Lecture 9: Instrumental Variables Methods

- Outline:

- Introduction

- Identifying assumptions, potential outcomes

- Discussion on nature of treatment effect (constant or heterogenous)

- Various estimators and their interpretation

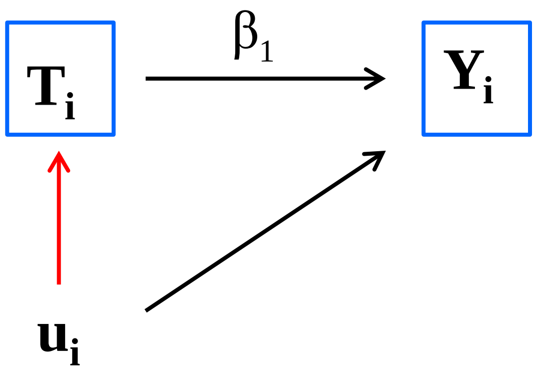
- Application: estimating the demand curve for fish at Fulton Fish Market

- Readings: A&P Chapter 4, I&W Lecture 5, Kendall (2015)

Motivation: Omitted variables bias

- The essence of OVB is a relationship between u_i and T_i

- In a path diagram:

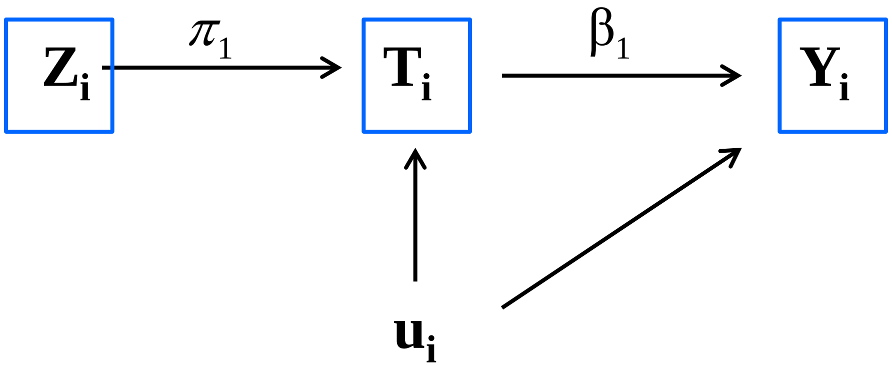


Red arrow is
a violation
of LSA#1

- Since we don't observe data on u_i, we cannot control for it's effect on T_i, and that influence confounds the measured relationship between Y_i and T_i

Instrumental variables in a path diagram

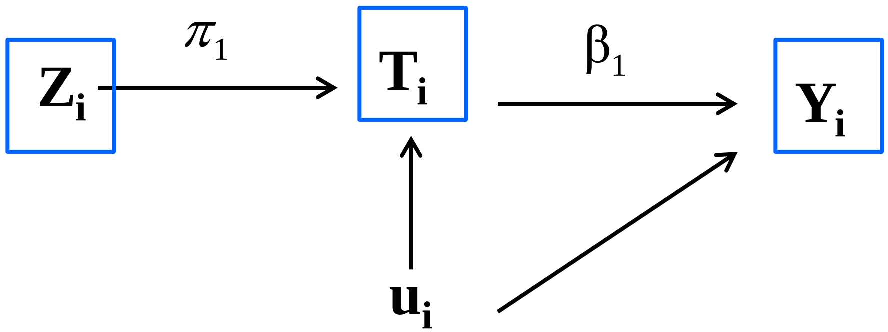
- Denote an instrumental variable as Z_i:



- Note the key assumption: <u>no link between u_i and Z_i</u>.
  - That assumption is valid if the natural experiment that generates Z_i operates through <u>exogenous</u> forces
  - Not really testable...

- Also note the assumption that <u>Z_i is correlated with T_i</u>
  - Testable...

How to Proceed?



- 1. Regression of Z_i on T_i identifies π_1
- 2. Regression of Z_i on Y_i identifies $\pi_1 \times \beta_1$
- Ratio is β_1 (basis for an IV estimator)
- Two requirements: **<u>i.</u>** Instrument predicts T_i ($\pi_1 \neq 0$) and **<u>ii.</u>** Instrument exogenous (uncorrelated with u_i). Plausible if $Z_i \sim$ (quasi) randomly assigned

Key Assumptions:

