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Lab 5

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1 Problem Descriptions

2 Analysis and Algorithms

$$\mu_i M_{i-1} + 2M_i + \lambda_i M_{i+1} = d_i, i = 1, 2, \cdots, n-1$$

其中

$$\mu_i = 1 - \lambda_i$$

$$\frac{h_i}{h_i + h_i}$$

$$d_i = \frac{6}{h_i + h_{i-1}} \left(\frac{y_{i+1} - y_i}{h_i} - \frac{y_i - y_{i-1}}{h_{i-1}} \right) = 6f\left(x_{i-1}, x_i, x_{i+1}\right)$$

给定 M_0, M_n 的值，此时 $n-1$ 个方程组有 $n-1$ 个未知量 $\{M_i, i = 1,\ 2, \cdots, n-1\}$. $M_0 = 0, M_n = 0$ 时，称为自然边界条件. $\begin{bmatrix} 2 & \lambda_1 & & & \\ \mu_2 & 2 & \lambda_2 & & \\ & \ddots & \ddots & \ddots & \\ & & \mu_{n-2} & 2 & \lambda_{n-2} \\ & & & \mu_{n-1} & 2 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ \\ M_{n-2} \\ \\ M_{n-1} \end{bmatrix} =$

$$\begin{bmatrix} d_1 - \mu_1 M_0 \\ d_2 \\ \vdots \\ d_{n-2} \\ d_{n-1} - \lambda_{n-1} M_n \end{bmatrix}$$

$$S(x) = \frac{(x_{i+1} - x)^3 M_i + (x - x_i)^3 M_{i+1}}{6h_i} + \frac{(x_{i+1} - x) y_i + (x - x_i) y_{i+1}}{h_i}$$
$$- \frac{h_i}{6}[(x_{i+1} - x)M_i + (x - x_i)M_{i+1}], \quad x \in [x_i, x_{i+1}]$$

3 Results

4 Conclusion