Exam 1

Name: ____

There are 5 problems worth a total of 40 points on the 4 page exam.

Problem 1. (6 points)

Let A and B be sets and let $f : A \to B$ be a function. State what it means to say that f is an *invertible function.*

Problem 2. (6 points)

Consider the regions

$$A = \{(x, y) \in \mathbb{R}^2 \mid x \geq 0\} \text{ and } B = \{(x, y) \in \mathbb{R}^2 \mid y \geq 0\}$$

Indicate whether each of the following statements is True or False by circling the correct choice. (Here you do not need to write an argument justifying your choice).

(a). **True** / **False** **:** $\forall t \geq -1, (t, 2) \in A \cap B$

(a). **True** / **False** **:** $\exists t \in \mathbb{R}, (t, -t) \in A \cup B$

Problem 3. (8 points)

Consider the proposition:

$$(\heartsuit) \quad \forall x \in \mathbb{N}, \exists y \in \mathbb{N} \text{ such that } xy \text{ is even.}$$

(a). Write the negation of ($\heartsuit$) so that the adjective "odd" is used instead of "even".

(b). Decide which is true: the statement ($\heartsuit$) or its negation. Justify your decision.

Problem 4. (10 points)

Let $F : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ be the function given by

$$F(a, b) = (2a + b, b + 1).$$

Prove that the function F is onto (i.e. surjective).

Problem 5. (10 points)

For $n \in \mathbb{N}$ define

$$S_n = \sum_{i=0}^{n} \frac{1}{3^i}.$$

Use mathematical induction to prove that

$$S_n = \frac{1}{2} \cdot \left(\frac{3^{n+1} - 1}{3^n} \right).$$