中国科学技术大学

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Lab 6

FFT 与 IFFT

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Lab 6

1 Problem Descriptions

通过快速傅里叶变换与快速傅里叶逆变换实现对给定函数的 Fourier 分析以及重建。

2 Analysis and Algorithms

2.1 Algorithm 1 FFT

$n \leftarrow length[f]$
if $n == 1$
  then return f
end if
$\omega_n \leftarrow e^{i2\pi/n}$
$\omega \leftarrow 1$
$\mathbf{f}^0 \leftarrow (f_0, f_2, \ldots, f_{n-2})$
$\mathbf{f}^1 \leftarrow (f_1, f_3, \ldots, f_{n-1})$
$\mathbf{g}^0 \leftarrow$FFT($\mathbf{f}^0$)
$\mathbf{g}^1 \leftarrow$FFT($\mathbf{f}^1$)
for $k \leftarrow 0$ to $n/2-1$**do**
  $\mathbf{g_k} \leftarrow \mathbf{g}_k^0 + \omega \mathbf{g}_k^1$
  $\mathbf{g_{k+n/2}} \leftarrow \mathbf{g}_k^0 - \omega \mathbf{g}_k^1$
  $\omega \leftarrow \omega\omega_n$
end for
return g

2.2 Algorithm 2 IFFT

2.2.1 方法 1

$n \leftarrow length[f]$
if $n == 1$
  then return f
end if
$\omega_n \leftarrow e^{-i2\pi/n}$
$\omega \leftarrow 1$
$\mathbf{f}^0 \leftarrow (f_0, f_2, \ldots, f_{n-2})$
$\mathbf{f}^1 \leftarrow (f_1, f_3, \ldots, f_{n-1})$
$\mathbf{g}^0 \leftarrow$IFFT($\mathbf{f}^0$)
$\mathbf{g}^1 \leftarrow$IFFT($\mathbf{f}^1$)
for $k \leftarrow 0$ to $n/2-1$**do**
  $\mathbf{g_k} \leftarrow \mathbf{g}_k^0 + \omega \mathbf{g}_k^1$
  $\mathbf{g_{k+n/2}} \leftarrow \mathbf{g}_k^0 - \omega \mathbf{g}_k^1$
  $\omega \leftarrow \omega\omega_n$
end for

$\mathbf{g} = \mathbf{g}/2$
return g

2.2.2 方法 2

$\mathbf{f}_{\text{temp}} \leftarrow (f_0, f_{n-1}, f_{n-2} \ldots, f_2)$
$\mathbf{g} \leftarrow \text{FFT}(\mathbf{f}_{\text{temp}})$
$\mathbf{g} = \mathbf{g}/n$
return g

3 Results

```
Function 1:
FFT of function for n = 16
Real part:
-2.44572e-15 7.18788e-16 2.17281e-15 3.24859e-16 -3.23092e-18 -4.75929e-15 -1.73518e-15 2.37211e-15 5.10379e-15 1.75304e-15 -4.91729e-16 -5.89239e-15 -3.23092e-18 1.01387e-15 9.29356e-16 9.42
148e-16
Imaginary part:
0 -1.69718e-16 5.6 -2.66454e-15 -1.77636e-15 8 1.33227e-15 3.14428e-15 0 -3.82709e-15 -1.77636e-15 -8 1.77636e-15 2.66454e-15 -5.6 8.52519e-16
```

