ERRATUM TO "THE CENTRALIZER OF A NILPOTENT SECTION"

GEORGE MCNINCH

The paper (McNinch 2008) contains an error concerning the smoothness of certain group schemes. Specifically, the statements (2.3.1) and (2.3.2) of *loc. cit.* – which give conditions for the smoothness of the stabilizer of a section – are incorrect. The conditions given by these statements fail to guarantee the *flatness* of the stabilizer group scheme, which is necessary for the smoothness..

Let $\mathcal{A}$ be a complete discrete valuation ring with fractions K and residue field k, and let $\mathcal{G}$ be a split reductive group scheme over $\mathcal{A}$. The results of the preceding paragraph were applied in *loc. cit.* §5.2 to conclude the smoothness over $\mathcal{A}$ of the centralizer $C = C_{\mathcal{G}}(X)$ of a nilpotent section $X \in \mathrm{Lie}(\mathcal{G}) = \mathrm{Lie}(\mathcal{G})(\mathcal{A})$ provided that C_{K} is a smooth group scheme over K, that C_{k} is a smooth group scheme over k, and that $\dim C_{\mathrm{K}} = \dim C_{\mathrm{k}}$. Because of the error noted above, that conclusion is not supported by the given arguments.

As a consequence, the proof of (McNinch 2008, Theorem B) is incorrect. That Theorem asserts that the component group of a nilpotent centralizer is – in a suitable sense – independent of "good" characteristic. The arguments of (McNinch 2008, §7) all depend on the smoothness of the *full* centralizer $C = C_{\mathcal{G}}(X)$, which has not been confirmed. Note that I am not aware of any cases where the concusion is known to be false; see (Booher 2016) where the smoothness is verified in some special cases.

The given proof of (McNinch 2008, Theorem A) also invoked the smoothness of C, but here the situation is repairable. Theorem A of the paper asserts that the root datum of the reductive quotient of the centralizer of a nilpotent element is in a suitable sense independent of "good" characteristic.

To repair the argument, one can use (McNinch 2021, Prop. 3.5.1). This result givesan open subgroup scheme $\mathcal{M}$ of the centralizer which is connected and reductive over $\mathcal{A}$, and for which $\mathcal{M}_{\mathrm{K}}$ and $\mathcal{M}_{\mathrm{k}}$ are, respectively, Levi factors of the connected centralizers C^0_{K} and C^0_{k}.

One can now repeat the proof of Theorem A given in (McNinch 2008, §5.8), using $\mathcal{M}$ rather than the full centralizer C.

REFERENCES

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DEPARTMENT OF MATHEMATICS, TUFTS UNIVERSITY, 503 BOSTON AVENUE, MEDFORD, MA 02155, USA
Email address: george.mcninch@tufts.edu

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