Math 51 **Homework 12** Spring 2022

Readings for the Week of April 11, 2022

Martin Guterman and Zbigniew Nitecki, *Differential Equations: A First Course*, 3rd edition. ISBN: 81-89617-20-6.

§5.2 The Laplace Transform: Definitions and Basic Calculations

§5.3 The Laplace Transform and Initial-Value Problems

Problem Set 12

(Due Monday, April 25, 2022, at 11:59 p.m.)

For a 10% penalty on your grade, you may hand in the problem set late, until Tuesday, April 26, 2022, 11:59 p.m.

1. **(Laplace transform from the definition)**
   Let $f(t) = te^{2t}$. Calculate the Laplace transform $F(s) = \mathcal{L}[f(t)]$ directly from the definition and indicate the values of s for which the integral defining $F(s)$ converges.

2. **(Laplace transform)**
   For each of the following functions, calculate its Laplace transform $F(s) = \mathcal{L}[f(t)]$ using the linearity of $\mathcal{L}$ together with the basic formulas summarized at the end of §5.2.

   (a) $f(t) = 2t + e^{-4t} - 3\cos 5t$.

   (b) $f(t) = e^{3t+2}$.

   (c) $f(t) = (t+2)(t+3)$.

3. **(Inverse transform)**
   For each of the following functions, calculate its inverse transform $f(t) = \mathcal{L}^{-1}[F(s)]$ using the linearity of $\mathcal{L}^{-1}$ together with the basic formulas summarized at the end of §5.2.

   (a) $F(s) = \dfrac{1}{3s+1}$.

   (b) $F(s) = \dfrac{2}{s^2+4} - \dfrac{10}{s^4} + \dfrac{1}{s}$.

4. **(First differentiation formula)**
Use the first differentiation formula to find an expression for the Laplace transform $\mathcal{L}[x]$, where x is the solution of the given initial-value problem.

   (a) $(D-1)x = e^{2t}, \quad x(0) = 2.$

   (b) $(D^2-1)x = e^{2t}, \quad x(0) = 0,\ x'(0) = 1.$

   (c) $(D^2+1)x = \cos 3t, \quad x(0) = 0,\ x'(0) = 3.$

5. **(Partial fraction decompostion)**
Find the inverse transform of $F(s) = \dfrac{s+4}{s^2+4s+3}$.

6. **(Initial-value problem)**
Use the Laplace tranform to solve the initial-value problem:

$$(D^2+4)x = t, \quad x(0) = -1,\ x'(0) = 0.$$

(End of Homework 11)