中国科学技术大学

University of Science and Technology of China

Homework 3

Bayesian Network

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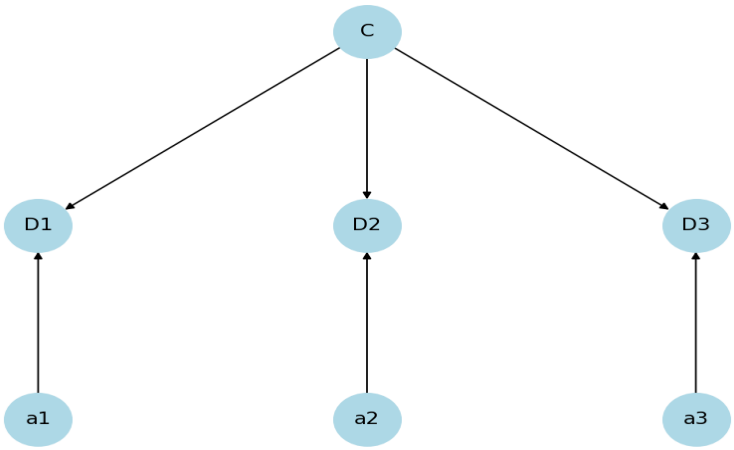
Homework 3

1 概率推断 [25%]

1.1



1.2

$\because D_t$ 只依赖于 C 和 a_t，

$\therefore P(C=c, D_1=d_1, D_2=d_2, D_3=d_3) = P(C=c)P(D_1=d_1|C=c, a_1)P(D_2=d_2|C=c, a_2)P(D_3=d_3|C=c, a_3)$

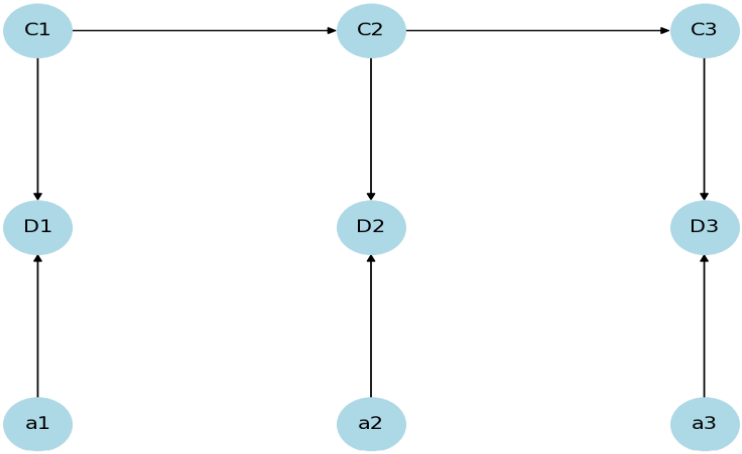
1.3

$$
\begin{aligned}
P(C=c|D_1=d_1,...,D_t=d_t) &= \frac{P(C=c, D_1=d_1, ..., D_{t-1}=d_{t-1}, D_t=d_t)}{P(D_1=d_1, ..., D_{t-1}=d_{t-1}, D_t=d_t)} \\
&= \frac{P(C=c)P(D_1=d_1|C=c)P(D_2=d_2|C=c)...P(D_t=d_t|C=c)}{P(D_1=d1, ..., D_{t-1}=d_{t-1}, D_t=d_t)} \\
&= \frac{P(C=c, D_1=d_1, ..., D_{t-1}=d_{t-1})P(D=d_t|C=c)}{P(D_1=d_1, ..., D_{t-1}=d_{t-1}, D_t=d_t)} \\
&= \frac{P(C=c|D_1=d_1, ..., D_{t-1}=d_{t-1})P(D=d_t|C=c)}{P(D_t=d_t|D_1=d_1, ..., D_{t-1}=d_{t-1})} \\
&\propto P(C=c|D_1=d_1, ..., D_{t-1}=d_{t-1})P(D=d_t|C=c)
\end{aligned}
$$

Homework 3

2 转移概率 [25%]

2.1



2.2

$\because D_t$ 依赖于 C_t 和 a_t，且 C_t 依赖于 C_{t-1}，

$\therefore P(C_1 = c_1, C_2 = c_2, C_3 = c_3, D_1 = d_1, D_2 = d_2, D_3 = d_3) = P(C_1 = c_1)P(D_1 = d_1|C_1 = c_1)P(C_2 = c_2|C_1 = c_1)P(D_2 = d_2|C_2 = c_2)P(C_3 = c_3|C_2 = c_2)P(D_3 = d_3|C_3 = c_3)$

2.3

$$
\begin{aligned}
P(C_{t+1} = c_{t+1}|D_1 = d_1, ..., D_t = d_t) &= \frac{P(C_{t+1} = c_{t+1}, D_1 = d_1, ..., D_{t-1} = d_{t-1}, D_t = d_t)}{P(D_1 = d_1, ..., D_{t-1} = d_{t-1}, D_t = d_t)} \\
&= \sum_{c_t} \frac{P(C_{t+1} = c_{t+1}, C_t = c_t, D_1 = d_1, ..., D_{t-1} = d_{t-1}, D_t = d_t)}{P(D_1 = d_1, ..., D_{t-1} = d_{t-1}, D_t = d_t)} \\
&= \sum_{c_t} \frac{P(C_t = c_t, D_1 = d_1, ..., D_t = d_t)P(C_{t+1} = c_{t+1}|C_t = c_t, ..., D_t = d_t)}{P(D_1 = d_1, ..., D_{t-1} = d_{t-1}, D_t = d_t)} \\
&= \sum_{c_t} \frac{P(C_t = c_t, D_1 = d_1, ..., D_t = d_t)P(C_{t+1} = c_{t+1}|C_t = c_t)}{P(D_1 = d_1, ..., D_{t-1} = d_{t-1}, D_t = d_t)} \\
&\propto \sum_{c_t} P(C_t = c_t|D_1 = d_1, ..., D_t = d_t)P(C_{t+1} = c_{t+1}|C_t = c_t)
\end{aligned}
$$

3 是哪辆车?[30%]

3.1

3.2

3.3

3.4

4 模型学习 [10%]

4.1

5 反馈 [10%]