Math 51 | Differential Equations | April 14–15, 2022

Worksheet for Week 12

Basic formulas

1. $\mathcal{L}[e^{\lambda t}] = \dfrac{1}{s-\lambda}, \quad s > \lambda.$
2. $\mathcal{L}[1] = \dfrac{1}{s}, \quad s > 0.$
3. $\mathcal{L}[t^n] = \dfrac{n!}{s^{n+1}}, \quad s > 0.$
4. $\mathcal{L}[\cos\beta t] = \dfrac{s}{s^2+\beta^2}, \quad s > 0.$
5. $\mathcal{L}[\sin\beta t] = \dfrac{\beta}{s^2+\beta^2}, \quad s > 0.$

First differentiation formulas

1. $\mathcal{L}[Dx] = s\mathcal{L}[x] - x(0), \quad s > 0.$
2. $\mathcal{L}[D^2x] = s^2\mathcal{L}[x] - s\,x(0) - x'(0), \quad s > 0.$
3. $\mathcal{L}[D^3x] = s^3\mathcal{L}[x] - s^2\,x(0) - s\,x'(0) - x''(0), \quad s > 0.$

Exercises

1. Find the Laplace transform of $f(t) = t^2 - 7 + \cos 2t$.
2. (a) Find the partial fraction decompostion of $\dfrac{2s}{(s-1)(s^2+1)}$.

   (b) Find the inverse transform of $\dfrac{2s}{(s-1)(s^2+1)}$.

   (c) Solve the inital-value problem

$$(D-1)x = 2\cos t; \quad x(0) = 0.$$

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3. Solve the initial-value problem

$$(D^2 + 4)x = e^t; \quad x(0) = x'(0) = 0.$$