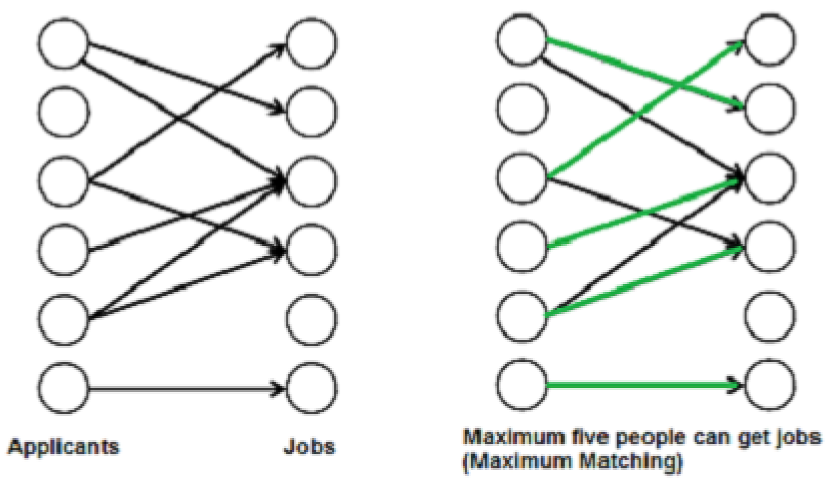
Maximum Bipartite Matching

A matching in a Bipartite Graph is a set of the edges chosen in such a way that no two edges share an endpoint. A maximum matching is a matching of maximum size (maximum number of edges). In a maximum matching, if any edge is added to it, it is no longer a matching. There can be more than one maximum matchings for a given Bipartite Graph.

Why do we care?

There are many real world problems that can be formed as Bipartite Matching. For example, consider the following problem:

There are M job applicants and N jobs. Each applicant has a subset of jobs that he/she is interested in. Each job opening can only accept one applicant and a job applicant can be appointed for only one job. Find an assignment of jobs to applicants in such that as many applicants as possible get jobs.

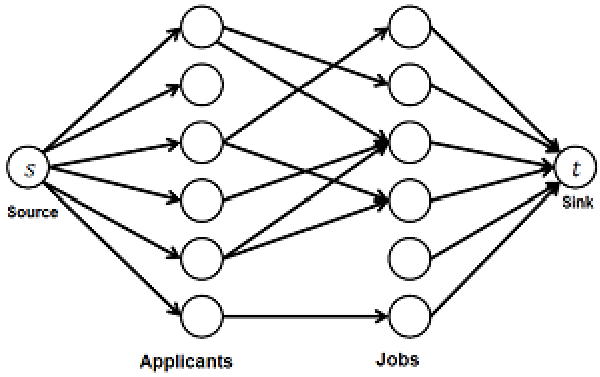


We strongly recommend to read the following post first.

Ford-Fulkerson Algorithm for Maximum Flow Problem

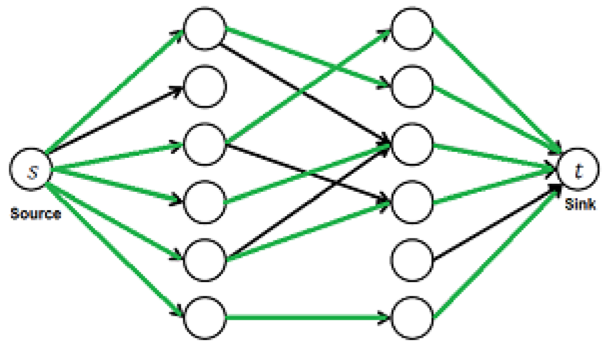
Maximum Bipartite Matching and Max Flow Problem

Maximum **B**ipartite **M**atching (**MBP**) problem can be solved by converting it into a flow network (See this video to know how did we arrive this conclusion). Following are the steps.



1) Build a Flow Network

There must be a source and sink in a flow network. So we add a source and add edges from source to all applicants. Similarly, add edges from all jobs to sink. The capacity of every edge is marked as 1 unit.



The maximum flow from source to sink is five units. Therefore, maximum five people can get jobs.

