1. (a). $\mathcal{L}\left[\frac{1}{2}t^2 \sin 5t\right]$

$$= \frac{1}{2}\mathcal{L}\left[t^2 \sin 5t\right]$$

$$= \frac{1}{2}\frac{d^2}{ds^2}\mathcal{L}\left[\sin 5t\right]$$

$$= \frac{5}{2}\frac{d^2}{ds^2}\left(1/(s^2+25)\right)$$

$$= \frac{5}{2}\frac{d}{ds}\left(-2s/(s^2+25)^2\right)$$

$$= \frac{5}{2}\left\{\left(-2(s^2+25)^2 + 4s(s^2+25)\cdot 2s\right\}\right/(s^2+25)^4$$

$$= 5(3s^2-25)/(s^2+25)^3$$

(b). $\mathcal{L}\left[t^n e^{mt}\right] = F(s-m)$,

$$\left(F(s) = \mathcal{L}\left[t^n\right] = \frac{n!}{s^{n+1}}\right.$$

$$= n!/(s-m)^{n+1}$$

(c). $\mathcal{L}\left[te^{2t}\sin 4t\right] = (-1)\frac{d}{ds}\mathcal{L}\left[e^{2t}\sin 4t\right]$

$$= (-1)\frac{d}{ds}\left(\frac{4}{(s-2)^2+16}\right)$$

$$= (-1)\frac{d}{ds}\left(\frac{4}{s^2-4s+20}\right)$$

$$= (-1)\cdot -8(s-2)/(s^2-4s+20)^2$$

$$= \frac{8s-16}{(s^2-4s+20)^2}.$$

2.(a) $\mathcal{L}^{-1}\left[\frac{s+1}{s^2+6s+9}\right] = \mathcal{L}^{-1}\left[\frac{s+1}{(s+3)^2}\right]$

$= \mathcal{L}^{-1}\left[\frac{A}{s+3} + \frac{B}{(s+3)^2}\right]$

$\Rightarrow A(s+3)+B = s+1$; $A=1$, $B=-2$

$\mathcal{L}^{-1}\left[\frac{s+1}{s^2+6s+9}\right] = \mathcal{L}^{-1}\left[\frac{1}{s+3}\right] + \mathcal{L}^{-1}\left[\frac{-2}{(s+3)^2}\right]$

$= e^{-3t} - 2\left(e^{-3t}\mathcal{L}^{-1}\left[\frac{1}{s^2}\right]\right)$

$= e^{-3t} - 2t\,e^{-3t} = (1-2t)e^{-3t}.$

(b). $\mathcal{L}^{-1}\left[\frac{1}{s^2+4s+13}\right] = \mathcal{L}^{-1}\left[\frac{1}{(s+2)^2+9}\right]$

$= e^{-2t}\cdot\mathcal{L}^{-1}\left[\frac{1}{s^2+9}\right]$

$= e^{-2t}\cdot\frac{1}{3}\mathcal{L}^{-1}\left[\frac{3}{s^2+9}\right]$

$= \frac{1}{3}e^{-2t}\sin(3t)$

3. $\mathcal{L}[D^2x]+2\mathcal{L}[Dx]+2\mathcal{L}[x]=0$

$$s^2\mathcal{L}[x] - s - 1 + 2s\mathcal{L}[x] - 2 + 2\mathcal{L}[x] = 0$$

$$\mathcal{L}[x] = \frac{s+3}{(s^2+2s+2)} = \frac{s+1}{(s+1)^2+1} + \frac{2}{(s+1)^2+1}$$

$$x = \mathcal{L}^{-1}\left[\frac{s+1}{(s+1)^2+1}\right] + 2\mathcal{L}^{-1}\left[\frac{1}{(s+1)^2+1}\right]$$

$$= e^{-t}\mathcal{L}^{-1}\left[\frac{s}{s^2+1}\right] + 2(e^{-t}\mathcal{L}^{-1}\left[\frac{1}{s^2+1}\right]$$

$$= e^{-t}\cos(t) + 2e^{-t}\sin(t)$$

4.(a). (i) $f(t) = t - 3 + u_3(t)(3-t)$

(ii). $\mathcal{L}[f(t)] = \mathcal{L}[t] - \mathcal{L}[3] + \mathcal{L}[u_3(t)(3-t)]$

$$= \frac{1}{s^2} - \frac{3}{s} + e^{-3s}\mathcal{L}[-t]$$

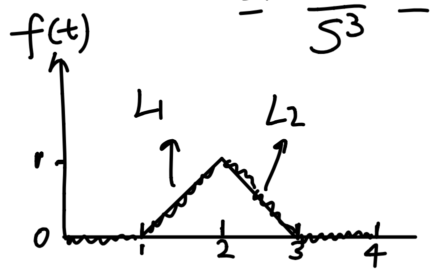
$$= \frac{1-e^{-3s}}{s^2} - \frac{3}{s}.$$

(b). (i) $f(t) = t^2 + u_1(t)(-1) + u_2(t)(-1)$

(ii). $\mathcal{L}[f(t)] = \mathcal{L}[t^2] - e^{-s}\mathcal{L}[1] - e^{-2s}\mathcal{L}[1]$

$$= \frac{2}{s^3} - \frac{e^{-s} - e^{-2s}}{s}$$

(c).



