Math 51 **Problem Set 9**

1. Given the matrix

$$A = \begin{bmatrix} 1 & 9 & 2 & 0 \\ 2 & 3 & 2 & 3 \\ 1 & -6 & 0 & 3 \end{bmatrix}$$

   find the general solution to the equation $B\mathbf{x} = 0$. Notice that the general solution will be a linear combination of vectors that satisfy the above property.

2. (a) Find the eigenvalues of the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

   (b) Notice that there's a multiplicity in one of the eigenvalues. Find the corresponding eigenvectors. If given a system $D\mathbf{x} = A\mathbf{x}$, are you currently able to compute a general solution? Why or why not?

3. Notice that the determinant of an upper triangular matrix is always the product of the diagonal entries of that matrix. Let A be the following upper triangular matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

   (a) Find the eigenvalues of A

   (b) One of the eigenvalues you found should have been 0. Find that eigenvalue's corresponding eigenvector.

4. Find the general solution to

$$D\mathbf{x} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 4 \\ 0 & 0 & 2 \end{bmatrix} \mathbf{x}$$

5. Solve the initial value problem for

$$D\mathbf{x} = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix} \mathbf{x}; \quad \mathbf{x}(0) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$