- Suppose you can observe a variable Z_i such that:
- **A1: Independence / exogeneity / exclusion**
- $Z_i \perp (Y_i(0), Y_i(1)) \mid X_i$
  - "Z_i independent of potential outcomes $Y_i(0)$ and $Y_i(1)$ conditional on X_i". [Remember u_i defined by $Y_i(0)$]
  - Typical interpretation as exclusion restriction "Z_i only affects Y_i through its effect on the endogenous regressor" is implied by A1
  - <u>A1 not really testable</u>
- **A2: First-stage / relevance**
- $\pi_1 \neq 0$ in $T_i = \pi_0 + Z_i'\pi_1 + X_i'\pi_2 + v_i$
  - <u>A2 testable</u>
- $\Rightarrow$ We can estimate ATE or LATE using IV methods

Constructing instruments with natural experiments

- Suppose you are interested in measuring the causal effect of military service on civilian earnings:

- Is there a correlation between u_i (unmeasured labor market skills) and T_i (military service indicator)? (i.e. LSA#1 valid?)
  - Depends on applications to military and screening process used by military administration

- Vietnam draft lottery: Between 1970-1972, the military drafted individuals according to "call numbers" (RSN) that were randomly assigned (each birth date was randomly assigned one)

- Men with RSN below a cutoff were eligible to be drafted while men with RSN above the cutoff could not
  - 1. RSN randomly assigned
  - 2. RSN imperfectly predicts military service: deferments (health, college), volunteering, draft-dodging

- $\Rightarrow$ Cohorts randomly assigned "incentive" to join military
  - In other words, a randomly assigned variable changes the probability that an individual serves in military

- Can use this to form an instrumental variables for military service for cohorts at risk of being drafted in 1970-72.
  - Define Z_i = $\mathbf{1}(RSN_i < cutoff)$. Then Z_i (~randomly determined) is plausibly independent of u_i (so A1 likely satisfied here)
  - If Z is correlated with T (A2) then can proceed with IV method

Not As Good Example of Natural Experiment

- County of residence at age 14 predicts college attendance
  - Individuals living near a college more likely to attend college than those living further from one (A2 holds)

- Location decisions made by parents, so it is exogenous to the teenager's college enrollment decision – but parents who value education more, have higher SES, etc may decide to live closer to a college
  - Individuals assigned "incentive" to attend college based on parental decisions, so not randomly determined

- If ability/motivation is transmitted across generations, this 'natural experiment' will not deliver a 100% valid estimate of the causal effect of college attendance
  - In other words, Cov(Z_i,u_i) may not equal zero (A1 fails)

What can we learn from the IV method?

- Unlike the models that rely on the assumption of treatment ignorability, what we can learn from the IV estimator depends on whether the effects of the treatment are <u>constant</u> (same for all) or <u>heterogeneous</u>

- Constant treatment effect: $Y_i(1) - Y_i(0) = \beta_1$ for all i.
  - In this case, a valid IV will identify ATE
  - We spend more time on this model

- Heterogenous treatment effect: $Y_i(1) - Y_i(0) = \beta_{1i}$
  - In this case, a valid IV will identify a new causal parameter called "LATE" for local ATE
  - *Requires some additional notation and assumptions*

Overview and Motivation:

- Several problems in applied econometrics are caused by a correlation between the regression error term and the regressors (violation of LSA#1):
  - Measurement error in T (see Kendall's paper)
  - Omitted variable bias
  - Simultaneous causality

- Instrumental variables regression provides a general approach to obtain a consistent estimator of regression coefficients in that case
  - But... you need to have a valid instrument in your data set to implement this approach
  - Often difficult to find a good one

Some terminology

- An <u>endogenous</u> variable is one that is <u>correlated</u> with u

- An <u>exogenous</u> variable is one that is <u>uncorrelated</u> with u

- Historical note: "Endogenous" literally means "determined within the system," that is, a variable that is jointly determined with Y, and so a variable subject to simultaneous causality problem

- We use the term *endogenous* more broadly since IV regression can be used to address omitted variables bias and errors-in-variable bias, not just to simultaneous causality bias

Basic Idea of IV Regression:

- Consider the basic linear regression model:

$$Y_i = \beta_0 + \beta_1 T_i + u_i$$

- Suppose T_i is correlated with u_i (T is endogenous)

- Consider decomposing T_i into two variables, P_{1i} and P_{2i}:

$$T_i = P_{1i} + P_{2i}$$

- Where P_{1i} is <u>uncorrelated</u> with u_i and P_{2i} is <u>correlated</u> with u_i

- The method of instrumental variables uses the "extra" information we have (the instrument) to isolate/extract the variation in T_i that is uncorrelated with u_i (i.e., the variation in P_{1i}) to estimate β_1

IV Regression in One Regressor Model

- Again, consider the basic linear regression model with constant treatment effect:

$$Y_i = \beta_0 + \beta_1 T_i + u_i$$

- If $E[u_i|T_i]\neq 0$, the OLS estimator of β_1 is not consistent

Two conditions for a valid instrument (Z_i) in this model:

