Homework 7

Math 87

Due: November 1, 2023

1 You can't handle the truth!

Let's look at the impact of a juror's likelihood to vote correctly and the chance of a wrongful conviction. A jury consists of 12 members, and each member must vote either guilty or not guilty. Assume that the vote of each member is independent. Suppose that the probability that a juror votes correctly (i.e. in line with the actual innocence/guilt of the person on trial) is 70%. *Disclaimer: these numbers are made up, though there are many studies done on this topic.*

1. What is the probability that there is an inconclusive (tied) voting outcome?
2. If there is an inconclusive verdict, then a judge breaks the tie. Suppose a judge votes correctly 80% of the time. What is the probability of a tie and a judge voting correctly?
3. A verdict is reached if at least 7 jury members vote the same way, or a tie is broken in the manner given above. What is the probability that the correct verdict is reached?
4. Of the times where the jury reaches the incorrect verdict, suppose 90% of people on trial are guilty but not convicted and 10% are innocent but convicted. What is the probability of a wrongful conviction?
5. What is the expected number of people wrongfully convicted in 2 million trials?
6. Write a `python` function that takes in the likelihood of a juror to vote correctly and returns the probability of a wrongful conviction. You may use `scipy.stats.binom` or compute your own binomial distribution. Include your code in your report.
7. Test your function on likelihoods 50%, 60%, 70%, 80%, and 90%. Report both the probability of a wrongful conviction and the expected number of people wrongfully convicted in 2 million trials. Discuss your results.
8. What are some flaws in the assumptions of the problem?

2 The Germinator

Suppose we are monitoring a population of bacteria in a lab. Initially there are 500 organisms. You've predicted that every day the population grows by 6.35%. Furthermore, suppose that you introduce an additional m organisms to the population every day.

1. Write a recurrence relation for the population after k days, p_k, as a function of p_{k-1} and m.
2. Solve this recurrence relation for p_k as a function of m and k (i.e. no explicit reference to p_{k-1}).

3. Suppose that you are breeding the bacteria for an experiment, and need a fixed number of organisms g. Find an expression for the day, k, in which your desired number of organisms g, as a function of m.

4. Write a `python` function that takes in goal population g and the added amount of organisms m and returns the number of days, k, to reach the goal. Suppose the goal balance is $200,000$ organisms. How many days does it take to reach the goal if we introduce 100 bacteria a day? 250? How many months does it take to reach a goal of $2,000,000$ in the case of introducing 100 bacteria and in the case of introducing 250?