Homework 13 will not be collected, but it will help you prepare for the final exam.

Readings

§**5.6:** Convolution
Video Slides: Integrating Factors

Upcoming Deadlines:

Tuesday, April 27, 11:59 p.m.:	Quiz 12 (§5.6 & Integrating Factors, Canvas)
Friday, April 30, 5 p.m.:	Homework 12 (Gradescope)

Homework Exercises:

In these exercises, you may use all the Laplace transform formulas, the convolution formulas # 8–10, pp. 462–463:

$$(\sin \alpha t) * (\cos \alpha t) = \frac{t}{2} \sin \alpha t,$$
$$(\sin \alpha t) * (\sin \alpha t) = \frac{1}{2\alpha} \sin \alpha t - \frac{t}{2} \cos \alpha t,$$
$$(\cos \alpha t) * (\cos \alpha t) = \frac{1}{2\alpha} \sin \alpha t + \frac{t}{2} \cos \alpha t,$$

as well as the integration formulas:

$$\int u \, \sin u \, du = \sin u - u \cos u + C,$$
$$\int u \, e^t \, du = e^u (u - 1) + C.$$

1. **Convolution**.
   Evaluate the convolution $t * e^{at}$.

2. **Convolution**.
   Evaluate the convolution $t * \sin \alpha t$.

3. **Inverse Laplace transform**.
   Find the inverse Laplace transform $\mathcal{L}^{-1}\left[\dfrac{3}{s(s^2+4)}\right]$ in two ways:
   (a) using the partial fraction decomposition.
   (b) using the convolution.

(*Continued on next page*)

4. **Initial-value problem**.
   Using the Laplace transform, solve the initial-value problem
   $$(D^2+1)^2 = 0, \quad x(0) = x'(0) = x''(0) = 0,\ x'''(0) = 1.$$

5. **Integrating factors**.
   Using an integrating factor, find the general solution of the differential equation on the indicated interval:
   $$x' + x\tan t = \cos t, \quad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

6. **Integrating factors**.
   Using an integrating factor, solve the initial-value problem:
   $$\frac{dx}{dt} - tx = t, \quad x(0) = \frac{1}{2}.$$

(End of Homework 13)