图 1

```
FFT of function for n = 128
Real part:
1.27353e-15 5.89064e-16 -4.91729e-16 3.00257e-15 1.42937e-15 -4.20628e-14 6.64774e-16 3.98279e-15 2.14217e-15 1.28272e-15 4.28494e-15 8.48911e-15 6.5888e-15 6.83043e-16 2.56785e-15 -4.45344e-
15 -1.86156e-15 1.09927e-15 1.56851e-15 2.3901e-15 5.15676e-15 -5.0123e-16 -1.67329e-15 -1.28695e-15 -4.93838e-15 4.16251e-15 -5.25202e-15 -2.25764e-14 -1.24117e-14 2.24828e-15 -1.51364e-15 -
1.46181e-16 -4.3331e-15 1.92642e-15 -2.93913e-15 -7.6406e-16 -7.00015e-15 1.11074e-15 -1.23243e-14 2.33531e-15 1.27908e-14 2.63615e-15 3.40783e-15 8.71302e-16 -1.12184e-15 -3.81005e-16 -2.407
76e-15 3.80564e-15 2.29919e-15 -2.01033e-15 2.90846e-15 2.43424e-15 6.68018e-16 2.38635e-15 -7.43873e-15 6.28316e-15 1.04206e-14 -2.1959e-15 8.98885e-15 3.43348e-15 6.66492e-15 -1.59195e-15 1
.11434e-14 5.13839e-15 -1.30484e-14 3.94276e-15 1.28309e-14 -2.10306e-15 6.66492e-15 5.69781e-16 7.71342e-15 -9.90771e-16 1.04206e-14 5.2522e-15 -6.08369e-15 -1.64486e-16 6.68018e-16 4.36596e
-15 3.34765e-15 -1.5031e-15 2.29919e-15 3.45386e-15 -5.70219e-15 -1.28223e-15 -1.12184e-15 2.58711e-15 1.24787e-15 2.14431e-15 1.27908e-14 1.16925e-15 -1.16327e-14 1.87852e-14 -7.00015e-15 4.
62238e-16 8.70142e-15 1.37091e-15 -4.3331e-15 -1.10093e-15 1.17256e-14 1.17738e-15 -1.24117e-14 -7.07661e-15 -6.5001e-15 5.02776e-15 -4.93838e-15 -1.95736e-15 -4.68667e-16 -2.56742e-15 5.1567
6e-15 1.83797e-15 -3.34302e-15 1.83865e-15 -1.86156e-15 -2.14981e-15 -2.51805e-15 -2.25608e-15 6.5888e-15 4.91232e-15 4.42897e-15 1.48125e-15 2.14217e-15 3.67477e-15 2.82167e-15 -5.34099e-14
1.42937e-15 4.86436e-16 -1.10611e-14 1.48631e-15
Imaginary part:
0 -2.99759e-15 44.8 -8.15579e-15 -5.98612e-15 64 9.25588e-15 8.2611e-15 9.10797e-15 2.63858e-15 7.33874e-15 -4.04822e-15 7.00174e-15 -4.7478e-15 -4.33098e-16 5.96891e-16 7.78312e-16 1.61851e-
15 5.47353e-15 -8.77356e-16 4.70971e-15 -9.04819e-15 -3.31085e-15 -6.78763e-16 5.38718e-15 -2.0183e-15 -1.47547e-15 1.38425e-14 2.43762e-15 1.79082e-14 1.31693e-14 -4.69077e-15 2.38698e-15 -1
.06773e-15 3.73161e-15 -4.18484e-16 2.2662e-15 -1.72144e-15 2.58773e-15 1.83129e-14 4.18037e-15 -1.89025e-15 4.38602e-15 -5.158e-15 1.64756e-15 1.86297e-15 -1.96604e-15 6.49025e-15 5.56267e-1
6 -3.28826e-16 3.61959e-15 -1.60461e-15 -1.90976e-15 2.01879e-15 4.97448e-15 7.10031e-15 4.34844e-15 8.60007e-16 1.1059e-15 2.13163e-14 2.20397e-15 -3.54308e-16 1.42109e-14 -3.16437e-15 0 3.6
5842e-15 7.10543e-15 -4.86904e-16 -2.20397e-15 0 -2.36993e-16 -1.71002e-15 -4.34844e-15 -7.06705e-15 -5.07268e-15 -5.20387e-15 1.90976e-15 1.03143e-15 -6.43944e-15 -1.13187e-15 -5.56267e-16 -
6.98559e-15 -1.43039e-15 -1.87117e-15 -1.64756e-15 2.61957e-15 -3.07094e-15 1.6231e-15 -4.18037e-15 -1.79701e-14 -3.60776e-15 1.81319e-14 -2.2662e-15 1.46805e-15 8.14702e-15 8.53806e-16 -2.38
698e-15 4.4442e-15 -1.78894e-16 -1.44064e-14 -2.43762e-15 1.72144e-15 3.82672e-15 1.40115e-15 -5.38718e-15 2.11946e-15 1.94995e-15 7.30468e-15 -4.70971e-15 1.92155e-16 -8.92498e-15 -1.68859e-
15 -7.78312e-16 3.00868e-16 -4.11002e-15 6.50448e-15 -7.00174e-15 4.40983e-15 -3.36787e-15 -9.15548e-15 -9.10797e-15 -7.28899e-15 -1.88356e-14 -64 5.98612e-15 3.95554e-15 -44.8 9.5548e-15
```

