

**Upcoming Deadlines:**

Friday, February 19, 5 p.m.: HW 3 (on §App.A, 2.3, Gradescope)

Sunday, February 21, 11:59 p.m.: Quiz 1 (on §2.4, 2.5, Canvas)

Friday, February 26, 5 p.m.: HW 4 (on §2.4, 2.5, Gradescope)

You can scan your solutions into a pdf file using Notes in an iPhone or other scanning apps in an Android phone. Submit your solutions as a single pdf file in Gradescope. Do not submit a pdf converted from a photo. Photos have uneven lighting and are often difficult to read. Before you submit, you need to indicate in Gradescope the pages that each question is on. You may submit your homework up to one day late, i.e., by 5 p.m., Saturday, February 27, but if you do, you lose 10% of your grade.

**Homework Exercises:**

1. Check the given functions for linear independence on  $-\infty < t < \infty$ :

(a)  $h_1(t) = 1, \quad h_2(t) = t, \quad h_3(t) = t^2.$

(b)  $h_1(t) = \sin^2 t, \quad h_2(t) = \cos^2 t, \quad h_3(t) = \cos 2t.$

(c)  $h_1(t) = e^t, \quad h_2(t) = te^t, \quad h_3(t) = e^{2t}, \quad h_4(t) = te^{2t}.$

(d)  $h_1(t) = e^{t^5}, \quad h_2(t) = e^{|t^5|}.$

2. Show that if  $h_1(t), \dots, h_n(t)$  are polynomials of degree  $1, 2, \dots, n$ , respectively, then they are linearly independent.

3. Show that if  $h_1(t), \dots, h_n(t)$  are linearly dependent, then  $h'_1(t), \dots, h'_n(t)$  are also linearly dependent.

4. Find the general solution for the given functions:

(a)  $x'' - 2x' - 15x = 0.$

(b)  $(D + 3)^2(D - 5)^3x = 0.$

*(More problems on next page.)*

5. Solve the initial-value problem:

$$x''' + 3x'' + 3x' + x = 0; \quad x(0) = x'(0) = x''(0) = 0$$

6. Use exponential shift to calculate  $Lf_i(t)$  in the following functions:

(a)  $L = D^2 + 3D - 2; \quad f_1(t) = e^{2t}, \quad f_2(t) = t^3 e^{2t}.$

(b)  $L = D^2 + 4D + 5; \quad f_1(t) = te^{-2t} \cos t.$

(End of Homework 4)