# Integrating Factors: Example

Loring Tu

**Ex.** Solve $2x' - x = e^{3t}$.

**Sol.** 1) Put in standard form:

$$x' - \tfrac{1}{2}x = \tfrac{1}{2}e^{3t}. \qquad (*)$$

2) An integrating factor is

$$\rho(t) = e^{-\int \frac{1}{2}\,dt} = e^{-\frac{1}{2}t}.$$

3) Multiply $(*)$ by $\rho(t)$:

$$\underbrace{e^{-\frac{1}{2}t}x' - \tfrac{1}{2}e^{-\frac{1}{2}t}x}_{(e^{-\frac{1}{2}t}x)'} = \tfrac{1}{2}e^{-\frac{1}{2}t}e^{3t} = \tfrac{1}{2}e^{\frac{5}{2}t}.$$

Integrating both sides, we get

$$e^{-\frac{1}{2}t}x = \int \tfrac{1}{2}e^{\frac{5}{2}t}\,dt = \tfrac{1}{5}e^{\frac{5}{2}t} + C.$$

$$\boxed{x = \tfrac{1}{2}e^{3t} + Ce^{\frac{1}{2}t}}$$

$\tfrac{1}{2}e^{3t}$: particular solution to the nonhomogeneous equation

$Ce^{\frac{1}{2}t}$: general solution to the related homogeneous equation