IV1. "Instrument relevance": $\mathrm{corr}(Z_i,T_i)\neq 0$

IV2. "Instrument exogeneity": $\mathrm{corr}(Z_i,u_i)=0$

- Under IV1 and IV2 you can estimate β_1 consistently using IV regression
  - IV2 implied by the assumption $Z_i \perp (Y_i(0), Y_i(1))$

Two Approaches to Estimate β_1 by IV Regression

- 1. Two Stage Least Squares (TSLS) Estimator

- 2. Instrumental Variables (IV) Estimator

- Under Assumptions IV1 and IV2 (plus a few technical ones):
  - Both estimators consistent and numerically equivalent in the 1 variable model
  - Both consistent (although not numerically equivalent) in the multivariate model (next lecture)

Two Stage Least Squares:

- **<u>First-stage:</u>** decompose T_i into the “good” and “bad” variation (i.e. isolate variation in T uncorrelated with u)

$$T_i = \underbrace{\pi_0 + \pi_1 Z_i}_{} + v_i$$

- **<u>Two components of T_i:</u>**
- <u>Exogenous component</u>: $\pi_0 + \pi_1 Z_i$. This is uncorrelated with u_i by assumption IV2 (the “good”, i.e. P_{1i} in previous slide)

- <u>Endogenous component</u>: v_i. This is correlated with u_i (the “bad”, i.e., P_{2i} in previous slide)

- The TSLS estimator only use $\pi_0 + \pi_1 Z_i$ (instead of the full $T_i = \pi_0 + \pi_1 Z_i + v_i$) to estimate β_1

- ...The TSLS estimator use $\pi_0 + \pi_1 Z_i$ to estimate β_1

- But we do not know the regression coefficients π_0 and π_1, so we need to consistently estimate them

- ***<u>First-stage regression</u>***: Estimate π_0 and π_1 by and OLS regression of T_i on Z_i, and construct the predicted value of T_i:

$$\hat{T}_i = \hat{\pi}_0 + \hat{\pi}_1 Z_i$$

- ***<u>Second-stage regression</u>***: Estimate β_1 by regressing Y_i on $\hat{T}_i$ by OLS:

$$Y_i = \beta_0 + \beta_1 \hat{T}_i + u_i$$

- Because $\hat{T}_i$ is uncorrelated with u_i (if n is large if IV1 and IV2 hold), then the first least squares assumption (LSA#1) holds

- Thus β_1 can be consistently estimated by OLS in the second-stage regression

- This argument relies on "n" being large (so π_0 and π_1 are well estimated in first stage regression)

- The resulting estimator is called the Two Stage Least Squares (TSLS) estimator ($\hat{\beta}_1^{TSLS}$)

"Instrumental Variable Estimator"

- Consider the 1 variable regression model:

$$Y_i = \beta_0 + \beta_1 T_i + u_i$$

- Suppose you have a valid IV, that satisfies the instrument relevance and instrument exogeneity assumptions

- Implications:
- $\text{Cov}(Y_i , Z_i) = \text{Cov}(\beta_0 + \beta_1 T_i + u_i , Z_i)$

$$= \text{Cov}(\beta_0, Z_i) + \text{Cov}(\beta_1 T_i, Z_i) + \text{Cov}(u_i , Z_i)$$

$$= \beta_1 \text{Cov}(T_i , Z_i)$$

- since $\text{Cov}(u_i , Z_i) = 0$ (by instrument exogeneity)

- ***Therefore:***
- $Cov(Y_i, Z_i) = \beta_1 \mathrm{Cov}(T_i, Z_i)$
- And so: $\beta_1 = \dfrac{Cov(Y_i, Z_i)}{Cov(T_i, Z_i)}$
- Note that instrument relevance assumption (IV1) is key
- The IV estimator replaces the population covariances with sample covariances:

$$\hat{\beta}_1^{IV} = \frac{S_{YZ}}{S_{TZ}}$$

- In the 1 variable regression model, TSLS and IV estimators are numerically the same

Classic Application of IV Regression:

- IV regression was originally developed to estimate <u>demand</u> elasticities for agricultural goods, for example coffee:

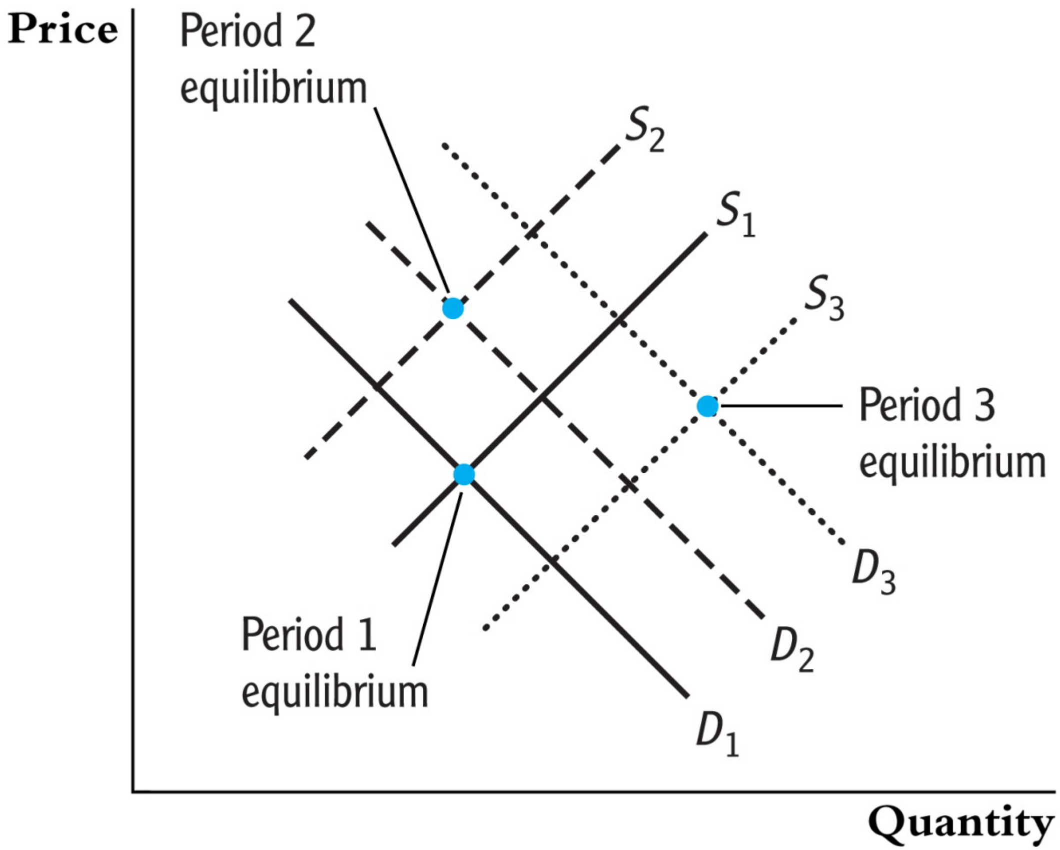
$$\log(Q_i^{coffee}) = \beta_0 + \beta_1 \log(P_i^{coffee}) + u_i$$

- β_1 = price elasticity of coffee demand = percent change in quantity for a 1% change in price (recall log-log specification in lecture 5)

- Data: observations on price and quantity of coffee for different years

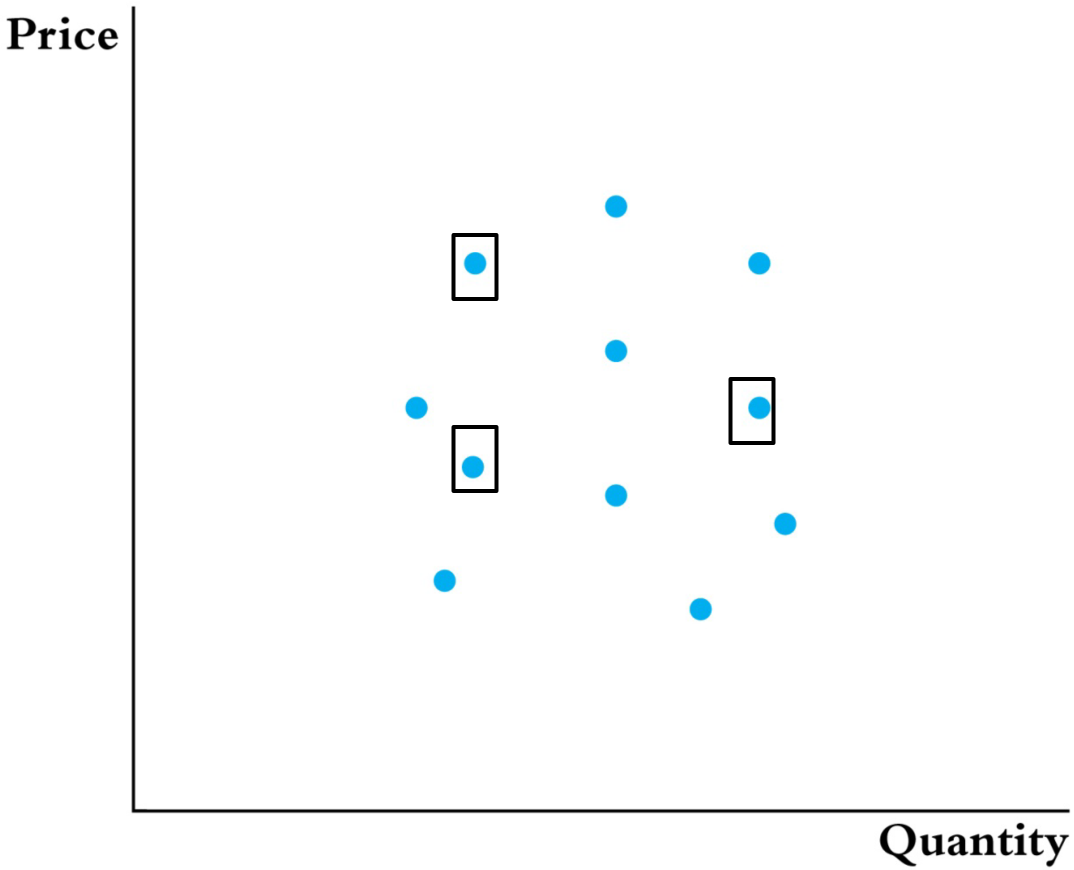
- $\Rightarrow$ This regression will suffer from simultaneous causality bias (LSA#1 does not hold)

