

# ProblemSet 1 – Optimization

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## 1. An optimization question in auto manufacturing

An automobile manufacturer makes a profit of \$1,500 per unit on the sale of a certain car model. It is estimated that for every \$100 of rebate, the number of units of this model sold in a given month will increase by 15%.

- What amount of rebate will maximize the manufacturers profit for the month? Model the question as a single-variable optimization problem.
- Compute the *sensitivity* of your answer to the 15% assumption. Consider both the amount of rebate and the resulting profit.
- Suppose that rebates actually generate only a 10% increase in sales per \$100. What is the effect? What if the response is somewhere between 10% and 15% per \$100 of rebate?
- Under what circumstances would an offer of a rebate cause a reduction in profit?

## 2. Computing yields with multi-variate optimization

A chemist is synthesizing a compound. In the last step, she must dissolve her reagents in a solution with a particular pH level  $H$ , for  $1.2 \leq H \leq 2.7$ , and heated to a temperature  $T$  (in degrees Celsius), for  $66 \leq T \leq 98$ . Her goal is to maximize her percent yield as a percentage of the initial mass of the reagents.

The equation determining the percentage  $F(H, T)$  is

$$F(H, T) = -0.038 \cdot T^2 - 0.223 \cdot T \cdot H - 10.982 \cdot H^2 + 7.112 \cdot T + 60.912 \cdot H - 328.898.$$

- Find the optimal temperature and pH level in the allowed range.
- Use `matplotlib` to produce a graph and a contour plot of the percentage of the powder function  $F(H, T)$ .

(To get a usable copy of your image, you can proceed in a few ways:

- if you produce the graph in colab you can right-click on the image and Save As a file on your file system.
- if you work in Python on your computer, you can save the image via a command like

```
> g.savefig("my_graph_image.png")
```

## 3. Blood typing

Human blood is generally classified in the “ABO” system, with four blood types: A, B, O, and AB. These four types reflect six gene pairs (genotypes), with blood type A corresponding to gene pairs AA and AO, blood type B corresponding to gene pairs BB and BO, blood type O corresponding to gene pair OO, and blood type AB corresponding to gene pair AB. Let  $p$  be the proportion of gene A in the population, let  $q$  be the proportion of gene B in the population, and let  $r$  be the proportion of gene O in the population. Observe that  $p + q + r = 1$ .

- The Hardy-Weinberg principle implies that:

(♣) The quantities  $p$ ,  $q$ , and  $r$  remain constant from generation to generation, as do the frequencies of occurrence of the different genotypes AA, AO, ...

Assuming the validity of (♣), what is the probability that an individual has genotype AA? BB? OO? What is the probability of an individual having two different genes? Express your response using the quantities  $p$ ,  $q$  and  $r$ .

- b. Still assuming the validity of (♣), find the maximum percentage of the population that can have two different genes. Perform this computation in two different ways:
- directly maximize a function of only two variables
  - use the method of Lagrange multipliers.
- c. Explain in words what the Lagrange multiplier represents in the second computation of part (b).

#### 4. Newton's method and root finding

- a. microprocessors

One of the uses of Newton's method is in implementing division on microprocessors, where only addition and multiplication are available as primitive operations. To compute  $x = a/b$ , first the root of  $f(x) = 1/x - b$  is found using Newton's method, then the fraction is computed with one last multiplication by  $a$ .

Find the Newton iteration needed to solve  $f(x) = 0$  and explain why it is well-suited to this purpose. (**Note:** We are trying to approximate division, so we shouldn't actually use division functions implemented in `python`...)

- b. experiments

Apply Newton's Method to compute  $1/b$ , where  $b$  is: (i) the last 3 digits of your student number; and (ii) the area code of your phone number. For these experiments, report the number of iterations required for the approximation to be consistent to 10 digits.

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