Midterm Project – Supply chain

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Instructions

Complete the project report described below in the style of a lab report – not that of a problem set. You may attach embed code in the submitted PDF. Take care to explain fully the formulation of linear programs and the associated flow diagrams – including all choices of nodes and edges. Write a summary of your solution, identifying the optimal shipping routes and how to deal with unexpected crises.

The problem

In this problem, consider yourself to be the logistics manager for a supply-chain company that makes and sells rubber ducks.

Your task is to minimize the shipping costs for your supply chain system for a given month, and then to maximize the overall profit for that month.

You have 3 main warehouses in Santa Fe, El Paso, and Tampa Bay. At each warehouse, you are given a certain number of rubber ducks that must be shipped to your stores in various cities across the US. The number of supplies (in units of ducks) for each warehouse is listed here:

Table 1: Supplies (in ducks)

Santa Fe	El Paso	Tampa Bay
700	200	200

You have 5 stores located across the US that will sell these ducks to your customers. The demands at each store for the given month are as follows (again in units of ducks):

Table 2: Demand (in ducks)

Chicago	LA	NY	Houston	Atlanta
200	200	250	300	150

In order to ship the rubber ducks to each of these cities, you use an air-freight service that charges different prices between different cities depending on how many ducks you ship. Some routes are not available. The following grid indicates the cost per duck (in dollars) to ship from a warehouse to a store (these routes are one-way; you can't ship from a store back to the warehouse):

Table 3: Shipping costs ($ per duck)

	Chicago	LA	NY	Houston	Atlanta
Santa Fe	6	3	-	3	7
El Paso	-	7	-	2	5
Tampa Bay	-	-	7	6	4

Thus e.g. it costs 6 dollars to ship a single duck from Santa Fe to Chicago.

Now, Houston and Atlanta are hubs that – in addition to accepting ducks to meet their own local demand – can also relay ducks to some other destinations. Those routes and their associated costs are indicated here:

Table 4: Relay route costs ($ per duck)

	Chicago	LA	NY	Houston	Atlanta
Houston	4	5	6	-	2
Atlanta	4	-	5	2	-

Finally, shipping on each route is restricted to a maximum of 200 units (ducks) for the month.

The basic problem is to determine an optimum shipping plan that minimizes the total cost of shipping while meeting all customer demands with available supplies. Your task will be to formulate a linear program to solve this problem – and some variations of this problem –, and to enter this linear program and solve it using `python/scipy`.

For simplicity, I'll accept answers involving partial ducks – you don't have to treat this as an integer programming problem.

Your tasks:

1. Draw a clearly labeled network-flow model describing this supply chain problem. You must explain all the constraints that you have included and why you have included them. You are strongly encouraged to include a node for the source of ducks (an `initial` node) and a node for the customers (a `terminal` node) even though these nodes are not really involved in the air-freight these nodes.

2. Use your network-flow model to formulate the linear program. Be sure to use descriptive variable names. Give the objective function, the equality constraints, and the inequality constraints.

3. Enter your model into `python` and use the `linprog` command (from `scipy.optimize`) to find an optimal solution. You are free to use `colab` or a `python` interpreter. Describe and include any code used in producing the matrices representing the equality and inequality constraints. Include the formulation of the call to `linprog` and the text of the output.

4. Next, consider the following variant problems. Assume that shipping workers in LA are unhappy and contemplating a strike. They demand that all shipping costs to LA be doubled; if their demand is not met they will strike and the maximum number of supplies that can be shipped on *all routes to LA* is cut in half (i.e., from 200 to 100). Model both scenarios and see which one increases the cost more.

5. Test the same scenarios contemplated in 4. on the hub city of Houston. Is the result more or less drastic? Which city (LA or Houston) would cause the most problems for us if a work stoppage occurred? Remark: Technically you are only asked to change values corresponding to routes *to Houston* and not from. However specifically in this problem that difference does not affect the result.

6. Finally, return to the non-strike scenario, and consider the value of the goods being made and sold. The following table shows the profit made at each city from selling 1 rubber duck. Note that in the warehouse cities, you are making the goods, which amounts to a cost rather than revenue. Now, use a related linear program to maximize the total profit, taking into account the shipping costs.

   Here assume that supply and demand are only upper bounds for the actual production and sales correspondingly. Then compare it with the profit based on the flow from part 3. Report which cities underproduced and which were undersupplied.

Table 5: Profit by city (in $ per duck)

Santa fe	El Paso	Tampa Bay	Chicago	NY	Houston	Atlanta	LA
-8	-5	-10	15	25	10	10	20