2) Find the maximum flow.

We use Ford-Fulkerson algorithm to find the maximum flow in the flow network built in step 1. The maximum flow is actually the MBP we are looking for.

How to implement the above approach?

Let us first define input and output forms. Input is in the form of Edmonds matrix which is a 2D array 'bpGraph[M][N]' with M rows (for M job applicants) and N columns (for N jobs). The value bpGraph[i][j] is 1 if i'th applicant is interested in j'th job, otherwise 0.
Output is number maximum number of people that can get jobs.

A simple way to implement this is to create a matrix that represents adjacency matrix representation of a directed graph with M+N+2 vertices. Call the fordFulkerson() for the matrix. This implementation requires O((M+N)*(M+N)) extra space.

Extra space can be be reduced and code can be simplified using the fact that the graph is bipartite and capacity of every edge is either 0 or 1. The idea is to use DFS traversal to find a job for an applicant (similar to augmenting path in Ford-Fulkerson). We call bpm() for every applicant, bpm() is the DFS based function that tries all possibilities to assign a job to the applicant.

In bpm(), we one by one try all jobs that an applicant 'u' is interested in until we find a job, or all jobs are tried without luck. For every job we try, we do following.
If a job is not assigned to anybody, we simply assign it to the applicant and return true. If a job is assigned to somebody else say x, then we recursively check whether x can be assigned some other job. To make

sure that x doesn't get the same job again, we mark the job 'v' as seen before we make recursive call for x. If x can get other job, we change the applicant for job 'v' and return true. We use an array maxR[0..N-1] that stores the applicants assigned to different jobs.

If bmp() returns true, then it means that there is an augmenting path in flow network and 1 unit of flow is added to the result in maxBPM().

Recommended: Please solve it on "*PRACTICE*" first, before moving on to the solution.

C++

```
// A C++ program to find maximal
// Bipartite matching.
#include <iostream>
#include <string.h>
using namespace std;

// M is number of applicants
// and N is number of jobs
#define M 6
#define N 6

// A DFS based recursive function
// that returns true if a matching
// for vertex u is possible
bool bpm(bool bpGraph[M][N], int u,
         bool seen[], int matchR[])
{
    // Try every job one by one
    for (int v = 0; v < N; v++)
    {
        // If applicant u is interested in
        // job v and v is not visited
        if (bpGraph[u][v] && !seen[v])
        {
            // Mark v as visited
            seen[v] = true;

            // If job 'v' is not assigned to an
            // applicant OR previously assigned
            // applicant for job v (which is matchR[v])
            // has an alternate job available.
            // Since v is marked as visited in
            // the above line, matchR[v] in the following
            // recursive call will not get job 'v' again
            if (matchR[v] < 0 || bpm(bpGraph, matchR[v],
                                     seen, matchR))
            {
                matchR[v] = u;
                return true;
            }
        }
```

```
    }
    return false;
}

// Returns maximum number
// of matching from M to N
int maxBPM(bool bpGraph[M][N])
{
    // An array to keep track of the
    // applicants assigned to jobs.
    // The value of matchR[i] is the
    // applicant number assigned to job i,
    // the value -1 indicates nobody is
    // assigned.
    int matchR[N];

    // Initially all jobs are available
    memset(matchR, -1, sizeof(matchR));

    // Count of jobs assigned to applicants
    int result = 0;
    for (int u = 0; u < M; u++)
    {
        // Mark all jobs as not seen
        // for next applicant.
        bool seen[N];
        memset(seen, 0, sizeof(seen));

        // Find if the applicant 'u' can get a job
        if (bpm(bpGraph, u, seen, matchR))
            result++;
    }
    return result;
}

// Driver Code
int main()
{
    // Let us create a bpGraph
    // shown in the above example
    bool bpGraph[M][N] = {{0, 1, 1, 0, 0, 0},
                          {1, 0, 0, 1, 0, 0},
                          {0, 0, 1, 0, 0, 0},
                          {0, 0, 1, 1, 0, 0},
                          {0, 0, 0, 0, 0, 0},
                          {0, 0, 0, 0, 0, 1}};

    cout << "Maximum number of applicants that can get job is "
         << maxBPM(bpGraph);

    return 0;
}
```

