

# Multivariate Analysis

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# Initial Dataset

ID	Reason for absence	Month of absence	Day of the week	Seasons	Transportation expense	...	Education	Son	Social drinker	Social smoker	Pet	Weight	Height	Body mass index	Absenteeism time in hours
11	26	7	3	1	289		1	2	1	0	1	90	172	30	4
3	23	7	4	1	179		1	0	1	0	0	89	170	31	2
7	7	7	5	1	279		1	2	1	1	0	68	168	24	4
11	23	7	5	1	289		1	2	1	0	1	90	172	30	2
3	23	7	6	1	179		1	0	1	0	0	89	170	31	2
10	22	7	6	1	361		1	1	1	0	4	80	172	27	8
20	23	7	6	1	260		1	4	1	0	0	65	168	23	4
14	19	7	2	1	155		1	2	1	0	0	95	196	25	40
1	22	7	2	1	235		3	1	0	0	1	88	172	29	8
20	1	7	2	1	260		1	4	1	0	0	65	168	23	8
20	1	7	3	1	260		1	4	1	0	0	65	168	23	8
20	11	7	4	1	260		1	4	1	0	0	65	168	23	8
3	11	7	4	1	179		1	0	1	0	0	89	170	31	1
3	23	7	4	1	179		1	0	1	0	0	89	170	31	4
24	14	7	6	1	246		1	0	1	0	0	67	170	23	8
3	23	7	6	1	179		1	0	1	0	0	89	170	31	2
3	21	7	2	1	179		1	0	1	0	0	89	170	31	8
6	11	7	5	1	189		1	2	0	0	2	69	167	25	8

- 21 variables
- 740 rows

# Data Preliminary Analysis

Division of the variable *Reason for Absence* in classes

- Disease **Reason for Absence = 1**
- Patient follow-up - **Reason for Absence = 2**
- Medical Consultation - **Reason for Absence = 3**
- Blood Donation - **Reason for Absence = 4**
- Laboratory Examination - **Reason for Absence = 5**
- Unjustified Absence - **Reason for Absence = 6**
- Physiotherapy - **Reason for Absence = 7**
- Dental Consultation - **Reason for Absence = 8**

# Data Preliminary Analysis

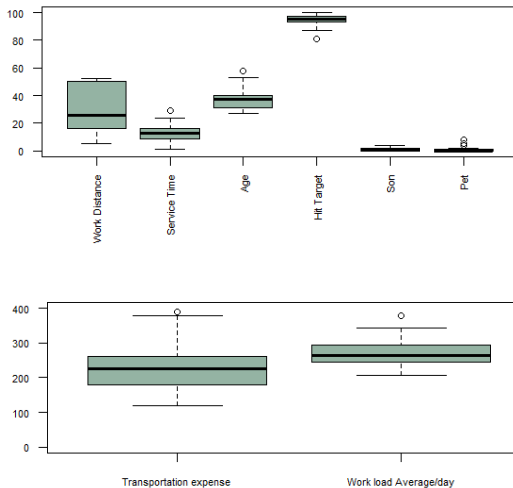
## Missing values

The rows in which the variable *Absenteeism* took the value 0 were removed.

## Division of the target variable *Absenteeism* in classes

- Number of hours  $< 8$  - **Absenteeism = 1**
- Number of hours = 8 - **Absenteeism = 2**
- $40 \leq$  Number of hours  $< 8$  - **Absenteeism = 3**
- Number of hours  $> 40$  - **Absenteeism = 4**

# Data Preliminary Analysis - Outliers



**Figure:** Boxplots of *Work Distance*, *Service Time*, *Age*, *Hit Target*, *Son*, *Pet*, *Work Load Average/day* and *Transportation Expense*