- Simultaneous causality bias in the OLS regression of $\log(Q_i)$ on $\log(P_i)$ arises because price and quantity are determined by the interaction of demand *and* supply



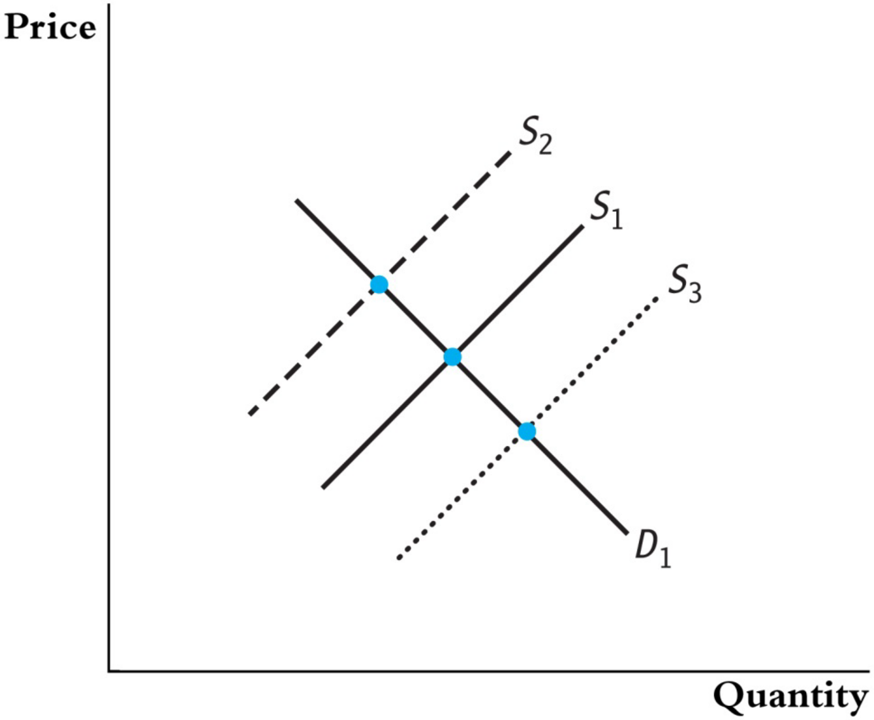
(a) Demand and supply in three time periods

- Can we identify demand elasticity from these data points?



(b) Equilibrium price and quantity for 11 time periods

- What if only supply curve shifted?



(c) Equilibrium price and quantity when only the supply curve shifts

- TSLS estimates the demand curve by isolating shifts in price and quantity that only arise from shifts in the supply curve

- $\Rightarrow$ here: Z is a variable that shifts supply but not demand

TSLS Approach:

$$\log(Q_i^{coffee}) = \beta_0 + \beta_1 \log(P_i^{coffee}) + u_i$$

- Let Z = measures of extreme weather (drought, flood, heat wave, frost, etc) in coffee-producing regions

- Is Z a valid instrument?
- (1) Exogenous? corr(Z_i,u_i) = 0?
  - *Plausible?*: whether there is a drought in coffee-producing regions should not affect residual demand <u>here</u>

- (2) Relevant? corr(Z_i,log(P_i))$\neq$ 0?
  - *Plausible?*: Drought means less coffee produced by coffee plants, and less available here, so expect positive correlation

