Midterm Project 1 – Supply chain *solutions*

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(You can get a full copy of the code I use for the solution here.

Description of the network flow

As the logistics manager for the rubber ducks company, we are first tasked with minimizing shipping costs for the duck supply chain.

We create a *network flow* to represent the distribution of ducks. The *nodes* of our Network flow are the `warehouse_cities` and the `store_cities`, together with a node representing the `source` of ducks and a node representing `demand` for ducks.

```
import numpy as np
from scipy.optimize import linprog
from math import inf
from itertools import product

warehouse_cities = [ 'Santa Fe',
                     'El Paso',
                     'Tampa Bay'
                    ]

store_cities = [ 'Chicago',
                 'LA',
                 'NY',
                 'Houston',
                 'Atlanta'
                ]

vertices=[ 'Source',
           *warehouse_cities,
           *store_cities,
           'Demand'
          ]
```

To formulate our *network flow*, we are going to describe edges between nodes. Ultimately, we describe a *linear program* using the network flow; the *variables* of the linear program correspond to the *edges* of our network flow. And the numerical quantities assigned to these variables represent *ducks*; in the case of an edge connecting city-nodes, the variable represents the quantity of ducks transported. In the case of an edge connecting the `source` node to a `warehouse_city`, the variable represents the *production* of ducks, and in the case of an edge connecting a `store city` to the `Demand` node, the variable represents the quantity of ducks *sold*.

We create an edge between `Source` and each `warehouse_city`, and between each `store_city` and `Demand`.

```
edges_source = [ { 'from': 'Source',
                   'to': c,
                  }
                for c in warehouse_cities ]
```

```python
edges_demand = [ { 'from': c,
                   'to': 'Demand',
                 }
                for c in store_cities
               ]
```

Now, we only create an edge between shipping when indicated.

Recall our shipping and relay costs:

Table 1: Shipping costs ($ per `duck`)

	Chicago	LA	NY	Houston	Atlanta
Santa Fe	6	3	-	3	7
El Paso	-	7	-	2	5
Tampa Bay	-	-	7	6	4

Table 2: Relay route costs ($ per duck)

	Chicago	LA	NY	Houston	Atlanta
Houston	4	5	6	-	2
Atlanta	4	-	5	2	-

We create functions in python representing these costs, as follows:

```python
def ship_costs(f,t):
    match (f,t):
        case 'Source',_:            # no shipping cost for "shipments" from source to warehouse
            return 0

        case _,'Demand':            # no shipping costs for "shipments" from store to customers
            return 0


        case 'Santa Fe','Chicago':
            return 6
        case 'Santa Fe','LA':
            return 3
        case 'Santa Fe','Houston':
            return 3
        case 'Santa Fe','Atlanta':
            return 7

        case 'El Paso','LA':
            return 7
        case 'El Paso','Houston':
            return 2
        case 'El Paso','Atlanta':
            return 5

        case 'Tampa Bay','NY':
            return 7
        case 'Tampa Bay','Houston':
            return 6
```

```
        case 'Tampa Bay','Atlanta':
            return 4

        case _:
            return inf

def relay_costs(f,t):
    match (f,t):
        case 'Houston','Chicago':
            return 4
        case 'Houston','LA':
            return 5
        case 'Houston','NY':
            return 6
        case 'Houston','Atlanta':
            return 2

        case 'Atlanta','Chicago':
            return 4
        case 'Atlanta','NY':
            return 5
        case 'Atlanta','Houston':
            return 2

        case _:
            return inf
```

Now we can create the remaining edges for our network flow. We only create an edge if the shipping costs are given above.

