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Home Work 1

Discrete Structures (CS 5333)

Page 15 - Exc.30: How many rows appear in a truth table for each of these compound propositions?

a) $(q \rightarrow \neg p) \vee (\neg p \rightarrow \neg q)$

b) $(p \vee \neg t) \wedge (p \vee \neg s)$

c) $(p \rightarrow r) \vee (\neg s \rightarrow \neg t) \vee (\neg u \rightarrow v)$

d) $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$

Answers:

a) 4 (2^2 – Two variables *p* and *q*)
b) 8 (2^3 – Three variables *p*, *s* and *t*)
c) 64 (2^6 – Six variables *p*, *r*, *s*, *t*, *u* and *v*)
d) 32 (2^5 – Five variables *p*, *q*, *r*, *s* and *t*)

Page 15 - Exc.32: Construct a truth table for each of these compound propositions.

a) $p \rightarrow \neg p$

d) $(p \wedge q) \rightarrow (p \vee q)$

e) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$

Answers:

a)

p	$\neg p$	$p \rightarrow \neg p$
T	F	F
F	T	T

d)

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

e)

p	q	$\neg p$	$q \rightarrow \neg p$	$p \leftrightarrow q$	$(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

Page 15 - Exc.34: Construct a truth table for each of these compound propositions.

d) $\neg p \oplus \neg q$

Answer:

p	q	$\neg p$	$\neg q$	$\neg p \oplus \neg q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Page 16 - Exc.30: Evaluate each of these expressions.

a) 1 1000 ∧ *(*0 1011 ∨ 1 1011*)*

b) *(*0 1111 ∧ 1 0101*)* ∨ 0 1000

c) *(*0 1010 ⊕ 1 1011*)* ⊕ 0 1000

d) *(*1 1011 ∨ 0 1010*)* ∧ *(*1 0001 ∨ 1 1011*)*

Answers:

a) 1 1000 ∧ (0 1011 ∨ 1 1011)
1 1000 ∧ 1 1011
1 1000

b) (0 1111 ∧ 1 0101) ∨ 0 1000
0 0101 ∨ 0 1000
0 1101

c) (0 1010 ⊕ 1 1011) ⊕ 0 1000
1 0001 ⊕ 0 1000
1 1001

d) (1 1011 ∨ 0 1010) ∧ (1 0001 ∨ 1 1011)
1 1011 ∧ 1 1011
1 1011

Page 35 – Exc 24: Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.

Answer:
For $(p \rightarrow q) \vee (p \rightarrow r)$ to be false both the conditional statements must be false. Which happens when p is true and both q and r are true. For all other cases it is true.

The conditional statement $p \rightarrow (q \vee r)$ is false when p is true and $(q \vee r)$ is false which implies both q and r must be false. It is true for all other cases. Therefore both the statements are logically equivalent.

$(p \rightarrow q) \vee (p \rightarrow r)$

$\equiv (\neg p \vee q) \vee (\neg p \vee r)$ as $(p \rightarrow q \equiv \neg p \vee q)$

$\equiv (\neg p \vee \neg p) \vee (q \vee r)$ Associative law

$\equiv \neg p \vee (q \vee r)$

$\equiv p \rightarrow (q \vee r)$ $(\neg p \vee s \equiv p \rightarrow s)$ *here* $s = (q \vee r)$

Page 36 - Exc.46: Construct a truth table for the logical operator *NAND*.

Answer:

p	q	p | q
T	T	F
T	F	F
F	T	F
F	F	T

Page 36 - Exc.58: How many of the disjunctions $p \vee \neg q$, $\neg p \vee q$, $q \vee r$, $q \vee \neg r$, and $\neg q \vee \neg r$ can be made simultaneously true by an assignment of truth values to p, q, and r?

Answer: All of them as they are independent on each other.

p	q	r	¬p	¬q	¬r	p ∨¬q	¬p ∨ q	q ∨ r	q ∨¬r	¬q ∨¬r
T	T	T	F	F	F	T	T	T	T	F
T	T	F	F	F	T	T	T	T	T	T

T	F	T	F	T	F	T	F	T	F	T
T	F	F	F	T	T	T	F	F	T	T
F	T	T	T	F	F	F	T	T	T	F
F	T	F	T	F	T	F	T	T	T	T
F	F	T	T	T	F	T	T	T	F	T
F	F	F	T	T	T	T	T	F	T	T

Page 125 - Exc.6: Suppose that $A = \{2, 4, 6\}$, $B = \{2, 6\}$, $C = \{4, 6\}$, and $D = \{4, 6, 8\}$. Determine which of these sets are subsets of which other of these sets.

Answer:

B and C are subsets of A
C is also a subset of D.

Page 152 – Exc.2: Determine whether f is a function from **Z** to **R** if

a) $f(n) = \pm n$.

b) $f(n) = \sqrt{(n^2 - 1)}$

c) $f(n) = 1/(n^2 - 4)$.

