gmduvvuri_problem3

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In [1]: import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        import seaborn as sns
        from scipy.stats import gaussian_kde
        sns.set_style('darkgrid')
        sns.set_context('talk')
        plt.rc('text', usetex=True)
        plt.rc('font', family='serif')
```

```
In [2]: # Returns (2, N) floats drawn from a 2D uncorrelated Gaussian
        # with mean=(0.0, 0.0) , var=(1.0, 1.0)
        def get_N_gaussian_samples(N=20, mu=0.0, var=1.0):
            return np.random.normal(loc=mu, scale=np.sqrt(var), size=(2, N))


        # Returns the Pearson product-moment correlation coefficient
        # assuming (2, N) structured sample array
        def get_r(sample_array):
            numerator = np.sum((sample_array[0, :]
                                - np.mean(sample_array[0, :]))
                               *(sample_array[1, :]
                                 - np.mean(sample_array[1, :])))
            denominator = np.sqrt(np.sum((sample_array[0, :]
                                          - np.mean(sample_array[0, :]))**2.0)
                                  *np.sum((sample_array[1, :]
                                           - np.mean(sample_array[1, :]))**2.0))
            return numerator/denominator


        # Calculate r for M samples of shape (2, N)
        def do_M_r_samples(M, N=20, mu=0.0, var=1.0):
            return np.array([get_r(get_N_gaussian_samples(N, mu, var))
                             for i in range(0, M)])
```

```
In [3]: def do_part_a(M=100000, N=20, mu=0.0, var=1.0, test_loc=0.975):
            r_array = do_M_r_samples(M, N, mu, var)
```

```
    sorted_r = np.sort(r_array)
    r_prob, prob_arr = np.unique(sorted_r, return_index=True) # Get CDF
    prob_arr = prob_arr/np.max(prob_arr)
    test_r = r_prob[np.where(prob_arr >= test_loc)][0] #Find where F(r) crosses test_l

    plt.hist(r_array, color='cornflowerblue', bins='auto')
    plt.xlabel(r'$r$')
    plt.suptitle(r'Histogram of $r$')
    plt.show()
    plt.clf()

    plt.plot(r_prob, prob_arr, label=r'$F(r)$',
             linestyle='-', color='cornflowerblue')
    plt.axhline(test_loc, label=r'$F(r) =$' + str(test_loc),
                linestyle=':', color='k')
    plt.axvline(test_r, label=r'$r = $' + str(test_r),
                linestyle='--', color='k')
    plt.legend(bbox_to_anchor=(1.05, 1.0))
    plt.ylabel(r'$F(r)$')
    plt.xlabel(r'$r$')
    plt.suptitle(r'Cumulative Distribution Function of $r$')
    plt.show()

    print('The value of r for which F(r)=0.975 is: ', test_r)
    return r_array
```

```
In [4]: def get_data(fname='hw5.dat'):
            return pd.read_csv(fname, delim_whitespace=True, names=['x', 'y'])


        def do_part_b(r_array, alpha_param=0.95):
            hw_dat = get_data()
            data_arr = np.empty((2, 20))
            data_arr[0, :] = np.array(hw_dat['x'])
            data_arr[1, :] = np.array(hw_dat['y'])
            r_data = get_r(data_arr)

            # Estimate PDF using Scipy Kernel Density Estimation
            # KDE smooths data over a window to estimate the PDF
            r_pdf = gaussian_kde(r_array)

