week13-03–scratch-work

April 7, 2025

[3]:
```
import numpy as np
import numpy.linalg as la

rng = np.random.default_rng()
```

[84]:
```
actual = { 'a': 10,
           'b': 15,
           'c': 13,
           'd': 20,
           'e': 25,
           'f': 30,
           'g': 5,
           'h': 15,
           'i': 17,
           'j': 20
         }

kl = actual.keys()

vv = np.array(list(actual.values()))

from itertools import chain,combinations

def nset(n,iterable):
    s = list(iterable)
    return list(combinations(s,n))

pairs = nset(2,kl)
```

[68]:
```
p_est = { (x,y):round(actual[x] + actual[y] + rng.uniform(-5,5),2) for (x,y) in␣
 ↪pairs }
p_est
```

[68]:
```
{('a', 'b'): 21.21,
 ('a', 'c'): 24.35,
 ('a', 'd'): 34.57,
 ('a', 'e'): 32.72,
 ('a', 'f'): 36.14,
```

```
 ('a', 'g'): 17.21,
 ('a', 'h'): 26.41,
 ('a', 'i'): 29.92,
 ('a', 'j'): 32.84,
 ('b', 'c'): 29.09,
 ('b', 'd'): 33.78,
 ('b', 'e'): 40.63,
 ('b', 'f'): 44.46,
 ('b', 'g'): 17.03,
 ('b', 'h'): 30.58,
 ('b', 'i'): 32.7,
 ('b', 'j'): 31.19,
 ('c', 'd'): 30.08,
 ('c', 'e'): 37.3,
 ('c', 'f'): 39.4,
 ('c', 'g'): 20.44,
 ('c', 'h'): 28.34,
 ('c', 'i'): 31.07,
 ('c', 'j'): 36.78,
 ('d', 'e'): 41.44,
 ('d', 'f'): 47.97,
 ('d', 'g'): 28.51,
 ('d', 'h'): 38.0,
 ('d', 'i'): 38.71,
 ('d', 'j'): 39.24,
 ('e', 'f'): 59.61,
 ('e', 'g'): 29.19,
 ('e', 'h'): 35.09,
 ('e', 'i'): 42.18,
 ('e', 'j'): 46.8,
 ('f', 'g'): 34.14,
 ('f', 'h'): 46.71,
 ('f', 'i'): 48.07,
 ('f', 'j'): 49.53,
 ('g', 'h'): 23.46,
 ('g', 'i'): 22.13,
 ('g', 'j'): 24.21,
 ('h', 'i'): 29.03,
 ('h', 'j'): 32.51,
 ('i', 'j'): 38.17}
```

[90]:
```python
def sbv(i,n):
    # return the ith standard basis vector of length n
    return np.array([1 if j == i else 0 for j in range(n)])

def sbv_list(elem,ls):
```

```
    # return the standard basis vector determined by the position of `elem` in␣
 ↪the list `ls`
    return sbv(list(ls).index(elem),len(ls))


M = np.array([sbv_list(x,k) + sbv_list(y,kl) for (x,y) in p_est.keys()])

b = np.array([p_est[x] for x in p_est.keys()])

(M,b)
```

