Game Theory 1

Exercise 2

The exercise is due 18/4/2021 at 22:00.

(1) Analyse the game of Nim with the new rule that the player who takes the last coin **loses**. Compute P and N and describe the winning strategy from each position. (Hint: start with the case where all piles contain one coin except from one pile).

(2) In the game Wythoff Nim we start with two piles of coins (n, m). Two players alternate taking a certain amount of coins from a chosen pile or an equal amount of coins from both piles. Recall the sequences a_i, b_i from the Tirgul notes and define the set

$$\hat{P} = \{(a_i, b_i) : i \in \mathbb{N}\} \cup \{(b_i, a_i) : i \in \mathbb{N}\} \cap X$$

where X is the set of all positions starting from (n, m).

(a) Let $(x, y) \in \hat{P}$ prove that any legal move ends in a position that is not in $\hat{P}$. Hint: Study each of the following cases separately

(i) Remove a certain number of coins from the first pile.

(ii) Remove a certain number of coins from the second pile.

(iii) Remove an equal amount of coins from both piles.

(b) Prove that if $(x, y) \notin \hat{P}$ then there exists a legal move which ends in $\hat{P}$.

(c) Conclude that $P = \hat{P}$. Namely, show that $(x, y) \in \hat{P}$ if and only if the second player has a winning strategy from this position.

(3) Two players play “chess in two steps”. The rules are the same as the original game but each player play two turns instead of one. Prove that the black (second) player can not have a winning strategy. (Hint: The knight can go forward and then backwards).

(4) Two players alternate placing a coin on a round table. The surface of the coin must touch the table (so it is not allowed to put one coin over another). The first player who can not place a coin loses. Which of the players has a winning strategy? Describe the winning strategy of that player. Hint: The answer is independent of the size of the table (we assume that the table is larger than a single coin).

(5) Starting with 1000 chips. Two players alternate removing one chip or half of the chips (Replacing N with $N - 1$ or $\lfloor \frac{N}{2} \rfloor$). The first player who removes the last chip wins. Which of the players has a winning strategy? (Hint: Solve the game first in the case where the number of chips is odd).