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Home Work 2

Discrete Structures (CS 5333)

Page 126 – Exc.32: Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

a) $A \times B \times C$.

c) $C \times A \times B$.

Answer:

a) {(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1),
(b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1),
(c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)}

c) {(0, a, x), (0, a, y), (0, b, x), (0, b, y), (0, c, x), (0, c, y)
(1, a, x), (1, a, y), (1, b, x), (1, b, y), (1, c, x), (1, c, y)}

Page 136 – Exc.20: Show that if A and B are sets with $A \subseteq B$, then

a) $A \cup B = B$.

b) $A \cap B = A$.

Answer:

a) If x ∈ A U B, then x ∈ A or x ∈ B or both by definition, Given A ⊆ B, so if x ∈ A then x ∈ B. This proves A U B ⊆ B. Now if x ∈ B, then by definition x ∈ A U B, too, so x ∈ A U B. This proves B ⊆ A ∩ B. Together this implies A U B = B.

b) Given A ⊆ B, If x ∈ A ∩ B, then x ∈ A and x ∈ B by definition, so in particular x ∈ A. This proves A ∩ B ⊆ A. Now if x ∈ A, then by assumption x ∈ B, too, so x ∈ A ∩ B. This proves A ⊆ A ∩ B. Together this implies A ∩ B = A.

Page 136 – Exc.23. Let A, B, and C be sets. Show that $(A - B) - C = (A - C) - (B - C)$.

Answer: First we will show that (A-B)-C is a subset of (A-C)-(B-C), and then we will show that (A-C)-(B-C) is a subset of (A-B)-C. Showing these two things is equivalent to showing equality between the two sides.

1. (A-B)-C subset of (A-C)-(B-C)

To show this, we choose an arbitrary x in (A-B)-C. Now we have to show that x is in (A-C)-(B-C). Since x is in (A-B)-C, it holds that x is in A, x is not in B, and x is not in C. Therefore, it is also true that x is in A-C and x is not in B-C, so x is in (A-C)-(B-C), as well.

2. (A-C)-(B-C) subset of (A-B)-C

We proceed as in the above case. Let x be an arbitrary element in (A-C)-(B-C). From this we can conclude that x is in A-C and x is not in B-C. Since x is not in C and x is not in B-C, x is also not in B. So we know: x is in A, x is not in B, and x is not in C. Putting this together gives us that x is in (A-B)-C.

Page 137 – Exc.48: Let Ai = {. . . , −2, −1, 0, 1 . . . i}. Find

a) $\bigcup_{i=1}^{n} A_i$ b) $\bigcap_{i=1}^{n} A_i$

Answer:

a) $\bigcup_{i=1}^{n} A_i = A_n$= {. . . , −2, −1, 0, 1 . . . n}.

b) $\bigcap_{i=1}^{n} A_i = A_1$= {..., −2, −1, 0, 1}

Page 153 – Exc.10: Determine whether each of these functions from {*a*, *b*, *c*, *d*} to itself is one-to-one.

a) *f (a) = b, f (b) = a, f (c) = c, f (d) = d*
b) *f (a) = b, f (b) = b, f (c) = d, f (d) = c*
c) *f (a) = d, f (b) = b, f (c) = c, f (d) = d*

Answer:

- **a)** One-to-one.
- **b)** Not one-to-one, *f (a)* and *f (b)* points to *b.*
- **c)** Not one-to-one, *f (a)* and *f (d)* points to *d.*

Page 153 – Exc.22: Determine whether each of these functions is a bijection from **R** to **R**.

a) $f(x) = -3x + 4$
b) $f(x) = -3x^2 + 7$
c) $f(x) = (x + 1)/(x + 2)$
d) $f(x) = x^5 + 1$

Answer:

a) It is a bijection; f is both one-to-one and onto.

b) Not a bijection; f is not one-to-one. Example. f(1)=f(-1)=4.

c) Not a bijection; f is not defined at x=-2.

d) It is a bijection; f is both one-to-one and onto.

Page 154 – Exc.34. If *f* and $f \circ g$ are one-to-one, does it follow that *g* is one-to-one? Justify your answer.

Answer: Suppose *g* is not one-to-one; let distinct x and y be in the domain of g, such that g(x) = g(y). Then f(g(x)) = f(g(y)) = (f ∘ g)(x) = (f ∘ g)(y). Thus (f ∘ g) is not one-to-one. So g not one-to-one implies (f ∘ g) not one-to-one. By the contrapositive, (f ∘ g) is one-to-one implies g is one-to-one.

Page 154 – Exc.38: Let $f(x) = ax + b$ and $g(x) = cx + d$, where *a*, *b*, *c*, and *d* are constants. Determine necessary and sufficient conditions on the constants *a*, *b*, *c*, and *d* so that $f \circ g = g \circ f$.

Answer:

$f \circ g$ = a(cx + d) + b = acx + ad + b.

$g \circ f$ = c(ax + b) + d = acx + cb + d.