图 2

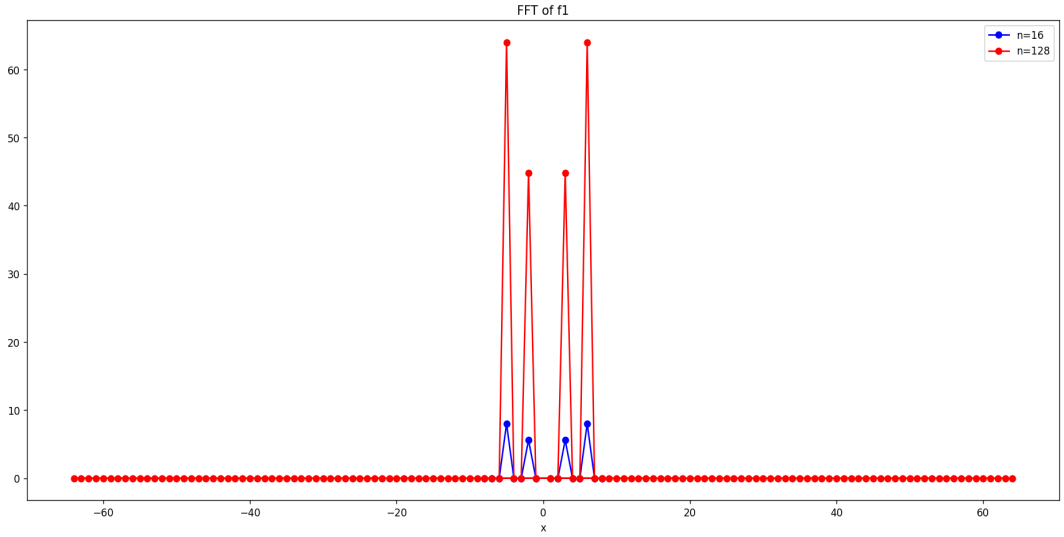

图 3

```
Function 2:
FFT of function for n = 128
Real part:
27.888 -1.1568e-16 -1.09912e-15 -4.15239e-16 2.63268e-15 -4.11778e-14 7.33691e-16 5.66975e-15 1.41002e-15 5.67371e-16 4.93446e-15 9.86427e-15 6.58252e-15 9.43666e-16 3.23234e-15 -3.5755e-15 -
1.88411e-15 -7.01818e-16 1.30766e-15 3.28758e-15 5.65296e-15 1.09969e-16 -2.21918e-15 -1.61502e-15 -6.37373e-15 3.8434e-15 -5.63003e-15 -2.54067e-14 -1.25669e-14 3.0653e-15 -7.88764e-16 -7.66
71e-16 -5.32907e-15 1.33809e-15 -2.63658e-15 -4.67211e-16 -6.91558e-15 1.32332e-15 -1.15073e-14 3.31455e-15 1.28734e-14 1.42558e-15 3.89932e-15 3.58519e-15 -6.25737e-16 -1.30605e-15 -3.39886
-15 4.53027e-15 1.88411e-15 -1.80528e-15 2.87349e-15 3.09985e-15 -4.49345e-17 1.2129e-15 -6.9107e-15 6.5672e-15 1.0742e-14 -2.54423e-15 9.60403e-15 1.6539e-15 7.06133e-15 -1.01345e-16 1.00919
e-14 4.43442e-15 -1.42109e-14 4.039e-15 1.22236e-14 -1.22559e-15 7.06133e-15 -3.21629e-16 8.08187e-15 -1.01888e-15 1.0742e-14 5.47784e-15 -5.92491e-15 2.30256e-16 -4.49345e-17 4.04695e-15 4.8
1214e-15 -1.52977e-15 1.88411e-15 4.37752e-15 -5.68131e-15 -9.02303e-16 -6.25737e-16 3.53605e-15 1.37012e-15 1.84922e-15 1.28734e-14 2.55767e-15 -1.10624e-14 1.98328e-14 -6.91558e-15 9.40705e
-16 9.4263e-15 9.59117e-16 -5.32907e-15 -8.66851e-16 1.19847e-14 -5.32909e-16 -1.25669e-14 -7.29565e-15 -6.6314e-15 4.5027e-15 -6.37373e-15 -1.98508e-15 -6.45304e-16 -3.51141e-15 5.65296e-15
1.33148e-15 -4.33411e-15 9.1466e-16 -1.88411e-15 -2.18165e-15 -2.89632e-15 -2.71144e-16 6.58252e-15 4.5127e-15 5.44774e-15 1.32263e-15 1.41002e-15 6.29709e-15 3.13731e-15 -5.34132e-14 2.63269
