1. Prove that the simplex method equipped with Bland's pivot rule always terminates.

Bland's rule: choose the column of minimum index j (among those with negative objective function coefficient).

choose the row so that the index k of the element that leaves the basis is minimum.

2. Complete the proof of the convergence rate of the ellipsoid method. Show that for the linear program $z = \max c^T x$ s.t. $x \in P$, where P is full dimensional and $B(x_0, r) \subset P_< \subset B(\vec{0}, R)$, after at most $2n(n+1)\left\lceil \ln \frac{2R^2 \|c\|}{r \cdot \varepsilon} \right\rceil$ steps we get a feasible solution $\hat{x}$ that satisfies $c^T \hat{x} \geq z - \varepsilon$.

3. Let $f: \mathbb{R}^n \to \mathbb{R}$ be convex and differentiable, with a gradient ∇f that is L-Lipschitz continuous. Show that the gradient descent iteration $x^{(i)} = x^{(i-1)} - \frac{1}{L} \nabla f(x^{(i-1)})$ satisfies, for all i, $f(x^{(i)}) - f^* \leq \frac{L}{2i} \|x^{(0)} - x^*\|_2^2$, where x^* is a minimizer of f and $f^* = f(x^*)$.

4. a. Show that for $\alpha \in (0,\infty)$, $x^{-\alpha}$ is not self-concordant on $[0,\infty)$.

b. Show that the set K of symmetric positive semidefinite $n \times n$ matrices is a cone in $\mathbb{R}^{n \times n}$.

c.* Show that $f(X) = -\ln \det(X)$ is an n-self concordant barrier function for K (the interior of K is the set of positive definite matrices in K).

* bonus question