Math 51 **Homework 10** **Spring 2021**

Readings

§3.9 Double Roots and Matrix Products.
§3.10 Multiple Roots, pp. 298–299, 301
§3.11 Nonhomogeneous Systems.

This homework will not be collected, but will be part of Exam 2. Solutions will be posted by Tuesday evening, April 6.

Upcoming Deadlines:

Sunday, April 4, 11:59 p.m.:	Quiz 9 (§3.9, 3.10, 3.11, Canvas)
Friday, April 9, 3–5 p.m.:	Exam 2

Homework exercises:

1. The matrix
$$A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$$
has eigenvalue $\lambda = 2$ with multiplicity 2, but only one independent eigenvector $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Find the general solution of $D\mathbf{x} = A\mathbf{x}$.

2. Suppose
$$\begin{aligned} x_1' &= \quad x_2, \\ x_2' &= -x_1 \quad -2x_2, \end{aligned} \qquad x_1(0) = 3, \quad x_2(0) = 4.$$
Find (a) the general solution and (b) the specific solution satisfying the initial condition.

3. Suppose A is an $n \times n$ matrix. Show that if $(A - \lambda I)^2\mathbf{v} = \mathbf{0}$ but $(A - \lambda I)\mathbf{v} \neq \mathbf{0}$, then $\mathbf{v}_1 := (A - \lambda I)\mathbf{v}$ is an eigenvector of A.

4. Given that the matrix
$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$
has eigenvalue 1 with multiplicity 3, find the general solution of $D\mathbf{x} = A\mathbf{x}$.

5. The matrix
$$A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{bmatrix}.$$
has eigenvalues $2i, 2i, -2i, -2i$. For the eigenvalue $2i$, an eigevector $\mathbf{v}$ and a generalized eigenvector $\mathbf{u}$ are
$$\mathbf{v} = \begin{bmatrix} -i \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} -\frac{1}{4}i \\ 0 \\ -i \\ 1 \end{bmatrix}.$$
Write down four linearly independent real solutions.

6. Let
$$A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} e^{2t} \\ 0 \end{bmatrix}.$$
It is known (if you did Question 1 correctly) that the general solution of the homogeneous system $D\mathbf{x} = A\mathbf{x}$ is
$$\mathbf{h} = c_1 e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 1-t \\ t \end{bmatrix}.$$
Find the general solution of $D\mathbf{x} = A\mathbf{x} + \mathbf{E}(t)$.

(End of Homework 10)