$f \circ g = g \circ f$

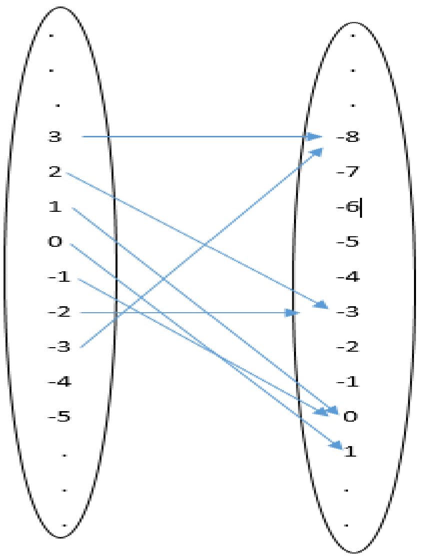
⇨ acx + ad + b = acx + cb + d

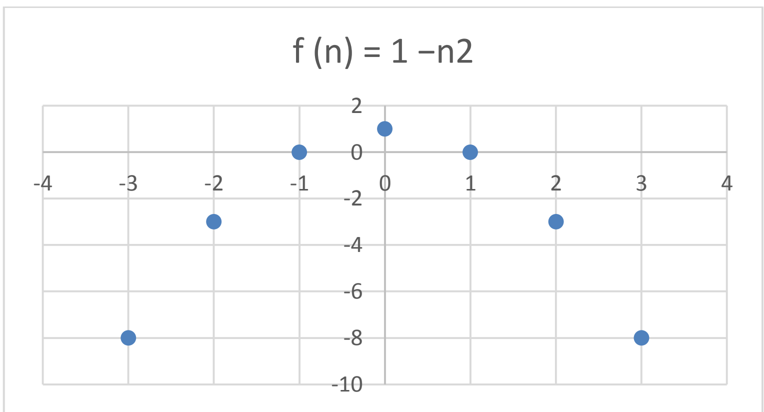
⇨ ad + b = cb + d.

Therefore the condition for $f \circ g = g \circ f$ is ad + b = cb + d.

Page 155 – Exc.62: Draw the graph of the function $f(n) = 1 - n^2$ from **Z** to **Z**.

Answer:





Page 169 – Exc.30: What are the values of these sums, where $S = \{1, 3, 5, 7\}$?

a) $\sum_{j \in S} j$ b) $\sum_{j \in S} j^2$ c) $\sum_{j \in S} \frac{1}{j}$ d) $\sum_{j \in S} 1$

Answer:

a) 1+3+5+7 = 16

b) $1^2 + 3^2 + 5^2 + 7^2 = 84$.

c) $\frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} = \frac{176}{105}$

d) 1+1+1+1 = 4

Page 169 – Exc.46: Find $\prod_{j=0}^{4} j!$.

Answer: $\prod_{j=0}^{4} j!$ = 0! x 1! x 2! x 3! x 4! = 1 x 1 x 2 x 6 x 24 = 288.

Page 819 – Exc.36: Show that in a Boolean algebra, every element x has a unique complement $\overline{x}$ such that $x \vee \overline{x} = 1$ and $x \wedge \overline{x} = 0$.

Answer: Assume $\overline{x}_1$ and $\overline{x}_2$ are both complements of x. Then $x \vee \overline{x}_1 = 1, x \vee \overline{x}_2 = 1, x \wedge \overline{x}_1 = 0$ and $x \wedge \overline{x}_2 = 0$.

$\overline{x}_1 = \overline{x}_1 \wedge 1$
- ⇨ $\overline{x}_1 \wedge (x \vee \overline{x}_2)$ since $x \vee \overline{x}_2 = 1$
- ⇨ $(\overline{x}_1 \wedge x) \vee (\overline{x}_1 \wedge \overline{x}_2)$ Distributive law
- ⇨ $0 \vee (\overline{x}_1 \wedge \overline{x}_2)$ as $\overline{x}_1 \wedge x = 0$ from above assumptions
- ⇨ $(x \wedge \overline{x}_2) \vee (\overline{x}_1 \wedge \overline{x}_2)$ as $x \wedge \overline{x}_2 = 0$
- ⇨ $(\overline{x}_2 \wedge x) \vee (\overline{x}_2 \wedge \overline{x}_1)$
- ⇨ $\overline{x}_2 \wedge (x \vee \overline{x}_1)$ Distributive law
- ⇨ $\overline{x}_2 \wedge 0$
- ⇨ $\overline{x}_2$

The two complements $\overline{x}_1$ and $\overline{x}_2$ of x are equal.
Therefore x has a unique complement $\overline{x}$.

Page 819 – Exc.40: Show that in a Boolean algebra, the **modular properties** hold. That is, show that $x \wedge (y \vee (x \wedge z)) = (x \wedge y) \vee (x \wedge z)$ and $x \vee (y \wedge (x \vee z)) = (x \vee y) \wedge (x \vee z)$.

Answer:

1) $x \wedge (y \vee (x \wedge z))$
- ⇨ $(x \wedge y) \vee (x \wedge (x \wedge z))$ Distributive law with $a = x$, $b = y$, $c = (x \wedge z)$
- ⇨ $(x \wedge y) \vee ((x \wedge x) \wedge z)$ Associative law
- ⇨ $(x \wedge y) \vee ((x \wedge z)$ as $(x \wedge x) = x$

2) $x \vee (y \wedge (x \vee z))$
- ⇨ $(x \vee y) \wedge (x \vee (x \vee z))$ Distributive law with $a = x$, $b = y$, $c = (x \vee z)$
- ⇨ $(x \vee y) \wedge (x \vee x) \vee z)$ Associative law
- ⇨ $(x \vee y) \wedge (x \vee z)$ as $(x \vee x) = x$