Supplementary Notes: Hypothesis Testing

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Inference: Hypothesis Testing

- Hypothesis tests for a single coefficient (t-statistic)
  - In large samples (always true in ESM 296) t-statistic is distributed like a normal random variable

- Hypothesis tests for group of coefficients (F-statistic)
  - In large samples F-statistic is distributed like a chi-square random variable

- Examples with house values and NOx data

Steps to Conduct Single Hypothesis Test

- Step 1. Set up regression model:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$$

- Y_i = *house value*, X_{1i} = *NOx*, X_{2i} = *Rooms*

- Step 2. Specify the null hypothesis to be tested

$$H_0: \beta_1 = 0 \qquad H_1: \beta_1 \neq 0$$

- Step 3. Estimate the <u>unrestricted</u> regression (under H_1)

- Step 4. Construct test-statistic and derive its sampling distribution under the null hypothesis

- Step 5. Conclude

Proceeding:

- Under the null hypothesis above, and under LSA1, LSA2, LSA3, and LSA4, the t-statistic has the following sampling distribution:

$$\hat{t} = \frac{\hat{\beta}_1 - 0}{SE(\hat{\beta}_1)} \overset{A}{\cong} N(0,1)$$

- More generally, if we want to test the hypothesis that β_1 equals a number different than zero, we need to replace "0" with the desired number in the t-statistic above

- Calculate the test-statistic, if its absolute value is greater than 2 (or 1.96), reject the null hypothesis at the 5% significance level

- The test-statistic testing the null that each $\beta_j = 0$, along with its p-value are automatically reported by STATA

- Recall that the p-value gives the probability of rejecting H_0 given that H_0 is "true" (Prob of Type I error)

- Another way to think about the p-value is as the smallest significance level at which the null hypothesis can be rejected

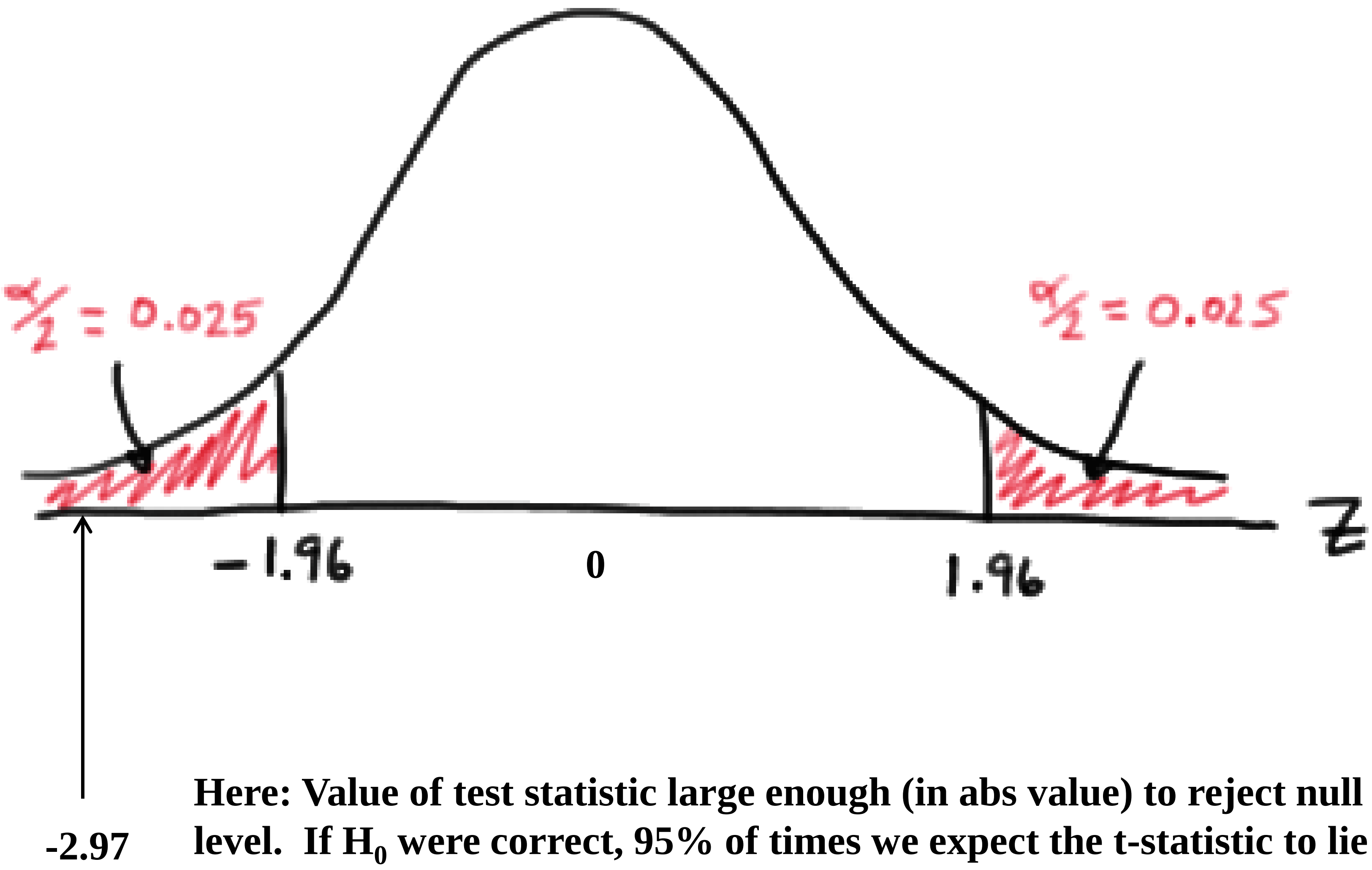
STATA Application

```
.  regress price nox rooms, robust;

Linear regression                                      Number of obs =     206
                                                       F(  2,   203) =   78.47
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.5923
                                                       Root MSE      =  6019.3

------------------------------------------------------------------------------
             |               Robust
       price |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         nox |  -1062.208   357.8614    -2.97   0.003    -1767.811   -356.6063
       rooms |   9836.748   924.3718    10.64   0.000     8014.146    11659.35
       _cons |  -33216.07   6655.565    -4.99   0.000    -46338.97   -20093.17
------------------------------------------------------------------------------
```

```
. test nox=-1000;

( 1)  nox = -1000
      F(  1,   203) =    0.03
           Prob > F =    0.8622
```

