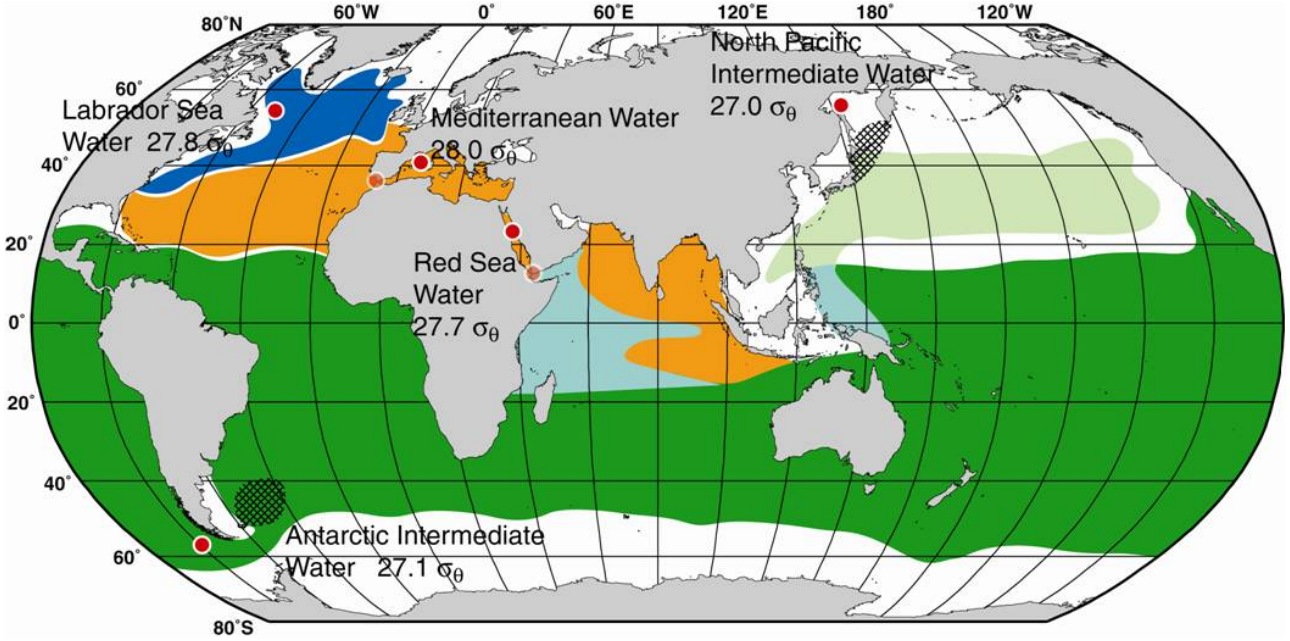
$$\mathbf{M} = \sum_{\{i_0,j_0\}} \mathbf{M}_{i_0,j_0} .$$

- The Transfer Operator contains all the possible journey from one location to an other.
- The non-stationary deterministic journeys of individual Argo float has been transformed into a stationary probabilistic transfer of Argo float density.

