ProblemSet 2 – Linear programs

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1. A linear program

   Consider the optimization problem: find the max of $f(x, y) = x + 2y$ subject to the following constraints:

$$\begin{aligned} y &\leq 9 \\ -y &\leq -1 \\ 2x + y &\leq 25 \\ -2x - y &\leq -9 \\ -2x + y &\leq 1 \\ 2x - y &\leq 15. \end{aligned}$$

   a. Draw the feasible region. Label the boundary curves and corner points.

   b. Find the maximum value of f and the point where it occurs.

   c. Verify your answer using `SciPy`.

2. Bakers

   A bakery wants to sell forty five Valentine's Day gift bags. They have decided to offer two types of bags:

   - Bags of type A will contain four cupcakes and two cookies, and will be sold for $12
   - bags of type B will contain two cupcakes and five cookies, and will be sold for $16

   The bakery has 90 cookies and 115 cupcakes in total. Write the bakery's optimization problem as a linear program. Solve this to determine how many baskets of both types should be made. If a fractional solution is obtained, round down to whole number solutions. What is the maximum profit?

   You may solve this by drawing the feasible region or using python.

3. A farmer owns 45 acres of land. This season, she will plant each acre with either wheat or corn. Each acre of wheat yields $200 in seasonal profits, whereas each acre of corn yields $300 in seasonal profits. Each acre of wheat requires 3 workers and 2 tons of fertilizer, while each acre of corn requires 2 workers and 4 tons of fertilizer. The farmer has 100 workers and 120 tons of fertilizer available. Determine how many acres of wheat and corn need to be planted to maximize profits for the season. (Non-integer acreage values are allowed in the solution.)