# Data Preliminary Analysis - Pearson's Correlation

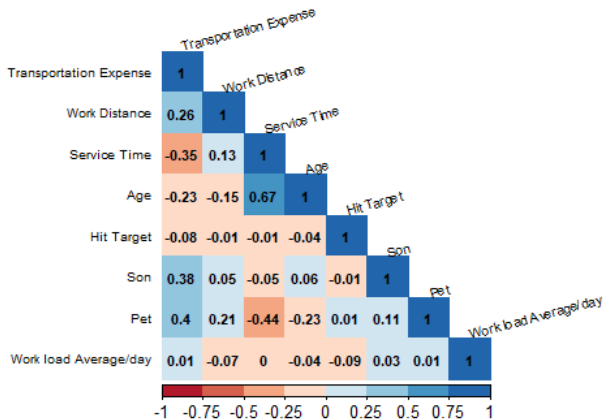
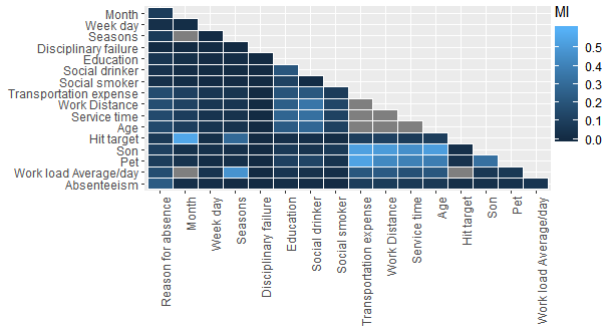


Figure: Correlation between the quantitative variables

# Data Preliminary Analysis - Mutual Information



- *Transportation Expense* and *Work Distance* - 0.854;
- *Transportation Expense* and *Age* - 0.834;
- *Service Time* and *Work Distance* - 0.868;
- *Age* and *Work Distance* - 0.869;
- *Age* and *Service Time* - 0.829;
- *Transportation Expense* and *Service Time* - 0.789.

# Data Preliminary Analysis - Categorical/Binary Variables

```
> table(data$Absenteeism)

 1  2  3  4
425 206 48 15

> table(data$`Reason for absence`)

 1  2  3  4  5  6  7  8
262 36 149 3 31 33 68 112

> table(data$Month)

 0  1  2  3  4  5  6  7  8  9 10 11 12
0 49 72 83 52 57 50 65 54 44 62 57 49

> table(data$`week day`)

 2  3  4  5  6
153 141 145 118 137

> table(data$Seasons)

 1  2  3  4
164 189 167 174

> table(data$`Disciplinary failure`)

 0  1
694 0
```

**Figure:** Table of Absenteeism, Reason for Absence, Seasons, Week Day, Month and Disciplinary Failure

- Most commonly used reasons for absence: Disease (38%), Medical Consultation (21%) and Dental Consultation (16%).
- The absenteeism time is mostly lower than 8 hours (61%), or equal to 8 hours (30%);
- 100% of the work absences are from people without disciplinary failures.



# Data Preliminary Analysis - Categorical/Binary Variables

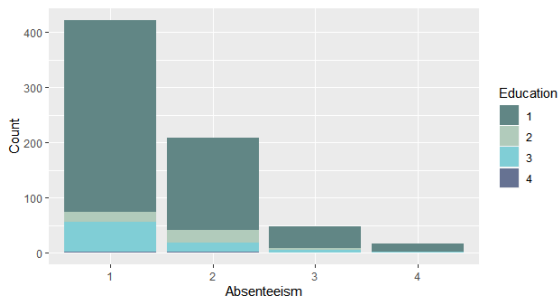
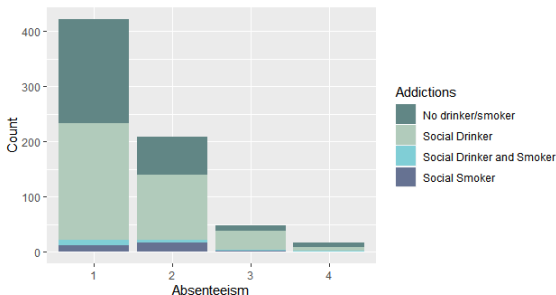


