

Readings for the Week of April 11, 2022

Martin Guterman and Zbigniew Nitecki, *Differential Equations: A First Course*, 3rd edition. ISBN: 81-89617-20-6.

§5.2 The Laplace Transform: Definitions and Basic Calculations

§5.3 The Laplace Transform and Initial-Value Problems

Problem Set 12

(Due **Monday, April 25**, 2022, at 11:59 p.m.)

For a 10% penalty on your grade, you may hand in the problem set late, until Tuesday, April 26, 2022, 11:59 p.m.

1. (Laplace transform from the definition)

Let $f(t) = te^{2t}$. Calculate the Laplace transform $F(s) = \mathcal{L}[f(t)]$ directly from the definition and indicate the values of s for which the integral defining $F(s)$ converges.

2. (Laplace transform)

For each of the following functions, calculate its Laplace transform $F(s) = \mathcal{L}[f(t)]$ using the linearity of \mathcal{L} together with the basic formulas summarized at the end of §5.2.

(a) $f(t) = 2t + e^{-4t} - 3 \cos 5t$.

(b) $f(t) = e^{3t+2}$.

(c) $f(t) = (t+2)(t+3)$.

3. (Inverse transform)

For each of the following functions, calculate its inverse transform $f(t) = \mathcal{L}^{-1}[F(s)]$ using the linearity of \mathcal{L}^{-1} together with the basic formulas summarized at the end of §5.2.

(a) $F(s) = \frac{1}{3s+1}$.

(b) $F(s) = \frac{2}{s^2+4} - \frac{10}{s^4} + \frac{1}{s}$.

4. (First differentiation formula)

Use the first differentiation formula to find an expression for the Laplace transform $\mathcal{L}[x]$, where x is the solution of the given initial-value problem.

(a) $(D - 1)x = e^{2t}, \quad x(0) = 2.$

(b) $(D^2 - 1)x = e^{2t}, \quad x(0) = 0, \quad x'(0) = 1.$

(c) $(D^2 + 1)x = \cos 3t, \quad x(0) = 0, \quad x'(0) = 3.$

5. (Partial fraction decomposition)

Find the inverse transform of $F(s) = \frac{s + 4}{s^2 + 4s + 3}.$

6. (Initial-value problem)

Use the Laplace transform to solve the initial-value problem:

$$(D^2 + 4)x = t, \quad x(0) = -1, \quad x'(0) = 0.$$

(End of Homework 11)