DejaVu Sans

Forms over a finite field

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Let's get started trying to formalize some results in linear algebra about bilinear forms!

As a first step, we should formalize the proof that there is – up to the right notion of conjugacy – only one non-degenerate alternating form on an even dimenisonal vector space (and that no such form exists on any odd dimensional vector space). This statement is true for any field whatsoever. (Corresponding statements for symmetric forms are more complicated, so the alternating case seems a good place to start).

1 Initial tasks

We are going to work with a finite vector space over a field k:

```
variable (k:Type*) [ Field k ]
variable (V:Type*) [ AddCommGroup V] [Module k V] [FiniteDimensional k V]
```

You can read about the `FiniteDimensional` typeclass in §12 of [Math in Lean].

You should read about how Lean represents a *basis* of our vector space:

```
varaible { : Type*} (B : Basis  k V)
```

Read this as: "B is a basis for the k-vector space V with index set ".