CS 403: Scanning and Parsing

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The Compilation Process

Character stream

Scanner (lexical analysis)

Token stream

Parser (syntax analysis)

Parse tree

Semantic analysis

Abstract syntax tree

Intermediate code optimization

Modified intermediate form

Target code generation

Target language

Target code optimization

Modified target language

Symbol table

The Lexical Analyzer

- Main role: split the input character stream into tokens
  - Usually even interacts with the symbol table, inserting identifiers in it (especially useful for languages that do not require declarations)
  - This simplifies the design and portability of the parser
- A token is a data structure that contains:
  - The token name = abstract symbol representing a kind of lexical unit
  - A possibly empty set of attributes
- A pattern is a description of the form recognized in the input as a particular token
- A lexeme is a sequence of characters in the source program that matches a particular pattern of a token and so represents an instance of that token
- Most programming languages feature the following tokens
  - One token for each keyword
  - One token for each operator or each class of operators (e.g., relational operators)
  - One token for all identifiers
  - One or more tokens for literals (numerical, string, etc.)
  - One token for each punctuation symbol (parentheses, commas, etc.)

Example of Tokens and Attributes

```
printf("Score = %d\n", score);
```

```
E = M * C ** 2
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Lexeme	Token	Attribute
`printf`	**id**	pointer to symbol table entry
`(`	**open_paren**	
`"Score = %d\n"`	**string**	
`,`	**comma**	
`score`	**id**	pointer to symbol table entry
`)`	**cls_paren**	
`;`	**semicolon**	

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Lexeme	Token	Attribute
`E`	**id**	pointer to symbol table entry
`=`	**assign**	
`M`	**id**	pointer to symbol table entry
`*`	**mul**	
`C`	**id**	pointer to symbol table entry
`**`	**exp**	
`2`	**int_num**	numerical value 2

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  - ε is the identity for concatenation ($\varepsilon w = w\varepsilon = w$)

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- Language: a countable set of strings
  - Abuse of notation: For $a \in \Sigma$ we write a instead of $\{a\}$
  - Useful elementary operations: union ($\cup$, $+$, $|$) and concatenation ($\cdot$ or juxtaposition): $L_1 L_2 = L_1 \cdot L_2 = \{w_1 w_2 : w_1 \in L_1 \wedge w_2 \in L_2\}$
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- An expression containing only symbols from Σ, ε, $\emptyset$, union, concatenation, and Kleene closure is called a regular expression
  - A language described by a regular expression is a regular language

Notation	Regular expression	
r^+	rr^*	one or more instances (positive closure)
$r?$	$r\|\varepsilon$ or $r+\varepsilon$ or $r \cup \varepsilon$	zero or one instance
$[a_1 a_2 \cdots a_n]$	$a_1\|a_2\|\cdots\|a_n$	character class
$[a_1 - a_n]$	$a_1\|a_2\|\cdots\|a_n$	provided that a_1, a_2, $\ldots a_n$ are in sequence
$[\hat{\ }a_1 a_2 \cdots a_n]$		anything except $a_1, a_2, \ldots a_n$
$[\hat{\ }a_1 - a_n]$		

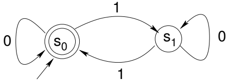
- The tokens in a programming language are usually given as regular definitions = collection of named regular languages

Examples of Regular Definitions

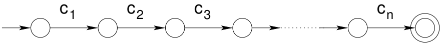
$$\begin{aligned}
\text{letter_} &= [A - Za - z_] \\
\text{digit} &= [0 - 9] \\
\text{id} &= \text{letter_}\ (\text{letter_} \mid \text{digit})^* \\
\text{digits} &= \text{digit}^+ \\
\text{fraction} &= .\ \text{digits} \\
\text{exp} &= E\ [+-]?\ \text{digits} \\
\text{number} &= \text{digits fraction? exp?} \\
\text{if} &= i\ f \\
\text{then} &= t\ h\ e\ n \\
\text{else} &= e\ l\ s\ e \\
\text{rel_op} &= < \mid > \mid <= \mid >= \mid == \mid !=
\end{aligned}$$

State Transition Diagrams

- In order for regular expressions to be used for lexical analysis they must be "compiled" into state transition diagrams
- Also called deterministic finite automata (DFA)
- Finite directed graph
- Edges (transitions) labeled with symbols from an alphabet
- Nodes (states) labeled only for convenience
- One initial state
- Several accepting states (double circles)



- A string $c_1 c_2 c_3 \dots c_n$ is accepted by a state transition diagram if there exists a path from the starting state to an accepting state such that the sequence of labels along the path is $c_1, c_2, \dots, c_n$



  - Same state might be visited more than once
  - Intermediate states might be final
- The set of exactly all the strings accepted by a state transition diagram is the language accepted (or recognized) by the state transition diagram

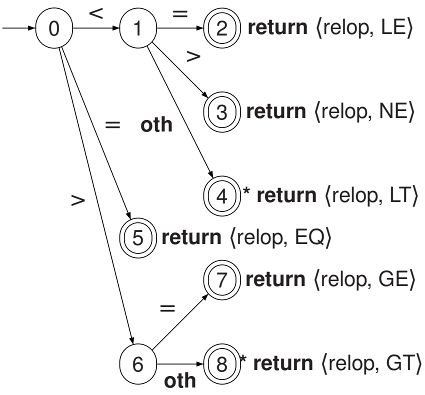
- Big practical advantages of DFA: easy and efficient implementation:
  - Interface to define a vocabulary and a function to obtain the input tokens

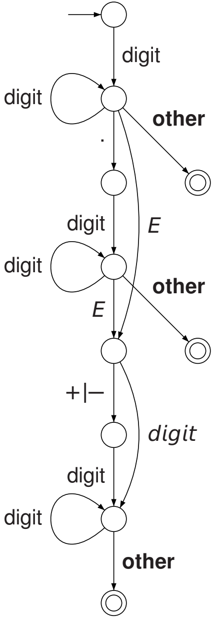
```
typename vocab;         /* alphabet + end-of-string */
const vocab EOS;        /* end-of-string pseudo-token */
vocab getchr(void);     /* returns next symbol */
```

  - Variable (state) changed by a simple `switch` statement as we go along

