Characters of irreducible representations

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The regular representation, again.

We write $\mathbb{C}[G]$ for the space of functions on G, viewed as a permutation representation of G (and we suppress the notation for the homomorphism $G \to \text{GL}(\mathbb{C}[G])$).

For functions $f_1, f_2 \in \mathbb{C}[G]$, we define their *convolution* by the formula

$$(f_1 \star f_2)(x) = \sum_{yz=x} f_1(y) f_2(z).$$

For the basis elements $\delta_g \in \mathbb{C}[G]$ (i.e. the *Dirac functions*), we have

$$\delta_g \star \delta_h = \delta_{gh}.$$

Proposition:

a. For $f \in \mathbb{C}[G]$, the action of an element $g \in G$ on f satisfies

$$g.f = \delta_g \star f.$$

b. Let $W \subseteq \mathbb{C}[G]$ be an invariant subspace. For any $f \in \mathbb{C}[G]$, we have

$$f \star f' \in W \quad \forall f' \in W.$$

Bibliography