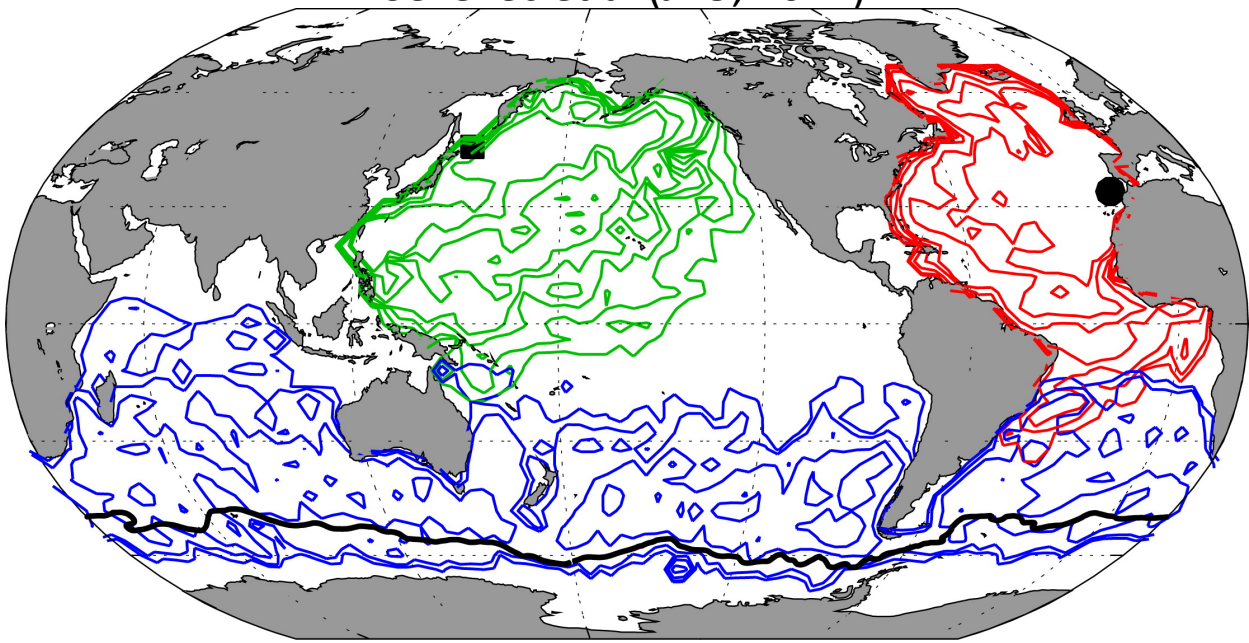
Transfer Operator and Markov Chain

The fate of Intermediate Water Masses:

Talley et al, descriptive Physical Oceanography



Sévellec et al (JPO, 2017)



Not bad...

Transfer Operator and Markov Chain

Intermediate Conclusions

- We have been able to track Intermediate Water Masses based on observations of Argo float journey at depth.

- The uncertainty on the exact location (and time) of the journey might be problematic to define the Markov Chain…
  - The 4x4 grid seems robust enough to define statistically the Argo float fate,
  - but this issue will be more problematic in the next example…

Transfer Operators and Analog Methods

Scientific Context:

- Attributing "forced" vs "natural" variations.
- On interannual to decadal timescales the "natural" variations controlled the GMT changes

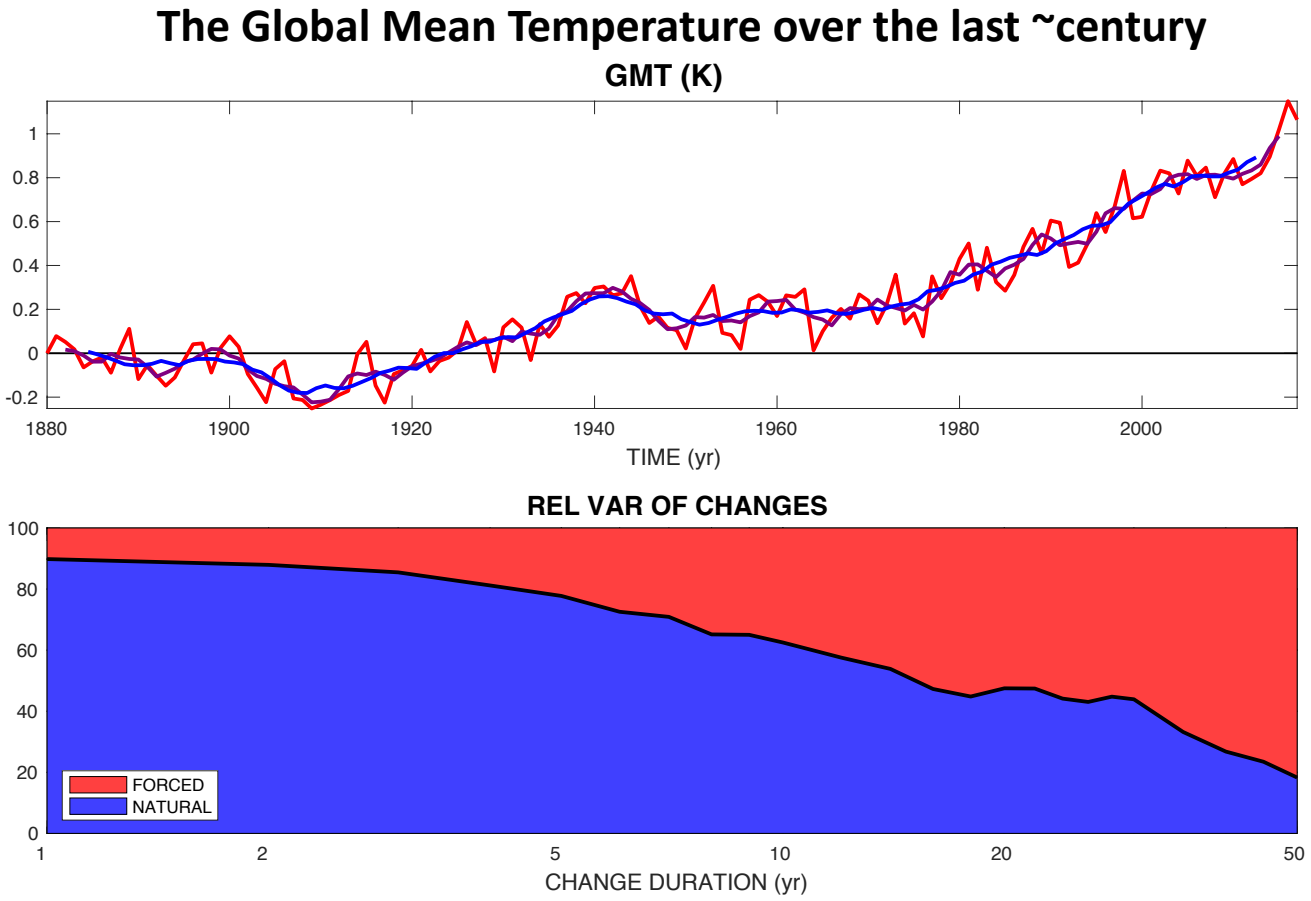
Scientific Questions:

