Math 51 **Homework 12** **Spring 2021**

Readings

§5.4 Further Properties of the Laplace Transform and Inverse Transform
§5.5 Functions Defined in Pieces

Upcoming Deadlines:

Tuesday, April 20, 11:59 p.m.: Quiz 11 (§5.4 §5.5, Canvas)
Friday, April 30, 5 p.m.: Homework 12 (Gradescope)

Homework Exercises:

1. **(Laplace transforms)**
   Calculate the Laplace transforms of the following functions.

   (a) $\frac{1}{2}t^2 \sin 5t$ (b) $t^n e^{mt}$ (c) $te^{2t} \sin 4t$

2. **(Inverse transforms)**
   Calculate the inverse transforms of the following functions.

   (a) $\dfrac{s+1}{s^2+6s+9}$ (b) $\dfrac{1}{s^2+4s+13}$

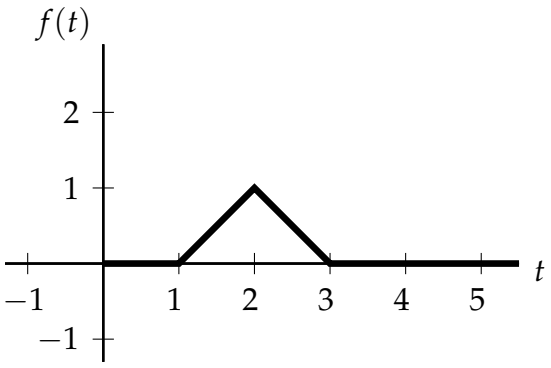
3. **(Initial-value problem)**
   Use the Laplace transform to solve the initial-value problem:

   $$(D^2+2D+2)x = 0, \quad x(0) = x'(0) = 1.$$

4. **(Laplace transforms of step functions)**
   For each of the following fuctions, (i) express $f(t)$ in step-function notation and (ii) find the Laplace transform $\mathcal{L}[f(t)]$.

   (a) $f(t) = \begin{cases} t-3 & \text{for } t < 3, \\ 0 & \text{for } t \geq 3. \end{cases}$ (b) $f(t) = \begin{cases} t^2 & \text{for } t < 1, \\ t^2 - 1 & \text{for } 1 \leq t < 2, \\ t^2 - 2 & \text{for } 2 \leq t. \end{cases}$

   (c) $f(t)$ with the graph below.



5. **(Inverse transforms involving step functions)**
   Find the inverse transform of the following functions.

   (a) $\dfrac{se^{-\pi s}}{s^2+2}$ (b) $\dfrac{e^{-(s+1)}}{s+1}$ (c) $\dfrac{e^{-2s}}{s(s+1)}$

6. **(Initial-value problem involving a step-function)**
   Use the Laplace tranform to solve the initial-value problem:

$$(D^3 - D)x = \begin{cases} 1 & \text{for } t < 2, \\ 0 & \text{for } t \geq 2, \end{cases} \quad x(0) = x'(0) = x''(0) = 0.$$

(End of Homework 12)