GAME THEORY 1

Exercise 9

The exercise is due 13.6.2021 at 22:00.

1. Find all of the Nash equilibrium points in the following games:

   (a)

2,2	0,0
0,0	1,1

   (b)

2,2	0,3
3,0	1,1

   (c)

1,8	1,2	0,0
1,4	2,12	9,3

2. Given a zero-sum strategic game A. We denote by V the value of the game.

   (a) A pair of mixed strategies (x^*, y^*) is a Nash Equilibrium if and only if x^* is optimal for player 1 and y^* optimal for player 2 .

   (b) (This question is optional, you do not have to submit a solution) Prove that if (x^*, y^*) and (z^*, w^*) are Nash-Equilibrium in a zero sum game then so are (x^*, w^*) and (z^*, y^*)

   (c) (optional) Does the same hold if the game is not zero-sum?

3. **(Ice-cream game)** Alice and Bob sell ice cream at the beach. The beach is divided into 10 sections $\{1, 2, ..., 10\}$ of the same size (ordered in a straight line, that is, in order to reach area 5 from area 2 one has to pass through areas 3 and 4). In each section there are 10 travelers who came to enjoy the beach and to buy ice cream. The ice cream sellers has to choose in which section they locate themselves in the beach. Then, each one of the travelers will go to the closest ice cream seller and buy ice cream. If the ice cream sellers are of equal distance from two points, then half will go to one seller and the other half to another. For example, if Bob is located at section 4 and Alice at section 2 then Bob will sell 75 ice creams while Alice will only sell 25. Alice and Bob wants to maximize the amount of ice cream they sell. Describe this as a strategic game and find the Nash equilibrium.
   Hint: Write the payoff matrix of the game and substitute a constant from all of the values of the matrix so it becomes a zero-sum game (why this does not change the Nash equilibriums?). Analyse this game using dominant strategies.

4. **(Corona-question).** In this question we try to analyse the situation of the Corona-Virus. For the sake of this question we go back in time (1 year backwards) to the time where Corona-Virus was dangerous and we did not have vaccines. In such days, we used to wear masks all the time, but not everyone followed the rules. We will try to analyse this situation from a game-theoretic point of view. First, instead of analysing this situation as a $9,000,000$ players game, we will assume that all players are equal and will focus only on two players that represents the public. We therefore have the following scenraio:

- Each player has two strategies. Wear a mask, or not.

- Nobody likes to wear a mask, a player who chose to wear a mask gets (-1).
- We assume that wearing a mask **does not** protect the person who wears the mask, but protects the other person (even though this is not the case, many people had this belief).
- Any player who stays near an unmasked person is in danger. Therefore, if one player chose not to wear mask, he causes a lose of (-10) to the other player. This (-10) adds up to (-11) if that player also chose to wear a mask.

Answer the following questions:
(a) Describe the payoff matrix of the game.
(b) Find the Nash Equilibrium (**Hint:** dominant strategies). From a non-mathematical point of view, is this scenario optimal?
(c) Back at that times, the Goverment decided to fine (-100) anyone who does not wear a mask. However, the police did not enforce the law because too many people did not wear masks. Write the new matrix of the game, assuming that the police enforce the rules only if one of the players wear a mask but the other is not. Find all of the Nash-Equilibriums in the new game.
(d) Due to a khamsin (a very hot day) the goverment decided not to enforce wearing masks. Then, when the khamsin was over the Goverment returned to the original law. Deduce from the previous questions that the players behavior is not expected to change (they will continue choosing not to wear masks). Since this was the case, the Goverment decided to increase the power of the police and enfore the rules regardless of the number of people who do not wear masks. Explain why the new strategy of the govermeant leads to an "optimal" situation for the people.