Methodologic

Can we predict global warming variations years ahead?

Pragmatic

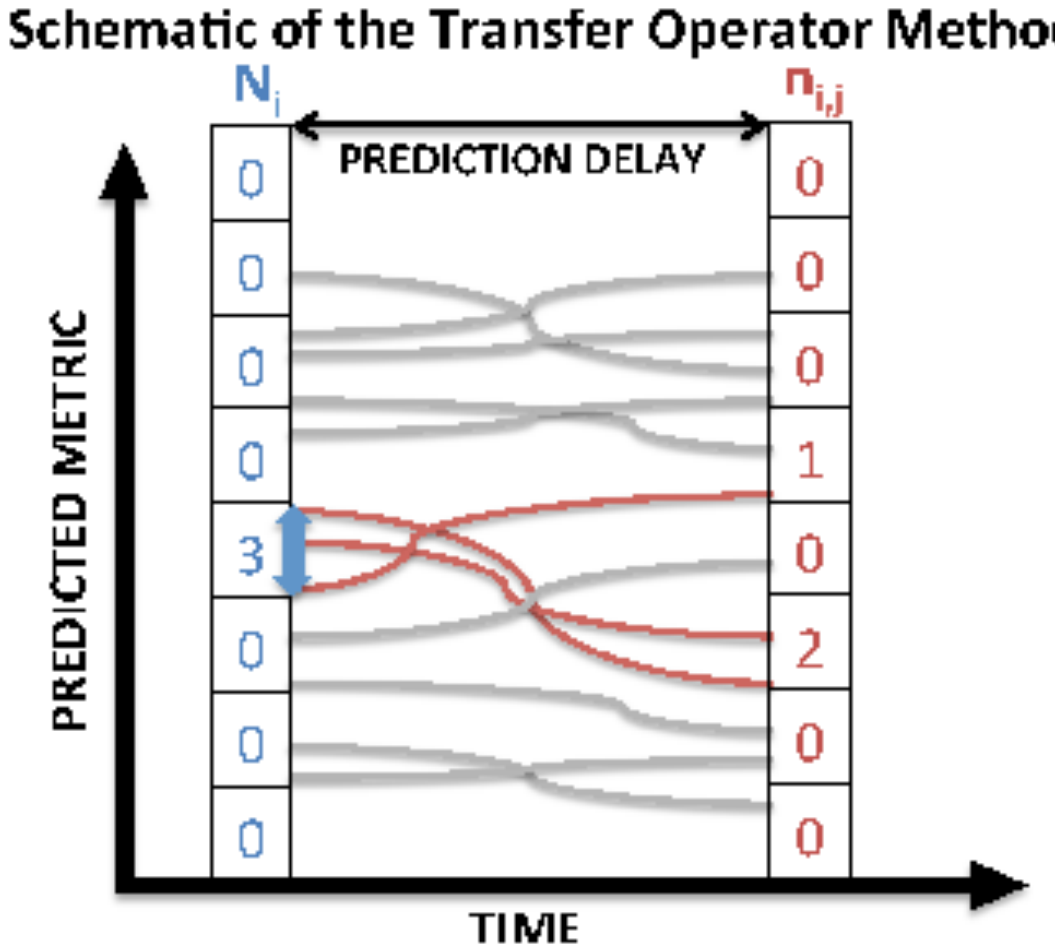
What will be the temperature in 2018-2022?



