On the 3×3 lemma

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Abstract. A regular category is said to be *normal* when it is pointed and every regular epimorphism is a normal epimorphism. Any abelian category is normal, and in a normal category one can define short exact sequences in a similar way as in an abelian category. Then, the corresponding 3×3 lemma is equivalent to the so-called subtractivity, which in universal algebra is also known as congruence 0-permutability. In the context of non-pointed regular categories, short exact sequences can be replaced with "exact forks" and then, the corresponding 3×3 lemma is equivalent, in the universal algebraic terminology, to congruence 3-permutability; equivalently, regular categories satisfying the 3×3 lemma are precisely the Goursat categories. We show how these two seemingly independent results can be unified in the context of star-regular categories recently introduced in a joint work of A. Ursini and the first two authors.

* Joint work with Marino Gran and Zurab Janelidze