[90]:
```
(array([[1, 1, 0, 0, 0, 0, 0, 0, 0, 0],
        [1, 0, 1, 0, 0, 0, 0, 0, 0, 0],
        [1, 0, 0, 1, 0, 0, 0, 0, 0, 0],
        [1, 0, 0, 0, 1, 0, 0, 0, 0, 0],
        [1, 0, 0, 0, 0, 1, 0, 0, 0, 0],
        [1, 0, 0, 0, 0, 0, 1, 0, 0, 0],
        [1, 0, 0, 0, 0, 0, 0, 1, 0, 0],
        [1, 0, 0, 0, 0, 0, 0, 0, 1, 0],
        [1, 0, 0, 0, 0, 0, 0, 0, 0, 1],
        [0, 1, 1, 0, 0, 0, 0, 0, 0, 0],
        [0, 1, 0, 1, 0, 0, 0, 0, 0, 0],
        [0, 1, 0, 0, 1, 0, 0, 0, 0, 0],
        [0, 1, 0, 0, 0, 1, 0, 0, 0, 0],
        [0, 1, 0, 0, 0, 0, 1, 0, 0, 0],
        [0, 1, 0, 0, 0, 0, 0, 1, 0, 0],
        [0, 1, 0, 0, 0, 0, 0, 0, 1, 0],
        [0, 1, 0, 0, 0, 0, 0, 0, 0, 1],
        [0, 0, 1, 1, 0, 0, 0, 0, 0, 0],
        [0, 0, 1, 0, 1, 0, 0, 0, 0, 0],
        [0, 0, 1, 0, 0, 1, 0, 0, 0, 0],
        [0, 0, 1, 0, 0, 0, 1, 0, 0, 0],
        [0, 0, 1, 0, 0, 0, 0, 1, 0, 0],
        [0, 0, 1, 0, 0, 0, 0, 0, 1, 0],
        [0, 0, 1, 0, 0, 0, 0, 0, 0, 1],
        [0, 0, 0, 1, 1, 0, 0, 0, 0, 0],
        [0, 0, 0, 1, 0, 1, 0, 0, 0, 0],
        [0, 0, 0, 1, 0, 0, 1, 0, 0, 0],
        [0, 0, 0, 1, 0, 0, 0, 1, 0, 0],
        [0, 0, 0, 1, 0, 0, 0, 0, 1, 0],
        [0, 0, 0, 1, 0, 0, 0, 0, 0, 1],
        [0, 0, 0, 0, 1, 1, 0, 0, 0, 0],
        [0, 0, 0, 0, 1, 0, 1, 0, 0, 0],
        [0, 0, 0, 0, 1, 0, 0, 1, 0, 0],
        [0, 0, 0, 0, 1, 0, 0, 0, 1, 0],
        [0, 0, 0, 0, 1, 0, 0, 0, 0, 1],
        [0, 0, 0, 0, 0, 1, 1, 0, 0, 0],
```

```
       [0, 0, 0, 0, 0, 1, 0, 1, 0, 0],
       [0, 0, 0, 0, 0, 1, 0, 0, 1, 0],
       [0, 0, 0, 0, 0, 1, 0, 0, 0, 1],
       [0, 0, 0, 0, 0, 0, 1, 1, 0, 0],
       [0, 0, 0, 0, 0, 0, 1, 0, 1, 0],
       [0, 0, 0, 0, 0, 0, 1, 0, 0, 1],
       [0, 0, 0, 0, 0, 0, 0, 1, 1, 0],
       [0, 0, 0, 0, 0, 0, 0, 1, 0, 1],
       [0, 0, 0, 0, 0, 0, 0, 0, 1, 1]]),
array([21.21, 24.35, 34.57, 32.72, 36.14, 17.21, 26.41, 29.92, 32.84,
       29.09, 33.78, 40.63, 44.46, 17.03, 30.58, 32.7 , 31.19, 30.08,
       37.3 , 39.4 , 20.44, 28.34, 31.07, 36.78, 41.44, 47.97, 28.51,
       38.  , 38.71, 39.24, 59.61, 29.19, 35.09, 42.18, 46.8 , 34.14,
       46.71, 48.07, 49.53, 23.46, 22.13, 24.21, 29.03, 32.51, 38.17]))
```

[91]:
```
x = la.lstsq(M,b,rcond=None)
x
```

[91]:
```
(array([10.63041667, 13.79291667, 13.31541667, 20.24666667, 24.32916667,
        29.46291667,  5.74916667, 14.97541667, 17.70666667, 20.11791667]),
 array([245.843925]),
 10,
 array([4.24264069, 2.82842712, 2.82842712, 2.82842712, 2.82842712,
        2.82842712, 2.82842712, 2.82842712, 2.82842712, 2.82842712]))
```

[92]:
```
(M @ x[0], b - M @ x[0])
```

[92]:
```
(array([24.42333333, 23.94583333, 30.87708333, 34.95958333, 40.09333333,
        16.37958333, 25.60583333, 28.33708333, 30.74833333, 27.10833333,
        34.03958333, 38.12208333, 43.25583333, 19.54208333, 28.76833333,
        31.49958333, 33.91083333, 33.56208333, 37.64458333, 42.77833333,
        19.06458333, 28.29083333, 31.02208333, 33.43333333, 44.57583333,
        49.70958333, 25.99583333, 35.22208333, 37.95333333, 40.36458333,
        53.79208333, 30.07833333, 39.30458333, 42.03583333, 44.44708333,
        35.21208333, 44.43833333, 47.16958333, 49.58083333, 20.72458333,
        23.45583333, 25.86708333, 32.68208333, 35.09333333, 37.82458333]),
 array([-3.21333333,  0.40416667,  3.69291667, -2.23958333, -3.95333333,
         0.83041667,  0.80416667,  1.58291667,  2.09166667,  1.98166667,
        -0.25958333,  2.50791667,  1.20416667, -2.51208333,  1.81166667,
         1.20041667, -2.72083333, -3.48208333, -0.34458333, -3.37833333,
         1.37541667,  0.04916667,  0.04791667,  3.34666667, -3.13583333,
        -1.73958333,  2.51416667,  2.77791667,  0.75666667, -1.12458333,
         5.81791667, -0.88833333, -4.21458333,  0.14416667,  2.35291667,
        -1.07208333,  2.27166667,  0.90041667, -0.05083333,  2.73541667,
        -1.32583333, -1.65708333, -3.65208333, -2.58333333,  0.34541667]))
```

[76]:
```
np.array(list(actual.values()))
```

[76]:
```
array([10, 15, 13, 20, 25, 30,  5, 15, 17, 20])
```

[93]:
```
len(x[0])
```

[93]:
```
10
```

[94]:
```
x[0] - vv
```

[94]:
```
array([ 0.63041667, -1.20708333,  0.31541667,  0.24666667, -0.67083333,
       -0.53708333,  0.74916667, -0.02458333,  0.70666667,  0.11791667])
```

[95]:
```
x[0],vv
```

[95]:
```
(array([10.63041667, 13.79291667, 13.31541667, 20.24666667, 24.32916667,
        29.46291667,  5.74916667, 14.97541667, 17.70666667, 20.11791667]),
 array([10, 15, 13, 20, 25, 30,  5, 15, 17, 20]))
```

[ ]:
```

```