Run on IDE

Java

```
// A Java program to find maximal
// Bipartite matching.
import java.util.*;
import java.lang.*;
import java.io.*;

class GFG
{
    // M is number of applicants
    // and N is number of jobs
```

```
    static final int M = 6;
    static final int N = 6;

    // A DFS based recursive function that
    // returns true if a matching for
    // vertex u is possible
    boolean bpm(boolean bpGraph[][], int u,
                boolean seen[], int matchR[])
    {
        // Try every job one by one
        for (int v = 0; v < N; v++)
        {
            // If applicant u is interested
            // in job v and v is not visited
            if (bpGraph[u][v] && !seen[v])
            {

                // Mark v as visited
                seen[v] = true;

                // If job 'v' is not assigned to
                // an applicant OR previously
                // assigned applicant for job v (which
                // is matchR[v]) has an alternate job available.
                // Since v is marked as visited in the
                // above line, matchR[v] in the following
                // recursive call will not get job 'v' again
                if (matchR[v] < 0 || bpm(bpGraph, matchR[v],
                                         seen, matchR))
                {
                    matchR[v] = u;
                    return true;
                }
            }
        }
        return false;
    }

    // Returns maximum number
    // of matching from M to N
    int maxBPM(boolean bpGraph[][])
    {
        // An array to keep track of the
        // applicants assigned to jobs.
        // The value of matchR[i] is the
        // applicant number assigned to job i,
        // the value -1 indicates nobody is assigned.
        int matchR[] = new int[N];

        // Initially all jobs are available
        for(int i = 0; i < N; ++i)
            matchR[i] = -1;

        // Count of jobs assigned to applicants
        int result = 0;
        for (int u = 0; u < M; u++)
        {
            // Mark all jobs as not seen
            // for next applicant.
            boolean seen[] =new boolean[N] ;
            for(int i = 0; i < N; ++i)
                seen[i] = false;

            // Find if the applicant 'u' can get a job
            if (bpm(bpGraph, u, seen, matchR))
                result++;
        }
        return result;
    }
```

```
    // Driver Code
    public static void main (String[] args)
                       throws java.lang.Exception
    {
        // Let us create a bpGraph shown
        // in the above example
        boolean bpGraph[][] = new boolean[][]{
                                {false, true, true,
                                 false, false, false},
                                {true, false, false,
                                 true, false, false},
                                {false, false, true,
                                 false, false, false},
                                {false, false, true,
                                 true, false, false},
                                {false, false, false,
                                 false, false, false},
                                {false, false, false,
                                 false, false, true}};
        GFG m = new GFG();
        System.out.println( "Maximum number of applicants that can"+
                            " get job is "+m.maxBPM(bpGraph));
    }
}
```

Run on IDE

Python

```
# Python program to find
# maximal Bipartite matching.

class GFG:
    def __init__(self,graph):

        # residual graph
        self.graph = graph
        self.ppl = len(graph)
        self.jobs = len(graph[0])

    # A DFS based recursive function
    # that returns true if a matching
    # for vertex u is possible
    def bpm(self, u, matchR, seen):

        # Try every job one by one
        for v in range(self.jobs):

            # If applicant u is interested
            # in job v and v is not seen
            if self.graph[u][v] and seen[v] == False:

                # Mark v as visited
                seen[v] = True

                '''If job 'v' is not assigned to
                   an applicant OR previously assigned
                   applicant for job v (which is matchR[v])
                   has an alternate job available.
                   Since v is marked as visited in the
                   above line, matchR[v]  in the following
                   recursive call will not get job 'v' again'''
                if matchR[v] == -1 or self.bpm(matchR[v],
                                               matchR, seen):
                    matchR[v] = u
                    return True
        return False
```

```
    # Returns maximum number of matching
    def maxBPM(self):
        '''An array to keep track of the
           applicants assigned to jobs.
           The value of matchR[i] is the
           applicant number assigned to job i,
           the value -1 indicates nobody is assigned.'''
        matchR = [-1] * self.jobs

        # Count of jobs assigned to applicants
        result = 0
        for i in range(self.ppl):

            # Mark all jobs as not seen for next applicant.
            seen = [False] * self.jobs

            # Find if the applicant 'u' can get a job
            if self.bpm(i, matchR, seen):
                result += 1
        return result


bpGraph =[[0, 1, 1, 0, 0, 0],
          [1, 0, 0, 1, 0, 0],
          [0, 0, 1, 0, 0, 0],
          [0, 0, 1, 1, 0, 0],
          [0, 0, 0, 0, 0, 0],
          [0, 0, 0, 0, 0, 1]]

g = GFG(bpGraph)

print ("Maximum number of applicants that can get job is %d " % g.maxBPM())

# This code is contributed by Neelam Yadav
```

