MATH 51 Differential Equations March 31–April 1, 2022

Worksheet Week 10

1. Suppose A is a square matrix and $\vec{v}$ a generalized eigenvector for the eigenvalue λ. Then $(A-\lambda I)^m\vec{v}=\vec{0}$ for some m. What might $e^{t(A-\lambda I)}\vec{v}$ mean? Compare with

$$\vec{v}+t(A-\lambda I)\vec{v}+\frac{t}{2}(A-\lambda I)^2\vec{v}+\cdots+\frac{t^{m-1}}{(m-1)!}(A-\lambda I)^{m-1}\vec{v}.$$

2. Read Note 2 on page 295 of the textbook. Then consider whether each of the following holds for all $n\times n$ matrices A and B—and accordingly give either a proof or a counterexample.

1. $(A+B)^2=A^2+AB+BA+B^2$,
2. $(A+B)^2=A^2+2AB+B^2$.

3. $\lambda=-1$ is a triple eigenvalue associated to $A=\begin{bmatrix}-1&-1&0\\1&-1&1\\0&1&-1\end{bmatrix}$. You do not have to verify this. Find the generalized eigenvectors associated to this eigenvalue.

4. Find **one** nonzero solution of $D\vec{x}=\begin{bmatrix}1&0&0\\2&3&0\\4&5&6\end{bmatrix}\vec{x}$.

5. Find the general solution of $D\vec{x}=\begin{bmatrix}0&0&0&0&1\\1&-1&0&-1&1\\1&0&-1&-1&0\\0&0&0&0&1\\1&0&0&-1&0\end{bmatrix}\vec{x}$.

You may use without checking that $\begin{bmatrix}1+\frac{t^2}{2}\\t\\1\\t^2/2\\t\end{bmatrix}$, $\begin{bmatrix}1\\0\\0\\1\\0\end{bmatrix}$, $\begin{bmatrix}t\\1\\0\\t\\1\end{bmatrix}$ *are linearly independent solutions.*

Recitation R

Name

6. Find the general solution of $D\vec{x} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \vec{x}$.