```
int main (void) {
    typedef enum {S0, S1, ... } state;
    state s = S0;    vocab t = getchr();
    while ( t != EOS ) {
        switch (s) {
            case S0: if (t == ...) s = ...; break;
                     if (t == ...) s = ...; break;
                     ...
            case S1: ...
            ...
        } /* switch */
        t = getchr();   } /* while */
    /* accept iff the current state s is final */
}
```

EXAMPLES OF STATE TRANSITION DIAGRAMS





When returning from *-ed states must retract last character

PRACTICAL EXAMPLE: LEX

- The LEX language is a programming language particularly suited for working with regular expressions
  - Actions can also be specified as fragments of C/C++ code
- The LEX compiler compiles the LEX language (e.g., `scanner.l`) into C/C++ code (`lex.yy.c`)
  - The resulting code is then compiled to produce the actual lexical analyzer
  - The use of this lexical analyzer is through repeatedly calling the function `yylex()` which will return a new token at each invocation
  - The attribute value (if any) is placed in the global variable `yylval`
  - Additional global variable: `yytext` (the lexeme)

- Structure of a LEX program:

  Declarations
  %%
  translation rules
  %%
  auxiliary functions

- Declarations include variables, constants, regular definitions
- Transition rules have the form

  Pattern { Action }

  where the pattern is a regular expression and the action is arbitrary C/C++ code

LEX BEHAVIOUR

- LEX compile the given regular expressions into one big state transition diagram, which is then repeatedly run on the input
- LEX conflict resolution rules:
  - Always prefer a longer to a shorter lexeme
  - If the longer lexeme matches more than one pattern then prefer the pattern that comes first in the LEX program
- LEX always reads one character ahead, but then retracts the lookahead character upon returning the token
  - Only the lexeme itself in therefore consumed

- A context-free grammar is a tuple $G = (N, \Sigma, R, S)$, where
  - Σ is an alphabet of terminals
  - N alphabet of symbols called by contrast nonterminals
    - Traditionally nonterminals are capitalized or surrounded by $\langle$ and $\rangle$, everything else being a terminal
  - $S \in N$ is the axiom (or the start symbol)
  - $R \subseteq N \times (N \cup \Sigma)^*$ is the set of (rewriting) rules or productions
    - Common ways of expressing $(\alpha, \beta) \in R$: $\alpha \rightarrow \beta$ or $\alpha \ ::= \ \beta$
    - Often terminals are quoted (which makes the $\langle$ and $\rangle$ unnecessary)
- Examples:

$$\begin{array}{lcl}
\langle \text{exp} \rangle & ::= & \mathit{CONST} \\
 & | & \mathit{VAR} \\
 & | & \langle \text{exp} \rangle \ \langle \text{op} \rangle \ \langle \text{exp} \rangle \\
 & | & (\ \langle \text{exp} \rangle \ ) \\
\langle \text{op} \rangle & ::= & + \mid - \mid * \mid /
\end{array}$$

$$\begin{array}{lcl}
\langle \text{stmt} \rangle & ::= & ; \\
 & | & \mathit{VAR} = \langle \text{exp} \rangle \ ; \\
