Problem Set 11
Non-homogeneous systems

Math 51 Spring 2022

2022-04-11 – because of Midterm 2, this assignment won't be collected

These problems concern (Nitecki and Guterman 1992, sec. 3.11).

1. The matrix $A = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$ has eigenvalues $\lambda = 1 + \sqrt{3}$, $\mu = 1 - \sqrt{3}$, and the general solution to $D\mathbf{x} = A\mathbf{x}$ is given by

$$\mathbf{x}(t) = c_1 e^{\lambda t} \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} + c_2 e^{\mu t} \begin{bmatrix} -1 \\ \sqrt{3} \end{bmatrix}.$$

   a. Find the general solution to the inhomogeneous equation $D\mathbf{x} = A\mathbf{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

   b. Solve the initial value problem $D\mathbf{x} = A\mathbf{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

2. The matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ has characteristic polynomial $\lambda^2 - 2\lambda$, and thus has eigenvalues $\lambda = 0, 2$.

   The general solution to $(H) \quad D\mathbf{x} = A\mathbf{x}$ is given by

$$\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

   Find the general solution to the inhomogeneous equation $D\mathbf{x} = A\mathbf{x} + \begin{bmatrix} 0 \\ t \end{bmatrix}$.

Bibliography

Nitecki, Zbigniew, and Martin Guterman. 1992. *Differential Equations: A First Course.* Saunders.