L_1 : $t-1$

L_2 : $-t+3$

(i). $f(t) = u_1(t)(t-1) + u_2(t)(-2t+4) + u_3(t)(t-3)$

(ii). $\mathcal{L}[f(t)] = \mathcal{L}[u_1(t)(t-1)] + \mathcal{L}[u_2(t)(-2t+4)] + \mathcal{L}[u_3(t)(t-3)]$

$= e^{-s}\mathcal{L}[t] + 2\mathcal{L}[u_2(t)(-t+2)] + e^{-3s}\mathcal{L}[t]$

$= e^{-s}/s^2 - 2e^{-2s}/s^2 + e^{-3s}/s^2$

5. (a). $\mathcal{L}^{-1}\left[\frac{se^{-\pi s}}{s^2+2}\right] = \mathcal{L}^{-1}\left[e^{-\pi s} \cdot \frac{s}{s^2+2}\right]$

$= u_\pi(t) \cdot \cos(\sqrt{2}(t-\pi))$

(b). $\mathcal{L}^{-1}\left[\frac{e^{-(s+1)}}{s+1}\right] = \mathcal{L}^{-1}\left[e^{-s} \cdot \frac{e^{-1}}{s+1}\right]$

$= u_1(t) \cdot \mathcal{L}^{-1}\left[\frac{e^{-1}}{s+1}\right]$

$= u_1(t) \cdot e^{-1} \cdot e^{-(t-1)}$ $\qquad \mathcal{L}^{-1}\left[\frac{1}{s+1}\right] = e^{-t} = f(t)$

$= u_1(t)\, e^{-t}$ $\qquad f(t-1) = e^{-(t-1)}$

(c) $\mathcal{L}^{-1}\left[\frac{e^{-2s}}{s(s+1)}\right] = \mathcal{L}^{-1}\left[e^{-2s} \cdot \frac{1}{s(s+1)}\right]$

$= \mathcal{L}^{-1}\left[e^{-2s} \cdot \left(\frac{1}{s} - \frac{1}{s+1}\right)\right]$

$\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) = e^{-t} = f(t)$ $\qquad = u_2(t)\left(1 - e^{-(t-2)}\right)$

$f(t-2) = e^{-(t-2)}$

6. $(D^3-D)x = 1-u_2(t)$, $x(0)=x'(0)=x''(0)=0.$

$$\mathcal{L}[D^3x]-\mathcal{L}[Dx]=\mathcal{L}[1]-\mathcal{L}[u_2(t)]$$

$$s^3\mathcal{L}[x]-s\mathcal{L}[x]=\frac{1}{s}-\frac{e^{-2s}}{s}$$

$$\Rightarrow x=\mathcal{L}^{-1}\left[{(1-e^{-2s})}/{s^4-s^2}\right]$$

$$=\mathcal{L}^{-1}\left[\frac{1}{s^4-s^2}-\frac{e^{-2s}}{s^4-s^2}\right]$$

$$\frac{1}{s^4-s^2}=\frac{1}{s^2(s^2-1)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{(s+1)}+\frac{D}{(s-1)}$$

$$A(s^3-s)=0,\quad A=0$$

$$Bs^2-Cs^2+Ds^2=0$$

$$-B=1$$

$$Cs^3+Ds^3=0$$

$$\begin{cases}-C+D=1\\ C+D=0\end{cases}$$

$$\Downarrow$$

$$C=-\frac{1}{2},\ D=\frac{1}{2}$$

$$\Rightarrow \mathcal{L}^{-1}\left[\frac{1}{s^4-s^2}\right]=\mathcal{L}^{-1}\left[-\frac{1}{s^2}-\frac{1}{2(s+1)}+\frac{1}{2(s-1)}\right]$$

$$=-t-\frac{1}{2}e^{-t}+\frac{1}{2}e^{t}=f(t)$$

$$\mathcal{L}^{-1}\left[\frac{e^{-2s}}{s^4-s^2}\right]=u_2(t)f(t-2)$$

$$\Rightarrow x=-t-\frac{1}{2}e^{-t}+\frac{1}{2}e^{t}-u_2(t)\left[-(t-2)-\frac{1}{2}e^{-(t-2)}+\frac{1}{2}e^{(t-2)}\right]$$