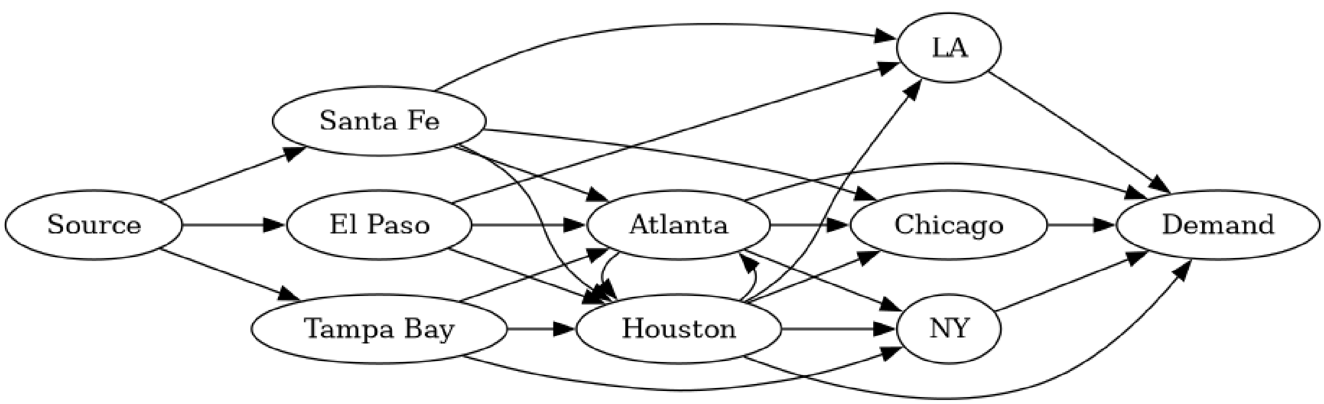
```
edges_ship = [ { 'from': source,
                 'to': dest,
                }
               for source,dest in product(warehouse_cities,store_cities)
               if ship_costs(source,dest) != inf
              ]

edges_relay = [ { 'from': source,
                  'to': dest,
                 }
                for source,dest in product(store_cities,store_cities)
                if relay_costs(source,dest) != inf
               ]

# we now get all the edges by concatenation of preceding lists
#
edges =  edges_source + edges_ship + edges_relay + edges_demand
```

And we can use the `vertices` and `edges` to produce a diagram of the network flow, using `graphviz`.

```
#--------------------------------------------------------------------------------
from graphviz import Digraph as GVDigraph

dot = GVDigraph("example",format='png')
dot.attr(rankdir='LR')

dot.node('Source')

with dot.subgraph(name='warehouse') as c:
```

```python
    c.attr(rank='same')
    for vertex in warehouse_cities:
        c.node(vertex)

with dot.subgraph(name='hubs') as c:
    c.attr(rank='same')
    for vertex in hubs:
        c.node(vertex)

with dot.subgraph(name='stores') as c:
    c.attr(rank='same')
    for vertex in store_cities:
        if not (vertex in hubs):
            c.node(vertex)

c.node('Demand')

# make an edge in the graph for each of our edges.
for e in edges:
#  dot.edge(e["from"],e["to"],label=f"costs {e['ship_costs']}")
  dot.edge(e["from"],e["to"])

dot.render('graph.png')
```

Here is the resulting network flow diagram. (We've chosen *not* to label it, since the labels can get a bit cluttered. We'll describe in words the constraints on the edge variables, below).



The objective function for shipping costs

Our next task is to identify the `objective function` for the linear program that we will use to minimize shipping costs.

```python
# return a standard basis vector
# these are "0-indexed" e.g. sbv(0,3) == [1,0,0]
def sbv(index,size):
    return np.array([1.0 if i == index else 0.0 for i in range(size)])

# we first create the vector for the costs for shipments from warehouse cities to store cities
ship_costs_obj = sum([ ship_costs(e['from'],e['to'])*sbv(edges.index(e),len(edges))
                       for e in edges_ship])

ship_costs_obj
=>
array([0., 0., 0., 6., 3., 3., 7., 7., 2., 5., 7., 6., 4., 0., 0., 0., 0.,
```

```
       0., 0., 0., 0., 0., 0., 0., 0.])
```

```
# we then create the vector for the relay costs
relay_costs_obj = sum([ relay_costs(e['from'],e['to'])*sbv(edges.index(e),len(edges))
                        for e in edges_relay])

relay_costs_obj
=>
array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 4., 5., 6., 2.,
       4., 5., 2., 0., 0., 0., 0., 0.])
```

```
# the objective function is then the vector sum of the previous two results
costs_obj = ship_costs_obj + relay_costs_obj

costs_obj
=>
array([0., 0., 0., 6., 3., 3., 7., 7., 2., 5., 7., 6., 4., 4., 5., 6., 2.,
       4., 5., 2., 0., 0., 0., 0., 0.])
```

