

ProblemSet 2 – Optimization

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1. Blood typing

Human blood is generally classified in the “ABO” system, with four blood types: A, B, O, and AB. These four types reflect six gene pairs, with blood type A corresponding to gene pairs AA and AO, blood type B corresponding to gene pairs BB and BO, blood type O corresponding to gene pair OO, and blood type AB corresponding to gene pair AB. Let p be the proportion of gene A in the population, let q be the proportion of gene B in the population, and let r be the proportion of gene O in the population. For example, the quantity p represents the ratio of the total number of *blood-type genes* of type A to the total number of *blood-type genes*. Since each blood-type gene is either A, B or O, it is clear that $p + q + r = 1$.

- a. The Hardy-Weinberg principle implies that:

(♣) The quantities p , q , and r remain constant from generation to generation, as do the frequencies of occurrence of the different genotypes AA, AO,

Assuming the validity of (♣), what is the probability that an individual has genotype AA? BB? OO? What is the probability of an individual having two different genes? Express your response using the quantities p , q and r .

- b. Still assuming the validity of (♣), find the maximum percentage of the population that can have two different genes. Perform this computation in two different ways:
- directly maximize a function of only two variables
 - use the method of Lagrange multipliers.
- c. Explain in words what the Lagrange multiplier represents in the second computation of part (b).

2. Newton’s method and root finding

- a. microprocessors

One of the uses of Newton’s method is in implementing division on microprocessors, where only addition and multiplication are available as primitive operations. To compute $x = a/b$, first the root of $f(x) = 1/x - b$ is found using Newton’s method, then the fraction is computed with one last multiplication by a .

Find the Newton iteration needed to solve $f(x) = 0$ and explain why it is well-suited to this purpose. (**Note:** The point here is to approximate division, so you shouldn’t use division functions implemented in `python`!)

- b. experiments

Apply Newton’s Method to compute $1/b$, where b is: (i) the last 3 digits of your student number; and (ii) the area code of your phone number. For these experiments, report the number of iterations required for the approximation to be consistent to 10 digits.