Transfer Operators and Analog Methods

Method: PRObabilistic forecast System

PROCAST



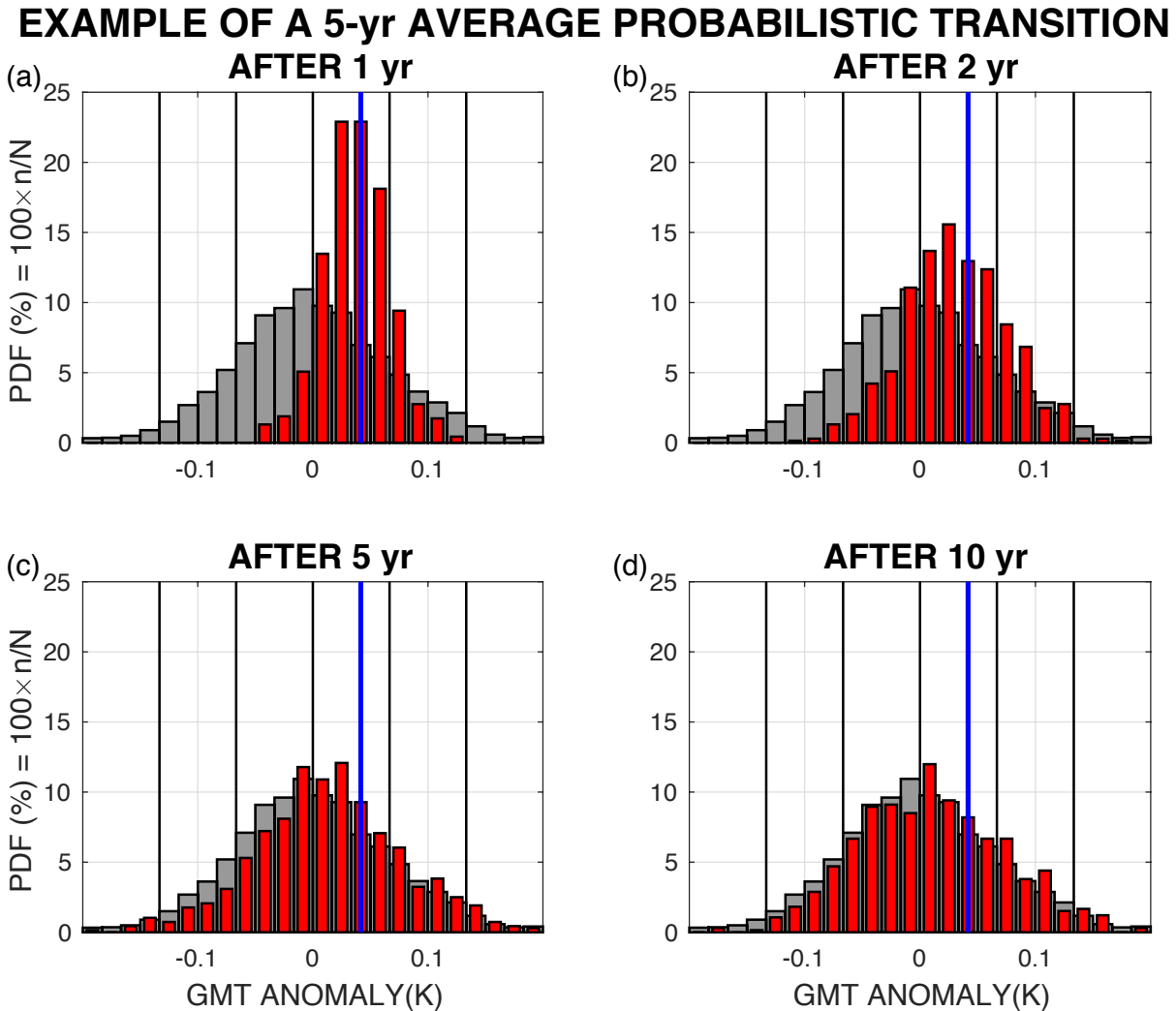
- The principle is the same as before.
- We use the CMIP5 database as the sudo-observations to train the Transfer Operator.
- Because we focus on a dramatically restricted sub-phase space (1D!), we cannot apply a Markov Chain…
  - Hence we built a different Transfer Operator for different Prediction Delay (τ).
  - The principle of PROCAST becomes fundamentally equivalent than an Analog Method.

