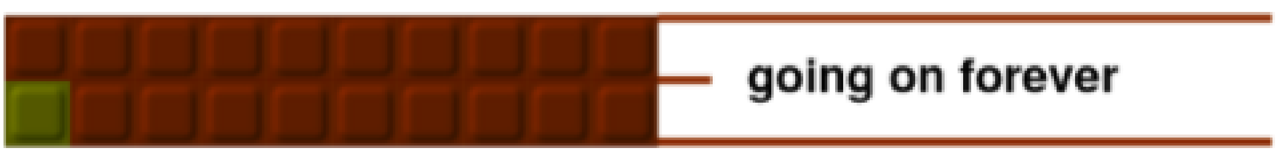
GAME THEORY 1

Exercise 3

The exercise is due 2/5/2021 at 22 : 00.

1. Let G_1 and G_2 be two progressively bounded impartial games and recall the definition of $G_1 \oplus G_2$ from the Tirgul.
   (a) Let G be the game of Nim with n piles. Let G_1 be the game of Nim which corresponds to the first m piles in G and G_2 be the game of Nim which corresponds to the last $n-m$ piles in G. Prove that $G = G_1 \oplus G_2$.
   (b) Suppose that $x \in N_{G_1}$ and $y \in P_{G_2}$. Prove by induction that $(x, y) \in N_{G_1 \oplus G_2}$.
   (c) Suppose that $x \in P_{G_1}$ and $y \in P_{G_2}$. Prove by induction that $(x, y) \in P_{G_1 \oplus G_2}$.
   (d) Give an example of games G_1 and G_2 and positions $x \in N_{G_1}$ and $y \in N_{G_2}$ such that $(x, y) \in P_{G_1 \oplus G_2}$.
   (e) Give anexample of games G_1 and G_2 and positions $x \in N_{G_1}$ and $y \in N_{G_2}$ such that $(x, y) \in N_{G_1 \oplus G_2}$.
2. Look at the game of Nim with piles of sizes $9, 10, 11, 12$.
   (a) Use the Nim sum and prove that this position is in N.
   (b) Describe a move which ends in a position in P. Are there any more moves like that?
   (c) Answer (a) and (b) with the new position $(9, 12, 14, 15)$.
3. Consider the game Chomp on a table of size $2 \times \infty$. With coordinates (i, n) for $1 \leq i \leq 2$ and $-\infty < n \leq 1$ (see the figure below).

   Answer the following questions:
   (a) Is this game finite?
   (b) Prove that the game is not progressively bounded and yet there is no infinite sequence of legal moves.
   (c) Which of the players has a winning strategy? Describe it.
   (d) If instead the size of the table is $m \times \infty$. Which of the players has a winning strategy? Describe it. (Hint: Use the case $2 \times \infty$).
4. Let G_1 be the game of Nim with one pile of two coins. Let G_2 be the game of chomp (without the poision - as in the Tirgul) of size 2×2. Draw the graph of $G_1 \oplus G_2$. Which player has a winning strategy?