Figure: Proportion of each level of Education by Absenteeism's class

- 82% of the work absences are from people with only high school level of education;

# Data Preliminary Analysis - Categorical/Binary Variables



**Figure:** Proportion of social smokers/drinkers and non drinkers and smokers by Absenteeism's class

- 93% of the work absences are from people that do not smoke;
- 39% of absences are from non smokers and non drinkers;

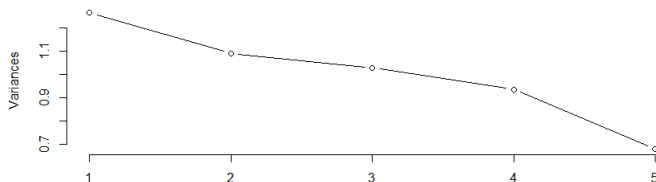
# Data Preliminary Analysis - Final dataset

Reason for absence	Month	Week day	Seasons	Education	Social drinker	Social smoker	Age	Hit target	Son	Pet	Work load Average/day	Absenteeism
3	10	5	4	1	1	0	40	93	1	1	253	1
3	10	4	4	1	1	0	38	93	0	0	253	2
3	10	4	4	1	0	0	28	93	1	2	253	1
1	10	5	4	1	1	0	36	93	4	0	253	3
1	10	3	4	1	1	0	40	93	1	1	253	2
8	10	3	4	1	0	0	28	93	1	2	253	1
6	10	4	4	1	1	0	33	93	2	1	253	2
3	10	6	4	1	1	0	28	93	1	4	253	1
8	10	6	4	1	1	0	36	93	4	0	253	1
3	11	5	4	1	1	0	38	93	0	0	306	1
3	11	4	4	1	0	0	28	93	1	2	306	1
1	11	5	4	1	1	0	38	93	0	0	306	2
1	11	5	4	2	0	1	40	93	2	0	306	2
3	11	5	4	1	1	0	40	93	1	1	306	1

- 13 variables
- 694 rows

# Dimensionality Reduction

## PCA standardized - Quantitative variables (5)



	PC1	PC2	PC3	PC4
Age	-0.67677819	0.13953744	-0.3482909	-0.07943722
Hlt target	0.05996430	-0.59364322	-0.3420561	0.71932955
Son	0.14552560	0.49310069	-0.7699853	0.07888582
Pet	0.71883886	0.06247886	-0.1549568	-0.17075880
work Load Average/day	0.02164617	0.61730473	0.3805287	0.66398283

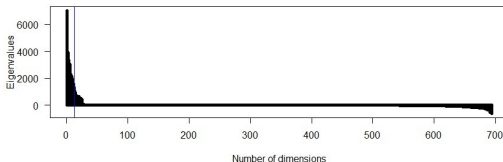
- The first 4 principal components explain **86.37%** of the variability

# Dimensionality Reduction

## Multidimensional Scaling - Categorical variables (7)

Reason for absence	Month	Week day	Seasons	Education	Social drinker	Social smoker	Week day=5	Week day=6	Seasons=1	Seasons=2	Seasons=3	Seasons=4	Education=1
6	7	3	1	1	1	0	0	0	1	0	0	0	1
3	7	4	1	1	1	0	0	0	1	0	0	0	1
1	7	5	1	1	1	1	1	0	1	0	0	0	1
3	7	5	1	1	1	0	1	0	1	0	0	0	1

Figure: Turning all variables into binary



- *Hamming distance* is good for binary variables
- The *Goodness of fit* is **0.805** using 12 dimensions

# Division of dataset

## Training and testing data

The dataset after PCA and MDS was divided into training (**70%**) and testing (**30%**) data.

## Cross Validation

- *k-fold* cross validation using training data, with  $k = 10$
- Testing the performance of each model using testing data

# Supervised Learning Methods for Classification

## Linear Algorithms

- Linear Discriminant Analysis

## Nonlinear Algorithms

- Regularized Discriminant Analysis
- Support Vector Machine
- Naive Bayes Classifier
- Neural Networks
- K-Nearest Neighbour

## Ensemble Algorithms

- Random Forest

# Supervised Learning Methods for Classification

Reference						Reference						Reference					
Prediction	1	2	3	4		Prediction	1	2	3	4		Prediction	1	2	3	4	
1	102	30	3	0		1	101	27	3	0		1	111	29	6	0	
2	23	31	9	4		2	24	28	9	3		2	16	29	4	2	
3	1	3	1	0		3	2	6	1	0		3	0	3	3	1	
4	1	0	0	0		4	0	3	0	1		4	0	3	0	1	

Figure: Confusion matrices using LDA, Neural Networks and Random Forest, respectively

- LDA (F1-Score = 0.779) does not classify any observation into class 4 - class with the lowest probability
- Neural Networks (F1-Score = 0.800) is a good model for this data
- Random Forest (F1-Score = 0.813) is the best algorithm for this dataset



# Supervised Learning Methods for Classification

	Accuracy	F1-score
LDA	0.6442	0.7786
RDA	0.6442	0.7663
SVM	0.6587	0.7865
Naive Bayes	0.6731	0.8263
Neural Networks	0.6587	0.7953
Random Forest	0.6923	0.8132
KNN	0.6442	0.7958

Figure: Overall performance of each method

# Cluster Analysis - First Approach

## Distance Metric

Gower's Distance:

$$d_G(\mathbf{x}, \mathbf{y}) = \sum_{k=1}^p \frac{|x_k - y_k|}{r_k}$$

## Cluster methods

- Hierarchical methods
- Partitioning method
- Density Based method

# Cluster Analysis - Second Approach

## Dimensionality Reduction

- Multidimensional Scaling and Principal Component Analysis

## Distance Metric

Euclidean Distance:

$$d_E(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_{k=1}^p (x_k - y_k)^2}$$

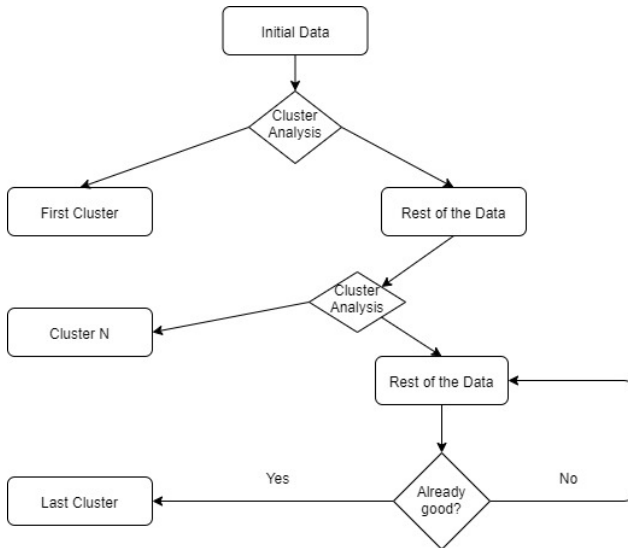
## Cluster methods

- Hierarchical methods
- Partitioning method
- Density Based method
- Mixture model method

# Cluster Analysis - Results

Method	Average Silhouette
Gower-Complete	0.1530
Gower-Average	0.2342
Gower-Ward	0.1350
Gower-PAM	0.1381
Gower-DBSCAN	-0.0613
Euclidean-Complete	0.1504
Euclidean-Average	0.2574
Euclidean-Ward	0.2038
Euclidean-PAM	0.1372
Euclidean-Mixture Models	0.1145