$$\rho(t + k\tau) = \mathbf{M}_{k\tau}\rho(t) \neq \mathbf{M}_{\tau}^{k}\rho(t).$$

Markov chain is ill-posed if the uncertainty in the "location" propagates faster than the information… A 1D subspace is unequivocally ill-posed…

Transfer Operators and Analog Methods

Example of the probability density function evolution



Transfer Operators and Analog Methods

PROCAST Hindcast performance

Prediction Skill:

$$\text{Coeff. Det.} = R^2 = 1 - \frac{\overline{\left(\overline{x_i p_i(t)}^{\,i} - o(t)\right)^2}^{\,t}}{\overline{o(t)^2}^{\,t}}.$$

~30%

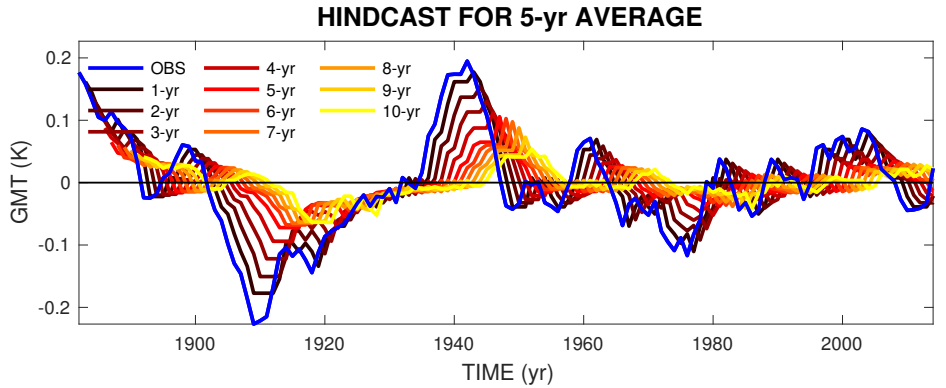
Acceptable skill

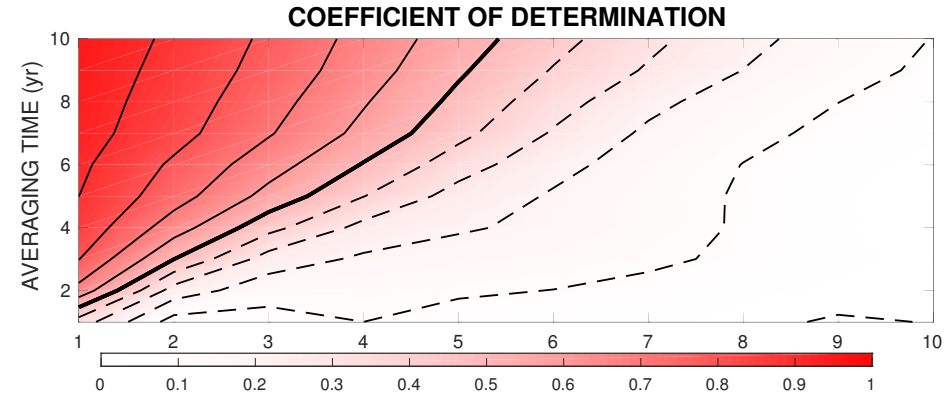
Prediction Reliability:

