

1. Prove that for $k=2$ (two commodities), in any undirected graph, the maximum concurrent flow equals the sparsest cut.
2. Show that for any instance on a tree, the minimum multicut is at most twice the maximum multiflow.
3. Design and analyze a randomized rounding-based algorithm for minimum set cover. (In particular, start with expressing the problem as an integer linear program.)
4. Suppose that at some time during the execution of the push-relabel algorithm there is $k \in \{1, \dots, n-1\}$ such that no vertex v has a distance label $d(v)=k$. Show that all $v \in V$ with $d(v) > k$ are on the side of the source s in a minimum s - t cut.