Readings

M. Guterman & Z. Nitecki: *Differential Equations: A First Course*, 3rd ed., Saunders Publ.,1984, reprinted by IA Books, 2006, ISBN 0-03-072878-9.

§3.7 Homogeneous Systems with Constant Coefficients: Real Roots.
§3.8 Homogeneous Systems with Constant Coefficients: Complex Roots.

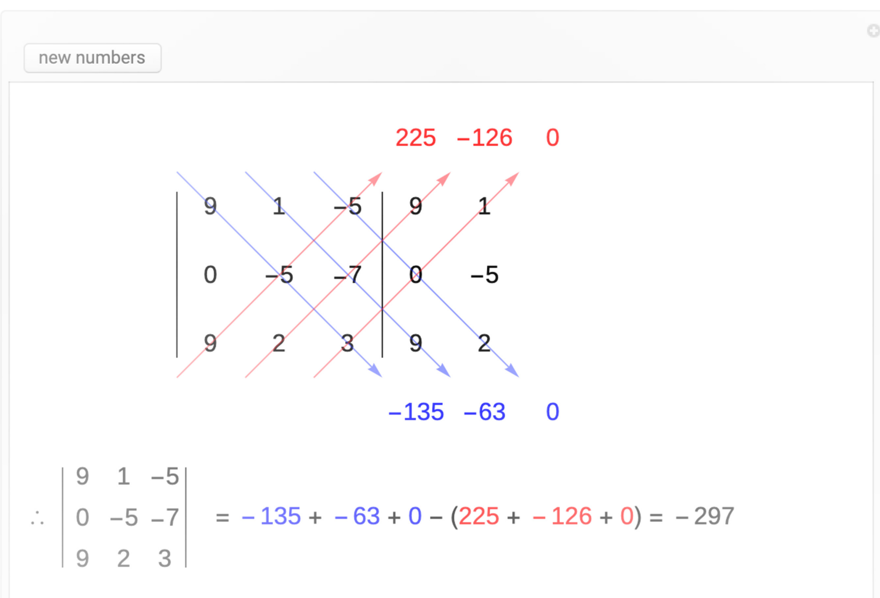
Upcoming Deadlines:

Sunday, March 28, 11:59pm: Quiz 8 (on §3.7, 3.8, Canvas)

Friday, April 2, 5:00pm: Homework 9 (Gradescope)

You may submit your homework up to one day late, i.e., by 5 p.m., Saturday, April 3, but if you do, you lose 10% of your grade.

3×3 Determinants Using Diagonals



Copy the first two columns of the matrix to its right. Multiply along the blue lines and the red lines. Add the numbers on the bottom and subtract the numbers on the top. The result is the value of the determinant.

This method does not work with 4×4 or higher-order determinants. For those, use expansion by minors or row reduction. Even when there are many zero entries, row reduction is more systematic, simpler, and less prone to error. (*Contributed by*: *George Beck* (March 2011), *Open content licensed under* CC BY-NC-SA.)

Homework Exercises:

1. Find the general solution to the system
$$\begin{aligned} x_1' &= \qquad -x_2 \\ x_2' &= 2x_1 + 3x_2. \end{aligned}$$
In this problem, write the answer not as a vector but in the form $x_1 = \cdots$, $x_2 = \cdots$.

2. Find a generating set of real solutions to the system $D\mathbf{x} = A\mathbf{x}$ where
$$A = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & 1 \\ -1 & -2 & 4 \end{bmatrix}.$$

3. Consider the system $D\mathbf{x} = A\mathbf{x}$ where
$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}.$$
By computing eigenvalues and eigenvectors, construct as many solutions as possible. Do they constitute a complete set of solutions?

4. Solve the following system of equations for $x, y,$ and z using row reduction on the related augmented matrix:
$$\begin{array}{rcrcrcr} x & + & iy & + & (-3+i)z & = & -1-i, \\ 2x & + & (1+3i)y & + & (4-2i)z & = & 2i, \\ 2ix & - & 2y & + & (-2-3i)z & = & -1+i. \end{array}$$
Note that the solutions may themselves be complex. Be sure to detail the row operations being performed.

5. Solve the initial value problem $D\mathbf{x} = A\mathbf{x}$ where
$$A = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}, \qquad \mathbf{x}(0) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

6. Let $\lambda_1, \ldots, \lambda_k$ be k distinct eigenvalues of an $n \times n$ matrix, with corresponding eigenvectors $\mathbf{v}_1, \ldots, \mathbf{v}_k$ respectively. This exercise proves that $\mathbf{v}_1, \ldots, \mathbf{v}_k$ are linearly independent for $k \geq 2$.

   Assume that $\mathbf{v}_1, \ldots, \mathbf{v}_k$ are linearly dependent, i.e., there is a nontrivial linear relation among them. We will try to get a contracdiction. Renumbering the eigenvectors if necessary, let
$$c_1\mathbf{v}_1 + \cdots + c_r\mathbf{v}_r = 0 \tag{1}$$
be the shortest nontrivial linear relation among $\mathbf{v}_1, \ldots, \mathbf{v}_k$.
   (a) Multiply equation (1) by A and simplify to get a new equation labelled as (2).
   (b) Multiply equation (1) by λ_1 to get a new equation labelled as (3).
   (c) Subtract equation (3) from equation (2) and explain why this results in a contradiction.

   The contradiction proves that $\mathbf{v}_1, \ldots, \mathbf{v}_k$ must be linearly independent.

(End of Homework 9)