- **TSLS in this context:**

- Stage 1: regress $\log(P_i^{coffee})$ on Z, get $\hat{\log}(P_i^{coffee})$
- $\hat{\log}(P_i^{coffee})$ isolates changes in log price that arise from supply shifts due to weather shocks (Z)

- Stage 2: regress $\log(Q_i^{coffee})$ on $\hat{\log}(P_i^{coffee})$

- The TSLS estimator in this context traces out the demand curve by exploiting shifts in supply curve caused by exogenous weather shocks

Application: Fulton Fish Market Data

Review of Economic Studies (2000) **67**, 499–527
0034-6527/00/0000000$02.00

The Interpretation of Instrumental Variables Estimators in Simultaneous Equations Models with an Application to the Demand for Fish

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Fulton Fish Market Data

- Daily data on whiting fish traded at the Fulton fish market in New York City

- Fish sold at the Fulton fish market by a small number of dealers to a large number of buyers

- Sample period:

111 days between December & 1991 and May 1992

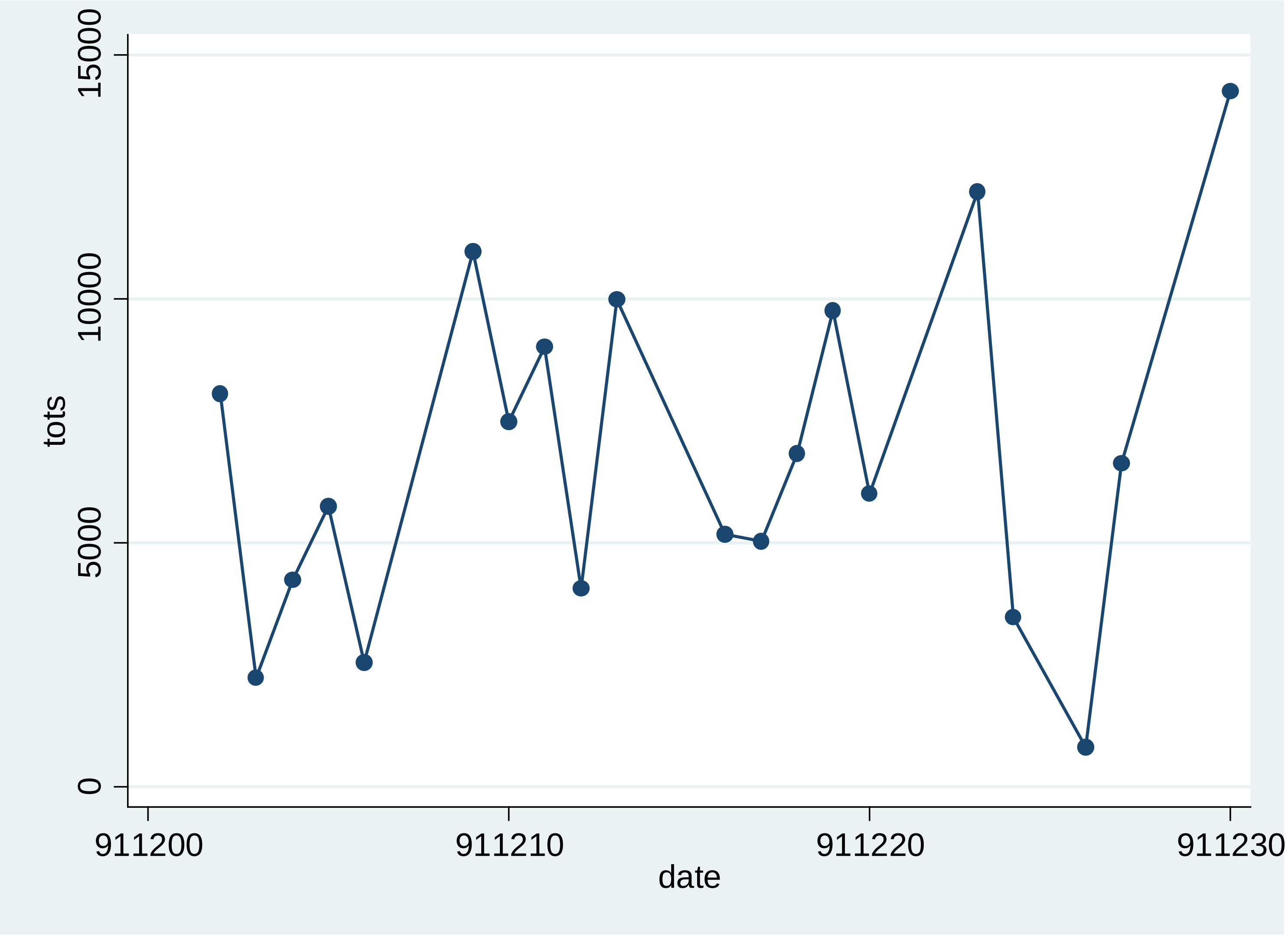
- Examine daily quantity sold (Q_t) and price (P_t)

