

week08-04-cycles

March 2, 2025

Consider a *cycle*

```
[3]: import numpy as np
from graphviz import Digraph

def cycle(n=5, labels=None):
    if labels==None:
        labels= n*[1]
    cyc = Digraph()
    cyc.attr(rankdir='LR')
    I = list(range(n))

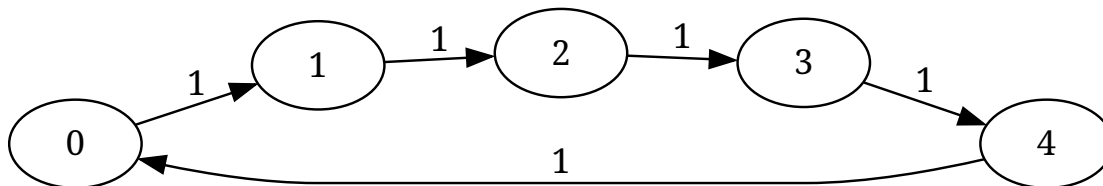
    for i in I:
        cyc.node(f"{i}")

    for i in I:
        cyc.edge(f"{i}", f"{np.mod(i+1,n)}", f"{labels[i]}")

    return cyc

cycle()
```

[3]:



What is the corresponding matrix P ?

Well, we can represent the a graph by a dictionary whose keys are pairs (a,b) and whose value is the probability of the corresponding state transition.

```
[18]: cd = { (n,(n+1) % 5):1 for n in range(5) }
cd
```

```
[18]: {(0, 1): 1, (1, 2): 1, (2, 3): 1, (3, 4): 1, (4, 0): 1}
```

```
[30]: def lookup(pair,dict):
        if pair in dict.keys():
            return dict[pair]
        else:
            return 0

    def mat(dict):
        return np.array([[ lookup((i,j),dict) for j in range(5)] for i in range(5)])

    M=mat(cd)
```

```
[15]: import numpy.linalg as npl
    e_vals, e_vects = npl.eig(M)
    e_vals
```

```
[15]: array([0., 0., 0., 0.])
```

Notice that every eigenvalue of the matrix M is 0. In particular, the conclusion of the *Frobenius-Perron Theorem* does not hold for M.

Adding an extra edge $4 \rightarrow 4$ fixes the problem!

```
[28]: acd = { (n,(n+1) % 5):1 for n in range(5) }
    acd[(4,4)] = .5
    acd[(4,0)]= .5
    acd
```

```
[28]: {(0, 1): 1, (1, 2): 1, (2, 3): 1, (3, 4): 1, (4, 0): 0.5, (4, 4): 0.5}
```

```
[32]: Ma=mat(acd)
    Ma
```

```
[32]: array([[0. , 1. , 0. , 0. , 0. ],
        [0. , 0. , 1. , 0. , 0. ],
        [0. , 0. , 0. , 1. , 0. ],
        [0. , 0. , 0. , 0. , 1. ],
        [0.5, 0. , 0. , 0. , 0.5]])
```

```
[33]: ae_vals,ae_vecs = npl.eig(Ma)
    ae_vals
```

```
[33]: array([ 1.          +0.j          ,  0.37103484+0.80377194j,
        0.37103484-0.80377194j, -0.62103484+0.50229651j,
        -0.62103484-0.50229651j])
```

```
[35]: [ abs(x) for x in ae_vals ]
```

[35]: [0.9999999999999989,
0.8852774620837582,
0.8852774620837582,
0.7987402949603679,
0.7987402949603679]

[]: