RELATIVE NON-ABELIAN HOMOLOGICAL ALGEBRA

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The direction of abelian relative homological algebra based on relative exactness properties was initiated by N. Yoneda, whose quasi-abelian categories [4] can be seen as additive categories equipped with a distinguished class E of epimorphisms, so that the short exact sequences $K \to A \to B$ with $A \to B$ in E have the same properties as all short exact sequences in an abelian category.

We consider the non-additive version, which is not immediately suggested by the Yoneda structure, but is based on it and is partly motivated by more recent developments in categorical algebra. In this talk we begin by recalling categorical approach to (absolute) non-abelian homological algebra based on the theory of protomodular [1], homological [2] , and semi-abelian [3] categories. Then we describe its relative version, where the ground semi-abelian/homological/protomodular category C is replaced with a pair (C, E), where C is a pointed category and E a class of regular epimorphisms in C satisfying suitable conditions. This includes:

(1) Revision of the axioms: In particular we show that similarly to the absolute case there are two equivalent sets of axioms that can be seen as "old style" and "new style" axioms.
(2) Removing completeness/cocompleteness conditions: We show how to modify the axioms above in order to develop the relevant constructions and results under no completeness/cocompleteness conditions on the category C.
(3) Obtaining relative versions of main homological lemmas: We present main formulations in comparison with the absolute case and indicate key arguments of the proofs; in particular we describe a suitable relative calculus of relations applicable here.
(4) Overview of examples, including iterated central extensions in semi-abelian categories.

References

[1] D. Bourn, Normalization equivalence, kernel equivalence and affine categories, *Lecture Notes in Mathematics* 1488, 1991, 43-62

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[4] N. Yoneda, On Ext and exact sequences, *J. Fac. Sci. Univ. Tokyo,* Sec. 1, 8 (1961) 507-576