 & | & \texttt{if} \ (\ \langle \text{exp} \rangle \ ) \ \langle \text{stmt} \rangle \ \texttt{else} \ \langle \text{stmt} \rangle \\
 & | & \texttt{while} \ (\ \langle \text{exp} \rangle \ ) \ \langle \text{stmt} \rangle \\
 & | & \{ \ \langle \text{seq} \rangle \ \} \\
\langle \text{seq} \rangle & ::= & \varepsilon \mid \langle \text{stmt} \rangle \ \langle \text{seq} \rangle
\end{array}$$

$$\begin{array}{lcl}
\langle \text{balanced} \rangle & ::= & \varepsilon \\
\langle \text{balanced} \rangle & ::= & 0 \ \langle \text{balanced} \rangle \ 1
\end{array}$$

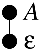
- $G = (N, \Sigma, R, S)$
- A rewriting rule $A ::= v' \in R$ is used to rewrite its left-hand side (A) into its right-hand side (v'):
  - $u \Rightarrow v$ iff $\exists\, x, y \in (N \cup \Sigma)^* : \exists\, A \in N : u = xAy, v = xv'y, A ::= v' \in R$
- Rewriting can be chained ($\Rightarrow^*$, the reflexive and transitive closure of $\Rightarrow$ = derivation)
  - $s \Rightarrow^* s'$ iff $s = s'$, $s \Rightarrow s'$, or there exist strings s_1, s_2, ..., s_n such that $s \Rightarrow s_1 \Rightarrow s_2 \Rightarrow \cdots \Rightarrow s_n \Rightarrow s'$
  - $\langle\text{pal}\rangle \Rightarrow 0\langle\text{pal}\rangle 0 \Rightarrow 01\langle\text{pal}\rangle 10 \Rightarrow 010\langle\text{pal}\rangle 010 \Rightarrow 0101010$

$$\langle\text{pal}\rangle \; ::= \; \varepsilon \mid 0 \mid 1 \mid 0\ \langle\text{pal}\rangle\ 0 \mid 1\ \langle\text{pal}\rangle\ 1$$

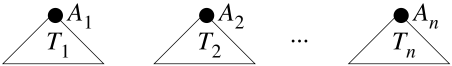
- The language generated by grammar G: exactly all the terminal strings generated from S: $\mathcal{L}(G) = \{w \in \Sigma^* : S \Rightarrow^* w\}$

Parse Trees

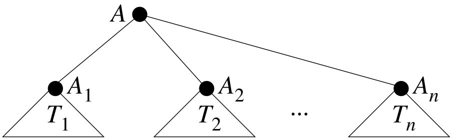
- Definition:
  1. For every $a \in N \cup \Sigma$ the following is a parse tree (with yield a): 

  2. For every $A ::= \varepsilon \in R$ the following is a parse tree (with yield ε): 

  3. If the following are parse trees (with yields $y_1, y_2, \ldots, y_n$, respectively):

  and $A ::= A_1 A_2 \ldots A_n \in R$, then the following is a parse tree (w/ yield $y_1 y_2 \ldots y_n$):

- Yield: concatenation of leaves in inorder

Derivations and Parse Trees

- Every derivation starting from some nonterminal has an associated parse tree (rooted at the starting nonterminal)
- Two derivations are similar iff only the order of rule application varies = can obtain one derivation from the other by repeatedly flipping consecutive rule applications
  - Two similar derivations have identical parse trees
  - Can use a "standard" derivation: leftmost ($A \overset{L}{\Rightarrow}{}^{*} w$) or rightmost ($A \overset{R}{\Rightarrow}{}^{*} w$)

Theorem

The following statements are equivalent:

- *there exists a parse tree with root A and yield w*
- $A \Rightarrow^{*} w$
- $A \overset{L}{\Rightarrow}{}^{*} w$
- $A \overset{R}{\Rightarrow}{}^{*} w$

- Ambiguity of a grammar: there exists a string that has two derivations that are not similar (i.e., two derivations with diferent parse trees)
  - Can be inherent or not — impossible to determine algorithmically

- Consider the following code:

```
int y;
template <class T> void g(T& v) {
    T::x(y);
}
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    - the declaration of `y` as a variable of type `T::x`.
  - Resolution: unless otherwise stated, an identifier is assumed to refer to something that is not a type or template.
    - If we want something else, we use the keyword `typename`:

```
T::x(y);             // function x of T applied to y
typename T::x(y);    // y is a variable of type T::x
```

- Interface to lexical analysis:

```
typename vocab;        /* alphabet + end-of-string */
const vocab EOS;       /* end-of-string pseudo-token */
vocab gettoken(void); /* returns next token */
```

- Parsing = determining whether the current input belongs to the given language
  - In practice a parse tree is constructed in the process as well
- General method: Not as efficient as for finite automata
  - Several possible derivations starting from the axiom, must choose the right one
  - Careful housekeeping (dynamic programming) reduces the otherwise exponential complexity to $O(n^3)$
  - We want linear time instead, so we want to determine what to do next based on the next token in the input

- Construct a function for each nonterminal
- Decide which function to call based on the next input token = linear complexity

```
vocab t;

void MustBe (vocab ThisToken) {
    if (t != ThisToken) { printf("reject"); exit(0); }
    t = gettoken();
}
void Balanced (void) {
    switch (t) {
      case EOS:
      case ONE: /* <empty> */
        break;
      default: /* 0 <balanced> 1 */
        MustBe(ZERO);
        Balanced();
        MustBe(ONE);
   }
} /* Balanced */
```

```
int main (void) {
    t = gettoken();
    Balanced();
    /* accept iff
       t == EOS   */
}
```

Recursive Descent Example

```
typedef enum { VAR, EQ, IF, ELSE, WHILE, OPN_BRACE, CLS_BRACE,
               OPN_PAREN, CLS_PAREN, SEMICOLON, EOS            } vocab;
vocab gettoken() {...}
vocab t;
void MustBe(vocab ThisToken) {...}

void Statement();
void Sequence();

int main() {
    t = gettoken();
    Statement();
    if (t != EOS) printf("String not accepted\n");
    return 0;        }
void Sequence() {
    if (t == CLS_BRACE) /* <empty> */ ;
    else { /* <statement> <sequence> */
        Statement();
        Sequence();
    }    }
```

Recursive Descent Example (cont'd)

```
void Statement() {
    switch(t) {
    case SEMICOLON: /* ; */
        t = gettoken();
        break;
    case VAR: /* <var> = <exp> */
        t = gettoken();
        MustBe(EQ);
        Expression();
        MustBe(SEMICOLON);
        break;
    case IF: /* if (<expr>) <statement> else <statement> */
        t = gettoken();
        MustBe(OPEN_PAREN);
        Expression();
        MustBe(CLS_PAREN);
        Statement();
        MustBe(ELSE);
        Statement();
        break;
```

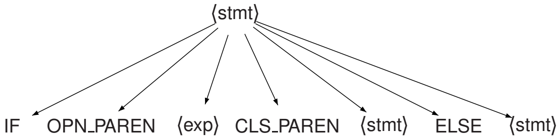
Recursive Descent Example (cont'd)

```
    case WHILE: /* while (exp) <statement> */
        t = gettoken();
        MustBe(OPEN_PAREN);
        Expression();
        MustBe(CLS_PAREN);
        Statement();
        break;
    default: /* { <sequence } */
        MustBe(OPN_BRACE);
        Sequence();
        MustBe(CLS_BRACE);
    } /* switch */
} /* Statement () */
```

Parse Trees vs. Abstract Syntax Trees

- In practice the output of a parser is a somehow simplified parse tree called abstract syntax tree (AST)
  - Some tokens in the program being parsed have only a syntactic role (to identify the respective language construct and its components)
  - Node information might be augmented to replace them
  - These tokens have no further use and so they are omitted form the AST
  - Other than this omission the AST looks exactly like a parse tree
- Examples of parse trees versus AST

Conditional (parse tree):



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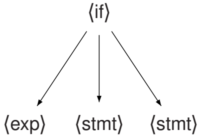