e-15 -8.35595e-16 -1.21125e-14 9.03708e-16
Imaginary part:
0 -4.17016e-15 44.8 -9.83882e-15 -6.12465e-15 64 9.71839e-15 9.54473e-15 9.31148e-15 2.07227e-15 7.1893e-15 -4.67348e-16 6.89155e-15 -4.52261e-15 -8.24151e-16 -1.96102e-16 1.25607e-15 8.503e-
16 5.08411e-15 -5.30192e-16 5.40899e-15 -9.92965e-15 -2.68349e-15 -4.45281e-16 5.44211e-15 -2.05506e-15 -1.50188e-15 1.34126e-14 2.3884e-15 1.78575e-14 1.49456e-14 -4.35794e-15 1.77636e-15 -1
.59634e-15 3.51063e-15 -1.99591e-16 2.37796e-15 -2.583e-15 2.38944e-15 1.81176e-14 3.94821e-15 -2.05972e-15 4.44677e-15 -6.18053e-15 2.1135e-15 3.1109e-15 -1.57499e-15 5.23378e-15 1.25607e-15
-5.17439e-16 3.6965e-15 -1.35317e-15 -2.43724e-15 3.29261e-15 5.14661e-15 7.66319e-15 5.15305e-15 2.87736e-15 5.08356e-16 2.13163e-14 1.15429e-15 2.16709e-16 1.06581e-14 -2.34084e-15 0 2.256
53e-15 7.10543e-15 -6.2702e-16 -1.15429e-15 0 1.17388e-15 -3.22221e-15 -5.15305e-15 -8.18369e-15 -4.55399e-15 -8.78475e-15 2.43724e-15 1.86314e-15 -6.8305e-15 -4.2368e-16 -1.25607e-15 -5.2631
9e-15 -1.47652e-15 -2.29466e-15 -2.1135e-15 4.3384e-15 -2.93433e-15 1.18984e-15 -3.94821e-15 -1.77753e-14 -3.33462e-15 1.85618e-14 -2.37796e-15 3.38736e-16 9.92338e-15 1.43566e-15 -1.77636e-1
5 3.44299e-15 4.20811e-17 -1.50753e-14 -2.3884e-15 2.583e-15 3.92797e-15 2.11645e-15 -5.44211e-15 1.81065e-15 1.51996e-15 8.3272e-15 -5.40899e-15 -1.12579e-15 -8.53393e-15 -7.7117e-16 -1.2568
7e-15 1.18942e-15 -3.75138e-15 5.04981e-15 -6.89155e-15 2.29864e-15 -2.52762e-15 -8.71346e-15 -9.31148e-15 -8.39972e-15 -1.84847e-14 -64 6.12465e-15 3.57764e-15 -44.8 1.06966e-14
```