Conservation laws

Now we are going to identify the *conservation laws* for our network flow. For each interior vertex of our diagram, we need that the sum of the *incoming flow* is equal to the sum of the *outgoing flow*.

```
def getIncoming(vertex,edges):
    return [ e for e in edges if e["to"] == vertex ]

def getOutgoing(vertex,edges):
    return [ e for e in edges if e["from"] == vertex ]

def isSource(vertex,edges):
    return getIncoming(vertex,edges) == []

def isSink(vertex,edges):
    return getOutgoing(vertex,edges) == []

def interiorVertices(vertices,edges):
    return [ v for v in vertices if not( isSource(v,edges) or isSink(v,edges) ) ]
```

Observe that this code indeed finds our interior vertices:

```
interiorVertices(vertices,edges)
=>
['Santa Fe', 'El Paso', 'Tampa Bay', 'Chicago', 'LA', 'NY', 'Houston', 'Atlanta']
```

Now we can create the *conservation laws matrix* for our network flow. This matrix has one row for each interior vertex of the network flow; this row expresses the relation that the sum of flow through edges *to* the vertex is equal to the sum of flow through edges *from* the vertex.

We use the following code:

```
def conservationLaw(vertex,edges):
    ii = sum([ sbv(edges.index(e),len(edges)) for e in getIncoming(vertex,edges) ])
    oo = sum([ sbv(edges.index(e),len(edges)) for e in getOutgoing(vertex,edges) ])
    return ii - oo

conservationMatrix =np.array([conservationLaw(v,edges) for v in interiorVertices(vertices,edges) ])
```

And we can inspect this matrix:

```
conservationMatrix
=>
array([[ 1.,  0.,  0., -1., -1., -1., -1.,  0.,  0.,  0.,  0.,  0.,  0.,
         0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  1.,  0.,  0.,  0.,  0.,  0., -1., -1., -1.,  0.,  0.,  0.,
         0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  0., -1., -1., -1.,
         0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,
         1.,  0.,  0.,  0.,  1.,  0.,  0., -1.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  1.,  0.,  0.,  1.,  0.,  0.,  0.,  0.,  0.,
         0.,  1.,  0.,  0.,  0.,  0.,  0.,  0., -1.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  1.,  0.,  0.,
         0.,  0.,  1.,  0.,  0.,  1.,  0.,  0.,  0., -1.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  1.,  0.,  0.,  1.,  0.,  0.,  1.,  0.,
        -1., -1., -1., -1.,  0.,  0.,  1.,  0.,  0.,  0., -1.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  1.,  0.,  0.,  1.,  0.,  0.,  1.,
         0.,  0.,  0.,  1., -1., -1., -1.,  0.,  0.,  0.,  0., -1.]])
```

The conservation matrix will be used to implement the the linear program we will use to minimize shipping costs; it will appear as part of the *equality constraints*.

Setting up the linear program

We have already described the *objective function*. It remains to describe the equality- and inequality- constraints that we will use.

Equality constraints

When minimizing the shipping costs, we ship all available ducks from the warehouses, and we meet demand in the store cities. Thus, we implement the `supply` and `demand` as *equality constraints* in our linear program.