## Cluster Analysis - Results(2)



# Cluster Analysis - Results(3)

## First Cluster

Reason for absence	Month	week day	Seasons	Education	Social	drinker	social smoker	Age
1 :247	3 : 78	2:146	1:158	1:563	0:279		0:648	Min. :27.0
3 :147	2 : 71	3:136	2:179	2: 23	1:389		1: 20	1st Qu.:30.0
8 :111	7 : 64	4:142	3:161	3: 78				Median :37.0
7 : 68	10 : 62	5:114	4:170	4: 4				Mean :36.1
2 : 34	11 : 54	6:130						3rd Qu.:38.0
6 : 33	5 : 52							Max. :58.0
(Other):28	(Other):287							
Hit target	Son	Pet	work load	Average/day	Absenteeism			
Min. :81.00	Min. :0.0000	Min. :0.0000	Min. :206.0	1:413				
1st Qu.:93.00	1st Qu.:0.0000	1st Qu.:0.0000	1st Qu.:243.2	2:193				
Median :95.00	Median :1.0000	Median :0.0000	Median :264.0	3: 47				
Mean :94.69	Mean :0.9671	Mean :0.7051	Mean :270.9	4: 15				
3rd Qu.:97.00	3rd Qu.:2.0000	3rd Qu.:1.0000	3rd Qu.:285.0					
Max. :100.00	Max. :4.0000	Max. :8.0000	Max. :379.0					

## Second Cluster

Reason for absence	Month	week day	Seasons	Education	Social	drinker	social smoker	Age
1 :15	3 : 5	2:7	1: 6	1: 6	0:26		0: 0	Min. :36.00
5 : 6	5 : 5	3:5	2:10	2:20	1: 0		1:26	1st Qu.:40.00
2 : 2	8 : 5	4:3	3: 6	3: 0				Median :40.00
3 : 2	1 : 4	5:4	4: 4	4: 0				Mean :40.92
8 : 1	11 : 3	6:7						3rd Qu.:40.00
4 : 0	2 : 1							Max. :48.00
(Other):0	(Other):3							
Hit target	Son	Pet	work load	Average/day	Absenteeism			
Min. :91.0	Min. :1.000	Min. :0.000	Min. :206.0	1:12				
1st Qu.:93.0	1st Qu.:2.000	1st Qu.:0.000	1st Qu.:239.5	2:13				
Median :96.0	Median :2.000	Median :0.000	Median :250.0	3: 1				
Mean :95.5	Mean :1.846	Mean :1.077	Mean :269.8	4: 0				
3rd Qu.:98.0	3rd Qu.:2.000	3rd Qu.:0.000	3rd Qu.:312.0					
Max. :99.0	Max. :2.000	Max. :5.000	Max. :343.0					

# Cluster Analysis - Problems and Recommendations

## Problems

- High Dimensionality
- Imbalanced Dataset

## Recommendations

- Oversampling
- Fuzzy Clustering

# Classification with new diagnosis variable

	Reason for absence	Month	Week day	Seasons	Education	Social drinker	Social smoker	Age	Hit target	Son	Pet	Work load Average/day	cluster
1	6	7	3	1	1	1	0	33	97	2	1	240	1
2	3	7	4	1	1	1	0	36	97	0	0	240	1
3	1	7	5	1	1	1	1	39	97	2	0	240	1
4	3	7	5	1	1	1	0	33	97	2	1	240	1
5	3	7	6	1	1	1	0	38	97	0	0	240	1
6	2	7	6	1	1	1	0	28	97	1	4	240	1
7	3	7	6	1	1	1	0	36	97	4	0	240	1
8	1	7	2	1	1	1	0	34	97	2	0	240	1
9	2	7	2	1	3	0	0	37	97	1	1	240	1
10	1	7	2	1	1	1	0	36	97	4	0	240	1
11	1	7	3	1	1	1	0	36	97	4	0	240	1
12	1	7	4	1	1	1	0	36	97	4	0	240	1
13	1	7	4	1	1	1	0	38	97	0	0	240	1
14	3	7	4	1	1	1	0	38	97	0	0	240	1
15	1	7	6	1	1	1	0	41	97	0	0	240	1
16	3	7	6	1	1	1	0	38	97	0	0	240	1
17	1	7	2	1	1	1	0	38	97	0	0	240	1
18	1	7	5	1	1	0	0	33	97	2	2	240	1
19	3	8	4	1	1	0	0	47	92	2	1	206	1
20	1	8	4	1	2	0	0	28	92	0	0	206	1
21	1	8	2	1	1	1	0	38	92	0	0	206	1
22	1	8	2	1	1	1	0	28	92	1	4	206	1