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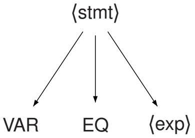

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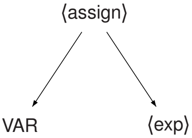

Constructing the Parse Tree

- The parse tree/AST can be constructed through the recursive calls:
  - Each function creates a current node
  - The children are populated through recursive calls
  - The current node is then returned

```
class Node {...};

Node* Sequence() {
    Node* current = new Node(SEQ, ...);
    if (t == CLS_BRACE) /* <empty> */ ;
    else { /* <statement> <sequence> */
        current.addChild(Statement());
        current.addChild(Sequence());
    }
    return current;
}
```

CONSTRUCTING THE PARSE TREE (CONT'D)

```
Node* Statement() {
    Node* current;
    switch(t) {
    case SEMICOLON: /* ; */
        t = gettoken();
        return new Node(EMPTY);
        break;
    case VAR: /* <var> = <exp> */
        current = new Node(ASSIGN, ...);
        current.addChild(VAR, ...);
        t = gettoken();
        MustBe(EQ);
        current.addChild(Expression());
        MustBe(SEMICOLON);
        break;
    case IF: /* if (<expr>) <statement> else <statement> */
        current = new Node(COND, ...);
    /* ... */
    }
    return current;
}
```

- Not all grammars are suitable for recursive descent:

$$
\begin{array}{rcl}
\langle \text{stmt} \rangle & ::= & \varepsilon \\
& | & \mathit{VAR} := \langle \text{exp} \rangle \\
& | & \texttt{IF}\ \langle \text{exp} \rangle\ \texttt{THEN}\ \langle \text{stmt} \rangle\ \texttt{ELSE}\ \langle \text{stmt} \rangle \\
& | & \texttt{WHILE}\ \langle \text{exp} \rangle\ \texttt{DO}\ \langle \text{stmt} \rangle \\
& | & \texttt{BEGIN}\ \langle \text{seq} \rangle\ \texttt{END} \\
\langle \text{seq} \rangle & ::= & \langle \text{stmt} \rangle \mid \langle \text{stmt} \rangle\ ;\ \langle \text{seq} \rangle
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\langle \text{seq} \rangle & ::= & \langle \text{stmt} \rangle \mid \langle \text{stmt} \rangle\ ;\ \langle \text{seq} \rangle
\end{array}
$$

  - Both rules for $\langle \text{seq} \rangle$ begin with the same nonterminal
  - Impossible to decide which one to apply based only on the next token

- Not all grammars are suitable for recursive descent:

$$
\begin{array}{rcl}
\langle\text{stmt}\rangle & ::= & \varepsilon \\
& | & \mathit{VAR} := \langle\text{exp}\rangle \\
& | & \texttt{IF}\ \langle\text{exp}\rangle\ \texttt{THEN}\ \langle\text{stmt}\rangle\ \texttt{ELSE}\ \langle\text{stmt}\rangle \\
& | & \texttt{WHILE}\ \langle\text{exp}\rangle\ \texttt{DO}\ \langle\text{stmt}\rangle \\
& | & \texttt{BEGIN}\ \langle\text{seq}\rangle\ \texttt{END} \\
\langle\text{seq}\rangle & ::= & \langle\text{stmt}\rangle \mid \langle\text{stmt}\rangle\ ;\ \langle\text{seq}\rangle
\end{array}
$$

  - Both rules for $\langle\text{seq}\rangle$ begin with the same nonterminal
  - Impossible to decide which one to apply based only on the next token
  - Fortunately concatenation is distributive over union so we can fix the grammar (left factoring):

$$
\begin{array}{rcl}
\langle\text{seq}\rangle & ::= & \langle\text{stmt}\rangle\ \langle\text{seqTail}\rangle \\
\langle\text{seqTail}\rangle & ::= & \varepsilon \mid ;\ \langle\text{seq}\rangle
\end{array}
$$

- Some programming constructs are inherently ambiguous

$$\begin{array}{lcl} \langle \text{stmt} \rangle & ::= & \texttt{if} \ (\ \langle \text{exp} \rangle \ ) \ \langle \text{stmt} \rangle \\ & | & \texttt{if} \ (\ \langle \text{exp} \rangle \ ) \ \langle \text{stmt} \rangle \ \texttt{else} \ \langle \text{stmt} \rangle \end{array}$$

