Math087 - Midterm Report 1 due 2025-02-14

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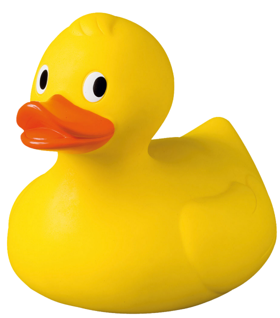
1 Instructions

Complete the project report described below in the style of a lab report. You should attach embedded code – and its output, as appropriate – in the submitted PDF. Take care to explain fully the formulation of linear programs and the associated flow diagrams – including all choices of nodes and edges. Write a summary of your solution, identifying the optimal shipping routes and how to deal with unexpected crises.

2 The Problem

In this problem, consider yourself to be the logistics manager for a supply-chain company that makes and sells rubber ducks.



Your task is to optimize the shipping of ducks through the country.

You have 3 main warehouses in Santa Fe, El Paso, and Tampa Bay. At each warehouse, you have a certain number of rubber ducks that must be shipped to your stores in various cities across the US. The number of supplies (in units of `ducks`) for each warehouse is listed here:

Table 1: Supplies (in `ducks`)

city	Santa Fe	El Paso	Tampa Bay
supply	700	200	200

You have 5 stores located across the US that will sell these ducks to your customers. The demands at each store for the given month are as follows (again in units of `ducks`):

In order to ship the rubber ducks to each of these cities, you use an air-freight service that charges different prices between different cities depending on how many ducks you ship. Some

Table 2: Demand (in `ducks`)

city	Chicago	LA	NY	Houston	Atlanta
demand	200	200	250	300	150

Table 3: Shipping costs ($ per `duck`)

	Chicago	LA	NY	Houston	Atlanta
Santa Fe	6	3	-	3	7
El Paso	-	7	-	2	5
Tampa Bay	-	-	7	6	4

routes are not available. The following grid indicates the cost per duck (in dollars) to ship from a warehouse to a store (these routes are one-way; you can't ship from a store back to the warehouse):

Thus e.g. it costs 6 dollars to ship a single duck from Santa Fe to Chicago.

Now, Houston and Atlanta are hubs that – in addition to accepting ducks to meet their own local demand – can also relay ducks to other destinations. Those routes and their associated costs are indicated here:

Table 4: Relay route costs ($ per `duck`)

	Chicago	LA	NY	Houston	Atlanta
Houston	4	5	6	-	2
Atlanta	4	-	5	2	-

Finally, shipping on each route is restricted to a maximum of 200 units (ducks) for the month.

The basic problem is to determine an optimum shipping plan that minimizes the total cost of shipping while meeting all customer demands with available supplies. Your task will be to formulate a linear program to solve this problem – and some variations of this problem –, and to enter this linear program and solve it using the `linprog` solver available via `python/scipy`.

For simplicity, I'll accept answers involving partial ducks – you shouldn't treat this as an integer programming problem.

3 Your tasks

a. Formulate and draw a network-flow model describing this supply chain problem. You must explain all the constraints that you have included and why you have included them. You are **strongly** encouraged to include a node for the source of ducks (an `initial` node) and a node for the customers (a `terminal` node) even though these nodes are not really involved in the air-freight.

b. Use your network-flow model to formulate the linear program.

   Make sure that you include a clear description (in addition to code) of the objective function, the equality constraints, and the inequality constraints and how this data is obtained from the network flow.

   You may adapt the code given in examples in class (`restaurant/tablecloths` example, and `grocery store` example), or you may "start from scratch" and write your own code to handle this model.

c. Enter your model into `python` and use the `linprog` command (from `scipy.optimize`) to find an optimal solution. As usual, you can use `colab` or a `python` interpreter.

   Be sure to include any code used in producing the specifications for the equality and inequality constraints. Include in your report the formulation of the arguments to `linprog` and the text of the output of that function.

d. Next, consider the following variant problems. Assume that shipping workers in LA are unhappy and contemplating a strike. They demand changes that would result in the doubling of all shipping costs to LA; if their demand is not met they will strike and the maximum number of supplies that can be shipped on *all routes to LA* is cut in half (i.e., from 200 to 100). Model both scenarios and see which one increases the cost more.

   Explain how you adapt your model to account for these scenarios.

e. Test the same scenarios contemplated in 4. on the hub city of Houston. Is the result more or less drastic? Which city (LA or Houston) would have the larger impact on costs if a work stoppage occurred?

f. Finally, return to the non-strike scenario, and consider the value of the goods being made and sold. The following table shows the profit made at each city from selling 1 rubber duck. Note that in the warehouse cities, you are manufacturing the ducks, which amounts to a cost rather than revenue. Now, use a related linear program to maximize the total profit, taking into account the shipping costs.

   > In contrast to the first part where you minimize the *shipping costs*, when you are maximizing profit you should view the number of available ducks in each warehouse, and demand for ducks at the stores, as *upper bounds* on the relevant variables.
   >
   > In words, this means that you ship **no more than** the available ducks from each warehouse. And you can sell **no more than** the indicated demand for ducks allows in each city.
   >
   > In this setting, you may well end up not shipping as many ducks to a given city as it might be possible to sell – the linear program will optimize profitability.

Table 5: Profit by city (in $ per duck)

city	Santa Fe	El Paso	Tampa Bay	Chicago	NY	Houston	Atlanta	LA
profit ($/duck)	-8	-5	-10	15	25	10	10	20