Math 51 **Homework 5** **Spring 2021**

Upcoming Deadlines:

Sunday, February 28, 11:59pm: Quiz 4 (on §2.6, 2.7, Canvas)

Friday, March 5, 3:00-5:00pm: Midterm 1

This homework will not be collected, however §2.6 and 2.7 will be included in Exam 1 and it therefore advised that this homework be seriously attempted. Solutions will be posted Tuesday, March 2.

Homework Exercises:

1. Find the general solution to
$$(D^4+2D^2+1)x=0.$$

2. Solve the initial value problem
$$(D^3-2D^2+2D-4)x=0, \quad x(0)=0,\ x'(0)=4,\ x''(0)=12.$$

3. Consider a nonhomogeneous linear differential equation, $Lx=E(t)$. In a few sentences, explain the idea of the annihilator method for solving this o.d.e. Some vocabulary that might be useful to include: homogeneous equation, nonhomogeneous equation, homogeneous solution, particular solution.

4. Find an annihilator of smallest possible order for:
(a) $e^t+\sin 2t-3$
(b) $t^2+e^t\sin 3t$

5. Make a *simplified guess* for a particular solution to the following equation (note: you do not need to solve for the coefficients),
$$(D-1)^2(D^2+1)^3(D+2)x=t^2e^{3t}+e^t+e^{-t}\sin 3t+t^4.$$

6. Let us define the constant coefficient differential operator $L=D^3-2D^2+D$, the function $f(t)=1+e^{-2t}$, and let x be a function of t.
(a) Solve the o.d.e. $Lx=0$ given the initial values
$$x(0)=0,\ \ x'(0)=0,\ \ x''(0)=1.$$
(b) Find an annihilator of f, i.e. find a polynomial $A(D)$ such that $A(D)f=0$.
(c) Find the general solution to the o.d.e. $Lx=f$.