图 4

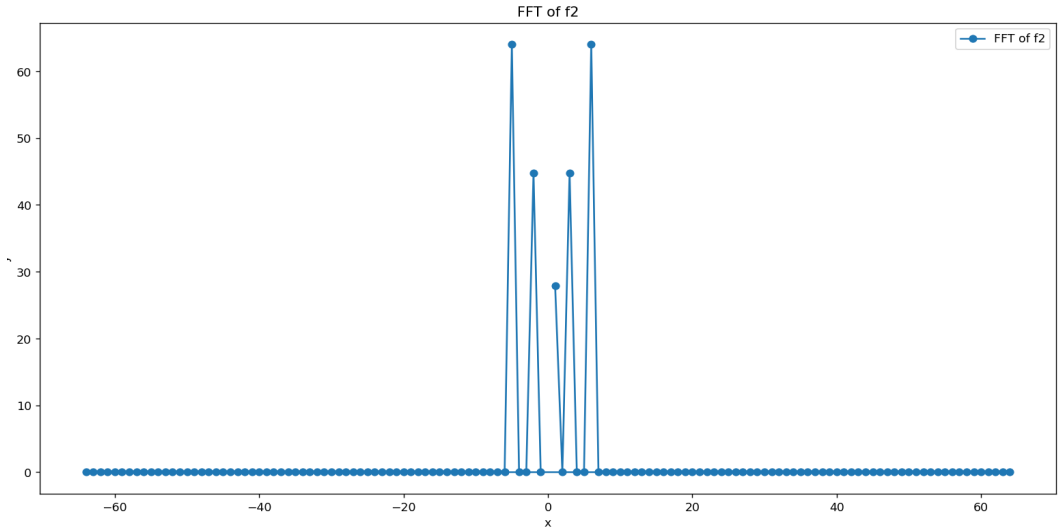

图 5

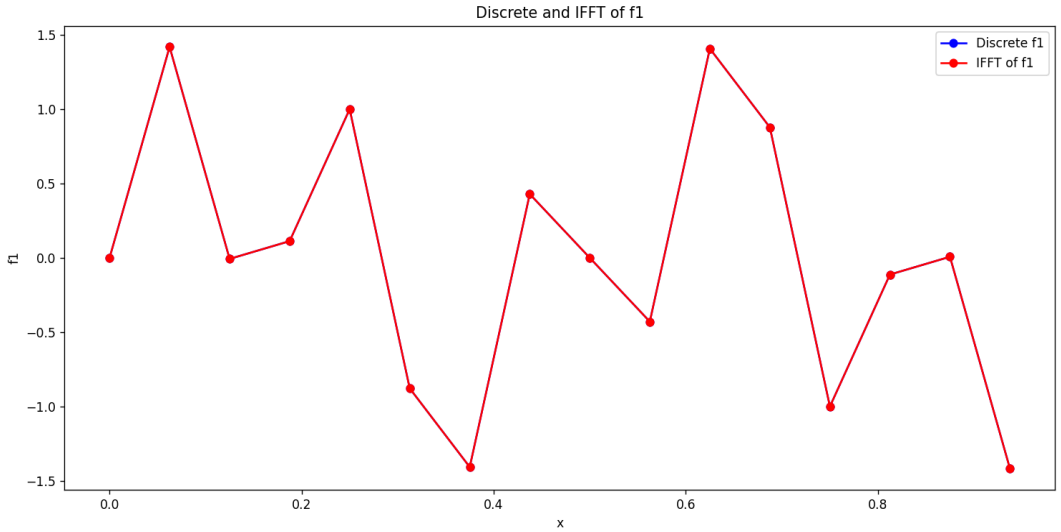

图 6

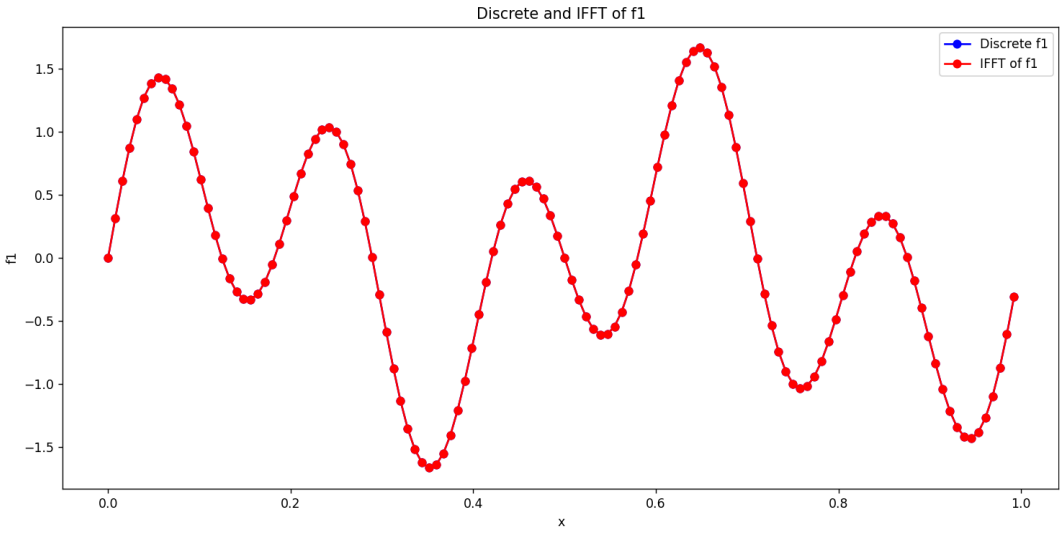
Discrete and IFFT of f1
Discrete f1
IFFT of f1
f1
x
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.0
0.2
0.4
0.6
0.8
1.0

