

**Exercises**

1. Calculate the Laplace transform of the following functions:

- (a)  $t^n e^{mt}$
- (b)  $te^{2t} \sin(t)$
- (c)  $(t^3 + 3)^2$

2. Compute the inverse transform of the following functions:

- (a)  $\frac{1}{s^2 + 2s + 5}$
- (b)  $\frac{1}{3s + 6}$
- (c)  $\frac{s + 3}{s^2 + 10s + 25}$

3. Using the Laplace Transform, solve the following initial value problems:

- (a)  $(D^2 + 2D + 2)x = 0; \quad x(0) = x'(0) = 0$
- (b)  $(D^2 + 4)x = t; \quad x(0) = -1; \quad x'(0) = 0$

4. Write the function

$$g(t) = \begin{cases} t^2 & t < 3 \\ e^{-t} & t \geq 3 \end{cases}$$

in step function notation, where  $u_a(t) = \begin{cases} 0 & t < a \\ a & t \geq a \end{cases}$

5. Find the inverse transform of the following functions:

- (a)  $\frac{se^{-2s}}{s^2 + 2}$
- (b)  $\frac{e^{-s}}{s(s + 3)}$

6. Solve the following initial value problem:

$$(D^3 - D)x = \begin{cases} 4 & t < 4 \\ 0 & t \geq 4 \end{cases}; \quad x(0) = x'(0) = x''(0) = 0$$