

1. Consider the system of ordinary differential equations

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 5 & -3 & 0 \\ 3 & -5 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}.$$

Consider the solutions: 
$$\begin{cases} x_1 = (6c_1 + 6c_3)e^{4t} + (-2c_2 + 2c_3)e^{-4t} \\ x_2 = (2c_1 + 2c_3)e^{4t} + (-6c_2 + 6c_3)e^{-4t} \\ x_3 = (c_1 + c_3)e^{4t} + (c_2 - c_3)e^{-4t} - 2 \end{cases}$$

- Describe these solutions in the form  $\mathbf{p} + c_1\mathbf{h}_1 + c_2\mathbf{h}_2 + c_3\mathbf{h}_3$

- Check directly that  $\mathbf{p}$  and  $\mathbf{h}_1$  are actually solutions.

- Decide whether this collection of solutions is *complete*.

2. For each of the following scenarios for vectors *in the plane* either draw a picture of such a scenario or explain why this can't be done.

(a) One linearly independent vector

(b) One linearly dependent vector

(c) Two linearly independent vectors

(d) Two linearly dependent vectors

(e) Three linearly independent vectors

(f) Three linearly dependent vectors

3. Consider vectors  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ .

- Show that if  $\mathbf{v}_1 = \mathbf{v}_4$ , then  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$  is linearly dependent.

- Show that if  $\mathbf{v}_3 = \mathbf{0}$ , then  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$  is linearly dependent.