            x_axis = np.linspace(np.min(r_array), np.max(r_array), 5000)
            y_axis = r_pdf.pdf(x_axis)
            max_loc = np.argmin(np.abs(x_axis))
            integral_values = np.array([r_pdf.integrate_box(x_axis[max_loc - i],
                                                            -x_axis[max_loc - i])
                                        for i in range(0, int(len(x_axis)/5))])
            r_bound = np.argmin(np.abs(integral_values - alpha_param))
```

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    r_low = x_axis[r_bound]
    r_diff = x_axis[np.argmin(np.abs(x_axis))] - r_low
    r_high = r_low + 2.0*r_diff

    plt.plot(x_axis, y_axis,
             linestyle='-', color='cornflowerblue', label=r'$P(r)$')
    plt.axvline(r_data, label=r'$r$ of data', color='k')
    plt.axvspan(r_low, r_high, color='g',
                alpha=0.3,
                label=r'Within $\alpha$ = ' + str(alpha_param*100) + r'$\%$ confidence
    plt.legend(bbox_to_anchor=(1.05, 0.5))
    plt.ylabel(r'PDF$(r)$')
    plt.xlabel(r'$r$')
    plt.suptitle(r'Null Hypothesis Cannot Be Rejected with 95$\%$ Confidence')
    plt.show()
    return r_low, r_high, x_axis, y_axis, r_pdf
```
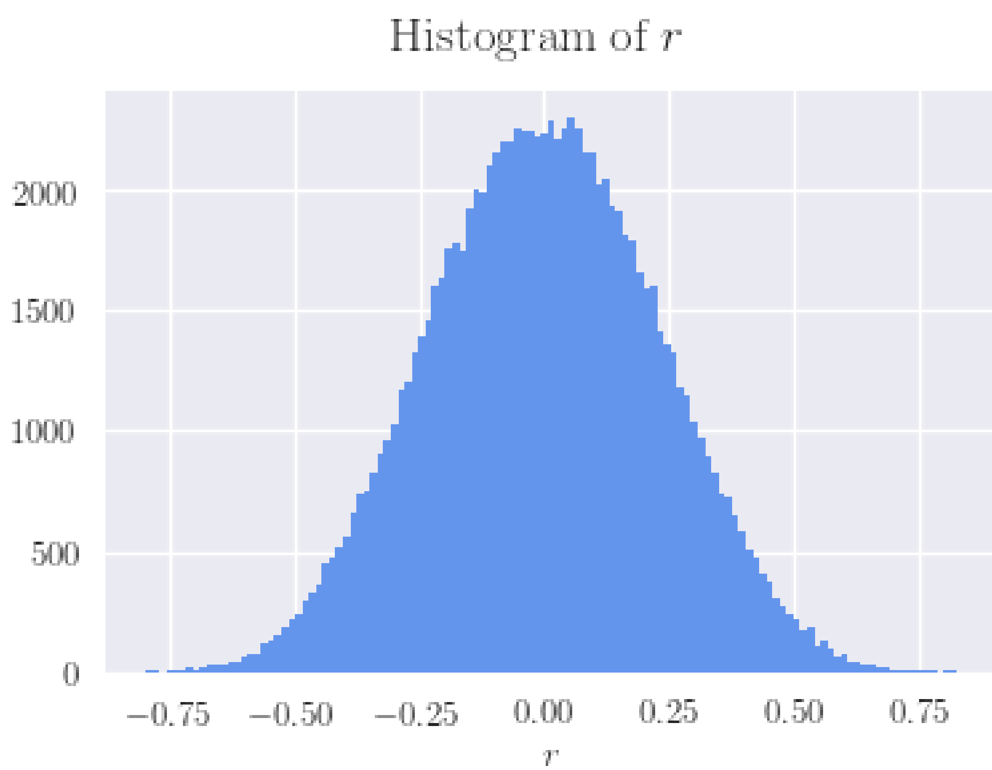
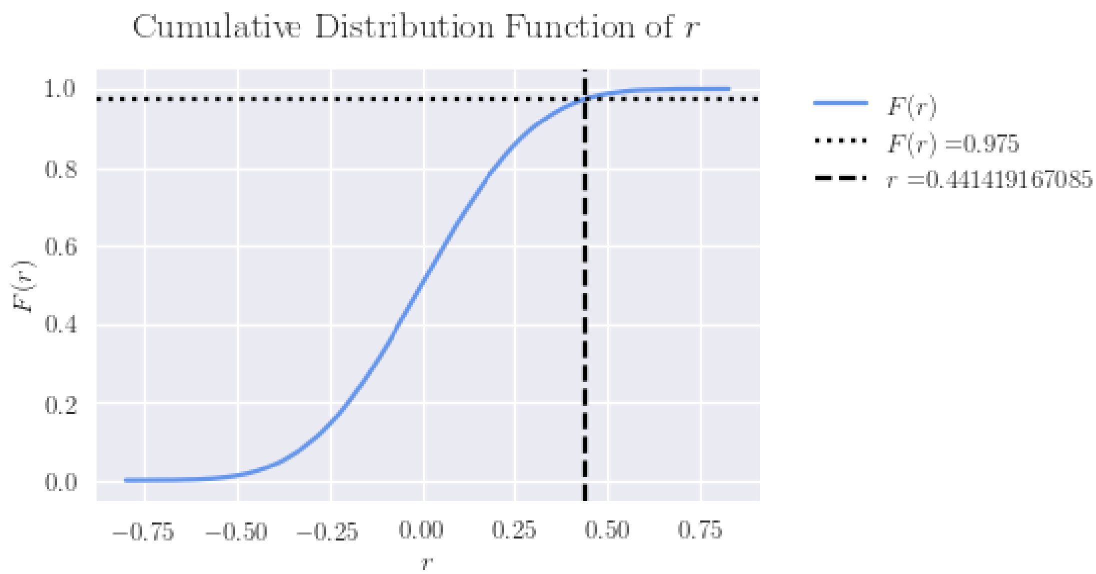

In [5]:
```
r_array = do_part_a()
r_low, r_high, x_axis, y_axis, r_pdf = do_part_b(r_array)
```






```
The value of r for which F(r)=0.975 is:  0.441419167085
```