## Readings for the Week of January 24, 2022

**§1.2:** Separation of Variables
**§1.3:** First-Order Linear Equations

## Problem Set 2

(Due Sunday, January 30, 2022, at 5 p.m.)

For a 10% penalty on your grade, you may hand in the problem set late, until January 31, 2022, 11 p.m.

1. **First-order nonlinear equation**
   Find the particular solution that satisfies the initial condition:
   $$x^2 \frac{dx}{dt} = t^2, \quad x(1) = 2.$$

2. **First-order nonlinear equation**
   Find the general solution of
   $$\frac{dx}{dt} = x^2 - 1, \quad x \geq 1.$$
   (*Hint*. Use partial fraction decomposition. Also remember that you cannot divide by 0.)

3. **Compound interest**
   When interest is compounded continuously, the rate of change of the principal is proportional to the principal. The constant of proportionality is called the *interest rate*.
   (a) Set up a differential equation to model the principal $x = x(t)$ in an account accruing interest at 8% per year, compounded continuously.
   (b) Solve the differential equation.
   (c) Calculate the percentage increase in such a deposit over one year. (This is called the *effective annual rate*.)

4. **Classification of ode's**
   In this problem, no work needs to be shown. Only the answers will be graded.
   (a) For each part, write down the order of the differential equation.
   (i) $r^4 \dfrac{d^3x}{dt^3} + t\dfrac{dx}{dt} - x = t^7$
   (ii) $\left(\dfrac{dx}{dt}\right)^5 + \dfrac{d^4x}{dt^4} - t^3x^7 + t^7 = 0$
   (iii) $x^8 \dfrac{dx}{dt} + \dfrac{d^7x}{dt^7} = x + t^9$
   (iv) $\left(x'\right)^2 x''' = x^4 x'' + t^5 x'$

(b) For each of the following o.d.e.'s, decide whether it is linear. If it is linear, is it homogeneous? In this problem, take $x$ to be the dependent variable; $x' = dx/dt$. Set up three columns, the first column for the equation number (i), (ii), (iii), ..., the second column for "linear," and the third for "homogeneous." In each entry, answer the question by writing "Y" or "N" for "Yes" or "No." If the question does not apply to an equation, leave the entry blank.

(i) $x' + x + t = 0$
(ii) $x' + xt = 0$
(iii) $x't + x = 0$
(iv) $x'x + t = 0$
(v) $x' + x + t^2 = 0$
(vi) $(x')^2 + x + t = 0$
(vii) $x' + x^2 + t = 0$
(viii) $x't + xt^2 + 1 = 0$

(c) For each of the following linear differential equations, decide whether it is normal on $0 < t < 2$. Answer "Yes" or "No."

(i) $(t-1)\dfrac{dx}{dt} - 5x = 3t$

(ii) $t\dfrac{dx}{dt} - e^t x = \sin t$

5. **First-order equation**.
Find the general solution of $x' + 3x = e^{-3t}\cos 2t$ on $(-\infty, \infty)$.

6. **Variation of parameter**.
Try variation of parameters on the differential equation $x' + x^2 = t$ as follows.
(a) Find the general solution of $x' + x^2 = 0$ by separating variables. There will be a parameter in the solution representing a constant.
(b) Let the parameter vary and substitute the solution back into the original nonhomogeneous equation. What goes wrong? What can you conclude about the method of variation of parameter?

(End of Homework 2)