Recall the supply and demand specifications:

Table 3: Supplies (in `ducks`)

Santa Fe	El Paso	Tampa Bay
700	200	200

Table 4: Demand (in `ducks`)

Chicago	LA	NY	Houston	Atlanta
200	200	250	300	150

We implement these with the following code:

```
supplies = { 'Santa Fe': 700,
             'El Paso': 200,
             'Tampa Bay': 200
             }

demand = { 'Chicago': 200,
           'LA': 200,
```

```
            'NY': 250,
            'Houston': 300,
            'Atlanta': 150
           }
```

More precisely, for each warehouse city `w`, the variable corresponding to edge `{'from':'Source','to':w}` is equated with the quantity `supplies[w]`.

Similarly, for each store city `w`, the variable corresponding to the edge `{'from':w,'to':'Demand'}` is equated with the quantity `demand[s]`.

We create the pair `Aeq_costs,beq_costs` determining the equality constraints using the following code:

```
# return the edge with 'from': f and 'to': t
#
def lookupEdge(f,t):
    r = list(filter(lambda x: x['from'] == f and x['to'] == t, edges))
    if r != []:
        return r[0]
    else:
        return "error"

# get the *index* (in our list of edges) of the edge with 'from': f and 'to': t
#
def lookupEdgeIndex(f,t):
    r = lookupEdge(f,t)
    return edges.index(r)


Aeq_costs = np.concatenate([ conservationMatrix,
                             [ sbv(lookupEdgeIndex('Source',w),len(edges))
                               for w in warehouse_cities ],
                             [ sbv(lookupEdgeIndex(s,'Demand'),len(edges))
                               for s in store_cities ]
                            ],axis=0)
beq_costs = np.concatenate([ np.zeros(len(conservationMatrix)),
                             [ supplies[w] for w in warehouse_cities ],
                             [ demand[s] for s in store_cities ]
                            ])
```

Thus the first rows of `Aeq_costs` are the `conservationMatrix` computer earlier. The next group of rows account for edges `Source -> warehouse_cities`, and the final group of rows account for edges `store_cities -> Demand`.

When running the linear program, the equality constraint `Aeq_costs,beq_costs` will thus enforce the conservation laws, require that we ship all available ducks, and require that we meet all demand.

Inequality constraints

Finally, we need to create inequality constraints reflecting the condition that we can't ship more than 200 ducks along any single route.

The pair `Aub_costs,bub_costs` implement these constraints; these quantities are created by the following code:

```
# create inequality constraint matrix
# initially the only thing to account for is "can't ship more than 200 ducks"

Aub_costs = np.array([ sbv(edges.index(e),len(edges)) for e in edges_ship ]
                     + [ sbv(edges.index(e),len(edges)) for e in edges_relay ])
```

```
bub_costs = np.array([ 200 for e in edges_ship]
                     + [ 200 for e in edges_relay ] )
```

Running the linear program.

We are now ready to run the linear program which minimizes shipping costs.

```
costs_result = linprog(costs_obj,
                       A_eq = Aeq_costs,
                       b_eq = beq_costs,
                       A_ub = Aub_costs,
                       b_ub = bub_costs
                       )
```

We see that the minimal shipping costs are $5,300.00:

```
costs_result.fun
=>
5300.0
```

And we can see the required shipping levels by inspecting `costs_result.x`. Let's view a report of this information:

```
def report(x):
    for (val,e) in zip(x,edges):
        print(f"{e['from']:10} -> {e['to']:10}:  {val: 7.2f}")
```

```
report(costs_result.x)
=>
Source     -> Santa Fe  :   700.00
Source     -> El Paso   :   200.00
Source     -> Tampa Bay :   200.00
Santa Fe   -> Chicago   :   200.00
Santa Fe   -> LA        :   200.00
Santa Fe   -> Houston   :   200.00
Santa Fe   -> Atlanta   :   100.00
El Paso    -> LA        :     0.00
El Paso    -> Houston   :   200.00
El Paso    -> Atlanta   :    -0.00
Tampa Bay  -> NY        :   200.00
Tampa Bay  -> Houston   :     0.00
Tampa Bay  -> Atlanta   :     0.00
Houston    -> Chicago   :     0.00
Houston    -> LA        :     0.00
Houston    -> NY        :    50.00
Houston    -> Atlanta   :    50.00
Atlanta    -> Chicago   :     0.00
Atlanta    -> NY        :     0.00
Atlanta    -> Houston   :     0.00
Chicago    -> Demand    :   200.00
LA         -> Demand    :   200.00
NY         -> Demand    :   250.00
Houston    -> Demand    :   300.00
Atlanta    -> Demand    :   150.00
```

