Carefully PRINT your full name:

Math 51 **Homework 8 (version 2)** **Spring 2021**

(This version differs from the first only in Question 3.)

Readings

§3.5 Homogeneous Systems, Eigenvalues, Eigenvectors.

§3.6 Systems of Algebraic Equations, Row Reduction.

Upcoming Deadlines:

Sunday, March 21, 11:59 p.m.:	Quiz 7 (§3.5, 3.6, Canvas)
Friday, March 26, 5 p.m.:	HW 8 (§3.5, 3.6 Gradescope)

To give you some practice with the format of exams, this homework is formatted like an exam. Print the homework and write your solutions to all the questions on the printed homework. Instead of printing the exam, you may also download it to a tablet and do your work there. When you are ready to submit, scan your homework as a single pdf file of ten pages and upload it to Gradescope.

This format is called the *fixed format*, where each question is on a predefined page, as opposed to the *variable format* of previous homeworks. Since the format of the homework is fixed, when you submit your file, you do not need to indicate the page that each question is on.

Students who do not have a printer or tablet may use their own paper. You need not copy down the questions, but your name and the answer for each question must be in the exact same area as the printed homework so that Gradescope can find them. For example, your name should be on the first page one inch from the top margin and 2 to 5 inches from the right margin. Question 1 should be on pages 2 and 3, Question 2 on pages 4 and 5, and so on. In the end you need to submit a pdf file of ten pages, with the questions in order. Do not use extra pages. If you have fewer or more than ten pages, you will not be able to submit.

1. Find the eigenvalues of A, and for each eigenvalue find a corresponding eigenvector.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(*Hints*: The diagonal method for calculating the determinant of a 3×3 matrix is often quicker than expansion along a row or column. To find the roots of a cubic (degree 3) polynomial, use the rational root test.)

(Continuation of Question 1)

2. Given a matrix A and an eigenvector of A, find

   (a) the eigenvalue λ to which $\mathbf{v}$ corresponds.

   (b) the associated solution of $D\mathbf{x} = A\mathbf{x}$.

$$A = \begin{bmatrix} 1 & -3 \\ -3 & 1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

(Continuation of Question 2)

3. (a) Calculate the determinant of the **upper triangular matrix**

$$A = \begin{bmatrix} 1 & 7 & 8 & -2 \\ 0 & 2 & -2 & 1 \\ 0 & 0 & -3 & 5 \\ 0 & 0 & 0 & 4 \end{bmatrix}.$$

(b) What can you say about the determinant of an upper triangular matrix?

(c) Find the eigenvalues of the matrix A in part (a).

(d) What can you say about the eigenvalues of an upper triangular matrix?

4. Reduce the matrix

$$A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ -1 & 1 & -2 & 0 \\ 2 & -1 & 3 & -1 \end{bmatrix}.$$

5. Find all solutions of the given system of equations (if they exist). express your answer (1) as separate parametric equations for the variables and (2) as a linear combination of vectors.

(a)

$$x_1 + 2x_2 + 3x_3 - 2x_4 = 0$$
$$3x_1 - 7x_2 - 4x_3 + 7x_4 = 0$$
$$4x_1 - 3x_2 + x_3 + 3x_4 = 0$$
$$x_1 + 3x_2 + 4x_3 - 3x_4 = 0$$

(b)

$$x_1 + 2x_2 + x_3 - x_4 - x_5 = 2$$
$$2x_1 + 2x_2 + 2x_3 - 3x_4 - 2x_5 = 1$$
$$-x_1 - x_3 + 2x_4 + x_5 = 0.$$

(Continuation of Question 5)

6. (a) Explain why a reduced matrix with fewer rows than columns ($n \times m, n < m$) must have some columns without corners (pivots).

   (b) Use part (a) to explain why a homogeneous system of linear equations with more unknowns than equations must have infinitely many solutions.

(End of Homework 8)