Partial Credit Question 3

$$4r^2 - 4r + 1 = 0$$

$\Leftrightarrow$

$$(2r-1)^2 = 0$$

$\Rightarrow$

$r = \frac{1}{2}$ with multiplicity 2,

So,

$$x(t) = c_1 e^{\frac{1}{2}t} + c_2 t e^{\frac{1}{2}t}.$$

$$x'(t) = \frac{1}{2}c_1 e^{\frac{1}{2}t} + c_2\left[\frac{1}{2}te^{\frac{1}{2}t} + e^{\frac{1}{2}t}\right].$$

Then,

$$x(2) = c_1 e + c_2 2e = e$$

$$x'(2) = c_1 \frac{1}{2}e + c_2[2e] = e$$

So, $c_1 = 0$ and $c_2 = \frac{1}{2}$.

$$\boxed{x(t) = \frac{1}{2}te^{\frac{1}{2}t}}$$