

# Homework 3

Math 87

Due September 27 2023

## 1 Work the fertilizer

A farmer owns 45 acres of land. They are planning to plant each acre with either wheat or corn. Each acre of wheat yields \$200 in profits, whereas each acre of corn yields \$300 in profits. Each acre of wheat requires 3 workers and 2 tons of fertilizer. Each acre of corn requires 2 workers and 4 tons of fertilizer. The farmer has 100 workers and 120 tons of fertilizer available.

1. Write down and solve the primal problem to determine how many acres of wheat and corn need to be planted to maximize profits. What is the maximum profit? Non-integer values are allowed. You may solve this by drawing the feasible region or using python.
2. Write down and solve the dual linear program. You may solve this by drawing the feasible region or using python.
3. Explain the meaning of the dual variables, objective function, and the constraints in the dual problem.

## 2 Hope for the best...

You've decided to build a doomsday shelter under your house. You have a barrel which can store seven gallons of food, and you decide to fill it with rice and dried beans. You estimate that each gallon of beans will provide enough nutrition for approximately 9 days of meals, whereas each gallon of rice would only provide around 5 days (Are these at all realistic? Asking for my doomsday shelter). Each gallon of beans costs \$12 and each gallon of rice costs \$5. You have \$60 to spend, and would like to calculate how many gallons of rice and beans to buy in order to maximize the number of days your food stores will last (fractional purchases are allowed).

1. Write this problem as a dual linear program.
2. Find the solutions to both the primal and the dual linear programs by plotting the feasible sets. Confirm that both the strong duality theorem and complementary slackness are satisfied. Write out the dual prices for each of your primal constraints.
3. Suppose you can increase the size of your barrel to hold an additional  $c$  gallons of food. Does the dual price for this modified constraint provide an accurate prediction of the increase in the primal objective function (i.e. the number of days of nutrition)? Answer this question for  $c = 1, 2, 4, 6$ .