图 7

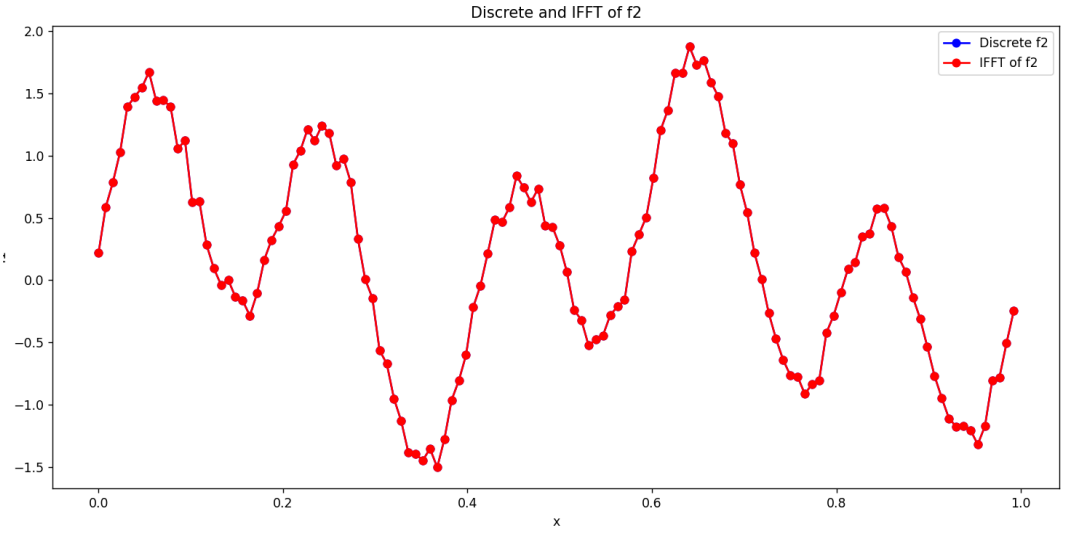
Discrete and IFFT of f2
Discrete f2
IFFT of f2
x
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.0
0.2
0.4
0.6
0.8
1.0

图 8

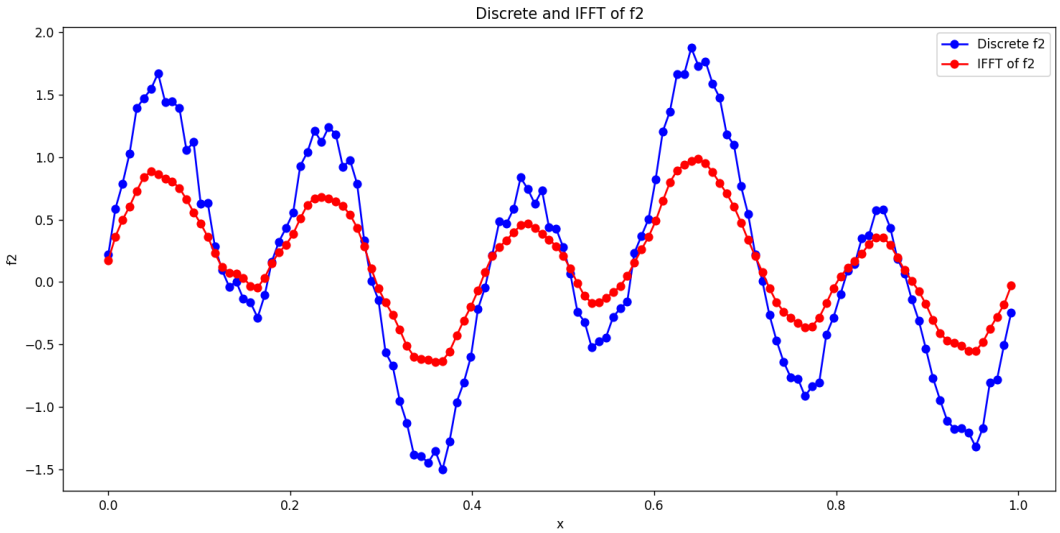

图 9: 只取前 25% 频率的结果

4 Conclusion

1. 采样数目为 128 时，抽样和重建的结果更加接近原函数，且 FFT 后频域上模长不为 0 的频率与采样数目为 16 时相同，但模长更大；

2. 对于 f2，分析去掉高频系数后 IFFT 的结果明显偏离于原来的抽样结果，因为去掉高频系数时会造成频率的损失。