Review for Final Exam

1. Remarks

The final exam is comprehensive, but will weight material since the second midterm exam more heavily.

You should refer to the review material for the midterms – and to the midterms themselves – for studying older material!

Since the second midterm, we have discussed:

- the construction of the real numbers
- probability
- graphs

For the newer material, you should review the homework problems on probability and on graphs (also, see a few further problems below).

2. terminology, definitions and results

- the definition of $\mathbb{R}$ as equivalence classes of Cauchy sequences of rational numbers
- the definition of operations on $\mathbb{R}$ (multiplication, addition, absolute value) described in terms of Cauchy sequences in $\mathbb{Q}$.
- the (mathematical) definition of a sample space and of the probability of an event
- conditional probability, independent events and Bayes Theorem
- random variables, expected value
- binomial coefficients, "stars and bars"
- definition of a (directed or un-directed) graph, incidence between edges and vertices, simple graph, degree of a vertex
- morphisms and isomorphisms between graphs, subgraphs
- the complete graph on n vertices, complete bipartite graph on n,m vertices
- connected graphs, Euler paths, condition for existence of Euler paths

3. problems

Problem 1: Suppose that there are only 4 kinds of donuts left in the shop: jelly, coconut, plain and chocolate, and the shop has at least a dozen of each of these 4 kinds.

- In how many ways can you choose a dozen donuts so that at least 4 are chocolate AND at most 3 are coconut?
- In how many ways can you choose 12 donuts so that at least 4 are chocolate, at most 3 are coconut AND at most one is jelly?

Problem 2:

a. Use the formula

$$\binom{n}{2} = n \cdot (n-1)/2$$

to prove algebraically that

$$\binom{n}{2} = \binom{k}{2} + k(n-k) + \binom{n-k}{2}.$$

b. Give a counting argument to confirm the equation in (a).

Problem 3:

a. Give the definition of for: the sequence a_n has limit L.

b. Give the definition of Cauchy sequence.

c. Prove that if a_n is not a Cauchy sequence, then a_n does not have a limit.

Problem 4: For a natural number $n \in \mathbb{N}$, the complete graph K_n on n vertices has $\binom{n}{2}$ edges. If Γ is any (undirected) graph with n vertices and $\binom{n}{2}$ edges, must Γ be isomorphic to K_n?

Problem 5: Let K_n be the complete graph with n vertices, and let Γ be the graph obtained from K_n by deleting one edge. If $n \geq 3$, prove that Γ is a connected graph.

Problem 6:

a. Consider the assignment

$$f : \mathbb{Q} \times \mathbb{Q} \to \mathbb{Q}$$

given by $(a/b, c/d) \mapsto (a+b)/(c+d)$. Prove or disprove that this is a well defined function.

b. Consider the assignment

$$f : \mathbb{Q} \times \mathbb{Q} \to \mathbb{Q}$$

given by $(a/b, c/d) \mapsto (ad+bc)/bd$. Prove or disprove that this is a well defined function.

Problem 7: Find an Eulerian path for the folowing graph, or prove that there can be no such path.