Los Angeles potential strike scenario

We must deal with restive workers in LA. We consider two outcomes: how are costs affected if we *meet workers demands* and if *the workers go on strike*?

Demand scenario

The workers demand would result in the doubling of all shipping costs to LA.

Using the following code, we can model the "demand scenario" shipping costs:

```
def LA_demand_ship_costs(f,t):
    match (f,t):
        case (_,'LA'):
            return 2*ship_costs(f,t)      ## double shipping costs to LA
        case _:
            return ship_costs(f,t)


def LA_demand_relay_costs(f,t):
    match (f,t):
        case (_,'LA'):
            return 2*relay_costs(f,t)      ## double shipping costs to LA
        case _:
            return relay_costs(f,t)
```

These changes modify the require *objective function* for the linear program which will tell us the impact of the demand-scenario on our shipping costs.

```
LA_demand_ship_costs_obj = sum([ LA_demand_ship_costs(e['from'],e['to'])*sbv(edges.index(e),len(edges))
                       for e in edges_ship])

LA_demand_relay_costs_obj = sum([ relay_costs(e['from'],e['to'])*sbv(edges.index(e),len(edges))
                        for e in edges_relay])

LA_demand_costs_obj = LA_demand_ship_costs_obj + LA_demand_relay_costs_obj
```

With the updated objective function, we can now run the linear program:

```
LA_demand_costs_result = linprog(LA_demand_costs_obj,
                                 A_eq = Aeq_costs,
                                 b_eq = beq_costs,
                                 A_ub = Aub_costs,
                                 b_ub = bub_costs
                                 )
```

We can see the outcome:

```
LA_demand_costs_result.fun
=>
5900.0
```

And we can see the details of how our shipping choices will be affected.

```
report(LA_demand_costs_result.x)
=>
Source     -> Santa Fe  :   700.00
Source     -> El Paso   :   200.00
Source     -> Tampa Bay :   200.00
Santa Fe   -> Chicago   :   200.00
Santa Fe   -> LA        :   200.00
Santa Fe   -> Houston   :   200.00
```

```
Santa Fe   -> Atlanta   :   100.00
El Paso    -> LA        :     0.00
El Paso    -> Houston   :   200.00
El Paso    -> Atlanta   :    -0.00
Tampa Bay  -> NY        :   200.00
Tampa Bay  -> Houston   :     0.00
Tampa Bay  -> Atlanta   :     0.00
Houston    -> Chicago   :     0.00
Houston    -> LA        :     0.00
Houston    -> NY        :    50.00
Houston    -> Atlanta   :    50.00
Atlanta    -> Chicago   :     0.00
Atlanta    -> NY        :     0.00
Atlanta    -> Houston   :     0.00
Chicago    -> Demand    :   200.00
LA         -> Demand    :   200.00
NY         -> Demand    :   250.00
Houston    -> Demand    :   300.00
Atlanta    -> Demand    :   150.00
```

Thus, our costs increase from \$5300 to \$5900 - an increase of \$600.

Strike scenario

Now we must model the strike scenario. If workers demands are not met, they will strike and the maximum number of supplies that can be shipped on *all routes to LA* is cut in half (i.e., from 200 to 100).

