

1. Let P be a polytope.
 - a. Prove that $x \in \text{vert}(P)$ iff x is an extremal point of P .
 - b. Prove that if $P = \text{cone}(Y)$ for some finite set $Y \subset \mathbb{R}^n$, then $\text{vert}(P) \subseteq \{\vec{0}\}$.
2. Consider a linear program in canonical form: minimize $c^T x$ s.t. $Ax \leq b$.
 - a. Show that unlike linear programs in standard form, here the program may be bounded, yet there is no optimal solution which is a vertex.
 - b. Convert your example to standard form and demonstrate that a vertex optimal solution appears.
3. Demonstrate that without convexity the Farkas lemma (i.e., $\forall x \notin P \exists \text{hyperplane separating between } x \text{ and } P$) does not necessarily hold.