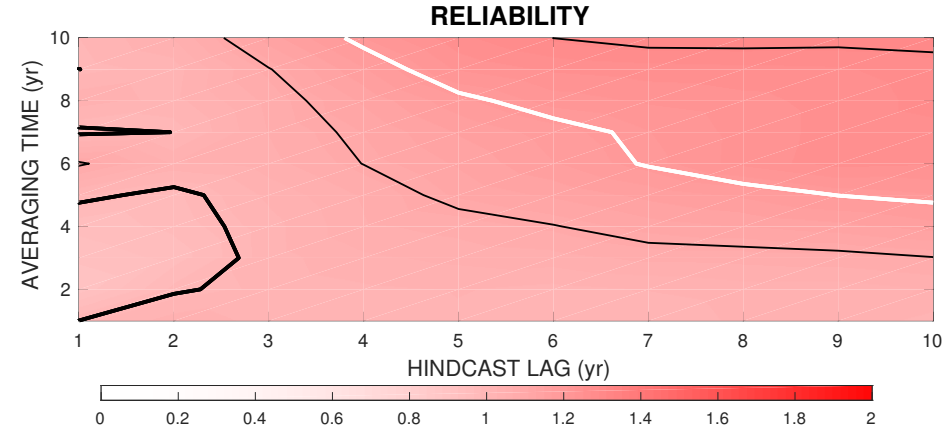
$$\text{Fiabilité} = \sqrt{\left\{ \frac{\left(\overline{x_i p_i(t)}^{\,i} - o(t)\right)^2}{\overline{\left[x_i - \overline{x_i p_i(t)}^{\,i}\right]^2 p_i(t)}^{\,i}} \right\}^t}.$$

1

Good statistical prediction of events







Transfer Operators and Analog Methods

Skill Comparison: PROCAST vs DePreSys3 (UK)

Skill on GMT prediction

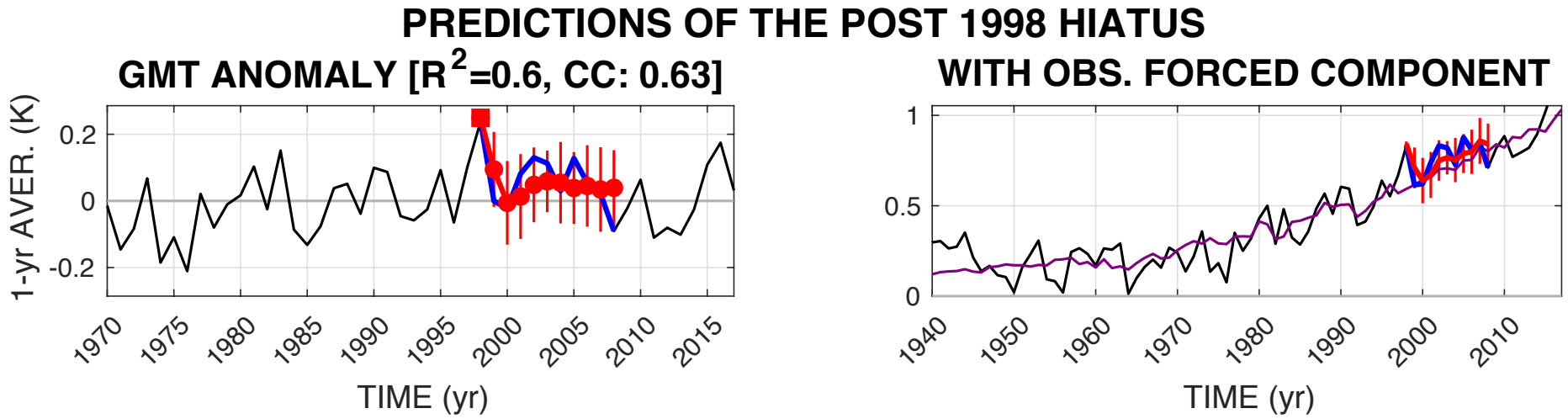
- 1-5-yr error
  - PROCAST – 0.104 K
  - DePreSys – 0.151 K
- Reliability
  - PROCAST – 1
  - DePreSys – 2.3
- Computational Performance
  - PROCAST – 22 ms on a laptop
  - DePreSys – 1 week on a supercomputer