Fulton Fish Market Data: Summary Statistics

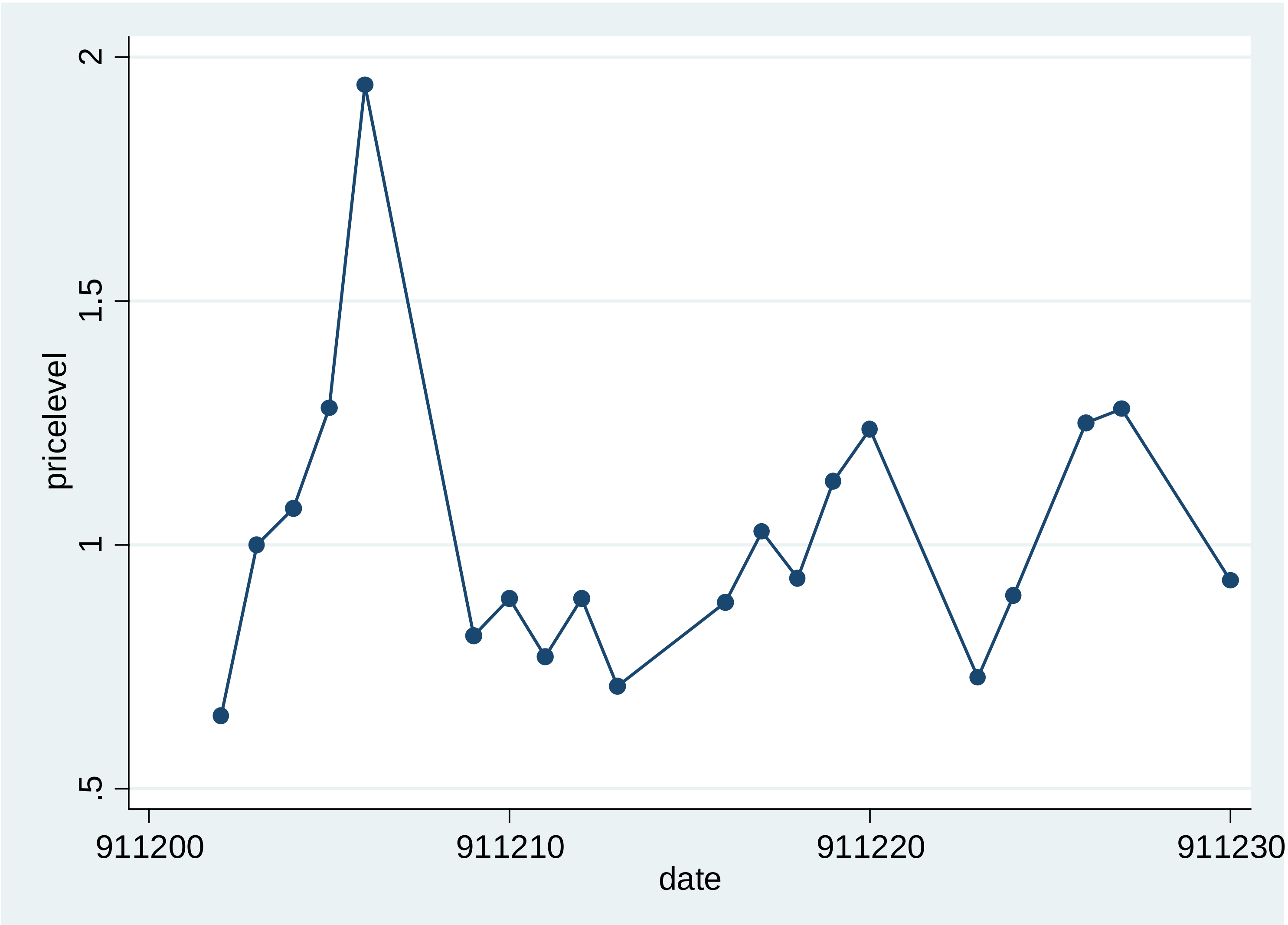
```
. summm tots pricelevel;

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
        tots |       111    6325.477    4050.279        490      21620
  pricelevel |       111    .8845243    .3351128    .330303   1.943182
```

Fulton Fish Market Data: Daily Quantity Sold (lbs)



Fulton Fish Market Data: Daily Avg Price ($/lbs)



