Homework 9

Math 87

Due: Nov 20 2023

1 Spline interpolation

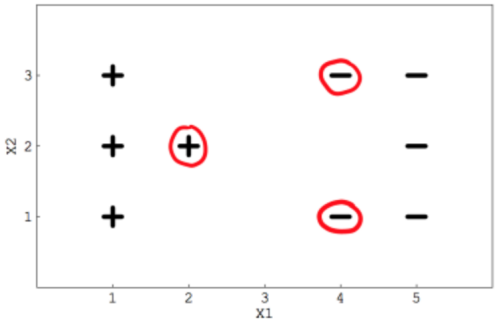
Manually form a matrix for cubic spline interpolation of $f(x)$ with natural and with clamped boundary conditions, if

$$f(0) = 0,\ f(1) = 2,\ f(2) = 1.$$

You may use numpy to solve the resulting linear systems for coefficients.

2 Comprehending SVM

Suppose we have data drawn from two different populations, shown in the figure below as (+) and (−). Support vector machines could be simply used to linearly separate such classes of data.



- Draw your approximation of the separating line SVM would generate to separate these two classes of data.
- Suppose that the (+) in the red circle was deleted from the data set. Would the supporting line change? If so, draw it.
- Suppose that all red circled data points were deleted. Would the supporting line change? If so, draw it.

3 Computing kernels

Let $\mathbf{x} = (x_1, x_2), \mathbf{y} = (y_1, y_2) \in R^2$. Suppose we use the kernel $K(\mathbf{x}, \mathbf{y}) = (\mathbf{x}.\mathbf{y} + c)^2$, where $\mathbf{x}.\mathbf{y}$ is the dot product between $\mathbf{x}$ and $\mathbf{y}$. Compute the higher dimensional embedding (i.e. the feature map) of $\mathbf{x}$ corresponding to this kernel.