Readings for the Week of March 28, 2022

§3.6–10: Homogeneous linear systems (real, complex, double, multiple roots)

Problem Set 10

(Due Monday, April 4, 2022, at 11:59pm.)

For a 10% penalty on your grade, you may hand in the problem set late, until Tuesday, April 5, 2022, 11:59 pm.

1. Let $A = \begin{bmatrix} 0 & 0 & 3 & 0 \\ 1 & 3 & 0 & -1 \\ 0 & 0 & 3 & 0 \\ 1 & 0 & -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 \\ 0 & -1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$, and $F = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$.

Find the indicated matrices or explain that this cannot be done.
(a) AB; (b) BA; (c) AC; (d) $A(B - C)$; (e) AF; (f) BF; (g) FB

2. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is a generalized eigenvector of $\begin{bmatrix} -1 & 1 & 4 \\ -2 & 2 & 4 \\ -1 & 0 & 4 \end{bmatrix}$ for the eigenvalue 2.
Find the associated solution of $D\vec{x} = A\vec{x}$.

3. Find the general solution of $D\vec{x} = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} \vec{x}$.

4. Find the general solution of $D\vec{x} = \begin{bmatrix} 1 & -1 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \vec{x}$.

5. Find the general solution of $D\vec{x} = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix} \vec{x}$.

6. Find the general solution of
$$\begin{aligned} x_1' &= 3x_1 - x_2 - 4x_4 \\ x_2' &= 3x_2 - 4x_4 \\ x_3' &= 2x_3 \\ x_4' &= x_2 - x_4 \end{aligned}$$
as well as the specific solution satisfying $x_1(0) = x_2(0) = x_3(0) = 1$, $x_4(0) = 0$.

7. Suppose A is a square matrix. Show that if $(A - \lambda I)^2\vec{v} = \vec{0}$ and $(A - \lambda I)\vec{v} \neq \vec{0}$, then $\vec{v}_1 = (A - \lambda I)\vec{v}$ is an eigenvector of A.

(End of Homework 10)