Problem Set 4
Linear Independence; Constant Coefficient Linear ODES (real roots)

Math 51 Fall 2021

This problem set won't be collected. You should complete it before the exam, though!

Reminders

- Midterm 1 is February 14 in the open block – 12:00-1:20 PM.

These problems cover (Nitecki and Guterman 1992, secs. 2.4, 2.5)

1. Decide whether the indicated functions are linearly independent on the interval $(-\infty, \infty)$. If the functions are linearly independent, show that this is the case using the definition, or using the Wronskian test. To show that the functions $f_1(t), f_2(t), \dots, f_n(t)$ are linearly dependent, you need to give explicit values $c_1, c_2, \cdots, c_n$ for which at least one c_i is non-zero and such that $0 = c_1 h_1(t) + c_2 h_2(t) + \cdots + c_n h_n(t)$ for every t.

   a. $h_1(t) = 1, \quad h_2(t) = t - 2, \quad h_3(t) = (t-2)^2.$

   b. $h_1(t) = t^5, \quad h_2(t) = |t^5|.$

   c. $h_1(t) = \sin^2(t) + 1, \quad h_2(t) = 2\cos^2(t), \quad h_3(t) = 10$

   d. $h_1(t) = e^t, \quad h_2(t) = e^{t+1}, \quad h_3(t) = 1.$

2. Find the general solution of each of the following ODEs:

   a. $(D^2 - 2)(D+4)^2 x = 0$

   b. $D(D^2 - 4)^2 x = 0.$

   c. $\dfrac{d^2x}{dt^2} - 2\dfrac{dx}{dt} - 4x = 0.$

3. Solve the initial value problem

$$(D+2)^2 D x = 0; \quad x(0) = x'(0) = 1, \quad x''(0) = 0.$$

4. Use the exponential shift formula (see the reminder below) to compute the function $Lf = L[f]$ in each case:

   a. $L = D^2 + D - 1, \quad f(t) = e^t \sin(t)$

   b. $L = (D-1)(D^2 + D + 1), \quad f(t) = te^{2t}.$

Exponential shift formula

Reminder: the exponential shift formula shows that for a polynomial $P(r)$, application of the corresponding differential operator $P(D)$ to the product $e^{\lambda t} y$ for a function y yields

$$P(D)[e^{\lambda t} y] = e^{\lambda t} P(D + \lambda)[y]$$

Bibliography

Nitecki, Zbigniew, and Martin Guterman. 1992. Differential Equations: A First Course. Saunders.