Math 51 **Homework 6** **Spring 2021**

Upcoming Deadlines:

Sunday, March 7, 11:59 p.m.: Quiz 5 (on §2.8, 3.2, Canvas)

Friday, March 12, 5 p.m.: HW 6 (on §2.8, 3.2, Gradescope)

You can scan your solutions into a pdf file using Notes in an iPhone or other scanning apps in an Android phone. Submit your solutions as a single pdf file in Gradescope. Do not submit a pdf converted from a photo. Photos have uneven lighting and are often difficult to read. Before you submit, you need to indicate in Gradesxope the pages that each question is on. You may submit your homework up to one day late, i.e., by 5 p.m., Saturday, March 13, but if you do, you lose 10% of your grade.

Homework Exercises:

1. **(Variation of parameters)** Using variation of parameters, find the general solution of
$$4x'' - 4x' + x = \frac{8}{t^2}e^{t/2}, \quad t > 0.$$

2. **(Tangent $E(t)$)** Find the general solution of
$$x'' + x = \tan t, \quad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$
(*Hints*: $\sin^2 t = 1 - \cos^2 t$,
$$\int \sec t \, dt = \int \sec t \left(\frac{\sec t + \tan t}{\sec t + \tan t} \right) dt.$$
)

3. **(Nonconstant coefficients)** Verify that t and e^t are solutions of the homogeneous equation corresponding to
$$[(t-1)D^2 - tD + 1]x = (t-1)^2 e^t, \quad t > 1. \tag{N}$$
and find the general solution of the nonhomogeneous equation (N).

(*More problems on next page.*)

4. **(Matrix product)** In this problem, you are given $A, \mathbf{E}(t)$ and $\mathbf{x}_i(t)$.

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad \mathbf{E}(t) = \begin{bmatrix} t \\ -1 \end{bmatrix}, \quad \mathbf{x}_1(t) = \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}, \quad \mathbf{x}_2(t) = \begin{bmatrix} 0 \\ -t \end{bmatrix}$$

   (a) Find $D\mathbf{x}_i(t)$.
   (b) Find $A\mathbf{x}_i + \mathbf{E}(t)$.
   (c) Determine whether $\mathbf{x}_i(t)$ is a solution of $D\mathbf{x} = A\mathbf{x} + \mathbf{E}(t)$.

5. **(Linear systems)** Determine which of the following systems are linear. For each linear system,

   (i) determine whether the system is homogeneous,
   (ii) find its order,
   (iii) write it in matrix form.

   (a)
$$\begin{aligned} x' &= -ty - z \\ y' &= -\frac{x}{t} - z + 1 \\ z' &= -x - ty + t^2 \end{aligned}$$

   (b)
$$\begin{aligned} x' &= 2x - 3y \\ y' &= 3xy + 4 \end{aligned}$$

   (c)
$$\begin{aligned} x' &= x + 2y \\ y' &= 3x + 4y. \end{aligned}$$

6. **(ODE as a linear system)** Consider the following differential equation,

$$(D-1)^2(D+1)x = t. \tag{N}$$

   (a) Find the equivalent linear system (S_N).
   (b) Find the general solution of (N) and use it to obtain the general solution of (S_N).
   (c) Write the linear system (S_N) in matrix form.
   (d) Write the general solution of the linear system (S_N) in the form

$$\mathbf{x} = c_1\mathbf{h}_1(t) + \cdots + c_n\mathbf{h}_n(t) + \mathbf{p}(t).$$

(End of Homework 6)