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Home Work 6

Discrete Structures (CS 5333)

Problem 1: Prove using general induction that:

$\forall m \geq 0 \, \forall l \geq m + 1 : f_l = f_{m+1} * f_{l-m} + f_m * f_{l-(m+1)}$; (1)

Where f_l is the l - th Fibonacci number, where f_0 = 0, f_1 = 1, etc.

Answer:

Let P(l)= $f_l = f_{m+1} * f_{l-m} + f_m * f_{l-(m+1)}$

Basic step: P(1)

$$f_1 = f_{0+1} * f_{1-0} + f_0 * f_{1-(0+1)} = f_1 * f_1 + f_0 * f_0 = 1, which\ is\ true.$$

Inductive step: In general mathematical induction , the induction step requires that if P(l) holds for all smaller than l, then it also holds for P(l).

Assume P(n) =$f_n = f_{m+1} * f_{n-m} + f_m * f_{n-(m+1)}$ is true, where n < l

$f_n = f_{n-1} + f_{n-2}$

- ⇨ $f_{m+1} * f_{n-1-m} + f_m * f_{n-1-(m+1)} + (f_{m+1} * f_{(n-2)-m} + f_m * f_{(n-2)-(m+1)})$
- ⇨ $f_{m+1}(f_{(n-m)-1} + f_{(n-m)-2}) + f_m(f_{(n-m)-2} + f_{(n-m)-3}$
- ⇨ $f_{m+1} * f_{n-m} + f_m * f_{(n-m)-1}$
- ⇨ $f_{m+1} * f_{n-m} + f_m * f_{n-(m+1)}$

Hence proved

Problem 2: Use Eqn. 1 to prove that:

$$\forall k \geq 1 \, \forall l \geq 1 \quad f_l \mid f_{k*l}.$$

Explain why for this problem, you did not use general induction.

Answer:

Basic step: k = 1, l = 1

P(1) is $f_1 | f_1$ = 1|1 which is true. Therefore P(1) is true.

From problem 1 let us consider m = (k-1)l, m>= 0

$$f_{kl} = f_{(k-1)l+1} * f_l + f_{(k-1)l} * f_{l-1}$$

From the above equation

$$f_{(k-1)l} = f_{(k-2)l+1} * f_l + f_{(k-2)l} * f_{l-1}$$

Substituting $f_{(k-1)l}$ in the first equation.

$$f_{kl} = f_{(k-1)l+1} * f_l + f_{(k-2)l+1} * f_l * f_{l-1} + f_{(k-2)l} * (f_{l-1})^2 \qquad \text{....... Eq 2}$$

From the above equation

$$f_{(k-2)l} = f_{(k-3)l+1} * f_l + f_{(k-3)l} * f_{l-1}$$

By substituting $f_{(k-2)l}$ in Eq2 we get

$$f_{kl} = f_l(f_{(k-1)l+1} + f_{(k-2)l+1} * f_{l-1} + f_{(k-3)l+1} * (f_{l-1})^2 + \ldots\ldots + (f_{l-1})^{k-2} f_{l+1}) + (f_{l-1})^{k-1} f_l$$

In the above equation f_l is common.

Therefore f_{kl} is divisible by f_l for all k>=1 and for all l>=1