中国科学技术大学

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# Lab 5

## 三次样条插值


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# Lab 5

## 1 Problem Descriptions

给定若干插值点，利用大 M 法计算三次样条插值函数，实现三次样条插值算法。

## 2 Analysis and Algorithms

$$\mu_i M_{i-1} + 2M_i + \lambda_i M_{i+1} = d_i, i = 1, 2, \cdots, n-1$$

其中

$$\mu_i = 1 - \lambda_i$$

$$\frac{h_i}{h_i + h_i}$$

$$d_i = \frac{6}{h_i + h_{i-1}} \left( \frac{y_{i+1} - y_i}{h_i} - \frac{y_i - y_{i-1}}{h_{i-1}} \right) = 6f\left(x_{i-1}, x_i, x_{i+1}\right)$$

给定 $M_0, M_n$ 的值，此时 $n-1$ 个方程组有 $n-1$ 个未知量 $\{M_i, i = 1, 2, \cdots, n-1\}$. 当 $M_0 = 0, M_n = 0$ 时，称为自然边界条件.

$$\begin{bmatrix} 2 & \lambda_1 & & & \\ \mu_2 & 2 & \lambda_2 & & \\ & \ddots & \ddots & \ddots & \\ & & \mu_{n-2} & 2 & \lambda_{n-2} \\ & & & \mu_{n-1} & 2 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ \\ M_{n-2} \\ \\ M_{n-1} \end{bmatrix} = \begin{bmatrix} d_1 - \mu_1 M_0 \\ d_2 \\ \vdots \\ d_{n-2} \\ d_{n-1} - \lambda_{n-1} M_n \end{bmatrix}$$

$$\begin{aligned} S(x) =& \frac{\left(x_{i+1} - x\right)^3 M_i + \left(x - x_i\right)^3 M_{i+1}}{6h_i} + \frac{\left(x_{i+1} - x\right) y_i + \left(x - x_i\right) y_{i+1}}{h_i} \\ & - \frac{h_i}{6}[(x_{i+1} - x)M_i + (x - x_i)M_{i+1}], \quad x \in [x_i, x_{i+1}] \end{aligned}$$

## 3 Results

```
原始数据插值结果:
M1[0]:0
M1[1]:-1.52544
M1[2]:-0.302627
M1[3]:1.21255
M1[4]:-0.252176
M1[5]:0.320552
M1[6]:-0.176831
M1[7]:-2.03243
M1[8]:3.41834
M1[9]:-1.91911
M1[10]:-0.700278
M1[11]:1.96983
M1[12]:-2.45882
M1[13]:2.02567
M1[14]:-0.0140509
M1[15]:-0.685465
M1[16]:-0.207489
M1[17]:0.77442
M1[18]:-1.14539
M1[19]:0.980548
M1[20]:0
```

```
S1[0]:-0.254241x^3-6.86449x^2-60.7398x-175.849
S1[1]:0.203803x^3+4.12854x^2+27.2045x+58.6696
S1[2]:0.25253x^3+5.15181x^2+34.3673x+75.3829
S1[3]:-0.244121x^3-3.7879x^2-19.2709x-31.8937
S1[4]:0.0954546x^3+1.30573x^2+6.19725x+10.5533
S1[5]:-0.0828972x^3-0.834491x^2-2.36364x-0.861222
S1[6]:-0.309266x^3-2.87181x^2-8.47559x-6.97317
S1[7]:0.90846x^3+4.43455x^2+6.13712x+2.76864
S1[8]:-0.889575x^3-0.959557x^2+0.743018x+0.9706
S1[9]:0.203139x^3-0.959557x^2+0.743018x+0.9706
S1[10]:0.445017x^3-1.68519x^2+1.46865x+0.728722
S1[11]:-0.738108x^3+5.41356x^2-12.7289x+10.1937
S1[12]:0.747415x^3-7.95615x^2+27.3803x-29.9154
S1[13]:-0.339953x^3+5.09227x^2-24.8134x+39.6762
S1[14]:-0.111902x^3+1.67151x^2-7.7096x+11.1698
S1[15]:0.0796627x^3-1.77666x^2+12.9794x-30.2082
S1[16]:0.163651x^3-3.54043x^2+25.3258x-59.0164
S1[17]:-0.319969x^3+8.06646x^2-67.5293x+188.597
S1[18]:0.354323x^3-10.1394x^2+96.3237x-302.962
S1[19]:-0.163425x^3+5.39301x^2-59.0007x+214.786
```

```
修改第十个压铁的坐标为(0,10)后:
M2[0]:0
M2[1]:-1.52776
M2[2]:-0.293369
M2[3]:1.17784
M2[4]:-0.122571
M2[5]:-0.163152
M2[6]:1.62838
M2[7]:-8.76957
M2[8]:28.5617
M2[9]:-41.579
M2[10]:24.4431
M2[11]:-4.76732
M2[12]:-0.653612
M2[13]:1.54196
M2[14]:0.115557
M2[15]:-0.720193
M2[16]:-0.198184
M2[17]:0.771928
M2[18]:-1.14473
M2[19]:0.980382
M2[20]:0
```

```
S2[0]:-0.254626x^3-6.87491x^2-60.8332x-176.126
S2[1]:0.205731x^3+4.16557x^2+27.5555x+59.5768
S2[2]:0.245201x^3+5.0303x^2+33.3575x+73.1148
S2[3]:-0.216734x^3-3.39909x^2-16.5315x-26.6632
S2[4]:-0.00676361x^3+0.0964704x^2-0.783675x-0.41684
S2[5]:0.298589x^3+2.77593x^2+13.8732x+19.1257
S2[6]:-1.73299x^3-12.9775x^2-40.9794x-35.7269
S2[7]:6.22188x^3+29.5779x^2+54.479x+27.912
S2[8]:-11.6901x^3-20.7895x^2+0.743018x+10
S2[9]:11.0037x^3-0.959557x^2+0.743018x+10
S2[10]:-4.8684x^3+1.68338x^2-46.8732x+25.8721
S2[11]:0.685617x^3+3.60835x^2+19.775x-18.5601
S2[12]:0.365929x^3-7.23059x^2+11.1434x-9.92847
S2[13]:-0.237734x^3+4.83306x^2-17.8325x+28.706
S2[14]:-0.139292x^3+1.75833x^2-10.4493x+16.4007
S2[15]:0.0870017x^3-1.80458x^2+13.9904x-32.4787
S2[16]:0.161685x^3-3.5317x^2+24.9689x-58.0952
S2[17]:-0.319443x^3+8.0638x^2-67.4076x+188.242
S2[18]:0.354185x^3-10.1387x^2+96.2838x-302.832
S2[19]:-0.163397x^3+5.39301x^2-58.9907x+214.75
```

## 4 Conclusion

在各个多项式中入相应插值点的值，发现曲线能较准确地拟合。将第十个点的坐标改为 (0,10) 后，计算出新的曲线中 M 值都改变了，每个区间内的三次函数也都发生了改变。