Math 51 **Homework 7** **Spring 2021**

Upcoming Deadlines:

Sunday, March 14, 11:59pm: Quiz 6 (on §3.3, 3.4, Canvas)

Friday, March 19, 5:00pm: Homework 7

This homework covers §3.3 and 3.4.

Homework Exercises:

1. Consider the following matrix A and list of vector-valued functions.

$$A = \begin{bmatrix} 5 & -3 \\ 3 & -5 \end{bmatrix}; \quad \mathbf{h}_1(t) = \begin{bmatrix} 3e^{4t} + e^{-4t} \\ e^{4t} + 3e^{-4t} \end{bmatrix}, \quad \mathbf{h}_2(t) = \begin{bmatrix} 3e^{4t} - e^{-4t} \\ e^{4t} - 3e^{-4t} \end{bmatrix}.$$

   (a) Are the functions $\mathbf{h}_1$ and $\mathbf{h}_2$ solutions to the equation $D\mathbf{x} = A\mathbf{x}$?
   (b) Of the functions that are solutions, do they generate the general solution? Explain why or why not.

2. Let

$$A = \begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix}, \quad \mathbf{E}(t) = \begin{bmatrix} 0 \\ -5e^t \end{bmatrix}.$$

   The formulas

$$\begin{cases} x_1 = c_1 \cos 2t + c_2 \sin 2t + e^t \\ x_2 = -2c_2 \cos 2t + 2c_1 \sin 2t - e^t \end{cases}$$

   describe a collection of solutions of the nonhomogeneous system $D\mathbf{x} = A\mathbf{x} + \mathbf{E}(t)$. Decide whether the collection is complete.

3. Check the following set of vectors for linear independence:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

4. Show that if $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are independent, then the set of vectors obtained by deleting $\mathbf{v}_i$ is also independent, where $1 \le i \le n$.

5. In words (though pictures may be drawn to help illustrate), describe what it means for vectors to be linearly independent and linearly dependent in both 2 and 3 dimensions.

6. (a) Suppose $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are n linearly independent n-vectors. Show that every n-vector $\mathbf{v}$ can be written as a linear combination of $\mathbf{v}_1, \ldots, \mathbf{v}_n$. (*Hint*: View this as a system of equations for the unknown coefficients in the linear combination. What can you say about the determinant of coefficients?)
   (b) Prove that any set $\mathbf{v}_1, \ldots, \mathbf{v}_{n+1}$ consisting of $n+1$ different n-vectors must be linearly dependent.
   (c) Does a set of $k \leq n$ different n-vectors have o be independent?