Answers:

a) f is a function from **Z** to **R.**

b) f is not a function from **Z** to **R.** It is not defined when $n = 0$.

c) f is not a function from **Z** to **R.** It is not defined when $n = 2$.

Page 53 - Exc.14: Determine the truth value of each of these statements if the domain consists of all real numbers.

a) $\exists x(x^3 = -1)$

b) $\exists x(x^4 < x^2)$

c) $\forall x((-x)^2 = x^2)$

d) $\forall x(2x > x)$

Answers:

a) True

b) True

c) False

d) False

Page 53 – Exc.18: Suppose that the domain of the propositional function *P(x)* consists of the integers −2, −1, 0, 1, and 2. Write out each of these propositions using disjunctions, conjunctions, and negations.

c) $\exists x \neg P(x)$

d) $\forall x \neg P(x)$

f) $\neg \forall x P(x)$

Answers:

c) $\neg P(-2) \vee \neg P(-1) \vee \neg P(0) \vee \neg P(1) \vee \neg P(2)$

d) $\neg P(-2) \wedge \neg P(-1) \wedge \neg P(0) \wedge \neg P(1) \wedge \neg P(2)$

f) $\neg (P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2))$

Page 55 - Exc.36: Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.

a) $\forall x(x^2 \neq x)$

b) $\forall x(x2 \neq 2)$

c) $\forall x(|x|>0)$

Answers:

a) $x = 1$
b) $x = \sqrt{2}$
c) $x = 0$

Page 56 - Exc. 50: Show that $\forall x P(x) \vee \forall x Q(x)$ and $\forall x(P(x) \vee Q(x))$ are not logically equivalent.

Answer:
If we can show that for a value *x* the truth values of the two statements are different then they are not logically equivalent.

For example if a is in the domain, such that $\exists$xP(a) is true, $\exists$xQ(a)is true and P(a) V Q(a) is true always. Then $\forall x(P(x) \vee Q(x))$ is true and $\forall x P(x) \vee \forall x Q(x)$ is false.

Therefore $\forall x P(x) \vee \forall x Q(x)$ and $\forall x(P(x) \vee Q(x))$ are not logically equivalent.

Page 67 - Exc.32: Express the negations of each of these statements so that all negation symbols immediately precede predicates.

a) $\exists z \forall y \forall x T(x, y, z)$

c) $\exists x \exists y(Q(x, y) \leftrightarrow Q(y, x))$

Answers:

a) $\forall z \exists y \exists x \neg T(x, y, z)$

b) $\forall x \forall y(Q(x, y) \oplus Q(y, x))$

Page 68 - Exc.40: Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

a) $\forall x \exists y(x = 1/y)$

b) $\forall x \exists y(y^2 - x < 100)$

c) $\forall x \forall y(x^2 \neq y^3)$

Answers:

a) $x = 3$. $\forall x \exists y(x = 1/y)$ this to be true, For every x there should be a y such that ($x = 1/ y$) is true. If $x = 3$ then there is no y in integers that can make this statement ($x = 1/ y$) true.

b) $y = 0$, $x = 100$. If $x < -100$ then what ever the y value be, the result is greater than 100.

c) $x = 1$, $y = 1$. For $\forall x \forall y(x^2 \neq y^3)$ to be true $(x^2 \neq y^3)$ should be true for every pair x, y.

Page 68 - Exc.42: Use quantifiers to express the distributive laws of multiplication over addition for real numbers.

Answer:
The distributive law of multiplication over addition says that if x, y and z are any numbers.

$x(y+z) = xy + xz$. It is true for all x,y and z. Therefore it can be expressed as

$\forall x \forall y \forall z(x (y + z)) = \forall x \forall y \forall (xy + xz)$.

Page 68 - Exc.48: Show that $\forall x P(x) \vee \forall x Q(x)$ and $\forall x \forall y(P(x) \vee Q(y))$, where all quantifiers have the same nonempty domain, are logically equivalent. (The new variable *y* is used to combine the quantifications correctly.)

Answer:

Suppose $\forall x P(x) \vee \forall x Q(x)$ is true, then either $P(x)$ is true $\forall x$ *or* $Q(y)$ is true $\forall y$. In the former case $P(x)$ v $Q(y)$ is true $\forall x$. so $\forall x \forall y(P(x) \vee Q(y))$. In the latter case, $Q(y)$ is true for all y, so $P(x)$ v $Q(y)$ is true for all y and consequently $\forall x \forall y(P(x) \vee Q(y))$ is true. Conversely suppose that second proposition is true. If $P(x)$ is true for all x, then the first proposition is true. If not, $P(x)$ is false for some x then there exists y such that $Q(y)$ is true. It follows that the first proposition must hold.