Here: a test of H_0: $\beta_1 = -1000$

Graphical representation:



Here: Value of test statistic large enough (in abs value) to reject null at 5% level. If H_0 were correct, 95% of times we expect the t-statistic to lie between -1.96 and 1.96

Testing a joint hypothesis

- Continuing with the same regression model:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$$

- Suppose you are interested in testing if the marginal effect of 5 extra units of NOx pollution (which is expected to be negative) is the same as the marginal effect of 1 extra room (positive)
  - (This is not a very compelling hypothesis, just for illustrating the method)

- The corresponding hypothesis test is:

H_0: $5*\beta_1 = -\beta_2$ H_1: $5*\beta_1 \neq -\beta_2$

F-statistic

- The F-statistic is used to test joint hypothesis about regression coefficients

- Let "U" denote the unrestricted regression (estimated under H_1) and "R" denote the restricted regression (estimated under H_0). The number of restrictions in denoted by "q"

- The F-statistic is: $\hat{F} = \dfrac{(SSR_R - SSR_U)/q}{SSR_U/(n-K)} \simeq F(q, n-K)$

- Where: $SSR = \sum_{i=1}^{n} \hat{u}_i^2$

F-statistic: notes

- 1. The formula above is technically correct only with homoskedastic errors

- In STATA applications, when using the “,robust” command, the theoretically (i.e. under heteroskedasticity) correct F-statistic is reported

- 2. In large samples (i.e., $n \rightarrow \infty$) the F-statistic becomes distributed like a chi-square random variable with “q” degrees of freedom

STATA Application

```
. regress price nox rooms, robust;

Linear regression                                      Number of obs =     206
                                                       F(  2,   203) =   78.47
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.5923
                                                       Root MSE      =  6019.3

------------------------------------------------------------------------------
             |               Robust
       price |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
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         nox |  -1062.208   357.8614    -2.97   0.003    -1767.811   -356.6063
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       _cons |  -33216.07   6655.565    -4.99   0.000    -46338.97   -20093.17
------------------------------------------------------------------------------

. test 5*nox=-rooms;

 ( 1)  5*nox + rooms = 0

       F(  1,   203) =    4.03
            Prob > F =    0.0461
```