Thus, the *inequality constraints* for the linear program must be modified.

```
## the Aub matrix is the same as befofe

LA_strike_Aub_costs = Aub_costs

## but we must change the bub vector

# the capacity for a route is 100 on any route `to` LA.
# otherwise the capacity remains 200

def LA_strike_capacity(e):
    match e['to']:
        case 'LA':
            return 100
        case _:
            return 200

LA_strike_bub_costs = np.array([ LA_strike_capacity(e) for e in edges_ship]
                               + [ LA_strike_capacity(e) for e in edges_relay ] )
```

We can now run the linear program modeling a strike in LA:

```
LA_strike_costs_result = linprog(costs_obj,
                                 A_eq = Aeq_costs,
                                 b_eq = beq_costs,
                                 A_ub = LA_strike_Aub_costs,
                                 b_ub = LA_strike_bub_costs
                                 )
```

In the strike scenario, our costs go up to \$6050.00. Indeed:

```
LA_strike_costs_result.fun
```

```
=>
6050.0
```

```
report(LA_strike_costs_result.x)
=>
Source     -> Santa Fe  :   700.00
Source     -> El Paso   :   200.00
Source     -> Tampa Bay :   200.00
Santa Fe   -> Chicago   :   200.00
Santa Fe   -> LA        :   100.00
Santa Fe   -> Houston   :   200.00
Santa Fe   -> Atlanta   :   200.00
El Paso    -> LA        :     0.00
El Paso    -> Houston   :   200.00
El Paso    -> Atlanta   :     0.00
Tampa Bay  -> NY        :   200.00
Tampa Bay  -> Houston   :     0.00
Tampa Bay  -> Atlanta   :     0.00
Houston    -> Chicago   :     0.00
Houston    -> LA        :   100.00
Houston    -> NY        :     0.00
Houston    -> Atlanta   :     0.00
Atlanta    -> Chicago   :    -0.00
Atlanta    -> NY        :    50.00
Atlanta    -> Houston   :     0.00
Chicago    -> Demand    :   200.00
LA         -> Demand    :   200.00
NY         -> Demand    :   250.00
Houston    -> Demand    :   300.00
Atlanta    -> Demand    :   150.00
```

In this case, the strike is $150 more costly – raising our costs to $6,050 – than the demand scenario – which only raises our costs to $5,900.

So unless there are relevant issues not considered here, we should probably agree to the LA workers demands.

Houston potential strike scenario

We now model the same situation as contemplated in LA, but instead for the city Houston.

Meeting worker demands

We model the shipping costs and the relay costs with new functions, as follows:

```
def Houston_demand_ship_costs(f,t):
    match (f,t):
        case (_,'Houston'):
            return 2*ship_costs(f,t)      ## double shipping costs to Houston
        case _:
            return ship_costs(f,t)


def Houston_demand_relay_costs(f,t):
    match (f,t):
        case (_,'Houston'):
            return 2*relay_costs(f,t)      ## double shipping costs to Houston
        case _:
            return relay_costs(f,t)
```

Using these new costs functions, we define the objective vector for the linear program minimizing costs if we meet worker demands, and we run the corresponding linear program:

```
Houston_demand_ship_costs_obj = sum([ Houston_demand_ship_costs(e['from'],e['to'])*sbv(edges.index(e),len(
                        for e in edges_ship])

Houston_demand_relay_costs_obj = sum([ relay_costs(e['from'],e['to'])*sbv(edges.index(e),len(edges))
                         for e in edges_relay])

Houston_demand_costs_obj = Houston_demand_ship_costs_obj + Houston_demand_relay_costs_obj

# results


Houston_demand_costs_result = linprog(Houston_demand_costs_obj,
                                        A_eq = Aeq_costs,
                                        b_eq = beq_costs,
                                        A_ub = Aub_costs,
                                        b_ub = bub_costs
                                        )
```

