Math 51 **Homework 8** Spring 2022

Readings for the Week of March 7, 2022

Martin Guterman and Zbigniew Nitecki, *Differential Equations: A First Course*, 3rd edition. ISBN: 81-89617-20-6.

§3.5 Homogeneous Systems, Eigenvalues, Eigenvectors.

§3.6 Systems of Algebraic Equations, Row Reduction.

Problem Set 8
(Due Monday, March 14, 2022, at 11:59 p.m.)

For a 10% penalty on your grade, you may hand in the problem set late, until Tuesday, March 15, 2022, 11:59 p.m.

1. Find the eigenvalues of A, and for each eigenvalue find a corresponding eigenvector.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(*Hints*: To find the roots of a cubic (degree 3) polynomial, use the rational root test.)

2. Given a matrix A and an eigenvector of A, find

   (a) the eigenvalue λ to which $\mathbf{v}$ corresponds.

   (b) the associated solution of $D\mathbf{x} = A\mathbf{x}$.

$$A = \begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

3. (a) Calculate the determinant of the **lower triangular matrix**

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 7 & -2 & -5 & 0 \\ -3 & 1 & 5 & 6 \end{bmatrix}.$$

   (b) What can you say about the determinant of a lower triangular matrix?

   (c) Find the eigenvalues of the matrix A in part (a).

   (d) What can you say about the eigenvalues of a lower triangular matrix?

   (Note. The same conclusions hold for an upper triangular matrix.)

4. Use row reduction to decide whether the following three vectors are linearly independent:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 4 \\ 1 \\ 3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ 2 \\ 2 \\ 1 \end{bmatrix}.$$

5. Find the general solution of $D\mathbf{x} = A\mathbf{x}$, where A is the matrix $\begin{bmatrix} 1 & -3 \\ -3 & 1 \end{bmatrix}$.

6. Find all solutions of the given system of equations (if they exist). express your answer (1) as separate parametric equations for the variables and (2) as a linear combination of vectors.

   (a)

$$\begin{array}{rcrcrl} x_1 & - & x_2 & + & x_3 & = 1 \\ 2x_1 & + & x_2 & + & 2x_3 & = 5 \\ x_1 & + & 2x_2 & + & x_3 & = 4. \end{array}$$

   (b)

$$\begin{array}{rcrcrcrcrl} x_1 & + & 2x_2 & + & x_3 & - & x_4 & - & x_5 & = 2 \\ 2x_1 & + & 2x_2 & + & 2x_3 & - & 3x_4 & - & 2x_5 & = 1 \\ -x_1 & & & - & x_3 & + & 2x_4 & + & x_5 & = 0. \end{array}$$

(End of Problem Set 8)