- Some programming constructs are inherently ambiguous

$$\begin{array}{lcl} \langle\text{stmt}\rangle & ::= & \texttt{if}\ (\ \langle\text{exp}\rangle\ )\ \langle\text{stmt}\rangle \\ & | & \texttt{if}\ (\ \langle\text{exp}\rangle\ )\ \langle\text{stmt}\rangle\ \texttt{else}\ \langle\text{stmt}\rangle \end{array}$$

- Solution: choose one path and stick to it (e.g., match the else-statement with the nearest else-less if statement)

```
case IF:
    t = gettoken();
    MustBe(OPEN_PAREN);
    Expression();
    MustBe(CLS_PAREN);
    Statement();
    if (t == ELSE) {
        t = gettoken();
        Statement();
    }
```

- Any left recursion in the grammar will cause the parser to go into an infinite loop:

$$\langle\text{exp}\rangle \quad ::= \quad \langle\text{exp}\rangle\ \langle\text{addop}\rangle\ \langle\text{term}\rangle \mid \langle\text{term}\rangle$$

Recursive Descent Parsing: Closure, etc.

- Any left recursion in the grammar will cause the parser to go into an infinite loop:

$$\langle \text{exp} \rangle \quad ::= \quad \langle \text{exp} \rangle \; \langle \text{addop} \rangle \; \langle \text{term} \rangle \mid \langle \text{term} \rangle$$

- Solution: eliminate left recursion using a closure

$$\begin{array}{rcl} \langle \text{exp} \rangle & ::= & \langle \text{term} \rangle \; \langle \text{closure} \rangle \\ \langle \text{closure} \rangle & ::= & \varepsilon \\ & \mid & \langle \text{addop} \rangle \; \langle \text{term} \rangle \; \langle \text{closure} \rangle \end{array}$$

  - Not the same language theoretically, but differences not relevant in practice
- This being said, some languages are simply not parseable using recursive descent

$$\langle \text{palindrome} \rangle \quad ::= \quad \varepsilon \mid 0 \mid 1 \mid 0 \; \langle \text{palindrome} \rangle \; 0 \mid 1 \; \langle \text{palindrome} \rangle \; 1$$

  - No way to know when to choose the ε rule
  - No way to choose between the second and the fourth rule
  - No way to choose between the third and the fifth rule

Recursive Descent Parsing: Sufficient Conditions

- *first*(α) = set of all initial tokens in the strings derivable from α
- *follow*$(\langle\text{N}\rangle)$ = set of all initial tokens in nonempty strings that may follow $\langle\text{N}\rangle$ (possibly including `EOS`)
- Sufficient conditions for a grammar to allow recursive descent parsing:
  - For $\langle\text{N}\rangle$ ::= $\alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$ must have $\mathit{first}(\alpha_i) \cap \mathit{first}(\alpha_j) = \emptyset$, $1 \leq i < j \leq n$
  - Whenever $\langle\text{N}\rangle \Rightarrow^* \varepsilon$ must have $\mathit{follow}(\langle\text{N}\rangle) \cap \mathit{first}(\langle\text{N}\rangle) = \emptyset$
- Grammars that do not have these properties may be fixable using left factoring, closure, etc.

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- Grammars that do not have these properties may be fixable using left factoring, closure, etc.
- Method for constructing the recursive descent function `N()` for the nonterminal $\langle N \rangle$ with rules $\langle N \rangle ::= \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$:
  1. For $\alpha_i \ne \varepsilon$ apply the rewriting rule $\langle N \rangle ::= \alpha_i$ whenever the next token in the input is in $\textsc{First}(\alpha_i)$
  2. For $\alpha_i = \varepsilon$ apply the rewriting rule $\langle N \rangle ::= \alpha_i$ (that is, $\langle N \rangle ::= \varepsilon$) whenever the next token in the input is in $\textsc{Follow}(\langle N \rangle)$
  3. Signal a syntax error in all the other cases

Steps to parse a programming language:

- Construct a scanner
  - Express the lexical structure of the language as regular expressions
  - Convert those regular expressions into a finite automaton (can be automated) = the scanner
- Construct a parser
  - Express the syntax of the language as a context-free grammar
  - Adjust the grammar so that it is suitable for recursive descent
  - Construct the recursive descent parser for the grammar (can be automated) = the parser
- Run the parser on a particular program
  - This implies calls to the scanner to obtain the tokens
  - The result is a parse tree, that will be used in the subsequent steps of the compilation process