Under the demand scenario in Houston, our costs go up to $6250.00.

```
Houston_demand_costs_result.fun
=>
6250.0


report(Houston_demand_costs_result.x)
=>
Source     -> Santa Fe  :   700.00
Source     -> El Paso   :   200.00
Source     -> Tampa Bay :   200.00
Santa Fe   -> Chicago   :   200.00
Santa Fe   -> LA        :   200.00
Santa Fe   -> Houston   :   100.00
Santa Fe   -> Atlanta   :   200.00
El Paso    -> LA        :     0.00
El Paso    -> Houston   :   200.00
El Paso    -> Atlanta   :    -0.00
Tampa Bay  -> NY        :   200.00
Tampa Bay  -> Houston   :     0.00
Tampa Bay  -> Atlanta   :     0.00
Houston    -> Chicago   :     0.00
Houston    -> LA        :     0.00
Houston    -> NY        :     0.00
Houston    -> Atlanta   :     0.00
Atlanta    -> Chicago   :     0.00
Atlanta    -> NY        :    50.00
Atlanta    -> Houston   :     0.00
Chicago    -> Demand    :   200.00
LA         -> Demand    :   200.00
NY         -> Demand    :   250.00
Houston    -> Demand    :   300.00
Atlanta    -> Demand    :   150.00
```

Strike in Houston

We now model the consequences on our shipping costs of a strike in Houston. As before, we have to modify the inequality constraints Aub,bub. We do this in the same manner as we did for the LA situation, and we run the resulting linear program.

```
strike_Aub_costs = np.array([ sbv(edges.index(e),len(edges)) for e in edges_ship ]
                            + [ sbv(edges.index(e),len(edges)) for e in edges_relay ])

def strike_capacity(e):
    match e['to']:
        case 'Houston':
            return 100
        case _:
            return 200

strike_bub_costs = np.array([ strike_capacity(e) for e in edges_ship]
                            + [ strike_capacity(e) for e in edges_relay ] )


Houston_strike_costs_result = linprog(costs_obj,
                                A_eq = Aeq_costs,
                                b_eq = beq_costs,
                                A_ub = strike_Aub_costs,
                                b_ub = strike_bub_costs
                                )
```

The result shows that our shipping costs go up to $6050 in the case of a strike in Houston:

```
Houston_strike_costs_result.fun
=>
6050.0

report(Houston_strike_costs_result.x)
=>
Source     -> Santa Fe  :   700.00
Source     -> El Paso   :   200.00
Source     -> Tampa Bay :   200.00
Santa Fe   -> Chicago   :   200.00
Santa Fe   -> LA        :   200.00
Santa Fe   -> Houston   :   100.00
Santa Fe   -> Atlanta   :   200.00
El Paso    -> LA        :    -0.00
El Paso    -> Houston   :   100.00
El Paso    -> Atlanta   :   100.00
Tampa Bay  -> NY        :   200.00
Tampa Bay  -> Houston   :     0.00
Tampa Bay  -> Atlanta   :     0.00
Houston    -> Chicago   :     0.00
Houston    -> LA        :     0.00
Houston    -> NY        :     0.00
Houston    -> Atlanta   :     0.00
Atlanta    -> Chicago   :    -0.00
Atlanta    -> NY        :    50.00
Atlanta    -> Houston   :   100.00
Chicago    -> Demand    :   200.00
LA         -> Demand    :   200.00
NY         -> Demand    :   250.00
Houston    -> Demand    :   300.00
```

```
Atlanta    -> Demand    :   150.00
>>>
```

Thus meeting the worker demands costs $200 more than allows the strike. Perhaps the best strategy is to continue to negotiate with the Houston workers…

Profit

In order to maximize profit, we need to create the appropriate objective function.