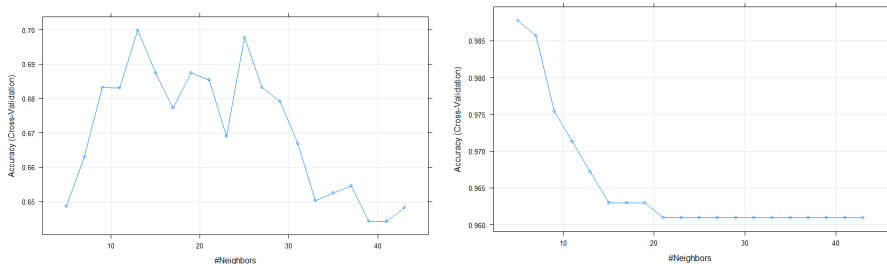


# Supervised Learning Methods for Classification

	Accuracy	F1-score		Accuracy	F1-score
LDA	0.6442	0.7786	LDA	0.976	0.9874
RDA	0.6442	0.7663	RDA	1	1
SVM	0.6587	0.7865	SVM	1	1
Naive Bayes	0.6731	0.8263	Naive Bayes	1	1
Neural Networks	0.6587	0.7953	Neural Networks	0.9904	0.995
Random Forest	0.6923	0.8132	Random Forest	1	1
KNN	0.6442	0.7958	KNN	0.9808	0.9901

**Figure:** Overall performance of each method with *Absenteeism* variable and clustering results

# Compare Performance: K- Nearest Neighbors



**Figure:** Relation between  $k$  values and accuracy of the respectively model, with *Absenteeism* variable on the left and clustering results on the right

# Compare Performance: Random Forest

## Confusion Matrix and Statistics

	Reference			
Prediction	1	2	3	4
1	107	27	6	0
2	19	31	4	2
3	0	3	3	1
4	1	3	0	1

## Overall Statistics

Accuracy : 0.6827  
 95% CI : (0.6148, 0.7453)  
 No Information Rate : 0.6106  
 P-value [Acc > NIR] : 0.01865

Kappa : 0.37

McNemar's Test P-value : 0.13630

## Statistics by class:

	Class: 1	Class: 2	Class: 3	Class: 4
Sensitivity	0.8425	0.4844	0.23077	0.250000
Specificity	0.5926	0.8264	0.97949	0.980392
Pos Pred Value	0.7643	0.5536	0.42857	0.200000
Neg Pred Value	0.7059	0.7829	0.95025	0.985222
Prevalence	0.6106	0.3077	0.06250	0.019231
Detection Rate	0.5144	0.1490	0.01442	0.004808
Detection Prevalence	0.6731	0.2692	0.03365	0.024038
Balanced Accuracy	0.7176	0.6554	0.60513	0.615196

## Confusion Matrix and Statistics

	Reference	
Prediction	1	2
1	201	0
2	0	7

Accuracy : 1  
 95% CI : (0.9824, 1)  
 No Information Rate : 0.9663  
 P-value [Acc > NIR] : 0.0008084

Kappa : 1

McNemar's Test P-value : NA

Sensitivity : 1.0000  
 Specificity : 1.0000  
 Pos Pred Value : 1.0000  
 Neg Pred Value : 1.0000  
 Prevalence : 0.9663  
 Detection Rate : 0.9663  
 Detection Prevalence : 0.9663  
 Balanced Accuracy : 1.0000  
 'Positive' Class : 1

Figure: RF's performance evaluators with *Absenteeism* variable and clustering results

# Conclusion

- Strongly imbalanced data
- Best obtained classification method for *Absenteeism*: Random Forest
- Few relations that allow group into clusters
- Generalized clusters which favors prediction

# Multivariate Analysis

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