Week01 – Optimization and modeling: overview

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In the first week (well, really just the one day…) we discuss *single-variable optimization problems*.

There are three associated `jupyter notebooks`. There are three ways to interact with this material: you can **[View]** the contents of the notebooks as a (non-interactive) web page, you can view the notebook **[via colab]**, or you can **[download]** the `ipynb` file for use with `jupyter` running on your computer. The latter two choices permit you to edit and execute code; for more information, see the course-resource discussion about `jupyter notebooks`.

- **jupyter demo** [view] [via colab] [download]
- **single-variable optimization** [view] [via colab] [download]
- **optimization & derivatives** [view] [via colab] [download]

Optimization Overview

Optimization is the most common application of mathematics. Here are some "real-world" examples:

- **Business optimization**. A business manager attempts to understand and control parameters in order to maximize profit and minimize costs.
- **Natural resource management**. Control harvest rates to maximize long-term yield, while conserving resources.
- **Environmental regulation**. Governments sets standards to minimize environmental costs, while maximizing production of goods.
- **IT management**. Computer system managers try to maximize throughput and minimize delays.
- **Pharmaceutical optimization**. Doctors and pharmacists regulate drugs to minimize harmful side effects and maximize healing.

In this first part of our modeling course, we are going to discuss some sorts of optimization problems and related matters:

- *single variable optimization* and *sensitivity analysis*
- *multivariable optimization*
- *multivariable optimization with constraints*

We begin this week with a discussion of *single variable optimization*.

Single Variable Optimization

- In this first section of our modeling class, we examine a few *single variable* optimization problems. In some sense, these amount to – perhaps complicated examples of! – *word problems* that you might have met in Calculus I (differential calculus).

The procedure to carry out a calculus based solution can then be described roughly as follows:

- find the function $f(x)$ that measures the quantity that you desire to optimize, and the relevant interval $[a, b]$ of values of independent variable x.
- find the critical points $c_1, c_2, \ldots, c_N$ of f in the interval (a, b).

- if f is a *differentiable* function, the maximum and minimum value of f will be found in the list $f(a), f(c_1), \ldots, f(c_N), f(b)$; remember that you must check the endpoints a, b!

Some remarks on modeling, in general

Here are some guidelines to follow when attempting to model a problem mathematically:

1. Ask the question:
   - The question should be phrased using correct mathematics; this will help make clear what must be found.
   - Make a list of all the variables and constants; include units as appropriate.
   - State all assumptions about these variables and constants; include equations and inequalities.
   - Check units to make sure things make sense.
   - State your objective in mathematical terms (i.e., the *oil spill* problem this week is a "minimization problem").
   - It may even be useful to make an educated guess at this point on what the answer should be.
2. Select the modeling approach.
   - Choose a general solution procedure to solve the mathematical problem (in our case first and second derivative tests).
   - This might be the most difficult part and to a large extent depends on just good experience. That's our goal…to get some experience.
3. Formulate the model.
   - Restate the question in terms of your model (in our example, what function are we taking the derivative of?).
   - You may need to relabel or redefine things to make it work. This is where the mathematical model and real physical model may start to differ…
4. Solve the model.
   - Apply Step 2 to Step 3.
   - Use any useful technologies, such as computation if necessary, but consider the errors that they may introduce.
5. Answer the question.
   - Rephrase the result of Step 4 in non-technical terms.
   - The goal is now to make the answer understandable to the person that posed it, keeping in mind that person may not be a mathematician.
   - Think about what the errors might be, or how realistic the answer actually is.
   - Assess to what extent the answer met expectations.

Of course this procedure is described in very general terms, and may need adaptation according to the problem at hand.

But: at least it describes our goals in modeling.
