Math 51 **Homework 11** **Spring 2021**

Readings

§5.2 The Laplace Transform: Definitions and Basic Calculations
§5.3 The Laplace Transform and Initial-Value Problems

Upcoming Deadlines:

Sunday, April 11, 11:59 p.m.:	Quiz 10 (§5.2 §5.3, Canvas)
Friday, April 16, 5 p.m.:	Homework 11 (Gradescope)

Homework Exercises:

1. **(Laplace transform from the definition)**
   Let $f(t) = te^{3t}$. Calculate the Laplace transform $F(s) = \mathcal{L}[f(t)]$ directly from the definition and indicate the values of s for which the integral defining $F(s)$ converges.

2. **(Laplace transform)**
   For each of the following functions, calculate its Laplace transform $F(s) = \mathcal{L}[f(t)]$ using the linearity of $\mathcal{L}$ together with the basic formulas summarized at the end of §5.2.
   (a) $f(t) = -3t + e^{-3t} - 5\sin 6t$
   (b) $f(t) = e^{2t+3}$
   (c) $f(t) = \sin\left(t + \frac{\pi}{6}\right)$ (*Hint*: Use trig identities.)

3. **(Inverse transform)**
   For each of the following functions, calculate its inverse transform $f(t) = \mathcal{L}^{-1}[F(s)]$ using the linearity of $\mathcal{L}^{-1}$ together with the basic formulas summarized at the end of §5.2.
   (a) $F(s) = \dfrac{1}{3s+1}$
   (b) $F(s) = \dfrac{3}{s^2+1} - \dfrac{20}{s^4} + \dfrac{3}{s}$

4. **(First differentiation formula)**
   Use the first differentiation formula to find an expression for the Laplace transform $\mathcal{L}[x]$, where x is the solution of the given initial-value problem.
   (a) $(D-1)x = e^{3t}, \quad x(0) = 3$
   (b) $(D^2-1)x = e^{2t}, \quad x(0) = 0,\ x'(0) = 1$
   (c) $(D^2+1)x = \cos 3t, \quad x(0) = 0,\ x'(0) = 3$

5. **(Partial fraction decompostion)**
   Find the inverse transform of $F(s) = \dfrac{s+4}{s^2+4s+3}$.

6. **(Initial-value problem)**
   Use the Laplace tranform to solve the initial-value problem:
   $$(D^2+4)x = t, \quad x(0) = -1,\ x'(0) = 0.$$

(End of Homework 11)