OLS regression of log(Q) on log(P)

```
. gen logQ=log(tots);
. gen logP=log(pricelevel);

. regress logQ logP, robust;

Linear regression                                      Number of obs =     111
                                                       F(  1,   109) =   11.86
                                                       Prob > F      =  0.0008
                                                       R-squared     =  0.0859
                                                       Root MSE      =  .71019

------------------------------------------------------------------------------
             |               Robust
        logQ |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        logP |  -.5675713    .164801    -3.44   0.001    -.8942015   -.2409411
       _cons |   8.412259   .0752359   111.81   0.000     8.263144    8.561374
------------------------------------------------------------------------------
```

Price elasticity = -0.57, so a 1% increase in the price of fish leads to a -0.57% decline in quantity of fish demanded. Likely biased due to simultaneous causality

First stage regression
Instrument = Z = wind speed

```
                T      Z
. regress logP windspd, robust;
Linear regression                                      Number of obs =     111
                                                       F(  1,    109) =   18.81
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.1641
                                                       Root MSE      =  .35079

------------------------------------------------------------------------------
             |               Robust
        logP |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     windspd |   .7416438   .1709903     4.34   0.000     .4027467    1.080541
       _cons |  -2.316781   .4876912    -4.75   0.000    -3.283369   -1.350193
------------------------------------------------------------------------------
```

Higher wind speeds lead to higher fish prices (sorry no units on data file)

⇒ "Instrument relevance" condition satisfied (t-ratio on windspd is 4.34). More on this next lecture

```
                 X-hat
. predict logP_hat;
```

Now we have the predicted values from the 1st stage

Second stage regression:

```
          Y    T-hat
. regress logQ logP_hat, robust;

Linear regression                                      Number of obs =     111
                                                       F(  1,   109) =    9.80
                                                       Prob > F      =  0.0022
                                                       R-squared     =  0.0668
                                                       Root MSE      =  .71758

------------------------------------------------------------------------------
             |               Robust
        logQ |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    logP_hat |    -1.2354    .394689    -3.13   0.002    -2.017661   -.4531391
       _cons |   8.282913    .112264    73.78   0.000     8.060409    8.505416
------------------------------------------------------------------------------
```

* These coefficients are the TSLS estimates. Price elasticity is now -1.24 (1% increase in price of fish reduces demand by 1.24%)

** The standard errors are wrong because they ignore the fact that the first stage coefficients are estimated (Stata has a way to adjust for this)

Combined into a single command: “IVREG”

```
          Y    T     Z
. ivreg logQ (logP=windspd), robust;
Instrumental variables (2SLS) regression               Number of obs =     111
                                                       F(  1,   109) =    6.47
                                                       Prob > F      =  0.0124
                                                       R-squared     =       .
                                                       Root MSE      =    .755

------------------------------------------------------------------------------
             |               Robust
        logQ |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        logP |    -1.2354   .4856181    -2.54   0.012    -2.197879   -.2729206
       _cons |   8.282913   .1269709    65.23   0.000     8.031261    8.534565
------------------------------------------------------------------------------
Instrumented:  logP                     This is the endogenous regressor
Instruments:   windspd                  This is the instrumental varible
------------------------------------------------------------------------------
```

⇒ **Price-elasticity -1.24 (unchanged)**

⇒ **Standard errors here are correct. Notice the previous ones were quite wrong**

⇒ **always use IVREG (or similar command) when implementing TSLS**

⇒ **“IVREGRESS” is currently most up-to-date command in Stata**

Sampling distribution of TSLS / IV Estimator

- The IV estimator of β_1 is consistent and asymptotically normally distributed under "instrument relevance" and "instrument exogeneity" assumptions
  - Other technical assumptions needed as well – to come

- In large samples, sample covariance converge in probability to population covariance (i.e. LLN), and so <u>IV estimator is consistent:</u>

$$\hat{\beta}_1^{IV} = \frac{S_{YZ}}{S_{TZ}} \xrightarrow{p} \frac{Cov(Y,Z)}{Cov(T,Z)} = \beta_1$$

- The "instrument relevance" assumption $Cov(T_i,Z_i) \neq 0$ rules out division by zero – we return to this in discussion of weak IV

TSLS/IV estimator is normally distributed in large samples

- The proof of this result is a bit more difficult
- What matters for us is the end result:

$$\hat{\beta}_1^{IV} \overset{A}{\cong} N(\beta_1, \sigma^2_{\hat{\beta}_1^{IV}})$$

- Where: $\sigma^2_{\hat{\beta}_1^{IV}} = \frac{1}{n} \frac{Var[(Z_i - \mu_Z)u_i]}{Cov(T_i, Z_i)^2}$ **Heteroskedasticity-robust…**

- Thus, we can conduct inference (hypothesis test, confidence intervals) in the same way as before