Worksheet Week 7

1. Consider the system of ordinary differential equations

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 5 & -3 & 0 \\ 3 & -5 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}.$$

Consider the solutions: $\begin{cases} x_1 = & (6c_1 + 6c_3)e^{4t} + & (-2c_2 + 2c_3)e^{-4t} \\ x_2 = & (2c_1 + 2c_3)e^{4t} + & (-6c_2 + 6c_3)e^{-4t} \\ x_3 = & (c_1 + c_3)e^{4t} + & (c_2 - c_3)e^{-4t} - 2 \end{cases}$

- Describe these solutions in the form $\mathbf{p} + c_1\mathbf{h}_1 + c_2\mathbf{h}_2 + c_3\mathbf{h}_3$

- Check directly that $\mathbf{p}$ and $\mathbf{h}_1$ are actually solutions.

- Decide whether this collection of solutions is *complete.*

2. For each of the following scenarios for vectors *in the plane* either draw a picture of such a scenario or explain why this can't be done.

   - One linearly independent vector

   - One linearly dependent vector

   - Two linearly independent vectors

   - Two linearly dependent vectors

   - Three linearly independent vectors

   - Three linearly dependent vectors

3. Consider vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$.

   - Show that if $\mathbf{v}_1 = \mathbf{v}_4$, then $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ is linearly dependent.

   - Show that if $\mathbf{v}_3 = \mathbf{0}$, then $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ is linearly dependent.