Intrinsic Limits

PROCAST do not yet predict where (region) and when (season) predicted GMT changes will manifest…

Transfer Operators and Analog Methods

Skill Example: The global warming Hiatus

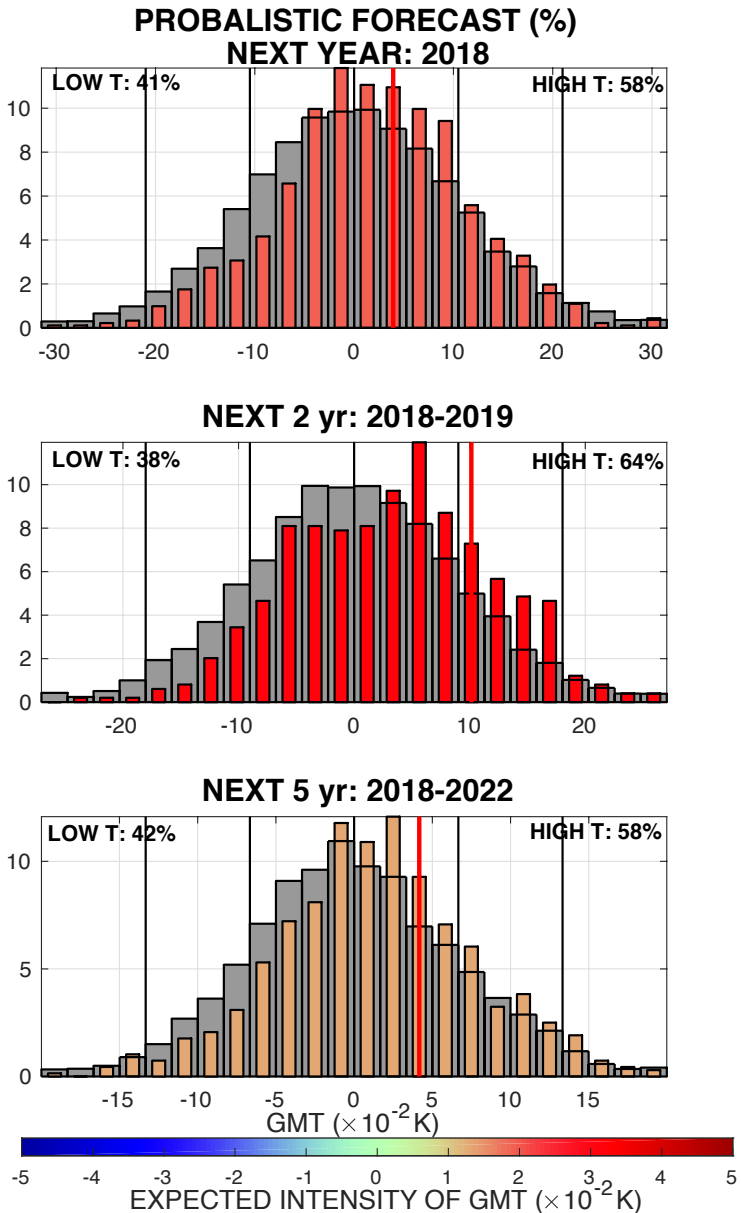


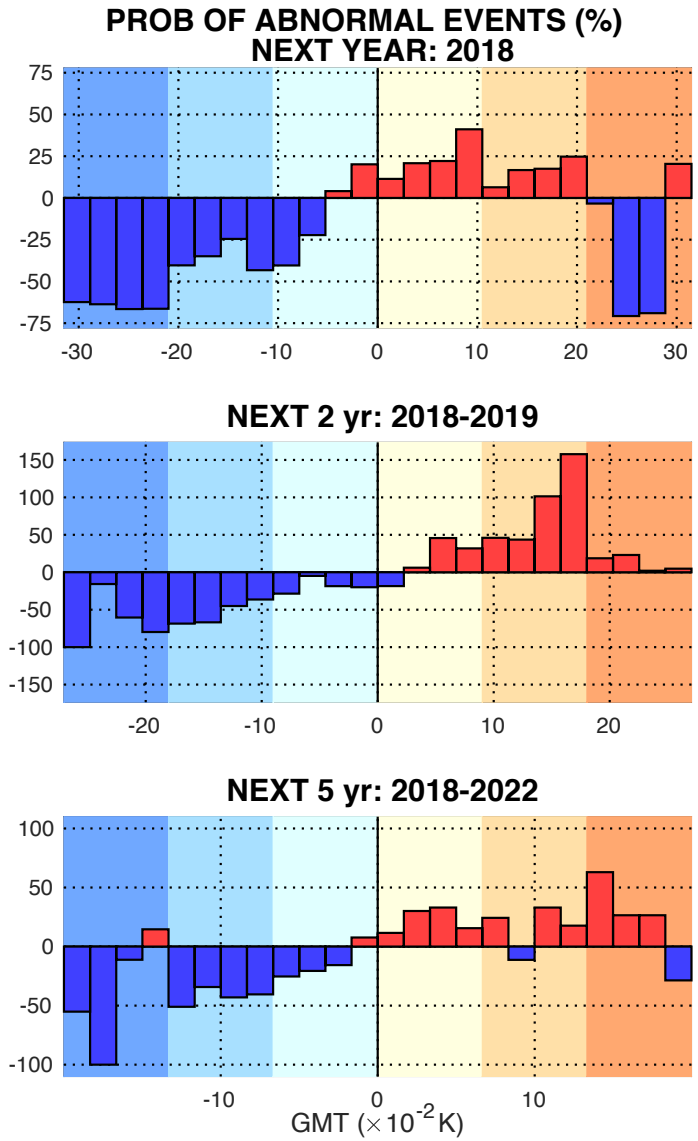
- Good qualitative and quantative prediction of the post-1998 hiatus of global warming.

- In hindsight, the post-1998 hiatus of global warming was predictable by PROCAST.

Transfer Operators and Analog Methods

Prediction for 2018-2022





Transfer Operators and Analog Methods

Final Conclusions

- Method
  - Fast and efficient method to predict GMT based on a range of Transfer Operators or Analog based on CMIP5 database
  - The dramatically reduced phase-space does not allow for Markov Chain.

- Result

  Toward anomalously warm years beyond the forced global warming

Literature

Transfer Operators:

- Froyland, G., K. Padberg, M. H. England, and A. M. Treguier, 2007: Detection of coherent oceanic structures via transfer operators. Phys. Rev. Lett., 98, 224503.
- Lucarini, V., 2016: Response operators for Markov processes in a finite state space: Radius of convergence and link to the re- sponse theory for Axiom A systems. J. Stat. Phys., 162, 312– 333.