We define a vector `sales` such that for a vector `x` of shipping values, `sales · x` returns the profit from sales of the corresponding ducks.

```
def profit(e):
    match e['from'],e['to']:
        case 'Santa Fe','Demand':
            return -8
        case 'El Paso','Demand':
            return -5
        case 'Tampa Bay','Demand':
            return -10
        case 'Chicago','Demand':
            return 15
        case 'NY','Demand':
            return 25
        case 'Houston','Demand':
            return 10
        case 'Atlanta','Demand':
            return 10
        case 'LA','Demand':
            return 20
        case _:
            return 0

sales = np.array([ profit(e) for e in edges])

sales
=>
array([ -8,  -5, -10,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
         0,   0,   0,   0,   0,   0,   0,  15,  20,  25,  10,  10])
```

Now the objective function for the profit linear program is given by `sales - ship_costs-obj`, where `ship_costs_obj` was the vector computing the shipping costs.

```
profit_obj = sales - ship_costs_obj
probit_obj
=>
array([ -8.,  -5., -10.,  -6.,  -3.,  -3.,  -7.,  -7.,  -2.,  -5.,  -7.,
        -6.,  -4.,   0.,   0.,   0.,   0.,   0.,   0.,   0.,  15.,  20.,
        25.,  10.,  10.])
```

Now, when maximizing profit, we no longer want to *require* that we use all available supplies, and we don't want to require that we meet demand in each store.

Thus, we will view the values in the `suplies` and `demand` variables as *upper bounds*.

Thus our equality constraints for the profit linear program will just be the *conservation laws*:

```
Aeq_profit = conservationMatrix
beq_profit = np.zeros(len(conservationMatrix))
```

And the inequality constraints will be determined by the pair `Aub_profit,bub_profit` where

```
Aub_profit = np.concatenate([ [ sbv(edges.index(e),len(edges)) for e in edges_ship ],
                              [ sbv(edges.index(e),len(edges)) for e in edges_relay ],
                              [ sbv(lookupEdgeIndex('Source',w),len(edges))
                                for w in warehouse_cities ],
                              [ sbv(lookupEdgeIndex(s,'Demand'),len(edges))
                                for s in store_cities]
                              ]
                            , axis=0)

bub_profit = np.concatenate([ [ 200 for e in edges_ship],
                              [ 200 for e in edges_relay ],
                              [ supplies[w] for w in warehouse_cities ],
                              [ demand[s] for s in store_cities ]
                             ])
```

We now run the linear program maximizing profit:

```
profit_result = linprog((-1)*profit_obj,
                        A_eq = Aeq_profit,
                        b_eq = beq_profit,
                        A_ub = Aub_profit,
                        b_ub = bub_profit)
```

This shows that the maximum profit is $13,450.00.

```
profit_result.fun
-6750.0

report(profit_result.x)
Source     -> Santa Fe  :   700.00
Source     -> El Paso   :   200.00
Source     -> Tampa Bay :   200.00
Santa Fe   -> Chicago   :   200.00
Santa Fe   -> LA        :   200.00
Santa Fe   -> Houston   :   200.00
Santa Fe   -> Atlanta   :   100.00
El Paso    -> LA        :     0.00
El Paso    -> Houston   :   200.00
El Paso    -> Atlanta   :    -0.00
Tampa Bay  -> NY        :     0.00
Tampa Bay  -> Houston   :    -0.00
Tampa Bay  -> Atlanta   :   200.00
Houston    -> Chicago   :    -0.00
Houston    -> LA        :     0.00
Houston    -> NY        :   200.00
Houston    -> Atlanta   :     0.00
Atlanta    -> Chicago   :     0.00
Atlanta    -> NY        :    50.00
Atlanta    -> Houston   :   100.00
Chicago    -> Demand    :   200.00
LA         -> Demand    :   200.00
NY         -> Demand    :   250.00
Houston    -> Demand    :   300.00
Atlanta    -> Demand    :   150.00
```

Notice that our profit was maximimized by using all available supplies (700 ducks in Santa Fe, 200 each in El Paso and in Tampa Bay) and by meeting demand in the stores (200 ducks in Chicago, 200 in LA, 250 in NY, 300 in Houston and 150 in Atl).