Math 51 **Homework 3** **Spring 2021**

Upcoming Deadlines:

Friday, February 12, 5 p.m.:	HW 2 (§1.3, 1.6, 1.7, 2.2, Gradescope)
Sunday, February 14, 11:59 p.m.:	Quiz 2 (App. A, §2.3, Canvas)
Friday, February 19, 5 p.m.:	HW 3 (App. A, §2.3, Gradescope)

You can scan your solutions to Homework 3 into a pdf file using Notes in an iPhone or other scanning apps in an Android phone. Submit your solutions as a single pdf file in Gradescope. Do not submit a pdf converted from a photo. Photos have uneven lighting and are often difficult to read. You may submit your homework up to one day late, i.e., by 5 p.m., Saturday, February 20, but if you do, you lose 10% of your grade.

Homework Exercises:

1. Which of the following ODEs are linear? Of the ones that are linear, rewrite them as linear operators, and say which ones are homogeneous.

   (a) $\dfrac{d^2x}{dt^2} + t^2\dfrac{dx}{dt} - t\sin(t) = 0$

   (b) $\dfrac{d^2x}{dt^2} + x\dfrac{dx}{dt} = 0$

   (c) $\dfrac{d^2x}{dt^2} + \sin(t)\dfrac{dx}{dt} = t^2x$

   (d) $\dfrac{d^3x}{dt^3} + e^t\dfrac{d^2x}{dt^2} + tx = te^t$

2. Calculate (and simplify) the determinant of the following matrix:

$$\begin{bmatrix} e^t & \sin t & \cos t \\ e^t & \cos t & -\sin t \\ e^t & -\sin t & -\cos t \end{bmatrix}.$$

3. Consider the differential equation $Lx = 0$ where $L = D^3 + D^2$. Using the Wronskian test, determine whether or not the given solutions generate the general solution:

$$h_1(t) = e^{-t}, \quad h_2(t) = t + 3e^{-t}, \quad h_3(t) = t.$$

4. For the differential equation $(D^3 + D^2 - D + 2)x = 0$,
   (a) Find all solutions of the form $e^{\lambda t}$ or t^{α}.
   (b) Determine whether the solutions found in (a) generate a complete collection of solutions.

5. Use Cramer's determinant test to determine whether or not the following system has solutions for all values of the right side.

$$\begin{aligned} x - y + 3z &= a \\ x + y - 3z &= b \\ 3x - y + 3z &= c \end{aligned}$$

(*Continued on next page*)

6. Prove that linear combinations of solutions to linear *nonhomogeneous* o.d.e.'s need not be solutions themselves. To do this, suppose that $x = \phi(t)$ is a solution to the o.d.e. $Lx = f(t)$ where $f(t) \neq 0$. Show that even though x is a solution, $y(t) = x + x$ is not a solution. Is this surprising?