- Sévellec, F., A. Colin de Verdière, A., and M. Ollitrault, 2017: Evolution of intermediate water masses based on argo float displacement. J. Phys. Oceanogr. 47, 1569–1586.
- Tantet, A., F. R. van der Burgt, and H. A. Dijkstra, 2015: An early warning indicator for atmospheric blocking events using transfer operators. Chaos, 25, 036406.
- van Sebille, E., M. H. England, and G. Froyland, 2012: Origin, dynamics and evolution of ocean garbage patches from observed surface drifters. Environ. Res. Lett., 7, 044040.

Analog Methods:

- Branstator, G. et al., 2012: Systematic estimates of initial-value decadal predictability for six AOGCMs. J. Clim. 25, 1827–1846.
- Hawkins, E., J. Robson, R. Sutton, D. Smith, and N. Keenlyside, 2011: Evaluating the potential for statistical decadal predictions of sea surface temperatures with a perfect model approach. Clim. Dyn. 37, 2495–2509.
- Ho, C. K., E. Hawkins, L. Shaffrey., and F. M. Underwood, 2013: Statistical decadal predictions for sea surface temperatures: a benchmark for dynamical gcm predictions. Clim. Dyn. 41, 917–935.
- Sévellec, F. and S. S. Drijfhout, 2018: A Novel Probabilistic Forecast System Predicting Anomalously Warm 2018-2022 Reinforcing the Long-Term Global Warming Trend, *Nature Communications*, 9, 3024.[