Math 51 **Homework 2** **Spring 2021**

Upcoming Deadlines:

Sunday, February 7, 5 p.m.:	HW 1 (on §1.1, 1.2)
Sunday, February 7, 11:59 p.m.:	Quiz 1 (on §1.1, 1.2, 1.3, 2.2)
Friday, February 12, 5 p.m.:	HW 2 (on §1.3, 1.6, 1.7, 2.2)

Homework 2 covers §1.3, 1.6, 1.7, 2.2 and is due on Friday, February 12, at 5 p.m. You can scan your solutions into a pdf file using Notes in an iPhone or other scanning apps in an Android phone. Submit your solutions as a single pdf file in Gradescope. Do not submit a pdf converted from a photo. Photos have uneven lighting and are often difficult to read. You may submit your homework up to one day late, i.e., by 5 p.m., Saturday, February 13, but if you do, you lose 10% of your grade.

Homework exercises:

1. Solve the initial-value problem:
$$\frac{dx}{dt} + 3x = 8e^t, \quad x(0) = 0.$$

2. Solve the initial-value problem:
$$tx' - x = t^3, \quad x(1) = 0.$$

3. §1.3, exercise 30: Try variation of parameters on the nonlinear equation $x' + x^2 = t$ as follows:
   (a) Find the general solution of $x' + x^2 = 0$ by separating variables.
   (b) Then replace the parameter with a variable and substitute back. What goes wrong?

4. §1.6, exercise 24: Show that for any nonnegative number a and b, the function
$$x(t) = \begin{cases} (t+a)^5, & t < -a \\ 0, & -a \le t \le b \\ (t-b)^5, & t > b \end{cases}$$
is a solution of $dx/dt = 5x^{4/5}$ and satisfies $x(0) = 0$. Why doesn't this contradict with the uniqueness part of the theorem?

5. Plot graph of solutions of the following o.d.e. in the region $-2 \le x \le 2$ and $-2 \le t \le 2$. Use at least five isoclines and sketch at least three different solutions
$$x' = x + 3t$$

6. In this problem you are given a nonhomogeneous equation, the general solution $x = H(t)$ of the related homogeneous equation, and a function $p(t)$ involving two unknown constants A and B:
$$(D^2+1)x = e^t + 1, \quad H(t) = c_1 \sin 2t + c_2 \cos 2t, \quad p(t) = Ae^t + B.$$

(a) Find the values of A and B for which $x = p(t)$ is a particular solution of the nonhomogeneous equation.
(b) Find the general solution of the nonhomogeneous equation.

(End of Homework 2)