Run on IDE

C#

```
// A C# program to find maximal
// Bipartite matching.
using System;

class GFG
{
    // M is number of applicants
    // and N is number of jobs
    static int M = 6;
    static int N = 6;

    // A DFS based recursive function
    // that returns true if a matching
    // for vertex u is possible
    bool bpm(bool [,]bpGraph, int u,
             bool []seen, int []matchR)
    {
        // Try every job one by one
        for (int v = 0; v < N; v++)
        {
            // If applicant u is interested
            // in job v and v is not visited
            if (bpGraph[u, v] && !seen[v])
            {
                // Mark v as visited
                seen[v] = true;
```

```
                // If job 'v' is not assigned to
                // an applicant OR previously assigned
                // applicant for job v (which is matchR[v])
                // has an alternate job available.
                // Since v is marked as visited in the above
                // line, matchR[v] in the following recursive
                // call will not get job 'v' again
                if (matchR[v] < 0 || bpm(bpGraph, matchR[v],
                                         seen, matchR))
                {
                    matchR[v] = u;
                    return true;
                }
            }
        }
        return false;
    }

    // Returns maximum number of
    // matching from M to N
    int maxBPM(bool [,]bpGraph)
    {
        // An array to keep track of the
        // applicants assigned to jobs.
        // The value of matchR[i] is the
        // applicant number assigned to job i,
        // the value -1 indicates nobody is assigned.
        int []matchR = new int[N];

        // Initially all jobs are available
        for(int i = 0; i < N; ++i)
            matchR[i] = -1;

        // Count of jobs assigned to applicants
        int result = 0;
        for (int u = 0; u < M; u++)
        {
            // Mark all jobs as not
            // seen for next applicant.
            bool []seen = new bool[N] ;
            for(int i = 0; i < N; ++i)
                seen[i] = false;

            // Find if the applicant
            // 'u' can get a job
            if (bpm(bpGraph, u, seen, matchR))
                result++;
        }
        return result;
    }

    // Driver Code
    public static void Main ()
    {
        // Let us create a bpGraph shown
        // in the above example
        bool [,]bpGraph = new bool[,]
                          {{false, true, true,
                            false, false, false},
                           {true, false, false,
                            true, false, false},
                           {false, false, true,
                            false, false, false},
                           {false, false, true,
                            true, false, false},
                           {false, false, false,
                            false, false, false},
                           {false, false, false,
                            false, false, true}};
        GFG m = new GFG();
```

```
    Console.Write( "Maximum number of applicants that can"+
                   " get job is "+m.maxBPM(bpGraph));
    }
}

//This code is contributed by nitin mittal.
```

Run on IDE

Output :

```
Maximum number of applicants that can get job is 5
```

You may like to see below also:

Hopcroft–Karp Algorithm for Maximum Matching | Set 1 (Introduction)

Hopcroft–Karp Algorithm for Maximum Matching | Set 2 (Implementation)

References:

http://www.cs.cornell.edu/~wdtseng/icpc/notes/graph_part5.pdf

http://www.youtube.com/watch?v=NlQqmEXuiC8

http://en.wikipedia.org/wiki/Maximum_matching

http://www.stanford.edu/class/cs97si/08-network-flow-problems.pdf

http://www.cs.princeton.edu/courses/archive/spring13/cos423/lectures/07NetworkFlowII-2×2.pdf

http://www.ise.ncsu.edu/fangroup/or766.dir/or766_ch7.pdf

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

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