Math 51 **Worksheet 8**

1. (a) Find the eigenvalues of the following matrices:

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

   (b) Describe how to obtain matrix B from matrix A. Are their their eigenvalues related? If so, how?

2. Find four linearly independent eigenvectors for the following matrix:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

3. The eigenvalues of the following matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are 1 and 2. The vector $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is an eigenvector for $\lambda = 1$ and $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ is an eigenvector for $\lambda = 2$.

(a) The eigenvalue $\lambda = 1$ has multiplicity 2. However, show that in this case, there are *not* two linearly independent eigenvectors for $\lambda = 1$.

(b) Let $\mathbf{h}_1(t) = e^t\mathbf{v}$ and $\mathbf{h}_2(t) = e^{2t}\mathbf{w}$. Explain why, for any c_1, c_2, the vector-valued function $\mathbf{x}(t) = c_1\mathbf{h}_1(t) + c_2\mathbf{h}_2(t)$ is a solution to $D\mathbf{x} = A\mathbf{x}$, but is not the *general* solution.

(c) Let $\mathbf{x}(t)$ be a solution to $D\mathbf{x} = A\mathbf{x}$ with $\mathbf{x}(0) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$. Find scalars c_1, c_2 such that $\mathbf{x}(t) = c_1\mathbf{h}_1(t) + c_2\mathbf{h}_2(t)$.

(d) Let $\mathbf{x}(t)$ be a solution to $D\mathbf{x} = A\mathbf{x}$ with $\mathbf{x}(0) = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$. Show that there are no scalars c_1, c_2 for which $\mathbf{x}(t) = c_1\mathbf{h}_1(t) + c_2\mathbf{h}_2(t)$.

4. Solve the initial value problem

$$D\mathbf{x} = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \mathbf{x}; \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$