

# LMie: Implementation optimized equations

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## Contents

<b>1</b>	<b>LMie core</b>	<b>2</b>
1.1	Forward . . . . .	2
1.2	Tangent linear . . . . .	3
1.3	Adjoint of tangent linear . . . . .	3

# 1 LMie core

## 1.1 Forward

$$x = 2\pi a_i / \lambda \quad (1)$$

$$z = mx \quad (2)$$

$$f = n(a_i)w_i \quad (3)$$

$$r_{n_1}(x) = 0 \quad (4)$$

$$r_i(x) = \left[ \frac{2i+1}{x} - r_{i+1}(x) \right]^{-1} \quad (5)$$

$$\Psi_0(x) = \sin x \quad (6)$$

$$\Psi_i(x) = r_i(x)\Psi_{i-1}(x) \quad (7)$$

$$\chi_{-1}(x) = \sin x \quad (8)$$

$$\chi_0(x) = \cos x \quad (9)$$

$$\chi_{i+1}(x) = \frac{2i+1}{x}\chi_i(x) - \chi_{i-1}(x) \quad (10)$$

$$\zeta_i(x) = \psi_i(x) + i\chi_i(x) \quad (11)$$

$$D_i(z) = \frac{i+1}{z} - \left[ D_{i+1}(z) + \frac{i+1}{z} \right]^{-1} \quad (12)$$

$$a_i = \frac{[D_i(z)/m + n/x] \psi_i(x) - \psi_{i-1}(x)}{[D_i(z)/m + n/x] \zeta_i(x) - \zeta_{i-1}(x)} \quad (13)$$

$$b_i = \frac{[mD_i(z) + n/x] \psi_i(x) - \psi_{i-1}(x)}{[mD_i(z) + n/x] \zeta_i(x) - \zeta_{i-1}(x)} \quad (14)$$

$$C_{\text{ext}} = \frac{\lambda^2}{2\pi} \sum_{i=1}^{n_{\text{size}}} \left[ \sum_{j=1}^{n_1} (2i+1) \text{Re}(a_j + b_j) \right] w_{\text{size},i} \quad (15)$$

$$C_{\text{sca}} = \frac{\lambda^2}{2\pi} \sum_{i=1}^{n_{\text{size}}} \left[ \sum_{j=1}^{n_1} (2i+1) (|a_i|^2 + |b_i|^2) \right] w_{\text{size},i} \quad (16)$$

$$C_{\text{bak}} = \frac{\lambda^2}{4\pi} \sum_{i=1}^{n_{\text{size}}} \left| \sum_{j=1}^{n_1} (2i+1)(-1)^i (a_j - b_j) \right|^2 w_{\text{size},i} \quad (17)$$

$$g = \frac{2}{C_{\text{sca}}} \sum_{i=1}^{n_{\text{size}}} \left\{ \sum_{i=1}^{n_1} \left[ \frac{i(i+2)}{i+1} \text{Re}(a_i a_{i+1}^* + b_i b_{i+1}^*) + \frac{2i+1}{i(i+1)} \text{Re}(a_i b_i^*) \right] \right\} w_{\text{size},i} \quad (18)$$

$$\pi_0(\mu) = 0 \quad (19)$$

$$\pi_1(\mu) = 1 \quad (20)$$

$$s = \mu \pi_i(\mu) \quad (21)$$

$$t = s - \pi_{i-1}(\mu) \quad (22)$$

$$\tau_i(\mu) = it - \pi_{i-1}(\mu) \quad (23)$$

$$\pi_{i+1}(\mu) = s + \left( \frac{i+1}{i} \right) t \quad (24)$$

## 1.2 Tangent linear

$$\mathcal{L}(x) = \frac{2\pi \mathcal{L}(a_i) - x \mathcal{L}(\lambda)}{\lambda} \quad (25)$$

$$\mathcal{L}(z) = \mathcal{L}(m)x + m \mathcal{L}(x) \quad (26)$$

$$\mathcal{L}(f) = \mathcal{L}[n(a_i)] w_i + n(a_i) \mathcal{L}(w_i) \quad (27)$$

## 1.3 Adjoint of tangent linear