Recitation handout ODEs via integration

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1. For a real number k consider the function

$$x(t) = kte^{5t}.$$

For what value(s) of k is $x(t)$ a solution to the following ODE?

$$x'' - 5x' = e^{5t}$$

2. Two species of rabbits – species X and species Y – live in a national park. Initially, the population consists of 30 thousand X rabbits (and no Y rabbits). Each month one thousand new Y rabbits (and no new X rabbits) arrive in the park.

However, 1 thousand of the rabbits also leave the park for good each month. Assume that the population of X and Y rabbits are thoroughly mixed, so that the proportion of X rabbits migrating out is the same as the total proportion of X rabbits in the population.

Note that the total number of rabbits in the park remains constant over time, at 30,000.

   a. Set up a differential equation to predict the number of X rabbits in the park after t months.

   (Note that t is a continuous variable; it can assume any 'real value ≥ 0.)

   b. Can you solve this ODE? Remember that $x' = \lambda x$ has solutions $x(t) = x_0 e^{\lambda t}$.

   Describe the population of X rabbits after 6 months.