Tufts University - Department of Mathematics
Math 87 Homework 2
Due: Wednesday, Sept 20

1 Root-finding methods

Consider the function $f(x) = e^x - xe^x$.

1. Find the root of f.

2. Test Newton's method, the secant method, and bisection for finding this root. For Newton, use initial points $x_0 = \frac{1}{2}, 2, 10, -\frac{1}{2}, -5$. For secant, use initial points $(x_0, x_1) = (0, 2)$, $(0, 10)$, $(-1, 2)$, $(-5, 5)$, $(-10, 2)$. For bisection, use $[x_L, x_R] = [0, 2]$, $[-5, 5]$, $[-10, 2]$, $[-1, 2]$, $[0, 1]$. Report the iteration count and approximation to the root for each method. You may use the SciPy optimization library.

3. How do the initial parameters impact the success in finding the root for the three methods?

4. Sketch or plot the function f. Use a geometric argument to explain why the negative initial parameters are not successful for some of the methods. What happens as $x \to -\infty$?

5. This is an example as to why a "stopping criteria" to stop iterating at the kth step should NOT be checking $f(x_k) \approx 0$. How does your answer from (d) justify that? What is a better termination condition to determine convergence?

2 Linear Program

Consider the optimization problem, $\max f(x, y) = x + 2y$ subject to:

$$\begin{aligned} y &\leq 9 \\ -y &\leq -1 \\ 2x + y &\leq 25 \\ -2x - y &\leq -9 \\ -2x + y &\leq 1 \\ 2x - y &\leq 15. \end{aligned}$$

1. Draw the feasible region. Label the boundary curves and corner points.

2. Find the maximum value of f and the point where it occurs.

3. Verify your answer using SciPy.

3 Some very Type A bakers

A bakery wants to sell forty five Valentine's Day gift bags. They have decided to offer two types of bags: Bags of type A will contain four cupcakes and two cookies, and bags of type B will contain two cupcakes and five cookies. Baskets of type A will be sold for \$12 and baskets of type B will be sold for \$16. The bakery has 90 cookies and 115 cupcakes in total.

Write the bakery's optimization problem as a linear program. Solve this to determine how many baskets of both types should be made. If a fractional solution is obtained, round down to whole number solutions. What is the maximum profit? You may solve this by drawing the feasible region or using python.